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Datta, Soumya

Faculty of Economics, South Asian University, New Delhi, India

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Soumya Datta*

Faculty of Economics, South Asian University, New Delhi, INDIA.
soumya@econ.sau.ac.in

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Abstract

The paper demonstrates possibilities of both convergence to the steady state and emergence of stable growth cycles around it in a simple macrodynamic model of debt-financed investment-led growth. The growth cycles are robust and are generated endogenously, either due to the existence of a supercritical Andronov-Hopf bifurcation, or due to the global stability condition through an application of the Poincaré-Bendixson theorem. The emergence of multiple limit cycles is also observed under certain conditions. The possibility of a deterioration of financial variables during a boom, with the resulting financial crisis providing an endogenous ceiling to a business cycle is examined in this context.

Keywords: Growth cycles, Financial crisis, Fisher, Minsky, Andronov-Hopf bifurcation, Limit cycles

JEL classification: C62; C69; E12; E32; E44; G01

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1 Introduction

The primary objective of this study is to investigate whether the macrodynamics of debt-financing investment can provide an endogenous explanation for emergence of growth cycles in demand-constrained closed economies. In addition, we also attempt to examine the possibilities of economic crises, especially of financial origins, emerging as a by-product of such growth cycles.

The basic motivation for this study comes from our observation of a two-way causality between the real and the financial sector. A simple interaction between the multiplier and the accelerator in a demand-constrained closed economy might lead to a monotonic movement of output and investment. Such models, therefore, would require exogenous ceilings and floors to stay bounded. In the presence of financial factors, however, an expansion of output and investment (or the rates of growth thereof) might, under certain conditions, lead to deterioration of certain financial variables. This, in turn, might lead to creation of conditions under which the initial increase in investment might be depressed. If suitably modeled, this might provide us with an endogenous explanation of growth cycles, with the real and the financial variables chasing each other.

One area of particular interest in the above story of growth cycles is the possibility of complications arising from the borrowers defaulting on their payment commitments. A substantial literature in this area suggests that the lenders, when faced with the possibility of the borrowers defaulting under conditions of market imperfections like incomplete and asymmetric information, might adopt non-market-clearing methods like red-lining and rationing credit and thus discriminate between various borrowers based on some assessment of their creditworthiness.[1] There is also a substantial literature, influenced by the contributions of Fisher (1932, 1933) and Minsky (1975, 1982, 1986, 1994), which argues that there is a general tendency for expansion of credit to lead to a deterioration of the financial variables in the economy during periods of boom and prosperity. A financial crisis follows, which is then followed with a contraction of the real sector as well, putting an end to the boom phase. The interaction between the real and the financial sector, therefore, leads us to an endogenous explanation for bounded systems and growth as well as financial cycles. Minsky's contribution, in particular, has influenced a huge literature on debt-deflation and financial crisis. Kindleberger's (1978) interesting and influential account of financial cycles, for instance, is influenced by Minsky's financial instability hypothesis. Similarly, there is a huge literature of economic models on financial fragility, which originates in an attempt to model

[1] See, for instance, Keynes (1930), Hodgman (1960), Catt (1965), Jaffee & Russell (1976), Stiglitz & Weiss (1981, 1983, 1992), Jaffee & Stiglitz (1990).

at least some aspect of Minsky's descriptive account.[2] However, the popularity and huge interest in Minsky's work notwithstanding, a critical component of Minsky's story, consisting of uncertainties regarding realization of profits and its consequent impact on repayment of debt commitments, is described at the microeconomic level. In a demand-constrained economy, a higher investment translates to a higher level of macroeconomic profits through the operation of the multiplier. Hence, there is no straightforward way to aggregate the above story of problems arising out of uncertainties faced by individual firms over realization of profits from investment to the macroeconomic level. An alternative story is required, therefore, to explain why during a prolonged boom there is a steady shift among firms from hedge towards speculative and ponzi financial postures, increasing the overall indebtedness and leverage in the economy and eventually leading to a financial crisis putting an end to the boom. This is one of the questions which we attempt to address in this paper.

We begin by introducing the model in section 2 before proceeding to discuss some of the preliminary results in section 3. In section 4 we explore cyclical possibilities. The main economic interpretations of these results are provided in section 5. Finally we reconsider the Fisher-Minsky hypothesis in light of these results in section 6.

2 Basic Model

2.1 Goods Market

We consider a simple continuous time model of a closed economy, consisting of the firm and the household sector. The household sector consists of two kinds of households – type 1 households consisting of workers, deriving income from wages, and type 2 and 3 households, deriving their income from two kinds of financial assets, debt and equities respectively. The aggregate demand at time t, $AD(t)$, is composed of the total expenditure on investment and consumption made by the firms and the households respectively, i.e. $AD(t) = C(t) + I(t)$. A firm finances its investment either internally out of retained earnings, or externally by issuing debt and equity instruments. The national income, Y, might be measured by income method as the sum of wages, W, and profits, P, i.e. $Y(t) = W(t) + P(t)$. In terms of various sectors in the economy, the total income might also be represented as $Y(t) = Y_f(t) + Y_{h1}(t) + Y_{h2}(t) + Y_{h3}(t)$, where Y_f, Y_{h1}, Y_{h2} and Y_{h3} is the income to firms (profits after paying outstanding debt commitments and dividends) and type 1, type 2 and type 3 households respectively. In other words, $Y_f(t) = \sigma P(t)$, where σ is the fraction of profits retained by firms; whereas

[2] See, for instance, Taylor & O'Connell (1985), Lavoie (1986-87), Foley (1987), Semmler (1987), Downe (1987), Franke & Semmler (1989), Greenwald & Stiglitz (1993), Palley (1994), Skott (1994, 1995), Keen (1995, 1996), Andresen (1996, 1999), Vercelli (2000), Asada (2001), Lagunoff & Schreft (2001), Arena & Raybaut (2001), Chiarella, Flaschel & Semmler (2001), Fazzari, Ferri & Greenberg (2001), Gatti & Gallegati (2001), Foley (2003), Setterfield (2004), Datta (2005), Meirelles & Lima (2006), Lima & Meirelles (2007), Fazzari, Ferri & Greenberg (2008), Taylor & von Arnim (2008), Charles (2008*b*, 2008*a*), Guilmi, Gallegati & Landini (2009).

$Y_{h1}(t) = W(t)$, where W represents the wages; so that $Y_{h2}(t) = (1-\sigma)P(t) - Y_{h3}(t)$, with Y_{h2} and Y_{h3} being the part of profits representing return to financial assets (debt and equities). If s_1, s_2 and s_3 represent the fraction of the respective incomes saved by type 1, 2 & 3 households respectively, with $s_1 < s_2 = s_3$, then we have

$$C(t) = (1-s_1)W(t) + (1-s_2)(1-\sigma)P(t) \tag{1}$$

Assuming a regime of mark-up pricing, where the price per unit is obtained by adding a fixed mark-up over the wage costs of production, we have

$$P(t) = \psi Y(t) \tag{2}$$

where ψ is the share of profits in national income. Following a simple algebraic manipulation, the consumption by the household sector can now be represented as $C(t) = (1-s)Y(t)$, where $s = 1 - \{(1-s_1)(1-\psi) + (1-s_2)(1-\sigma)\psi\}$ is the propensity to save out of national income.

Let the potential output or the rate of capacity of production in the economy, $Y^\star$, be defined as the maximum output that can possibly be produced, given the existing constraints of factors and a given technology. Assuming the availability of capital as the binding constraint on production, we have $Y^\star(t) = \beta K(t)$, where β is the output-capital ratio determined by the existing technology. The actual level of output or the national income, Y, can now be represented as $Y(t) = \min(AD(t), Y^\star(t))$. In other words, for all $AD \leq Y^\star$, aggregate demand acts as the main constraint on the level of production and the output is determined by the aggregate demand.

At the goods market equilibrium, the level of output measured by the income method equals the aggregate demand, i.e. $Y(t) = AD(t)$ so that $W(t) + P(t) = C(t) + I(t)$. Substituting the value of C from (1), we have

$$Y(t) = \frac{1}{s} I(t) \tag{3}$$

Let the rate of capacity utilization be defined as the ratio of actual to potential output, i.e. $u(t) = Y(t)/Y^\star(t)$. We define the rate of investment,

$$g(t) \equiv \frac{I(t)}{K(t)} \tag{4}$$

From the definition of u, $Y^\star$ and g, and the goods market equilibrium condition given in (3), we have

$$g(t) = s\beta u(t) \tag{5}$$

with a feasibility condition $0 \leq u \leq 1 \Leftrightarrow 0 \leq g \leq g_{\max}$, where $g_{\max} \equiv s\beta$ represents the rate of investment corresponding to full capacity utilization.

Let $g^\star$, the desired rate of investment, depend directly and linearly on the rate of capacity utilization, i.e. $g^\star(t) = \bar{\gamma} + \gamma(t)\,u(t)$. Substituting from (5), we have

$$g^\star(t) = \bar{\gamma} + \frac{\gamma(t)\,g(t)}{s\beta} \tag{6}$$

where γ is the 'financial accelerator' or the sensitivity of the desired rate of investment, $g^\star$ to the rate of capacity utilization, u, and is determined by financial factors. $\bar{\gamma}$, on the other hand, due to reasons given by Duménil & Lévy (1999, page 686), comprises the exogenous component of investment. Next, we turn our attention to the financial sector.

2.2 Dynamics of Debt

Consider a simple model of debt dynamics. The total stock of outstanding debt commitment in any given period, t, is given by a history of borrowing, B, at a rate of interest, r, and repayment, R. If the rate of interest, r, as mentioned above, is given exogenously by the Central Bank, then the stock of debt in period t is given by

$$D(t) = \int_{\tau=0}^{t} \left(B(\tau) - R(\tau)\right) e^{r(t-\tau)} d\tau \tag{7}$$

which, with simple algebraic manipulation and differentiation with respect to t, reduces to

$$\dot{D}(t) = B(t) - R(t) + rD(t) \tag{8}$$

Equation (8) provides us with the basic accounting identity describing the growth in stock of debt. Next, we proceed to construct a macroeconomic index of financial fragility or gearing ratio, in the form of a ratio of the level of indebtedness to the ability to pay for all the debtors, i.e. the firm sector together.

In any time period, t, the firm sector's total payment commitment consists of principal and interest commitments. However, since the debt stock is accumulated over a period of time, the debtors are expected to pay only a part of the total principal in a given period. For each borrower, the minimum part of principal that is expected to be paid back in each period would differ, and would, among other things, depend on a credit rating of the borrower by the lenders. A borrower who is considered relatively safe (i.e. less likely to default) by the lenders would be expected to pay a smaller fraction of the principal in each period than a borrower who is considered relatively unsafe. In other words, borrowers with higher credit ratings will have access to loans with longer terms, resulting in a proportionally smaller minimum repayment requirements each period.

At the macroeconomic level, however, the lenders as a whole expect, in each time period, an exogenously given minimum fraction of the total debt stock as repayment towards the principal. Let this fraction be q of the total outstanding debt commitments. The interest commitments, on the other hand, are accumulated within the time period, and hence, are expected to be fully paid. In any given period t, therefore, the total minimum payment commitment of debtors is given by $(q + r) D(t)$, where $qD(t)$ and $rD(t)$ is the principal and interest component respectively. These payments are to be paid by the debtors out of their current retained profits or the internal finance. In other words, current retained profits are used to repay current payment commitments, and the residual determines the level of retained profits in the next period. The macroeconomic index of financial fragility or gearing ratio can now be represented as

$$\lambda(t) = \frac{(q + r) D(t)}{\sigma P(t)} \tag{9}$$

We define

$$d(t) \equiv \frac{D(t)}{K(t)} \tag{10}$$

as the stock of debt in intensive form. Substituting from (2), (3), (4) and (10) into (9), we have

$$\lambda(t) = \frac{(q + r) sd(t)}{\sigma \psi g(t)} \tag{11}$$

The actual repayment in period t, denoted by $R(t)$, however, is independent of $(q + r) D(t)$. It might either exceed or fall short of it, depending on the profile of the borrowers and repayment by individual borrowers. Let us consider a situation where a fraction $\phi(t)$ of the total outstanding debt stock is repaid in period t, i.e.

$$R(t) = \phi(t) D(t) \tag{12}$$

This fraction, $\phi(t)$ depends on:

1. The ability of the firms to repay, given by the ratio of retained profits to the capital stock, $\sigma P/K$. A higher ratio of retained profits to capital stock would enable the borrowers to repay a larger fraction of the outstanding debt commitments without altering it's capital structure (i.e. without taking recourse to additional external finance); and,

2. The level of the index of financial fragility, λ. Higher level of λ is associated with a borrower profile where firms, in general, have higher gearing ratios, and hence, are forced to repay back a higher fraction of outstanding debt stock. Thus, in aggregate, a higher fraction of outstanding debt stock will actually be repaid back.

Based on these considerations, we suggest the following functional form for $\phi(t)$:

$$\phi(t) = \phi\left(\frac{\sigma P(t)}{K(t)}, \lambda(t)\right) \ ; \quad \phi_{\sigma P/K} > 0, \ \phi_\lambda > 0 \tag{13}$$

which, taking a linear functional form, might be expressed as

$$\phi(t) = m \frac{\sigma P(t)}{K(t)} \lambda(t)$$

where m is constant. Substituting for the value of $P(t)$ from (2) & (3), and for the value of $\lambda(t)$ from (9), we have $\phi(t) = m(q+r)(D(t)/K(t))$, or,

$$\phi(t) = m(q+r)\, d(t) \tag{14}$$

Next, we turn to the borrowing function, $B(t)$. In any given period t, let a fraction $a(t)$ of the total investment made by the firm sector be financed by fresh borrowing, i.e.

$$B(t) = a(t)\, I(t) \tag{15}$$

The fraction, $a(t)$, will be determined by the financial structure of the firms, i.e. the manner in which the firms decide to finance fresh investments. To arrive at a particular level of $a(t)$ the firms need to take two kinds of decisions: (a) the decision on distribution of the cost of investment between internal (i.e. retained profits) and external (i.e. debt and outside equities) sources of finance; and, (b) the decision on how to distribute the proportion of investment costs marked for external source between debt and equity financing. We first note the following:

Proposition 1. *For a given level of profits, a higher rate of investment would necessarily mean a higher level of outside sources of finance.*

Proof. Following a flow of funds approach, we note that the firm sector receives its funds from retained profits, borrowing and equity financing, and uses these funds in making planned investment, paying out outstanding debt commitments, and in unplanned accumulation of inventories, i.e. $\sigma P(t) + B(t) + E(t) \equiv I(t) + R(t) + \Delta N(t)$, where $\Delta N(t)$ represents the unplanned accumulation of inventories by the firm sector in period t. Substituting from (2), (3), (12) and (14), we have

$$B(t) + E(t) \equiv \left(1 - \frac{\psi}{s}\right) I(t) + m(q+r)\{d(t)\}^2 K(t) + \Delta N \tag{16}$$

$$\Rightarrow \frac{\partial (B(t) + E(t))}{\partial I(t)} \equiv \left(1 - \frac{\psi}{s}\right) > 0 \tag{17}$$

In other words, for a given level of profits, higher the level of investment higher would be the use of outside sources of finance like debt and outside equities. □

Further, though a detailed analysis of equity financing is beyond the scope of our analysis, we note the following:

Remark 1. *Between two sources of external finance, there might be an increasing preference for debt as the rate of investment increases.*

Remark 1 could be explained by the following:

1. The main difference between debt and equities is with regard to the resulting payment commitments. While the payment commitments arising out of debt commitments, consisting of the principal and the interest, is independent of profits, the payment commitments arising out of equity financing, consisting of dividends, $(1-\sigma) P(t)$, depend directly on profits. Hence, in periods of prosperity, characterized by a high rate of both investment and profits (related through the multiplier from (3)), cost of equity financing would be higher. In other words, any increase in investment would increase the cost of equity financing faster than the cost of debt financing.

2. Further, as increases in investment leads to increased recourse to external financing from proposition 1, the managers of the firms might be averse to continue increasing the dilution of shareholding from equity financing. Since a dilution of shareholding, by changing the ownership structure, increases the threat of hostile takeovers and change in corporate controls (provided, of course, such markets exist), managers might prefer debt financing when the requirement of external financing is higher.

It should be pointed out that proposition 1 and remark 1, taken together, establishes a direct relationship between the fraction of investment cost in any period, $a(t)$, financed by debt, $B(t)$. In addition, we also note the following:

Remark 2. *An increase in the level of financial fragility, λ, might necessitate financing a higher proportion of the cost of investment through debt.*

We should note that remark 2 is motivated by the relationship implied in (13). A higher level of financial fragility, λ, from (13), will imply that a higher fraction outstanding debt commitments will have to be repaid in the current period. This will require a higher level of borrowing, to be used not only towards meeting the cost of investment but also towards repaying outstanding debt commitments.

From proposition 1 and remark 1 and 2, we suggest the following functional form for $a(t)$:

$$a(t) = a(g(t), \lambda(t)) \ ; \quad a_g > 0,\ a_\lambda > 0 \tag{18}$$

which, taking a linear functional form and substituting from (2), (3) and (11), might be expressed as

$$a(t) = \frac{k(q+r)s}{\sigma\psi} d(t) \tag{19}$$

where k is a constant. Substituting from (12), (14), (15) and (19) into (8), we have

$$\dot{d}(t) = \left[\left\{\frac{k(q+r)s}{\sigma\psi} - 1\right\} g(t) - m(q+r)d(t) + r\right] d(t) \tag{20}$$

2.3 Financial Determinants of Investment

We now turn our attention to the financial determinants of the rate of investment. Consider the process of assessment of loan application by lenders. Any decision on such an application, in the form of an approval or lack of it, would involve a detailed analysis of the creditworthiness of the loan application. While the actual process of an assessment of creditworthiness can be quite complicated[3], we consider a simple version of this process here. Broadly, the quantitative factors determining the creditworthiness of a loan application might be categorized into two classes: those which remain unchanged across various stages of a business cycle; and, those which vary as an economy moves through a business cycle. In the first category, which might be considered as a preliminary assessment by the lending institutions, we might include permanent factors like the credit history and reputation of an individual, or a group of individuals. Based on these factors, the lending institutions assign a credit rating or score to the borrowers. A borrower might be classified as either *prime* or *sub-prime* through such a process. Once classified, the identity of a borrower does not change across various stages of the business cycle; in other words, a change in the rate of capacity utilization will have no impact on this identity of the borrower. However, the final decision on creditworthiness, in addition to above, is also likely to consider an additional component that includes current determinants. This would include, for instance, the current income of the loan applicant and an assessment of the expected future income. Assessment of future income might include, among other things, the expected profitability and risk associated with the investment project for which the borrower seeks a loan. As would be evident, these factors would vary across various stages in a business cycle; in particular, it would depend on the current rate of capacity utilization.

We begin by attempting to formalize the first, i.e. the fixed component of creditworthiness. As we noted above, this depends on an individual credit rating of each borrower. Consequently, consider the portfolio of a lender; this portfolio will be characterized by a certain spread of prime or safe, and sub-prime or risky borrowers. This might be formalized by introducing η, an indicator of the proportion of borrowers with high perceived risk of default in the overall debt portfolio, such that $\eta \in [0, 1]$. A higher value of η would imply a greater proportion of borrowers with high perceived risk of default in the macroeconomic distribution of debt.

Here we recall that one of the main arguments made in the Fisher-Minsky story described earlier was that periods of relative prosperity might be accompanied with a gradual worsening of the profile of borrowers, leading to inclusion of borrowers with higher perceived risk of default (i.e. the sub-prime borrowers). This inclusion of sub-prime borrowers would be quite evident if the prudential norms followed by the lenders are fixed at an absolute level. For instance, if having access to a particular value of loan requires furnishing a fixed amount of

[3]See, for instance, Kalapodas & Thomson (2006) and Abrahams & Zhang (2009) for a discussion of the process of credit risk assessment.

collateral, it is clear that a greater number of potential borrowers would be able to provide the required collaterals, and hence, have access to loan in periods of prosperity. In other words, those excluded by the debt market during periods with lower levels of economic activity would be included during periods of prosperity. The prudential norms, however, typically do not remain fixed but, in fact, are relaxed during periods of prosperity, because of optimistic expectations. Apart from a direct relaxation, financial innovation and predatory lending practices by organized lenders during a boom and emergence of new financial instruments might aid such relaxation of prudential norms during periods of prosperity (see, for instance, Kregel 2008, Shiller 2008, Abrahams & Zhang 2009, Reinhart & Rogoff 2009, Akerlof & Shiller 2010). This reinforces the impact of a phase of prosperity in increasing the proportion of risky borrowers in the macroeconomic distribution of debt.

Next, we formalize the above argument. Since the period of prosperity, as defined throughout our analysis, is characterized by an increase in u, Y and g, we suggest the following functional formulation for the proportion of risky borrowers, η in the portfolio:

$$\eta(t) = \eta_g g(t) \tag{21}$$

where η_g is a constant such that $\eta_g \in \left]0, \frac{1}{g_{\max}}\right]$.

We now construct a cumulative index of risk of default by including the impact of η, as defined above in (21), and the macroeconomic indicator of financial fragility, λ, as defined in (9), as follows:

$$\Lambda(t) = \Lambda_\eta \eta(t) + \Lambda_\lambda \lambda(t) \tag{22}$$

where Λ_η and Λ_λ represent the sensitivity of the cumulative index of risk of default to η and λ respectively.

One should note that the cumulative index of risk of default, Λ, consist of two separate risk components. These two components might be interpreted as emerging from two different kinds of risks involved in credit expansion. The first, or the proportion of risky borrowers in the macroeconomic distribution of debt or η, might be considered an indicator of risk involved in credit widening, i.e. inclusion of new borrowers, some of whom might be considered subprime. The second, the macroeconomic indicator of financial fragility or λ, on the other hand, might be considered a more conventional financial ratio that takes into account both credit deepening and credit widening. Hence, taken together, Λ might be considered a more comprehensive macroeconomic indicator of risk of default than some of the conventional indicators, since it takes into account both credit deepening and credit widening.

There are two ways the rate of investment might be affected by the risk of default. Firstly, as we have argued before, the managers are concerned with the risk of default, since in case of a default, a firm might face a hostile takeover, leading to a change in corporate control threatening the job of the managers. Hence, an increase in Λ might prompt the managers to respond by reducing the sensitivity of the rate of investment to the capacity utilization, i.e. the accelerator. Secondly, the lenders are concerned with the risk of default. An increase in a macroeconomic indicator of the risk of default like Λ is likely to make them more cautious about lending. In light of a substantial literature in this area (see, for instance Kalecki 1937, Hodgman 1960, Catt 1965, Stiglitz & Weiss 1981, Stiglitz & Weiss 1983, Jaffee & Stiglitz 1990, Stiglitz & Weiss 1992), we might note that a rationing and red-lining of credit might be one of the possible responses from the lenders under such a situation. While such a rationing and red- lining will directly affect only a section of borrowers, all borrowers are likely to take steps to reduce the possibility of being rationed and red-lined. Since individual or firm-level gearing ratio is one of the deciding factors on which firms are rationed or red-lined, an increase in Λ is likely to induce individual firms to respond by trying to reduce their gearing ratios. Since this logic applies to all the firms, an increase in Λ will have a negative impact on the accelerator of the investment function. We formalize this argument by introducing the following formulation for the accelerator:

$$\gamma(t) = \bar{\mu} - \hat{\mu}\Lambda(t) \tag{23}$$

where $\hat{\mu}$ is the sensitivity of the accelerator to the cumulative risk of default, and $\bar{\mu}$ represents the maximum possible level of the accelerator, when there is no risk of default. Substituting the values of $\lambda(t)$ and $\eta(t)$ from (11) and (21) into (22), and then substituting the resultant expression into (23), we have

$$\gamma(t) = \bar{\mu} - \hat{\mu}\Lambda_\eta \eta_g g(t) - \frac{\hat{\mu}\Lambda_\lambda (q+r)\, s}{\sigma\psi}\frac{d(t)}{g(t)} \tag{24}$$

Substituting the value of accelerator, $\gamma(t)$, from (24) into the investment function in (6), we have

$$g^\star(t) = \frac{\bar{\mu}}{s\beta} g(t) - \frac{\hat{\mu}\Lambda_\eta\eta_g}{s\beta}\{g(t)\}^2 - \frac{\hat{\mu}\Lambda_\lambda(q+r)}{\sigma\psi\beta} d(t) + \bar{\gamma} \tag{25}$$

Let the rate of investment be continuously adjusted so as to meet a fraction, h, of the gap between the actual and the desired rate of investment, i.e.

$$\frac{\dot{g}(t)}{g(t)} = h\left(g^\star(t) - g(t)\right) \tag{26}$$

subject to the feasibility condition $0 \leq g \leq g_{\max}$, where h represents the speed of adjustment of the actual investment to the desired level by the investors. Substituting the value of $g^\star(t)$ from (25) into (26), we have the following equation of motion to represent the dynamics of the rate of investment:

$$\dot{g}(t) = \left[\left(\frac{\bar{\mu}}{s\beta} - 1\right) g(t) - \frac{\hat{\mu}\Lambda_\eta\eta_g}{s\beta}\{g(t)\}^2 - \frac{\hat{\mu}\Lambda_\lambda(q+r)}{\sigma\psi\beta} d(t) + \bar{\gamma}\right] h g(t) \tag{27}$$

subject to the feasibility condition, $0 \leq g(t) \leq g_{\max}$.

3 Complete Model

From (27) and (20), we get the following $\mathbf{2}\times\mathbf{2}$ dynamical system:

$$\begin{aligned}\dot{g}(t) &= \left[\left(\frac{\bar{\mu}}{s\beta}-1\right)g(t)-\frac{\hat{\mu}\Lambda_\eta\eta_g}{s\beta}\{g(t)\}^2-\frac{\hat{\mu}\Lambda_\lambda(q+r)}{\sigma\psi\beta}d(t)+\bar{\gamma}\right]hg(t)\\ \dot{d}(t) &= \left[\left\{\frac{k(q+r)s}{\sigma\psi}-1\right\}g(t)-m(q+r)d(t)+r\right]d(t)\end{aligned}\tag{28}$$

which we rewrite as follows:

$$\begin{aligned}\dot{g}(t) &= \left[a_1g(t)-a_2\{g(t)\}^2-a_3d(t)+a_4\right]hg(t)\\ \dot{d}(t) &= \left[b_1g(t)-b_2d(t)+b_3\right]d(t)\end{aligned}\tag{29}$$

where $a_1\equiv\frac{\bar{\mu}}{s\beta}-1,\ a_2\equiv\frac{\hat{\mu}\Lambda_\eta\eta_g}{s\beta},\ a_3\equiv\frac{\hat{\mu}\Lambda_\lambda(q+r)}{\sigma\psi\beta},\ a_4\equiv\bar{\gamma},\ b_1\equiv\frac{k(q+r)s}{\sigma\psi}-1,\ b_2\equiv m(q+r),\ b_3\equiv r$, with $a_1,a_2,a_3,a_4,b_1,b_2,b_3\in\,]0,\infty[$. It might be noted that the dynamics in (29) resembles that of the generalized predator-prey or Kolmogorov-Lotka-Volterra class of models. The debt-capital ratio, d is the predator that feeds on the rate of investment, g. We should point out here that (29) contains at least two financial dampeners to mitigate the positive impact of the accelerator on the rate of investment, g, or the prey: firstly, the debt-capital ratio, which works through the indicator of financial fragility, λ; and secondly, the rate of investment itself for all $g>a_1/2a_2$, which works through the index of risk of default. We should also note that the rate of investment here plays a dual role; it has a positive role on itself through the accelerator, on the other hand, it also has a self-limiting negative role on itself through the risk of default. The self-limiting role originates in the arguments found in the Fisher-Minsky hypothesis described in section 1. Solving for the steady state of the dynamical system (29), we have:

$$E_1:\left(\bar{g}_1,\bar{d}_1\right)=(0,0)\tag{30a}$$

$$E_2:\left(\bar{g}_2,\bar{d}_2\right)=\left(-\frac{\sqrt{4\,a_2\,a_4+a_1^2}-a_1}{2\,a_2},0\right)\tag{30b}$$

$$E_3:\left(\bar{g}_3,\bar{d}_3\right)=\left(\frac{\sqrt{4\,a_2\,a_4+a_1^2}+a_1}{2\,a_2},0\right)\tag{30c}$$

$$E_4:\left(\bar{g}_4,\bar{d}_4\right)=\left(0,\frac{b_3}{b_2}\right)\tag{30d}$$

$$\begin{aligned}E_5:\left(\bar{g}_5,\bar{d}_5\right)=\Bigg(&-\frac{\sqrt{4\,a_2\,b_2^2\,a_4-4\,a_2\,b_2\,a_3\,b_3+b_1^2\,a_3^2-2\,a_1\,b_1\,b_2\,a_3+a_1^2\,b_2^2}+b_1\,a_3-a_1\,b_2}{2\,a_2\,b_2},\\ &-\frac{b_1\sqrt{4\,a_2\,b_2^2\,a_4-4\,a_2\,b_2\,a_3\,b_3+b_1^2\,a_3^2-2\,a_1\,b_1\,b_2\,a_3+a_1^2\,b_2^2}-2\,a_2\,b_2\,b_3+b_1^2\,a_3-a_1\,b_1\,b_2}{2\,a_2\,b_2^2}\Bigg)\end{aligned}\tag{30e}$$

$$\begin{aligned}E_6:\left(\bar{g}_6,\bar{d}_6\right)=\Bigg(&\frac{\sqrt{4\,a_2\,b_2^2\,a_4-4\,a_2\,b_2\,a_3\,b_3+b_1^2\,a_3^2-2\,a_1\,b_1\,b_2\,a_3+a_1^2\,b_2^2}-b_1\,a_3+a_1\,b_2}{2\,a_2\,b_2},\\ &\frac{b_1\sqrt{4\,a_2\,b_2^2\,a_4-4\,a_2\,b_2\,a_3\,b_3+b_1^2\,a_3^2-2\,a_1\,b_1\,b_2\,a_3+a_1^2\,b_2^2}+2\,a_2\,b_2\,b_3-b_1^2\,a_3+a_1\,b_1\,b_2}{2\,a_2\,b_2^2}\Bigg)\end{aligned}\tag{30f}$$

It would be evident that $E_2\notin\Re^2_{++}$ since $\bar{g}_2<0$. Hence we do not discuss E_2 any further in the following sections. Further, E_3 and E_4 are non-negative and lie on the g and d axis respectively. Regarding E_5 and E_6, we note the following:

Remark 3. *Whenever E_5 and E_6 are real and distinct, $\dot{d}/d = 0$ must intersect $\dot{g}/g = 0$ from above at E_5 and from below at E_6. If E_5 and E_6 are not distinct, then $\dot{d}/d = 0$ is a tangent to $\dot{g}/g = 0$ at the point representing the unique non-trivial steady state.*

Remark 4. *$a_3b_3 < a_4b_2$ is a sufficient (though not necessary) condition for the non-trivial steady state E_6 to be inside the real positive orthant, $\Re^2_{++}$.*

Remark 5. *For $g(t) \geq \bar{g}_3$, we have $\dot{g}(t) \leq 0$ for all $d(t) \in \Re^+$; in other words, if $\bar{g}_3 \leq g_{\max}$, then the feasibility condition $0 \leq g(t) \leq g_{\max}$ is always satisfied.*

For any $(g^\circ, d^\circ) \in \text{int } \Re^2_{++}$ as the initial point, let the solution to (29) be represented by $\Theta(t) = (g(t), d(t);\ g^\circ, d^\circ)$. From (29), we can conclude the following about the behavior of trajectories in case the initial point is on one of the axes:

$$\begin{array}{l}
\text{(a) } \dot{g} > 0,\ \dot{d} = 0\ \forall\ \{(g^\circ, d^\circ):\ g^\circ \in]0, \bar{g}_3[,\ d^\circ = 0\} \text{ as the initial point.} \\
\text{(b) } \dot{g} < 0,\ \dot{d} = 0\ \forall\ \{(g^\circ, d^\circ):\ g^\circ \in]\bar{g}_3, \infty[,\ d^\circ = 0\} \text{ as the initial point.} \\
\text{(c) } \dot{g} = 0,\ \dot{d} > 0\ \forall\ \left\{(g^\circ, d^\circ):\ g^\circ = 0,\ d^\circ \in \left]0, \bar{d}_4\right[\right\} \text{ as the initial point.} \\
\text{(d) } \dot{g} = 0,\ \dot{d} < 0\ \forall\ \left\{(g^\circ, d^\circ):\ g^\circ = 0,\ d^\circ \in \left]\bar{d}_4, \infty\right[\right\} \text{ as the initial point.}
\end{array} \tag{31}$$

i.e. both the g-axis and the d-axis are trajectories. Since trajectories cannot cross each other, this would make the real positive orthant invariant, i.e. trajectories starting from an initial point in the real positive orthant will always remain within it. Given that only dynamics strictly within the real positive orthant is economically meaningful, we focus our attention on only such trajectories and ignore other trajectories in the rest of our discussion. In other words, among the steady states listed in (30), we only consider E_5 and E_6 for discussion, and do not discuss the other steady states in the rest of this study.

Next we turn our attention to the trajectories starting from an initial point inside the real positive orthant. For $g, d \neq 0$, from (29) we have

$$\begin{array}{l}
\dot{g}(t) \lesseqqgtr 0 \quad \Leftrightarrow \quad d(t) \gtreqqless \dfrac{a_1}{a_3} g(t) - \dfrac{a_2}{a_3} \{g(t)\}^2 + \dfrac{a_4}{a_3} \\
\dot{d}(t) \lesseqqgtr 0 \quad \Leftrightarrow \quad d(t) \gtreqqless \dfrac{b_1}{b_2} g(t) + \dfrac{b_3}{b_2}
\end{array} \tag{32}$$

Depending on the configuration of parameters, we can list four different possibilities exhibiting qualitatively different dynamics (See figure 1):

1. Case 1: Here, $a_4b_2 - a_3b_3 > 0$, i.e. intercept of $\dot{g}/g = 0$ is greater than that of $\dot{d}/d = 0$, and $b_1/b_2 > (a_1 - 2a_2\bar{g}_6)/a_3 > 0$, i.e. $\dot{d}/d = 0$ intersects $\dot{g}/g = 0$ from below in the positively sloped section of the latter curve. $E_6 \in \text{int}\Re^2_{++}$ is the only steady state in this case inside the real positive orthant.

2. Case 2: Here, $a_4b_2 - a_3b_3 > 0$, i.e. intercept of $\dot{g}/g = 0$ is greater than that of $\dot{d}/d = 0$, but unlike case 1, $(a_1 - 2a_2\bar{g}_6)/a_3 < 0 < b_1/b_2$, i.e. $\dot{d}/d = 0$ intersects $\dot{g}/g = 0$ from below in the negatively sloped section of the latter curve. $E_6 \in \text{int}\Re^2_{++}$ is the unique steady state inside the real positive orthant.

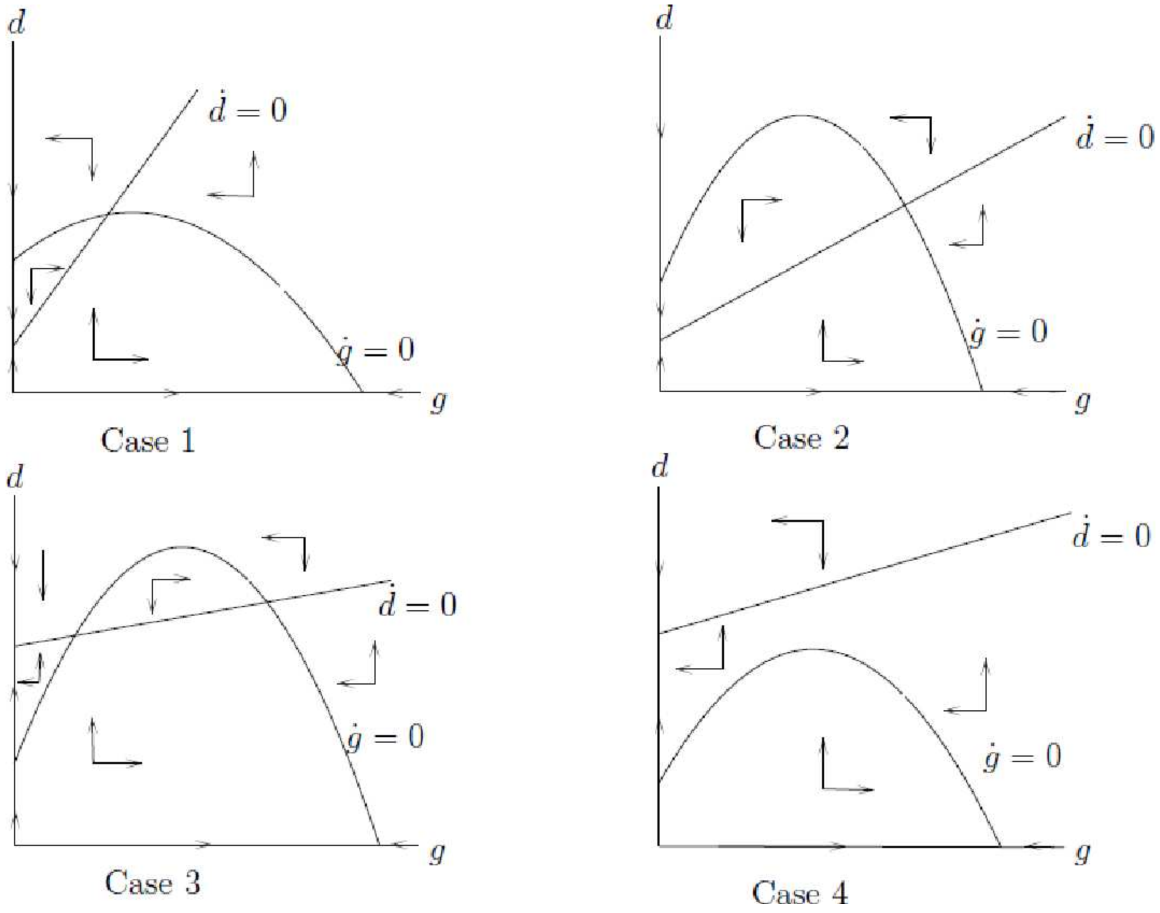

Figure 1: Phase diagram of (29): Four cases

3. Case 3: Here, $a_4b_2 - a_3b_3 < 0$, i.e. intercept of $\dot{g}/g = 0$ is less than that of $\dot{d}/d = 0$, and $(a_1 - 2a_2\bar{g}_5)/a_3 > b_1/b_2 > 0 > (a_1 - 2a_2\bar{g}_6)/a_3$, i.e. $\dot{g}/g = 0$ intersects $\dot{d}/d = 0$ from below at E_5 when the former is sloping upward, and from above at E_6 when the former is sloping downward. In this case, $E_5, E_6 \in \mathrm{int}\Re^2_{++}$, i.e. $\dot{d}/d = 0$ intersects $\dot{g}/g = 0$ twice in the interior of the real positive orthant.

4. Case 4: Here, $a_4b_2 - a_3b_3 < 0$, i.e. intercept of $\dot{g}/g = 0$ is less than that of $\dot{d}/d = 0$, and, unlike case 3, $E_5, E_6 \notin \mathrm{int}\Re^2_{++}$ so that there does not exist any steady state in the interior of the real positive orthant. Since we are interested in only the real positive orthant, we do not discuss case 4 any further in the rest of our discussion.

Further, performing the Routh-Hurwitz test for local stability on the two economically meaningful steady states, E_5 and E_6, we note that (a) *whenever the non-trivial steady state solution, E_5 exists and is distinct from E_6 and lies in the interior of real positive orthant, it is a saddle-point;* and, (b) *depending on the configuration of the parameters, the non-trivial steady state solution, E_6, whenever it exists and is distinct from E_5 and lies within the interior of the real positive orthant, is either a source or a sink.* We further note that E_6 is always a sink in case 2 and 3.

4 Possibilities of Cyclical Behavior

Next, we investigate possibilities of growth cycles emerging from an interaction between the investment function and debt dynamics. For this purpose, we restrict our attention to case 1 of figure 1 since, from above, this is the only case where cyclical possibilities exist. We recall that this is the case where E_6 is the unique steady state in the interior of positive orthant, and at E_6, the positive impact of g on $\dot{g}/g$ outweighs its negative impact.

Cyclical possibilities in dynamical systems of the type represented by (28) or (29) have been investigated extensively in Datta (2012). Here we present a summary of these results:

1. For the dynamical system represented by (28) or (29), we define a critical value of the parameter h given by $\hat{h}$, where h represents the rate of adjustment of the actual rate of investment to its desired rate by the private investors and $\hat{h}$ is defined as follows:

$$\hat{h} = \frac{b_2 \bar{d}_6}{\left(a_1 - 2a_2\bar{g}_6\right)\bar{g}_6} > 0 \tag{33}$$

which, by substituting the values of $\bar{g}_6$ and $\bar{d}_6$ from (30), might be expanded as

$$\hat{h} = \frac{b_1\, b_2 \sqrt{4\, a_2\, b_2^2\, a_4 - 4\, a_2\, b_2\, a_3\, b_3 + b_1^2\, a_3^2 - 2\, a_1\, b_1\, b_2\, a_3 + a_1^2\, b_2^2} + 2\, a_2\, b_2^2\, b_3 - b_1^2\, b_2\, a_3 + a_1\, b_1\, b_2^2}{\left(2\, b_1\, a_3 - a_1\, b_2\right)\sqrt{4\, a_2\, b_2^2\, a_4 - 4\, a_2\, b_2\, a_3\, b_3 + b_1^2\, a_3^2 - 2\, a_1\, b_1\, b_2\, a_3 + a_1^2\, b_2^2} - 4\, a_2\, b_2^2\, a_4 + 4\, a_2\, b_2\, a_3\, b_3 - 2\, b_1^2\, a_3^2 + 3\, a_1\, b_1\, b_2\, a_3 - a_1^2\, b_2^2} \tag{34}$$

At $h = \hat{h}$, we have a point of non-degenerate Andronov-Hopf bifurcation, leading to emergence of limit cycles.

2. Depending on the values of various parameters, the Andronov-Hopf bifurcation is either supercritical or subcritical, leading to emergence of either stable or unstable limit cycles respectively. Whether the Andronov-Hopf bifurcation is stable or unstable can be determined if we are provided with information on the values of various parameters.[4]

3. In case the limit cycle emerging from Andronov-Hopf bifurcation is unstable, we have another stable limit cycle enclosing the unstable limit cycle.

4. For $h > \hat{h}$, we have a stable limit cycle from an application of Poincaré-Bendixson theorem.

In other words, there exists a unique stable limit cycle for all $h \geq \hat{h}$.

[4]For instance, if the parameters have values as follows:

$$\begin{array}{lllllll} s = 0.3, & \sigma = 0.4, & \psi = 0.3, & r = 0.1, & q = 0.6, & m = 0.6, & k = 0.7, \\ \beta = 0.8, & \bar{\mu} = 0.3, & \hat{\mu} = 0.4, & \eta_g = 0.1, & \Lambda_\eta = 0.1, & \Lambda_\lambda = 0.63, & \bar{\gamma} = 0.5. \end{array}$$

then at the non-trivial steady state, E_6, the rate of investment, $\bar{g}_6$, is at 8.49% and the debt-capital ratio is at 28.36%. The Poincaré-Andronov-Hopf bifurcation for this steady state occurs at $h = 5.67$, leading to the emergence of limit cycles. The first lyapunov exponent at this point can be calculated to be -1.02×10^{-5}, which is negative; hence, the Andronov-Hopf bifurcation is supercritical and the limit cycles are stable.

We should further note, given that the limit cycle is to be interpreted as a growth cycle in the rate of investment, g, and the debt-capital ratio, d, we have a robust result for existence of a growth cycle. The growth cycle emerges under a wide range of conditions, i.e. for all $h \geq \hat{h}$, irrespective of the values of other parameters, and for a very wide range of initial conditions.

We next turn to the implication of this result, by looking closely at some of the properties of this growth cycle.

5 Business and Financial Cycles

We noticed in section 4 a variety of cyclical possibilities. We now turn our attention to the behavior of the economy through various stages of a business cycle.

5.1 Business Cycles

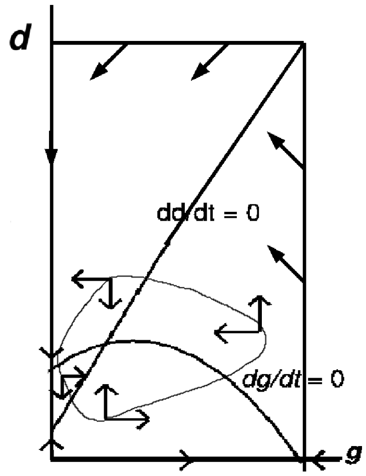

Figure 2: An Economic Cycle

5.1.1 Stage 1: Period of high growth

In this stage, there is an increase in both the rate of investment, g and the debt-capital ratio, d. Following a recent history of high growth phase (see figure 2), this phase is also accompanied with an all-round optimistic expectations. However, this phase also contains conditions for a worsening of financial variables in the following ways:

1. An increase in d, ceteris paribus, leads to an increase in financial fragility, λ. This leads to an increase in the cumulative index of risk of default, Λ.

2. An increase in g, as argued above, will lead to an increased inclusion of risky or subprime borrowers, resulting in a fall in the profile of borrowers. In our model, this is captured by an increase in the proportion of risky borrowers, η. This will further put an upward pressure on the cumulative risk of default, Λ.

As we have argued above, an increase in Λ creates a negative impact on the rate of investment, g, in our model. This negative impact occurs because of two sets of reasons. Firstly, as we have argued before, the managers are concerned with the risk of default, since in case of a default, a firm might face a hostile takeover, leading to a change in corporate control threatening the job of the managers. Hence, an increase in Λ might prompt the managers to respond by reducing the sensitivity of the rate of investment to the capacity utilization, i.e. the accelerator. Secondly, the lenders are concerned with the risk of default. An increase in a macroeconomic indicator of the risk of default like Λ is likely to make them more cautious about lending, possibly leading to a rationing and red-lining of credit. While such a rationing and red-lining will directly affect only a section of borrowers, all borrowers are likely to take steps to reduce the possibility of being rationed and red- lined. Since individual or firm-level gearing ratio is one of the deciding factors on which firms are rationed or red-lined, an increase in Λ is likely to induce individual firms to respond by trying to reduce their gearing ratios. Since this logic applies to all the firms, an increase in Λ will have a negative impact on the accelerator of the investment function.

The negative impact, however, will be offset in this stage by the positive impact of an increase in g. Primarily this will operate through an increased demand having a positive impact on investment through a combination of the multiplier and the accelerator. There will also be an indirect positive impact: an increase in g, by increasing retained earnings, ceteris paribus, will have a negative impact on financial fragility and risk of default, which in turn will have a positive impact on the rate of investment, g.

5.1.2 Stage 2: Onset of a financial crisis

This stage begins when the negative factor discussed above starts dominating the positive factors, resulting in a fall in the rate of investment, g. A fall in g would lead to a reduction in borrowing, imparting a negative impact on the debt-capital ratio, d. The negative impact on d will be further reinforced by an increase in λ forcing an increase in repayment of debt. However, the negative impact on d will lead to an actual decrease in d only with a lag. Till that happens, the economy will be characterized by classis features of onset of an economic crisis: a fall in the rate of investment along with an increase in debt-capital ratio.

5.1.3 Stage 3: Full-blown recession

In this stage, g continues to fall. The negative factors on d discussed above finally results in a fall in d. In other words, both the rate of investment and the debt-capital ratio falls in this stage. Conditions for a turnaround and recovery, however, are also created in this stage.

This would primarily operate through an improvement in financial variables in the following manner:

1. A decrease in d, cateris paribus, leads to a decrease in the financial fragility, captured by λ. This leads to a reduction in the cumulative index of risk of default, Λ.
2. A decrease in g, implying a recession, will lead to fall in the proportion of risky or subprime borrowers, leading to a rise in η. This would primarily operate through a process of exclusion from the debt market. In other words, recession would lead to exclusion of those borrowers who might have had access to loans during better times. This would lead to a fall in the cumulative index of risk of default, Λ.

One should expect a fall in Λ to have a positive impact on the rate of investment, g. Such an expectation should follow from a straightforward and symmetric application of the logic provided above in our discussion of stage 1. Firstly, a decrease in the risk of default would reduce fear of defaults and takeover for the managers of the firms, allowing them to invest more aggressively. Secondly, the lenders, faced with a reduced risk of default, might eventually reduce credit rationing and red-lining, allowing the firms to borrow and invest more.

The positive impact on g, however, is offset by the negative impact of a decrease in g. This will primarily operate through a situation of reduced demand having a negative impact through multiplier and the accelerator. Further, a fall in g, by reducing profits and retained earnings, will also tend to increase the financial fragility, λ (where g appears in the denominator), which, through the investment function, will have further negative impact on g. The negative effect will dominate in this stage.

One also needs to exercise a bit of caution here in a symmetric application of the logic provided in stage 1. Unlike the process of inclusion of risky borrowers in stage 1, (which leads to an immediate impact), their exclusion is not as straightforward. This is because despite their exclusion from fresh borrowing, the risky borrowers who have already borrowed will still remain in the market. Further, unlike the process of inclusion, the process of exclusion might also lead to these borrowers facing a payment crisis, leading to various complications beyond the scope of analysis of our model. In other words, there is an element of assymmetry in an increase and a decrease in η - a fact which is not captured in our model.

5.1.4 Stage 4: Recovery

This stage begins when the factors having a positive impact on g discussed above starts dominating, leading to an increase in g. The debt-capital ratio, d, however will continue to fall. An increase in g, by increasing borrowing will have a positive impact on d. This will be further reinforced by a fall in d reducing λ, and hence repayments. However these effects will lead to an actual increase in d only with a lag. Till that happens, the economy will be in a purely recovery path, with the rate of investment, g increasing along with a continuing fall in the debt-capital ratio, d. Once there is a turnaround in d, the economy leaves stage 4 and re-enters stage 1.

The dynamics of g and d through various stages of cycle are shown below in figure 3 and 4 respectively.

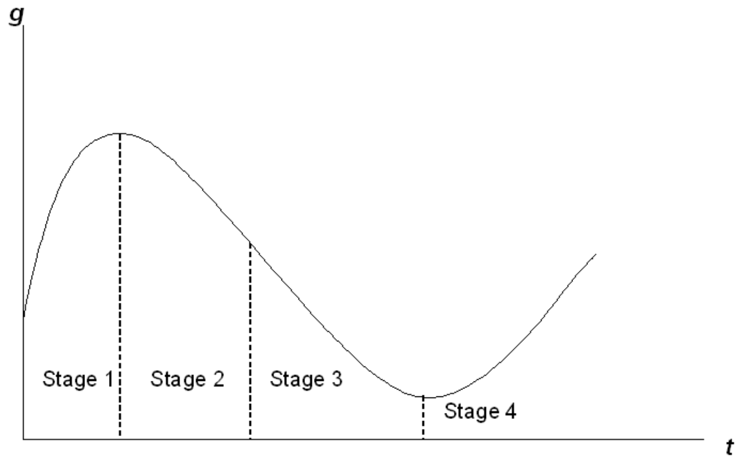

Figure 3: The rate of investment through a business cycle

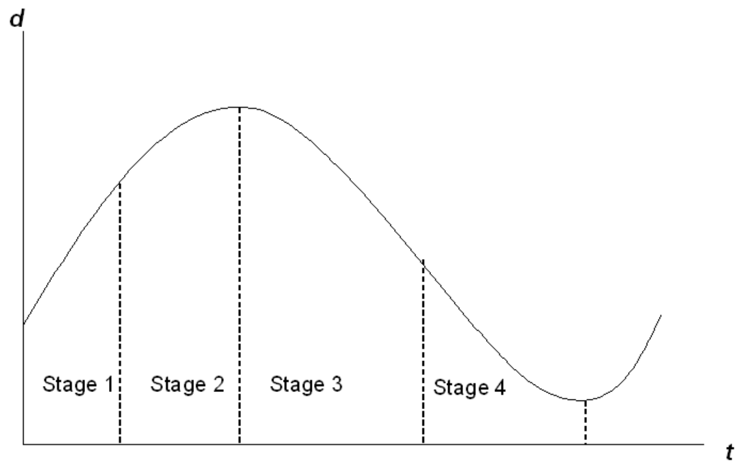

Figure 4: The debt-capital ratio through a business cycle

5.2 Financial Cycles

It would be evident from above discussion that the financial sector, in the form of debt market, plays an important role in the business cycle. Hence, we would expect a financial cycle to accompany the business cycle. However, as we find out below, the financial cycle is not synchronized with the business cycle (i.e. the cycle in g) but in fact precedes the latter.

This is best captured by the index of financial fragility, λ in our model. We recall from (11) that the index of financial fragility, λ is given by

$$\lambda = \frac{k\,(q+r)\,sd}{\sigma\psi g}$$

Taking logarithmic differentiation of both sides, we have

$$\frac{\dot{\lambda}}{\lambda} = \frac{\dot{d}}{d} - \frac{\dot{g}}{g} \tag{35}$$

From (35), it would be clear that λ starts stage 1 by decreasing till it reaches a trough, and then starts increasing within stage 1. It continues to increase through stage 2 and beginning of stage 3. Within stage 3, it reaches a peak and then starts declining. This decline in λ continues through stage 4 into stage 1. This is shown in figure 5.

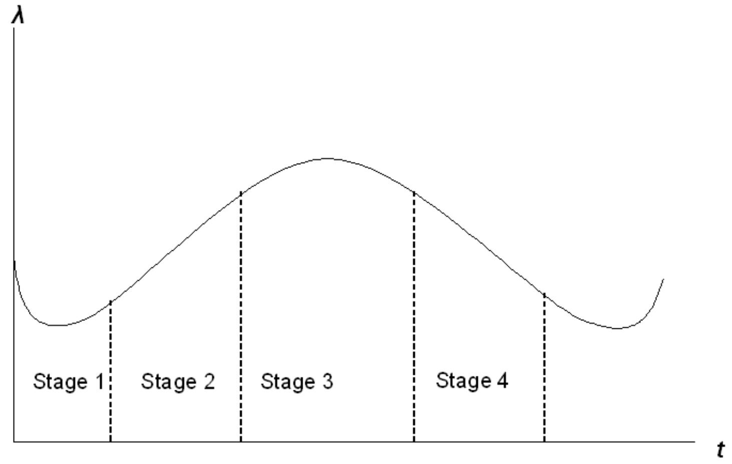

Figure 5: A financial cycle

It would be clear that the cycle in λ precedes the cycle in g. For instance, in stage 1, the turnaround in λ occurs when it starts increasing in the middle of stage 1. However, the turnaround in g occurs only at the end of stage 1 when g starts falling. Similarly, while the next turnaround in λ occurs in the middle of stage 3 when it starts, falling, the turnaround in g occurs only at the end of stage 3. The lag between two cycles is shown in figure 6, where both the business and the financial cycles are superimposed on each other. This also seems to fit in well with the general observation that a financial crisis typically works as a precursor to a general economic crisis.

6 Concluding Remarks: A Reconsideration of Fisher-Minsky Hypothesis

The model developed in this study includes the primary contention of the Fisher-Minsky hypothesis, that there is a deterioration of financial variables during boom, captured by

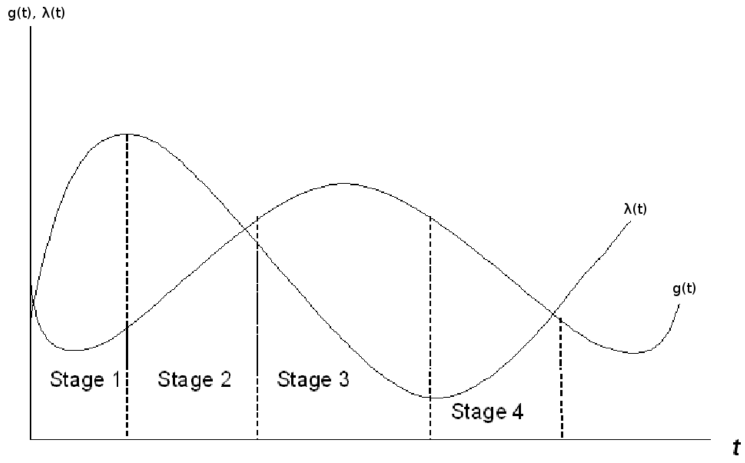

Figure 6: Lag between financial and business cycles

an increase in Λ. In addition, it also offers a macroeconomic mechanism by which this deterioration of financial variables might put an end to the boom. We find that such a macroeconomic mechanism, in addition to providing endogenous bounds, also leads to growth cycles, involving cyclical behavior in rate of investment and debt-capital ratio. Further, we find that a financial cycle would typically precede cycles in the rate of investment and output. Thus, our model offers an endogenous explanation for turnarounds in business cycles driven by financial factors, and hence, preceded by a financial cycle. A boom will end, for instance, when a deterioration in financial variables will induce a cutback in the rate of investment. The end to the boom, therefore, will be preceded by a financial crisis. In this sense, we might offer our model as providing a more complete story of finance-led growth cycles than the existing literature around the Fisher-Minsky hypothesis. Since it explains some of the missing links without resorting to some of the less than convincing routes often found in some of the literature, we might consider this as a substantial contribution to the literature in this area.

We should, however, exercise a bit of caution while drawing conclusions from our model developed above. Firstly, we should note that, while in our model the turnaround at the peak and the trough of the growth cycle is treated in a symmetric manner, in real world a number of complications might make such a symmetric treatment unwarranted. As we noted earlier, the exclusion of subprime borrowers, unlike their inclusion, often involves a time-lag. While the lenders might exclude new borrowers from having access to fresh borrowing, existing subprime borrowers can be excluded only with a time-lag, i.e. only after the existing debt contracts have expired. Further, exclusion of these borrowers might trigger off an all-round payment crisis in the economy, creating further complications. Thus, typically in a real world economy, while the end to the boom might occur endogenously, the end to a recession often requires state intervention in the form of writing off existing loans or playing the role of lender-of-last-resort. Secondly, the model developed above does not include income distribution considerations and the role of expectations leading to changes in asset prices. In light of a substantial literature in this area, we should note that such considerations might play a significant role in these growth cycles. This is an area we reserve for future research.

In other words, the model presented in this paper should primarily be looked upon as an investigation into the nature of macroeconomic feedback mechanism between an investment function and debt dynamics. Such a feedback mechanism is able to provide an endogenous bound to the rate of investment, and in this sense, fills a gap in the existing literature attempting to model the Fisher-Minsky hypothesis.

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