Forever Minus a Day? Some Theory and Empirics of Optimal Copyright

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ABSTRACT. The optimal level for copyright has been a matter for extensive debate over the last decade. This paper contributes several new results on this issue divided into two parts. In the first, a parsimonious theoretical model is used to prove several novel propositions about the optimal level of protection. Specifically, we demonstrate that (a) optimal copyright is likely to fall as the production costs of ‘originals’ decline (for example as a result of digitization) and that (b) the optimal level of copyright will, in general, fall over time. The second part of the paper focuses on the specific case of copyright term. Using a simple model we characterise optimal term as a function of a few key parameters. We estimate this function using a combination of new and existing data on recordings and books and find an optimal term of around fifteen years. This is substantially shorter than any current copyright term and implies that existing copyright terms are too long.

Keywords: Copyright, Intellectual Property, Copyright Term, Protection

JEL Classification: O31 O34 L10

Corresponding author: Rufus Pollock, Faculty of Economics, Cambridge University, Sidgwick Avenue, Cambridge, CB3 9DD. Email: rp240@cam.ac.uk. This paper is licensed under Creative Commons attribution (by) license v3.0 (all jurisdictions). I thank my advisors Rupert Gatti and David Newbery, participants at the 2007 SERCI conference as well as those individuals who emailed or posted comments and suggestions. All remaining errors are mine.
1. Introduction

The optimal level of copyright, and in particular, copyright term have been matters of some importance to policymakers over the last decade. For example, in 1998 the United States extended the length of copyright from life plus 50 to life plus 70 years, applying this extension equally to existing and future work.1 More recently in the EU generally, and particularly in the UK, there has been an extensive debate over whether to extend the term of copyright in sound recordings.

Using a parsimonious framework based on those already in the existing literature (see e.g. Landes and Posner (1989); Watt (2000)) we analyze various questions related to the optimal level of copyright protection, deriving, under a simple set of assumptions, several novel results. In particular, we show that (a) optimal protection is likely to decrease as the cost of production falls (and vice-versa); and (b) the level of optimal protection, in general, declines over time.

Note that costs are divided into those related to ‘production’, ‘reproduction’ and ‘distribution’ with the distinction between the first two being that production costs are those relating to the creation of the first instance of a work while reproduction relates to the costs of producing subsequent copies. However in this particular case we take ‘production’ costs to include all expenditures, fixed as well as variable, related to the creation and distribution of the first version of the work and all authorised reproductions thereof (these are often termed ‘originals’ in the literature in opposition to ‘copies’: unauthorised – though not necessarily illegal – reproductions of the work in question).

This first result is of particular interest because recent years have witnessed a dramatic, and permanent fall, in the costs of production of almost all types of copyrightable subject matter as a result of rapid technological advance in ICT and related fields. With the growth of the Internet costs of distribution have plummeted and will continue to do so as both the capacity and the level of uptake continue to increase. Similarly, cheaper

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1It was in a congressional speech prior to the enactment of the Copyright Term Extension Act (CTEA) that Mary Bono, widow of the musician Sonny Bono, famously referred to the proposal of Jack Valenti, president of the Motion Picture Association of America, to have copyright last for ‘Forever minus a day’: “Actually, Sonny wanted the term of copyright protection to last forever. I am informed by staff that such a change would violate the Constitution, . . . As you know, there is also Jack Valenti’s proposal for the term to last forever less one day. Perhaps the Committee may look at that next Congress.” (CR.144.H9952)
computers, cameras, and software have had a significant impact on basic production costs in both the low and high end market.

One caveat needs to be mentioned here. As discussed, there is a distinction to be drawn both between authorised and unauthorised reproduction. The move to a digital environment reduces the costs of both of these types of activities – formally, there is a high degree of correlation between the changes in the costs of producing ‘originals’ and ‘copies’. As a variety of authors have pointed out, a reduction in the cost of making ‘copies’, that is in the cost of unauthorised reproduction, may or may not necessitate an increase in the optimal level of protection – see e.g. (Johnson, 1985), (Novos and Waldman, 1984), Liebowitz (1985) and Peitz and Waelbroeck (2006). Our result, by contrast, deals with the case of a reduction in costs related to ‘originals’.

The second result, that optimal protection falls over time, also has importance for policy. In most systems of law, it is extremely difficult to remove or diminish rights once they have been granted. Thus, once a given level of protection has been awarded it will be all but impossible to reduce it. However, according to our result, the optimal level of protection will decline over time (as the amount of work available grows). This being the case, a prudent policy-maker faced with uncertainty would want to be especially careful about increasing the level of copyright.

Finally, in the last section of the paper we turn to the specific case of copyright ‘term’ – that is, the duration of the copyright. Building on the framework already developed, we derive a single simple equation which defines optimal copyright term as a function of the key exogenous variables: the discount rate, the rate of ‘cultural decay’, the supply function for creative work and the associated welfare (and deadweight-loss) associated with new works. Combining this with empirical data we are able to provide one of the first theoretically and empirically grounded estimates of optimal copyright term.2

2As Png (2006) notes, there is a lack of empirical work on copyright generally. Existing estimates of optimal term are very sparse. Boldrin and Levine (2005) calibrate a macro-oriented model and derive a figure of 7 years for optimal term in the United States. (Akerlof et al., 2002) in an examination of the US Copyright Term Extension Act argue, simply on the basis of the discount rate, that a term of life plus seventy years must be too long. By contrast, Liebowitz and Margolis (2005), argue that the current US term of life plus 70 years might not be too long – though they too do not provide an explicit model.
2. A Brief Note on Copyright Law

The reader should be aware that the term of copyright varies both across jurisdictions and across types of protected subject matter. The right in a recording – as opposed to the underlying composition – is considered a ‘neighbouring right’ and is treated differently from a normal ‘copyright’. In particular, signatories to the Berne convention (and its revisions) must provide for an ‘authorial’ copyright with a minimal term of life plus 50 years, recordings need only be protected for 50 years from the date of publication.

Furthermore, and rather confusingly, works can sometimes be moved from one category to the other as was the case with film in the UK following the implementation of the 1995 EU Directive on ‘Harmonizing the Term of Copyright Protection’ (which ‘harmonised’ copyright term up to life plus 70 years). Prior to this UK law had treated the copyright in the film itself as a neighbouring right and therefore accorded it a 50 year term of protection. Following the implementation of the Directive, the copyright in a film became an ‘authorial’ copyright and subject to a term of protection of life plus 70 years.\(^3\)

3. Framework

In this section we introduce a minimal framework but one which is still rich enough to allow the derivation of our results.

The strength of copyright (also termed the level of protection) is represented by the continuous variable \(S\) with higher values implying stronger copyright. For our purposes here it will not matter exactly what \(S\) denotes but the reader might keep in mind, as examples, the length of copyright term and the breadth of the exclusions (conversely the narrowness of the exceptions from the monopoly right that copyright affords its owner).

Many possible possible works can be produced which may be labelled by 1,2,3, ... Let \(N = N(S)\) denote the total number of works produced when the strength is \(S\).\(^4\) Note that \(N\) may also depend on other variables such as the cost of production, the level of demand.

\(^3\)That was not all, as Cornish and Llewelyn (2003, para. 10-45) note, ‘the very considerable investment which goes into major film productions was held to justify a special way of measuring lives. To guard against the consequences of the director’s early death, the longest life among “persons connected with the film” is taken; and these include not only the principal director but the author of the screenplay, the author of the dialogue and the composer of any specifically created film score.’

\(^4\)Throughout we shall gloss over the fact that \(N\) is discrete and allow the differential both of \(N\) and with respect to \(N\) to exist.
etc. however we have omitted these variable from the functional form for the time being for the sake of simplicity.

**Assumption 1.** (The form of the production function for copyrightable work)

1. At low levels of protection, increasing protection increases the production of works: \[ \lim_{S \to 0} N'(S) > 0. \]
2. Diminishing returns to protection: \[ N''(S) < 0. \]
3. (optional) Beyond some level increasing protection further reduces production: \[ \lim_{S \to \infty} N'(S) < 0. \]

Each work created generates welfare for society, and we denote by \( w_i \) the welfare generated by the \( i \)'th work. The welfare deriving from a given work (once produced) depends on the strength of copyright, so \( w_i = w_i(S) \) and it is assumed that increasing copyright reduces the welfare generated from a work so \( w'_i(S) < 0. \)

Total welfare, denoted by \( W = W(N, S) \), is then the aggregation of the welfare from each individual work. This need not be a simple sum as we wish to allow for interactions between works – for example we would expect that as there are more and more works the value of new work declines. We shall discuss this further below, but for the time being we may leave the exact form of aggregation opaque.

**Assumption 2.** Using subscripts to indicate partial differentials:

1. Welfare is increasing in the number of works produced: \( W_N > 0. \)
2. Keeping the number of works produced fixed, welfare is decreasing in the strength of copyright: \( W_S < 0 \) (this follows immediately from the assumption of diminishing welfare at the level of individual works).
3. Diminishing marginal welfare from new works: \( W_{NN} < 0. \)

Since the number of works produced is itself a function of the level of copyright we may eliminate \( N \) as an argument in \( W \) and write:

\[
W = W(S) = W(N(S), S)
\]

\footnote{This assumption is based on a very similar one in Landes and Posner (1989). Unless otherwise stated this assumption will not be used when deriving any of the results below.}
Where it is necessary to distinguish the different forms of the welfare function we shall denote this version as the ‘reduced form’. As \([0, \infty]\) is compact and \(W(S)\) is a continuous function \(W\) has a unique maximum somewhere in this range. As this is the welfare maximizing level of protection we term this the \textit{optimal} level.

Finally before commencing on the derivation of results we require the technical assumption that all functions are continuous and at least twice continuously differentiable.

4. The Relation of the Production and Welfare Maximising Levels of Protection

\textbf{Lemma 3.} Under assumptions 1.1 and 1.2 there exists a unique level of protection which maximizes the production of creative work. We denote this by \(S^p\). Furthermore, EITHER there exists a finite solution to \(N'(S) = 0\) and this is \(S^p\) OR no such solution exists and \(S^p = \infty\). With assumption 1.3 only the first option is possible.

\textit{Proof.} By Assumption 1.1 \(N\) is increasing when the level of protection is 0 (the lowest possible) thus 0 cannot be a maximum. By Assumption 1.2 if a finite maximum exists it must be unique and this maximum must be a solution of \(N'(S) = 0\) (if there is such a solution then \(N'\) is negative from that solution onwards so infinity is not a solution). If no such solution exists then for all \(S > 0\) we have \(N'(S) > 0\) and the maximizing level of protection is infinite. \(\Box\)

\textbf{Theorem 4.} If the level of protection which maximizes the production of copyrightable work, \(S^p\), is finite then the optimal level of protection, \(S^o\), is strictly less than \(S^p\).

\textit{Proof.} If \(S^p\) is finite then \(N'(S^p) = 0\) and since \(N''(S) < 0\) we have \(N'(S) \leq 0, \forall S \geq S^p\).

Marginal welfare is:

\[ W'(S) = \frac{dW(S)}{dS} = \frac{dW(N(S), S)}{dS} = N_S W_N + W_S \]

Now \(W_S < 0, \forall S\), so combining this with the properties of the work production function, \(N(S)\), we have that:

\[ \forall S \geq S^p, W'(S) < 0 \]
Hence, welfare is already declining at $S_p$ and continues to decline thereafter. Thus, the optimal, that is welfare maximizing, level of protection, $S^o$, must lie in the range $[0, S_p)$. □

Remark 5. If the level of protection which maximizes the production of copyrightable work, $S_p$, is infinite then no immediate statement can be made as to whether the optimal level of protection, $S^o$, will be finite (and hence less than $S_p$) or infinite.\textsuperscript{6}

From this point on we make the following assumption:

\textbf{Assumption 6.} The optimal level of protection is finite, and is the unique level of protection, $S^o$, satisfying $W'(S^o) = 0, W''(S^o) < 0$.

5. \textsc{Production Costs and the Optimal Level of Protection}

Let us now introduce production costs by writing write $N = (C, U, S)$ where $C$ is a variable denoting production costs of ‘originals’ (authorised reproductions) and $U$ a variable denoting the production cost of ‘copies’ (unauthorised reproductions) (we do not need to be specific here as to their form so these may be marginal costs or fixed costs or both).\textsuperscript{7} We assume that:

1. For any given level of protection as the costs of ‘originals’ increase (decrease) production decreases (increases): $N_C < 0$

2. For any given level of protection as the costs of ‘copies’ (unauthorised reproductions) increase (decrease) production increases (decreases): $N_U > 0$.\textsuperscript{8}

\textsuperscript{6}For example, consider a very simple multiplicative structure for total welfare of the form: $W(S) = f(N(S))w(S)$ with $f(N)$ any functional form with $f' > 0, f'' < 0$ (e.g. $N^a, a \in (0, 1)$). Then taking any function $g(S)$ with $g' > 0, g'' < 0$ and defining $N(S) = g(S), w(S) = g(S)^{1-a+\epsilon}, \epsilon \in (0, a)$ we have a setup satisfying Assumptions 1 (excluding 1.3) and 2 and with $W(S) = g(S)^a - a$ a welfare function whose maximising level of protection is clearly infinite. Finally note that this does not require that the number of works produced be infinite, for example we could have $g(S) = 1 + K - K/(1 + S)$ in which case there is a finite upper bound on the number of works produced.

\textsuperscript{7}Note that we would usually assume that the cost of making ‘copies’ is itself, at least partially, a function of the level of protection. However here we prefer to keep the effect of the level of protection and of the cost of making ‘copies’ distinct. Thus, it is perhaps better to think of $U$ as encapsulating copying costs as determined purely by exogenous factors such as technology.

\textsuperscript{8}The assumption that decreases in the cost of unauthorised copying are unambiguously bad for the producers of copyrightable works is a standard one. However, there are at least two factors which operate in the opposite direction. First, ‘copiers’ still need to purchase ‘originals’ and thus producers of ‘originals’ may still be able to extract rents from ‘copiers’ by raising the price of originals much in the way that the price of a first-hand car takes account of its resale value on the second-hand market (see Liebowitz (1985)). Second, greater dissemination of a work due to unauthorized copying may lead to increase in
We also need to take account of the impact of costs on welfare. To reflect this we rewrite welfare as a function of both the level of protection and the level of costs: \( W = W(S, C, U) \).

**Lemma 7.** Take any exogenous variable \( X \) which affects the welfare function (whether directly and/or via its effect on production \( N \)). Assuming that the initial optimal level of protection, \( S^o \), is finite, if \( d^2W(S^o)/dXdS \) is positive then an increase (decrease) in the variable \( X \) implies an increase (decrease) in the optimal level of protection.

**Proof.** Denote the initial optimum level of protection, where \( X \) is at its initial value, by \( S^o \). Since we are a finite optimum we have that at \( S^o \):

\[
W'(S^o) = N_S W_N + W_S = 0 \tag{5.1}
\]
\[
W''(S^o) < 0 \tag{5.2}
\]

Suppose, \( X \) now increases. Since \( d^2W/dXdS \) is positive we must now have: \( W'(S^o) > 0 \). For small changes in \( X \), \( W''(S^o) \) is still negative and thus protection must increase to some \( S^{o2} > S^o \) in order to have \( W'(S^{o2}) = 0 \); and \( S^{o2} \) is the new optimum level of protection. \( \square \)

5.1. **Production Costs.** Let us consider first, what occurs is there is an increase (or conversely a decrease) in the costs of producing ‘originals’ with all other exogenous variables, including the cost of producing ‘copies’, unchanged. Substituting \( C \) for \( X \) we have:

**Corollary 8.** If \( d^2W(S^o)/dCdS > 0 \) then an increase (decrease) in costs of ‘originals’ implies an increase (decrease) in the optimal level of protection.

Given the importance of signing \( d^2W/dCdS \) let us explore further by working through the differential:

\[
\frac{d^2W}{dCdS} = \frac{d}{dC}(N_S W_N + W_S) = N_{CS} W_N + N_S W_{NN} N_C + N_S W_{CN} + W_{SN} N_C + W_{CS}
\]

Demand for ‘originals’ or for complementary goods, particularly if ‘copies’ and ‘originals’ are not perfect substitutes. For a recent theoretical model see Peitz and Waelbroeck (2006). Empirical work, mainly centred on the impact of unauthorised file-sharing on music sales has, as yet, provided no decisive answer as to whether ‘sampling’ may outweigh ‘substitution’ (see, for example, the contradictory results of Oberholzer and Strumpf (2007) and Blackburn (2004)). Given these uncertainties, we feel it prudent to stick with the straightforward, and conservative, assumption that decreases in the cost of unauthorised copying decreases the production of creative work.
Now:

1. \( W_C < 0 \) – welfare declines as costs rise.

2. \( W_{NS} < 0 \) – increasing \( S \) for a given work reduces welfare (which is why \( W_S < 0 \)) and thus increasing the number of works increases the negative effect on total welfare.

3. \( W_{CS} \geq 0 \) – the marginal effect of increasing protection declines as costs rise (remember \( W_S \) is negative).

4. \( W_{CN} \leq 0 \) – increasing production costs reduces the marginal benefit of new work (as each new work provides less welfare).

5. \( N_{CS} > 0 \) – the marginal impact of protection declines with lower costs.

The last inequality is the least self-evident of these. One justification for it is as follows: the level of production is a function of the level of (average) profit, \( \pi \), per work: \( N = g(\pi) \). With diminishing returns we would expect \( g'' < 0 \). Profits can be broken up into income and costs, \( \pi = I - C \), with the level of protection only affecting income and not costs. In that case we have \( N_{CS} = g''\pi S \pi C > 0 \).

Furthermore, by prior assumption or analysis we have: \( W_N > 0, N_S > 0, W_{NN} < 0, N_C < 0, W_S < 0 \). Thus, four of the five terms in the equation for the mixed second-order derivative for welfare are positive while one, \( N_S W_{CN} \) is not.

This means, that we cannot unambiguously say whether an increase or decrease in the costs of ‘originals’ implies an increase or decrease in the level of protection. In some ways this is somewhat surprising. Increased costs reduces the number of works and reduces the deadweight loss per work from protection so we might expect that increasing protection would unambiguously improve welfare.

The reason this is not necessarily so is that increased costs also reduce the welfare per work and hence while the number of works falls, which increases the marginal value of a new work, the increase in costs provides a countervailing effect (\( W_{CN} \)). As a result it is possible that the reduction in welfare per work due to higher costs is so dramatic as to outweigh all the other effects which favour an increase in term. Thus, a general statement based on theory alone is not possible.

That said, all of the reduction in welfare comes via a reduction in producer surplus due to higher costs. Hence the proportional reduction in income, and hence output, is likely to
be substantially higher than the proportional reduction in welfare. As a result one would expect the effect of a reduction in output \((N)\) to outweigh the effect of a reduction in welfare and therefore for \(d^2W/dCdS\) to be negative. Formalizing this condition we have:

**Proposition 9.** Assuming an initial finite optimal level of copyright, a sufficient condition for a reduction in the cost of ‘originals’ (leaving other variables unchanged) to imply a reduction in the strength of copyright is that an increase in costs \(C\), results in an increase in the marginal value of new work: \(\frac{d}{dC}W_N > 0\).

5.2. **Technological Change.** Let us now introduce ‘technological’ change explicitly as a variable \(T\). We shall assume that \(T\) has no direct effect on welfare but only operates through its impact on the costs of ‘originals’ and ‘copies’ \((C\) and \(U\)), and does so by reducing both types of costs (so \(C_T < 0, U_T < 0\)). Substituting \(T\) for \(X\) in Lemma 7 we have:

**Corollary 10.** If, at the current optimal level of protection, \(d^2W/dTdS < 0\) then technological change implies a reduction in the level of copyright. Conversely if \(d^2W/dTdS > 0\) then an increase in the level of copyright is required.

Turning again to an explicit consideration of the second derivative we have:

\[
\frac{dW^2}{dTdS} = \frac{d}{dT}(N_SW_N + W_S) = N_{TS}W_N + N_SW_{NN}N_T + N_SW_{TN} + W_{NS}N_T + W_{TS}
\]

Focusing on the effect on the output of works: \(N_T = N_CC_T + N_UU_T\), the effect of technological change will be ambiguous: the first term is positive since improvements in technology reduce the costs of originals \((C_T < 0)\), while the second is negative since production goes up (down) as the cost of unauthorised copying decreases (increases): \(N_U > 0\). However unlike welfare, \(N\) is (easily) observable, and it seems that recent years have seen an increase in the amount of work available. Thus, let us assume \(N_T > 0\). We then have:

(1) \(N_{TS} < 0\) – as costs drop value of increasing protection diminishes (as the number of works is increasing).

(2) \(W_{TN} > 0\) – marginal value of new work increases as \(T\) increases (a reduction in both types of costs increases welfare: \(W_U, W_C < 0\)).
(3) $W_{NS} < 0$ – see above.

(4) $W_{TS}$ is ambiguous – increasing $T$ reduces both $C$ and $U$ and while a reduction in the costs of ‘originals’ increases deadweight losses a reduction in $U$ reduces them with the overall effect ambiguous.

Thus, we have:

$$\frac{dW^2}{dTdS} = -\text{ve} + -\text{ve} + +\text{ve} + -\text{ve} + ?$$

In many ways this is similar to the previous situation. However the ambiguities here are more pronounced. In particular, one term can not be signed unambiguously from theory alone ($W_{TS}$) and it is less likely that the ‘contrary’ term here, $N_{SW_{TN}}$, will be small relative to the others. The key trade-off then is similar to the one discussed above.

On the one hand technological change reduces costs and thereby increases output which diminishes the value of new work (implying a reduction in copyright). However, at the same time, by reducing costs technological change increases the value of new work. These two effects operate in opposite directions and it is not a priori clear which will be the stronger. Again one might argue that the proportional increase in incomes for producers is likely to be at least as large as the increase in welfare and hence the increase in output will more than offset the impact on welfare per work. However, one must be cautious here because technological change may also reduce deadweight losses via a reduction in the cost of unauthorised copying and overall it would seem impossible to draw unambiguous conclusions from theory alone.

5.3. Discussion. Examples of cost-reducing technological change are ubiquitous in recent years arising, in the main, from the move to a digital environment. As discussed, focusing on the case of ‘originals’ alone, it seems likely that such changes would imply a reduction in the optimal level of copyright. However, this gives only half the story – technological change is likely to reduce the costs of both ‘originals’ and ‘copies’ at the same time. While it is unclear whether technological advance has reduced the costs of one faster than the other – the reductions in both cases seem dramatic – it appears that the overall level of output has risen. Using this fact, we examined whether optimal term should rise or fall as technological progress reduces costs. While based on theory alone, it was not possible
for an unambiguous answer to be given, we were able to characterise (and sign) most of the main factors impacting on welfare.

This ambiguous result is not surprising given the contrary effects at play. Furthermore our work highlights the key terms in need of empirical estimation in order to obtain an unambiguous conclusion regarding the implications of technological change for copyright.\(^9\)

We also think it important in demonstrating that care must be taken when drawing ‘obvious’ conclusions for copyright policy from changes in the external environment. Much of the motivation for strengthening copyright in recent years, whether by extending term or by the addition of legal support for technological protection measures (TPMs) – as in the WIPO Copyright Treaty of 1996 and its subsequent translation into national laws such as the DMCA (1998) and the EUCD (2001) – has been based on the implicit assumption that the move to a digital environment necessitated an increase in the strength of copyright because technological change made unauthorised copying (‘piracy’) easier. But focusing only on the reduction in the costs of unauthorised copies ignores the impact of technology on authorised production and distribution. As we have shown, such an approach omits a major part of the overall picture and may lead to erroneous conclusions regarding both the necessity and direction of policy changes.

6. **Optimal Copyright in a Dynamic Setting**

Our previous analysis has dealt only with a static setting in which all production could be aggregated into a single figure, $N$. In this section we will need to enrich this basic approach by introducing ‘time’. To do this let us define $n_t$ as the number of works produced in time period $t$ and $N_t$ as the number of works available to society in period $t$.\(^{10}\) $N_t$ will be the ‘real’ or ‘effective’ amount of work available, that is it takes account of cultural depreciation and obsolescence – which represent the fact that many works are ‘of their time’ and are, or at least appear to be, of little value to future generations. Specifically we expect $N_t$ not to be the absolute amount of past and present work available but rather an

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\(^9\)This is very similar to situation regarding copyright term which we address below. There too theory cannot tell us what level of term is optimal but can help us pinpoint the key variables in need of empirical estimation.

\(^{10}\)Both numbers will have the same set of arguments as the static $N$ we had before so we will have $n_t = n_t(S, C)$, $N_t = N_t(S, C)$ though note that if the arguments can vary over time then the arguments would have be modified appropriately (those to $n$ would need to include future values and those for $N$ both past and future values).
‘equivalent’ amount denominated in the same terms as \( n_t \). Formally, if we let \( b(i) \) be the ‘rate of cultural decay’ after \( i \) time periods \( (b(0) = 1) \), then the ‘effective’ amount of work in period \( T \) is the sum of the production of all previous periods appropriately weighted by the level of cultural decay:

\[
N_t = \sum_{i=0}^{\infty} b(i) n_{t-i}
\]

Then total welfare calculated at time \( t \) is:

\[
W^\text{Tot}_t(S) = \sum_{i=0}^{\infty} d(i) W(N_{t+i}(S), S)
\]

We shall assume this is single-peaked and differentiable (so the first-order condition is necessary and sufficient).\(^{11}\)

**Theorem 11.** Assume that at time \( t = 0 \) production is approximately zero (this could be for several reasons the most obvious being that this type of work only comes into existence at this point, e.g. film around 1900, sound recordings in late 19th century). Then, assuming that sequence of works produced per year, \( n_i \) is such that \( N(t) = \sum_{i=0}^{t} b(t-i) n_i \) is non-decreasing, optimal protection declines over time asymptoting towards what we term the ‘steady-state’ level.

\(^{11}\)This dynamic problem has substantial similarities with the standard optimal control problems of dynamic growth models. Specifically, let \( b(i) \) takes a standard exponential form \( b(i) = \beta^i \) and allow \( S \) to be set anew each time period (it can then take the role of a standard control variable). Then:

\[
\begin{align*}
N_t &= \beta N_{t-1} + n_t \\
n_t &= f(S_t, S_{t+1}, ..., N_t, N_{t+1}, ...) \\
W_t &= W(N_t, S_t) \\
W^\text{Tot}_t &= W_t + \beta \sum_{i=0}^{\infty} \beta^i W_{t+i+1} = W_t + \beta W^\text{Tot}_{t+1}
\end{align*}
\]

Then, comparing to growth models, \( N_t \) is \( K_t \) (capital), \( n_t \) is \( Y_t \) (production), \( S_t \) is \( c_t \) (the control variable – usually consumption), \( W_t \) is \( U(c_t) \) (utility from consumption) and \( W^\text{Tot}_t \) is the value function (overall welfare). Of course our setup is more complex than the standard growth framework since output (the number for works produced) depends not just on current values for the control variable but on future values of the control variable and future levels of output (this is because creative works are durable).

We note that these sorts of problems have been extensively analyzed – see Stokey, Robert E., and Prescott (1989) for a mathematical survey – and while it is relatively straightforward to ensure the existence of an equilibrium it is hard to state any general results about the time paths of the state and control variables (see e.g. the ‘anything goes’ result of Boldrin and Montrucchio (Stokey, Robert E., and Prescott, 1989, Thm 6.1) which demonstrates that any twice-differentiable function \( g \) can be obtained as the policy function of a particular optimal dynamic growth problem).

Thus, here we restrict to the case where the control variable may only be set once \( (S \) is given forever) and we also assume, when stating our result, that the time path of the number of works (‘capital’) is non-decreasing – a result obtained in many, though not all, growth models and which, in the case of copyright, appears to fit well with the available data.
Proof. We first provide an informal justification for this result before turning to a formal, mathematical, ‘proof’.

No works are produced before time zero so, as time increases, the backlog of work will grow. As the backlog grows a) the value of producing new work falls and b) the welfare losses from increased protection are levied not just on new works but on the backlog as well.

To illustrate consider the situation with respect to books, music, or film. Today, a man could spend a lifetime simply reading the greats of the nineteenth century, watching the classic movies of Hollywood’s (and Europe’s) golden age or listening to music recorded before 1965. This does not mean new work isn’t valuable but it surely means it is less valuable from a welfare point of view than it was when these media had first sprung into existence. Furthermore, if we increase protection we not only restrict access to works of the future but also to those of the past.

As a result the optimal level of protection must be lower than it was initially in fact it must fall gradually over time as our store of the creative work of past generations gradually accumulates to its long-term level. We now turn to the formal argument.

Optimal protection, $S^t$, at time $t$ solves:

$$\max_S W^T_{tot}(S)$$

The first-order condition is:

$$\frac{dW^T_{tot}(S^t)}{dS} = 0$$

Consider this at time $t$ then:

$$\sum_{i=0}^{\infty} d(i) \frac{dW(N_{i+t}(S^t), S^t)}{dS} = 0$$

Recall that $\frac{\partial}{\partial N} \frac{dW}{dS} < 0$ (the marginal value of protection goes down as the number of works increases and the total deadweight loss increases) so that, if $N^1 > N^2$:

$$\frac{dW(N^1, S)}{dS} < \frac{dW(N^2, S)}{dS}$$

Now, by assumption on the structure of $n_i$, $\forall i, N_{i+t+1} > N_{i+t}$. Thus, we must have:
\[ \frac{dW_{T_{t+1}}(S^t)}{dS} = \sum_{i=0}^{\infty} d(i) \frac{dW(N_{i+t+1}(S^t), S^t)}{dS} < \sum_{i=0}^{\infty} d(i) \frac{dW(N_{i+t}(S^t), S^t)}{dS} = \frac{dW_{T_{t}}(S^t)}{dS} = 0 \]

So we have that:

\[ \frac{dW_{T_{t+1}}(S^t)}{dS} < 0 \]

Since \( W_{T_{t+1}} \) is single-peaked this implies that the level of protection which maximizes \( W_{t+1} \) must be smaller than \( S^t \). That is the optimal level of protection at \( t+1, S^{t+1} \), is lower than the optimal level of protection at \( t, S^t \).

Finally, we show that the optimal level of protection will tend to what we term the steady-state level. We have just proved that \( S^t \) is a declining sequence. Since values for \( S \) are bounded below by 0 by Bolzano-Weierstrass we immediately have that the sequence must converge to a unique \( S = S^\infty \). By analogous arguments associated with this ‘steady-state’ level of protection will be a steady-state level of output per period \( n^\infty \) and effective number of works \( N^\infty \).

\( \square \)

6.1. Remarks. The preceding result has important implications for policy. In most systems of law, it is extremely difficult to remove or diminish rights once they have been granted. Thus, in most circumstances, once a given level of protection has been granted it will be all but impossible to reduce it. However, according to the preceding result, in general the optimal level of protection will decline over time.

In many ways this is a classic ‘dynamic inconsistency’ result: the preferences of a welfare-maximizing policy-maker at time zero are different from those at some future point T. Furthermore, it is clear that no particular point in time has any more validity over any other point as regards being chosen as a reference point. Moreover, from the perspective of any given point in time the ability to ‘commit’ to a given level of protection is extremely valuable.\(^{12}\) That said the result is still important for two reasons.

First, whether because of a paucity of data or disagreement about the form of the model, there is frequently significant uncertainty about the optimal level of protection. But one

\(^{12}\)It is precisely concerns over the ability of a policy-maker to credibly commit to a particular macroeconomic target that animates many of the traditional models of dynamic inconsistency.
thing we do know from the preceding result is that, whatever optimal level of protection currently, it will be lower in the future. Combined with the asymmetry in decision-making already mentioned – namely, that it is much harder to reduce protection than to extend it – this implies it is prudent for policy-makers to err on the low side rather than the high side when setting the strength of copyright.

Second, and more significantly this result provokes the question: if optimal protection should decline over time why does the history of copyright consists almost entirely of the opposite, that is to say, repeated increases in the level of protection over time (duration, for example, has been increased substantially in most jurisdictions since copyright was first introduced\(^{13}\)). After all, while one can argue that for ‘commitment’ reasons a policy-maker would not reduce the level of protection over time, our result certainly runs counter to the repeated increases in protection, many of which have taken place in recent years (when the stock of copyrightable works was already large).

The obvious answer to this conundrum is that the level of protection is not usually determined by a benevolent and rational policy-maker but rather by lobbying. This results in policy being set to favour those able to lobby effectively – usually groups who are actual, or prospective, owners of a substantial set of valuable copyrights – rather than to produce any level of protection that would be optimal for society as a whole. Furthermore, on this logic, extensions will be obtained precisely when copyright in existing, and valuable, material is about to expire. In this regard it is interesting to recall that many forms of copyrightable subject matter are of relatively recent origin. For example, the film and recording industry are only just over a hundred years old with the majority of material, in both cases, produced within the last fifty years. In such circumstances, and with copyright terms around 50 years, it perhaps not surprising that the last decade has seen such a flurry of extensions and associated rent-seeking activities.

7. Optimal Copyright Term

We now turn to the case of optimal copyright term. By interpreting the level of protection, \(S\), as the length of copyright the framework set out above can be re-applied directly. At the same time, because we are now dealing with a more specific case we can add greater

\(^{13}\text{Most prominently in recent times in the United States in 1998 and in the EU in 1995.}\)
structure to the model and, by so doing, obtain some sharper predictions. Our aim here
is to derive a numerical, quantitative estimate, for the length of copyright term. Clearly
this will be an empirical task and the main use of theory in this section will be in charac-
terising optimal term as a function of underlying variables that can be feasibly estimated
from available data.

As the reader will recall, the basic trade-off inherent in copyright is between increasing
protection to promote the creation of more work and reducing protection so as to gain
more from existing work. The question of term, that is the length of protection, presents
these two countervailing forces particularly starkly. By extending the term of protection
the owners of copyrights receive revenue for a little longer. Anticipating this, creators of
work which were nearly, but not quite, profitable under the existing term will now produce
work, and this work will generate welfare both current and future welfare for society. At
the same time, the increase in term applies to all existing works – those which would
have been created under the initial level of copyright. Since extending term on these
works prolongs the copyright monopoly it reduces total welfare as a result of the extra
deadweight loss.

It is these two, contrary, effects which will form the main focus of our investigation here.
Together they will already provide with plentiful matter for theoretical and empirical
efforts but we should note that in confining ourselves in this we we will be ignoring a
variety of further issues. For example, much creative endeavour builds upon the past and
an extension of term may make it more difficult or costly do so – were Shakespeare’s work
still in copyright today it is likely that this would substantially restrict the widespread
adaptation and reuse that currently occurs. However we make no effort to incorporate
this into our analysis despite its undoubted importance (it is simply too intractable from
a theoretical and empirical perspective to be usefully addressed at present). We will also
ignore questions of ex post investment, that is investment by a copyright owner after
creation of the work, as well as inefficient exploitation, that is a failure by a copyright
owner to maximize the value of the work in their possession.\footnote{See e.g. Landes and Posner (2003) on ex post investment and Brooks (2005) for evidence on inefficient exploitation. We should note that, in our opinion, both of these effects are likely to be relatively limited, and hence we believe their omission, unlike that of ‘reuse’, is unlikely to have a serious impact on the overall results.}
7.1. Theory. Our first step then is to link the two main effects to a common set of underlying variables. We begin by introducing explicit consideration of the revenue from a work. We shall assume that without copyright revenue is zero. Let revenue (under copyright) on the jth work in the ith period after a work’s creation be given by \( r_j(i) \) and present value of total revenue to period \( T \) be \( R_j(T) \) (where implicitly we assume that \( T \) is less than the current copyright term). Let \( d(i) \) be, as above, the discount factor up to time \( i \), then:

\[
R_j(T) = \sum_{t=0}^{T} d(t)r_j(t)
\]

Revenue decays over time due to ‘cultural decay’. We specify cultural decay by \( b(t) \), with \( r_j(t) = b(t)r(0) \) (cultural decay is assumed to occur at the same rate independent of the work), so that we have:

\[
R_j(T) = \sum_{t=0}^{T} d(t)b(t)r_j(0)
\]

As was shown above, the level of optimal copyright will not be constant over time even if all underlying parameters stay constant. This variation is not our focus here. Instead we are interested in how the basic parameters – the discount rate, the level of cultural decay etc – affect the optimal level of copyright. Thus, when comparing two terms here we shall compare them at their long-run, steady-state, level.\(^{15}\) Formally, the following assumptions will be made in what follows:

1. All calculations will be of a comparative static nature with the level of production taken at its long run equilibrium value. Thus we take the amount of work produced per period \( n_t \) to be the constant and equal to the steady-state level which we will denote by \( n \). Similarly the ‘effective’ amount of work available per period will be constant and will denote it by \( N \).

2. Discount factors are the same for producers and for society (i.e. we discount welfare at the same rate we discount income for producers).

\(^{15}\)It is in this assumption that we differ most significantly from previous analyses such as that of Landes and Posner (1989). Their model implicitly assumes no work already exists and therefore, in the formulation of the previous section, maximizes welfare from the perspective of a social planner at time \( t = 0 \) rather than at the steady-state. We believe that the steady-state analysis presented here, which includes the prospective and retrospective effects of changes in copyright term, is the more appropriate – particularly since today most forms of copyrightable work have been produced for decades if not centuries.
(3) Revenue and welfare (and dead-weight loss) per work experience the same rate of cultural decay. Thus total welfare per period may be obtained by summing over all vintages of works weighted by the relevant cultural decay.

Since we evaluate welfare at the long run equilibrium, production per period and welfare per period may be taken to be constant and equal to their long run equilibrium values. Therefore in what follows we focus on welfare per period (converting to total welfare is a trivial matter). We have the following result:

\[ \frac{dW(S^1)}{dS} = ns(n)y(n)b(S^1) \left( d(S^1) \frac{\sum_{i=0}^{\infty} b(i)}{\sum_{i=0}^{\infty} d(i)b(i)} + d(S^1) \frac{z(n)}{y(n)} \frac{\sum_{i=S^1}^{\infty} b(i)}{\sum_{i=0}^{\infty} b(i)d(i)} - \theta(n) \right) \]

Where:

\[ d(t) = \text{Discount factor to time } t \]
\[ b(t) = \text{Cultural decay to period } t \]
\[ y(j) = \text{Welfare (under copyright) from an extra } j \text{th new work} \]
\[ z(j) = \text{Deadweight-loss under copyright on the } j \text{th new work} \]
\[ s(n) = \text{Elasticity of supply of works with respect to revenue when there are } n \text{ works} \]
\[ \theta(n) = \text{Ratio of avg. d/w loss to welfare from new works} = \frac{\sigma(n)}{s(n)y(n)} \]

In particular define the ‘determinant’, \( \Delta \) as the bracketed term above, i.e.

\[ \Delta = d(S^1) \frac{\sum_{i=0}^{\infty} b(i)}{\sum_{i=0}^{\infty} d(i)b(i)} + d(S^1) \frac{z(n)}{y(n)} \frac{\sum_{i=S^1}^{\infty} b(i)}{\sum_{i=0}^{\infty} b(i)d(i)} - \theta(n) \]

Then the optimal copyright term is determined by reference to the ‘determinant’ alone and is the solution of \( \Delta = 0 \).
Proof. As we are going to take derivatives we shall take all necessary variables (number to works, time etc) to be continuous rather than discrete (and we therefore have integrals rather than sums). Note that the conversion back to the discrete version is straightforward (but would make the notation and proof substantially more cumbersome).

We can express welfare per period as:

\[ W = \text{Welfare under infinite copyright} + \text{Extra welfare on works out of copyright} \]

\[ = \int_{j=0}^{n(S)} \int_{i=0}^{\infty} y(j)b(i) + \int_{j=0}^{n(S)} \sum_{i=S}^{\infty} z(j)b(i) \]

With the first sum being over the \(n\) works produced each period and the second being over past periods (\(i = 1\) corresponding to the period previous to this one, \(i = 2\) to two periods ago etc). With the double sum we cover all works ever produced, bringing them up to the present in welfare terms by multiplying by a suitable amount of ‘cultural decay’.

Differentiating we have:

\[ \frac{dW}{dS} = n'y(n) \int_{i=0}^{\infty} b(i) + n'z(n) \int_{i=S}^{\infty} b(i) - b(S) \int_{j=0}^{n} z(j) \]

\[ = \text{Gain in welfare from new works} - \text{Extra deadweight loss on existing works} \quad (7.1) \]

Let us re-express the increase in the number of works, \(n'(S^1)\), in terms of the change in revenue for a marginal (nth) work \(R_n\) (note we suppress the \(n\) subscript for terminological simplicity):

\[ n' = \frac{dn}{dS} = n \frac{n'(R(S))}{n} = n \frac{dn}{R} \frac{R'}{R} \]

The middle term of the final expression is the elasticity of supply with respect to revenue, \(s(n)\), while the last is the percentage increase in revenue. Total revenue on the marginal work itself equals (remember that once out of copyright revenue per period is zero):

\[ R(S) = \int_{i=0}^{S} d(i)b(i)r(0) \implies R'(S) = d(S)b(S)r(0) \]

Thus, substituting, using \(z\), for average \(z\) and converting back to summations we have:
\[
\frac{dW(S^1)}{dS} = ny(n)d(S^1)b(S^1)r(0)\left(\frac{\sum_{i=0}^{\infty} b(i)}{\sum_{i=0}^{\infty} b(i)d(i)r(0)} + \frac{z(n)}{y(n)} \frac{\sum_{i=0}^{\infty} b(i)}{\sum_{i=0}^{\infty} b(i)d(i)r(0)}\right) - b(S^1)n\bar{z}(n)
\]

\[
= ns(n)y(n)b(S^1)\left(d(S^1)\frac{\sum_{i=0}^{\infty} b(i)}{\sum_{i=0}^{\infty} b(i)d(i)} + d(S^1)\frac{z(n)}{y(n)} \frac{\sum_{i=0}^{\infty} b(i)}{\sum_{i=0}^{\infty} b(i)d(i)} - \frac{\bar{z}(n)}{\bar{y}(n)\bar{y}(n)}\right)
\]

\[\square\]

Having now obtained an expression which characterises the optimal copyright term (\(\Delta\)) our next task is to obtain estimates for its various component variables (\(b, d, \theta\) etc). We go through each of the variables in turn, starting with the simplest to estimate (the discount rate) and progressing to the hardest (\(\theta\)).

7.2. The Discount Rate. We assume a standard geometric/exponential form for the discount function. The relevant discount factor to use here is that related to those producing works so a plausible range is a discount rate in the range 4-9%. For example, CIPIL (2006) in considering a similar issue report that: Akerlof et al. (2002) use a real discount rate of 7%, Liebowitz in his submission to the Gowers review on behalf of the IFPI (International Federation of the Phonographic Industry) uses a figure of 5%, while PwC’s report to the same review on behalf of the BPI (British Phonographic Industry) use the figure of 9%. Where we need to use a single value we will by default use a rate or 6% (corresponding to a discount factor of 0.943).

7.3. The Rate of Cultural Decay. We assume an exponential form for the cultural decay so that \(b(i) = b(0)^i\) with \(b(0)\) the cultural decay factor.\(^{16}\) A plausible range for this cultural decay rate is 2-9% and by default we will use 5% (corresponding to a factor of 0.952). Since values for these variables are less well-established than those for the discount rate the evidence on which they are based merits discussion.

The prime source is CIPIL (2006), which reports estimates made by PwC based on data provided by the British music industry which indicate decay rates in the region of 3-10%. As these come from the music industry itself, albeit indirectly, these have substantial

\(^{16}\)It is likely that an exponential distribution is not a perfect fit for the cultural decay rate. In general, it appears that the rate of decay is sharper than an exponential for young works but flatter than an exponential for old works. This suggests that hyperbolic cultural decay might be a better model (just as hyperbolic discounting may be more accurate than exponential discounting for income). However, an exponential form appears to be a reasonable approximation and it is substantially more tractable. Thus we retain it here rather than using the more complex hyperbolic approach (just as an exponential form is regularly used for time discounting for analogous reasons).
authority. To check these we have performed our own calculations using data on the UK music and book industry and obtain estimates for the rate of decay that are similar (in the case of music) or even higher (in the case of books).

Evidence from elsewhere includes the Congressional Research Service report prepared in relation to the CTEA (Rappaport, 1998). This estimates projected revenue from works whose copyright was soon to expire (so works from the 1920s to the 1940s). Rappaport estimates (p.6) that only 1% of books ever had their copyright renewed and of those that had their copyright renewed during 1951 to 1970 around 11.9% were still in print in the late 1990s. The annual royalty value of books go from $46 million (books from 1922-1926) to $74 million (books from 1937-1941). Turning to music, Rappaport focuses on songs (early recordings themselves have little value because of improvements in technology) and finds that 11.3% of the sample is still available in 1995. Annual royalty income rises from $3.4 million for works from 1922-1926 to $15.2 million for works from 1938-1941.

These figures correspond, in turn, to cultural decay rates of 3.2% and 10.5% respectively. However these are far from perfect estimates since we only have two time points. Furthermore these time points correspond to different ‘cohorts’ of work – which makes it difficult to disentangle decay effects from cohort effects, and both these cohorts are of fairly old works – which, as explained in a previous footnote means that the decay rate is likely to be underestimated. One might also want to cautious about extrapolating to the behaviour of current and future creative output from data of such elderly vintage.\(^{17}\)

Liebowitz and Margolis (2005) argues that overall decay rates may be misleading and presents evidence that books that are popular upon release as measured by being best-sellers survive well (for example the table on p. 455 indicates that of the 91 bestsellers in their sample from the 1920s 54% are still in print 58 years later compared to only 33% of non-bestsellers. However it is not clear how one should interpret this sort of evidence.

Simple ‘in-print’ status of a book only places a lower-bound on sales (furthermore a lower bound that is dropping with advances in technology) and does not allow us to compare

\(^{17}\)The issue of technological change is clearly an important one here: one might argue that with improvements in technology, both in production but also in distribution and discovery, the decay rate will fall in future. For example, it has been argued recently that technologies such as the Internet have made it easier to discover and access more obscure works leading to the growing importance of the ‘long-tail’ and a flattening of the distribution of sales (traditionally sales for most types of copyrightable goods have been dominated by a top 10-20% of works. The ‘long-tail’ then refers to the tail of this sales distribution). Here we do not explicitly consider the impact of technological change but we note that an earlier section dealt specifically with this issue.
the sales of a book today compared to when it was first released. More fundamentally, much heterogeneity is eliminated by the aggregation of copyrights into portfolios by the investors in creative work such as publishers, music labels and movie studios. In this case returns will tend to the average. Furthermore, were such aggregation not to occur it would require a substantial increase in the discount rate to take account of the increased uncertainty due to the reduction in diversification of the portfolio.\footnote{In these circumstances the issue of serial correlation would also become important. With high serial correlation, that is older successful works are those that were successful when young (and vice versa), the revenue when one extends term goes primarily to the owners works which have already generated substantial revenue (think here of a group like The Beatles). If one makes the standard assumption of diminishing marginal returns to creative output with respect to revenue, then serial correlation implies a very low elasticity of supply with respect to revenue – the revenue from extending term goes to those whose incomes are already high and therefore from whom little extra ‘creation’ can be expected when their incomes increase.}

### 7.4. Deadweight-Loss, Welfare Under Copyright and θ(n)

Our preference would be to estimate all of these values directly from empirical data. However, this is a daunting task given currently available datasets as it requires us to determine: the full demand system for copyright goods and the supply function for creative work. Because this task presents such insurmountable difficulties given present data availability we instead take a ‘reduced-form’ approach where we supply particular functional forms for the various quantities of interest (the average deadweight loss, marginal welfare etc). Where possible we calibrate these using existing data and we also perform robustness checks to ensure these results are reasonably robust. We begin by making the following assumptions:

1. The elasticity of production with respect to revenue, $s(n)$, is constant, equal to $s$.
2. The ratio of deadweight-loss to welfare under copyright on any given work is constant. This constant will be termed $\alpha$.
3. The ratio of marginal welfare, $y(j)$, to marginal sales is constant. That is welfare follows the same trend as sales. This constant will be termed $\beta$.

Assumption 1: little if anything is known about how the elasticity of supply with respect to revenue varies with the number of works produced. Furthermore we are already allow changes in welfare per work so it seems reasonable to take elasticity as constant.

Assumption 2: this assumption is questionable as one might expect that deadweight losses relative to welfare (under copyright) increase as the welfare (and revenue) from a
work decline. If this were so then this assumption would be incorrect and would result in an underestimate of the costs of copyright – and hence an overestimate of optimal copyright term. Nevertheless, we shall make this assumption for two reasons. First, it is difficult to derive estimates of this ratio from existing data. Second, as we shall see below, even with it (and the associated upward bias) we find that optimal term is well below the copyright terms found in the real world.

Assumption 3: this requires that the ratio welfare (under copyright) arising from a new work to the sales of that work does not vary over works. Again this is almost certainly not an accurate description of reality but as a first order approximation we believe it is not that bad. Furthermore, this assumption is crucial for our empirical strategy since it is relatively easy to obtain sales data compared to welfare data (which requires information on large segments of the demand curve).

Now, to proceed with the empirics, first let us switch to total welfare for notational convenience and define \( Y(j) \) to be total welfare under copyright from \( j \) new works so that \( y(j) = Y'(j) \). Also define \( Q(j) \) as total sales and \( q(j) = Q'(j) \) as marginal sales (i.e. sales from the \( n \)th work).

What form does \( Q(j) \) take? We shall assume it takes a ‘power-law’ form:

\[
Q(j) = A j^\gamma
\]

This functional form appears to represent a reasonably good fit for sales of cultural goods and is frequently used in the literature. We then have:

**Lemma 13.** \( \theta(n) \) has the following simple form:

\[
\theta(n) = \frac{\alpha}{s^\gamma}
\]

**Proof.** Recall that \( \theta(n) = \frac{\bar{z}(n)}{sy(n)} \). Now \( y(n) = \beta q(n) \), \( z(n) = \alpha y(n) \) so average deadweight loss, \( \bar{z}(n) \) equals \( \alpha \beta Q(n) \). Hence:

\[
\theta(n) = \frac{\alpha}{s} \frac{\beta Q(n)}{n} = \frac{\alpha}{s^\gamma}
\]

---

19 For example, this would be the case if there was some fixed lower bound to transaction costs.

20 See e.g. Goolsbee and Chevalier (2002); Ghose, Smith, and Telang (2004); Deschatres and Sornette (2004)
Thus, one very convenient aspect of using a ‘power-law’ form is that $\theta(n)$ is not a function of $n$ – it is ‘scale-free’. In this case calculations of optimal copyright term do not depend on, $n$, the production function for works but only on $\alpha$, $\gamma$ and $s$.

7.5. **Optimal Copyright Term: Point Estimates.** Combining estimates of the ratio of deadweight losses to welfare under copyright ($\alpha$) and the rate of diminishing returns ($\gamma$) with those provided above for cultural decay ($b$) and the discount factor ($d$) we will obtain point estimates for optimal copyright term.

7.5.1. $s$. There is an almost total lack of data which would allow us to estimate the elasticity of supply with respect to revenue. Landes and Posner (2003) who point out that there is no discernible impact on output of work from the US 1976 extension of term. Hui and Png (2002) find a similar result when looking at movies and the CTEA in the US though more recent work with a cross-country dataset, Png and Hong Wang (2007), does find an impact. Given this uncertainty and lack of information the best we can do is to posit what we feel is a plausible range for $s$ of $[0.5, 1.5]$ with an average value of 1.0.

7.5.2. $\gamma$. Ghose, Smith, and Telang (2004) list a whole range of estimates for $\gamma - 1$ (all derived from Amazon) ranging from -0.834 to -0.952 with the best estimate being -0.871. These imply $\gamma$ in the range 0.048 to 0.166 with best estimate at 0.129. We shall proceed using this estimate of 0.129.

7.5.3. $\alpha$. Estimating $\alpha$ is harder because of the paucity of data which would permit estimation of off-equilibrium points on the demand curve. However the available evidence though scanty suggests that the ratio could be quite large. For example, Rob and Waldfogel (2004) investigate file-sharing among college students and estimate an implicit value for deadweight-loss of around 36% of total sales. Converting this to welfare ratio requires some assumption about the ratio of welfare (under copyright) to sales. A linear demand structure (with zero marginal costs) would give a deadweight loss to sales ratio of 50% and deadweight loss to welfare under copyright ratio of a third. Increasing marginal costs would reduce the ratio to sales but keep the ratio to welfare constant at a third. Being more conservative, assuming producer surplus were around 50% of sales and consumer surplus to be two to five times would give a value for $\alpha$ of between 0.24 and 0.12. Other
<table>
<thead>
<tr>
<th>Cultural Decay Rate (%)</th>
<th>Discount Rate (%)</th>
<th>$\alpha$</th>
<th>Optimal Term</th>
</tr>
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<td>4.53</td>
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</table>

Table 1. Optimal Term Under Various Scenarios. $\alpha$ is the ratio of deadweight loss to welfare under copyright and $s$ (the elasticity of supply) is set to 1 and $\gamma$ (sales curve exponent) to 0.129.

papers, such as Le Guel and Rochelandet (2005); Ghose, Smith, and Telang (2004), while not providing sufficient data to estimate deadweight loss, do suggest it is reasonably substantial. Thus, we feel a plausible, and reasonably conservative, range for $\alpha$ would be from $[0.05, 0.2]$, that is deadweight loss per work is, on average, from a twentieth to a fifth of welfare derived from a work under copyright. When required to use a single value we will use the halfway point of this range 0.12.

7.6. A Point Estimate for Optimal Copyright Term. With $\alpha = 0.12, s = 1.0, \gamma = 0.129$ then $\theta \approx 0.93$. With our defaults of a discount rate of 6% and cultural decay of 5% this implies an optimal copyright term of around 15 years.

7.7. Robustness Checks. Given the uncertainty over the values of some of the variables it is important to derive optimal copyright term under a variety of scenarios to check the robustness of these results. Table 1 presents optimal term under a range of possible parameter values including those at the extreme of the ranges suggested above.

With variables at the very lower end of the spectrum (the first row) optimal term comes out at 52 years which is substantially shorter than authorial copyright term in almost all jurisdictions and roughly equal to the 50 years frequently afforded to neighbouring rights (such as those in recordings). However as we move to scenarios with higher levels for the exogenous variables optimal term drops sharply. For example, with cultural decay at 3%, the discount rate at 5% and the ratio of deadweight loss to welfare under copyright at 7% we already have an optimum term of just over 30 years. At the very highest end of the spectrum presented here, with deadweight losses at 20% of welfare under copyright (recall that a linear demand curve corresponds to a 33% ratio) and cultural decay and the discount rate both at 8% optimal term is around four and a half years.
We can also plot a probability density function under the assumed variable ranges. This has the advantage that incorporates the interrelations of the various variables – by contrast, Table 1, by nature of its form, implicitly gives the inaccurate impression that each of the outcomes listed is equally likely. We present the distribution function in Figure 1. As this shows, the mode of the distribution is around 20 years and the median is just under 15 years. From the underlying cumulative distribution function we can calculate percentiles and find the 95th percentile at just under 31 years, the 99th percentile at 39 years and the 99.9th percentile at just over 47 years. This would suggest, that at least under the parameters ranges used here, one can be extremely confident that copyright term should be 50 years or less – and it is highly like that term is under 30 years (95th percentile).

7.7.1. An Inverse Approach. An alternative approach to estimating underlying parameters and using that to find the optimal term is to look at the inverse problem of calculating the ‘break-even’ value for a particular variable for a given copyright term. The ‘break-even’ value is the level of that variable for which that term is optimal. Here we will focus on $\alpha$, the ratio of deadweight loss to welfare under copyright – so if the actual value $\alpha$ is higher...
Figure 2. Break-even alpha as a function of copyright term. $b$ is the cultural decay factor and $d$ the discount factor.

than this break-even level then term is too long and if actual $\alpha$ is below it then term is too short. This provides a useful robustness: derive the break-even $\alpha$ corresponding to the copyright term currently in existence and then compare this value to whatever is a plausible range for $\alpha$. If the value is outside this range one can be reasonably certain that current copyright term is too long.

Given our assumption on the form of the discount factor and the rate of cultural decay theta takes the following form:

$$\alpha^{-1}(S) = \frac{dS(1 - bd)}{1 - b} bS(1 - bd)$$

Figure 2 provides a plot of this inverse, ‘break-even’, function. Under the Berne convention minimal terms of protection for most types of work is life plus 50 years (and many countries including the US and all of those in the EU now provide for life plus 70). This in turn will correspond to a copyright length of somewhere between 70 and 120 years (assuming the work is created between the ages of 20 and 70). Let us take a low value in this range, say 80 years. We summarize the ‘break-even’ $\alpha$ corresponding to term of this length in Table 2 focusing on a set of very conservative parameter values. As can be
seen there, even with a cultural decay rate of 2%, a discount rate of 4% and elasticity at its uppermost value the break-even $\alpha$ is 2.5% – so for any $\alpha$ above that term is too long. With a slightly higher decay and discount rate (3% and 5% respectively) break-even $\alpha$ falls to 1.3%. Thus, even with low values for the discount and cultural decay rate the level of $\alpha$ required for current copyright terms to be optimal seem too low to be plausible.

### 8. Conclusion

In this paper we have developed a simple framework for analysing copyright based on those in the existing literature. Using it, we obtained two sets of separate, but complementary, results. In the first sections, which was entirely theoretical, we demonstrated in substantial generality that (a) optimal protection falls with a decline in the costs of production and distribution of ‘originals’ (b) optimal protection falls over time.

In the second section we turned our attention to one specific aspect of optimal copyright, namely the term of protection. In Theorem 12 we used our model to derive a single equation that defined optimal term as a function of key exogenous variables. Using the estimates for these variables derived from the available empirical data we obtained an estimate for optimal copyright term of approximately 15 years. To our knowledge this is one of the first estimates of optimal copyright term which is properly grounded, both theoretically and empirically, to appear in the literature.

All our results have significant implications for policy. In recent times technological change has substantially reduced the costs of production and distribution of most copyrightable goods. Much of the existing policy discussion has focused, almost exclusively, on reductions in the costs of ‘unauthorised’ (‘pirate’) copies and has tended to assume that this necessitates an increase in the level of protection. However, as we pointed out, the costs of ‘originals’ have also fallen dramatically, and this change is likely to require...
a reduction in the strength of protection. Looking more generally at the case of technological change which reduces the production costs of both ‘originals’ and ‘copies’, the implications for copyright policy were ambiguous – not surprising given the two contrary effects at play – and we highlighted the key terms in need of empirical estimation if an unambiguous answer were to be obtained.

Moving on we came to the question of optimal copyright term – probably the most important aspect of the overall ‘level’ of copyright. Our estimate of optimal term (15 years) is far below the length copyright in almost all jurisdictions and we confirmed this general fact – that current copyrights are likely too long – using several robustness checks. This implies that there is a significant role for policymakers to improve social welfare by reducing copyright term as well as indicating that existing terms should not be extended. Such a result is particularly importance given the degree of recent debate on this precise topic.

Finally, there remains plentiful scope to extend and build upon the work here. In particular, there is room for further empirical work on all aspects of these results. For example, it would be valuable to calibrate the production costs model to investigate what changes in the level of copyright would be implied by the recent reductions in the cost of production and distribution. Similar work could be done in relation to changes of copyright over time where one would need to collect data on the level of production and the form of the welfare function.

Regarding the derivation of optimal term, the main challenge would be to improve the estimates for the key parameters, especially that of the ratio of deadweight loss to welfare under copyright. As discussed above, the perfect approach would involve estimating the demand-system for the copyrightable goods under consideration. This is a non-trivial task but one of great value – and not simply in relation to the problems considered here.

REFERENCES


