Project Screening with Tiered Evaluation

Andrei Barbos

University of South Florida


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Project Screening with Tiered Evaluation

Andrei Barbos*
University of South Florida, Tampa, FL 33620

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Abstract

We study a Bayesian game of two-sided incomplete information in which an agent, who owns a project of unknown quality, considers proposing it to an evaluator, who has the choice of whether or not to accept it. There exist two distinct tiers of evaluation that differ in the benefits they deliver to the agent upon acceptance of a project. The agent has to select the tier to which the project is submitted for review. Making a proposal incurs a cost on the agent in the form of a submission fee. We examine the effect of a change in the submission fees at the two tiers of evaluation on the expected quality of projects that are implemented by the evaluator.

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*E-mail: andreibarbos@gmail.com; Address: 4202 East Fowler Ave, BSN 3403, Tampa, FL 33620-5500; Phone: 813-974-6514; Fax: 813-974-6510; Website: http://sites.google.com/site/andreibarbos/
1 Introduction

We study a Bayesian game in which an agent, who owns a project of unknown quality, considers proposing it to an evaluator, who upon receiving a project for review, has the choice of whether or not to accept it. Prior to taking their actions, each player performs an assessment of the project that yields a private signal of quality. Making the proposal incurs a cost on the agent in the form of a submission fee.¹ There exist two distinct tiers of evaluation in which a project can be accepted, an upper tier and a lower tier. If the agent submits a project, he has to select the tier to which to submit it. The upper tier entails higher submission costs, delivers higher benefits to the agent upon acceptance, and higher losses to the evaluator upon acceptance of a low-quality project.

A real world application of this framework is that of a prosecutor deciding on the charges to file, if any, against a defendant in a trial. Filing more severe charges incurs higher costs on the prosecution in the form of resources spent on collecting evidence, and induces a more stringent burden of proof, but the rewards following a favorable ruling by the jury are also higher.²

The main objective of this paper is to study the effects of submission fees, in their role of instruments of control of the level of self-screening exerted by the agent, on the expected quality of projects implemented by the evaluator.

Several recent papers, starting with Leslie (2005), investigate the optimal submission fee problem and show that these optimal fees are strictly positive³ because they reduce the burden on the evaluators by discouraging long-shot submissions.⁴ Departing from earlier papers, in a framework

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¹ This fee may be a payment toward the evaluator or a third entity, or it may take a non-monetary form, such as a cost incurred by the agent in terms of time or resources spent on preparing the application or in terms of time by which the evaluator’s decision is delayed.

² Another example would be that of a firm interested in developing an economic activity in an environmentally sensitive area. Different extents of intrusion in this area may entail different assessments of the trade-offs between its social benefits and costs. Finally, another example would be that of a manufacturer of a new regulated product, who has to decide on the strength of claims to make regarding its benefits and risks.

³ The optimal fees are not unboudedly high because in these models, the evaluators need to accept a minimum number of articles. In our paper, we discard this requirement on the evaluator so as to capture situations of project screening beyond that of the academic articles evaluation examined in those papers.

⁴ Heintzelman and Nocetti (2009) confirm the insight from Leslie (2005) in a search theoretical model where an author facing multiple journals has to decide on the optimal submission path. See also Azar (2007) and the references therein. Cotton (2012) distinguishes between monetary costs and time delays and shows that when authors of academic articles are heterogenous, the optimal fee structure implies a combination of these monetary and non-monetary fees. Boleslavsky and Cotton (2011) study a model in which an evaluator has to select one of several competing proposals of unknown quality, and investigate the effect of the limited capacity of the evaluator to accept proposals on the incentives of the proposers to produce information.
with one tier of evaluation, Barbos (2012) considers the case of two-sided incomplete information where not only the agent’s, but also the evaluator’s assessment of the project is imperfect. Under this specification, while a higher submission fee does increase the quality of projects that the agent submits, it may not always be beneficial, as it also induces the evaluator to weaken his standards of acceptance. In particular, if the elasticity of the likelihood of a high-quality project as inferred from the evaluator’s minimum quality standard is lower than the elasticity of the likelihood of a high-quality project as inferred from the fact that the evaluator accepted a project, a higher submission fee will decrease the expected quality of projects that are implemented.

The model with multiple tiers examined in this paper unveils four main insights.

1. We first investigate the assortative matching between the agent’s signal of quality and the rank of the tier to which he submits the project. We show that if the submission fee at the upper tier is low enough so as to induce the agent to exert insufficient self-screening at that tier, then negative assortative matching may emerge in equilibrium, where projects with high signals of quality are submitted to the lower tier while projects of lower quality are submitted to the upper tier.

2. Focusing on equilibria with positive assortative matching, we argue that in contrast to the earlier literature, with a tiered system of evaluation, higher fees are not unequivocally beneficial for the quality of projects submitted for review. In particular, a higher submission fee at the upper tier decreases the quality of projects submitted between the two tiers. A higher fee at the lower tier does discourage marginal submissions to that tier, and therefore increases the quality of projects submitted between the two tiers, but may lower the quality of projects submitted to the upper tier.

3. For the same class of information structures identified in the case of a system of evaluation with a single tier, a higher submission fee at the upper tier increases the expected quality of projects that are implemented by each of the two tiers. On the other hand, since a higher submission fee at the lower tier may decrease the quality of projects submitted at either of the two tiers, its effect on the expected quality of projects that are implemented depends not only on the underlying information structure, but also on its impact on the evaluator’s equilibrium strategy.

4. The last main result of the paper compares the equilibria of the games with one and two tiers, and provides support for a tiered system of evaluation as an efficient project-screening mechanism.

As with the case of the main finding from Barbos (2012), the insights from this paper hinge on
the assumption of imprecise evaluation, which renders the evaluator a strategic player who adjusts his acceptance policy in response to the increase in the quality of projects submitted for review that is induced by a higher submission fee. From a policy perspective, this suggests that in those situations where there is reason to believe that the evaluator may infer information from the agent’s decision, such as if a prosecutor’s choice of the charges to file in a trial were likely to influence the jury’s beliefs, then when studying the optimal submission fee problem, one has to account for that fact that an increase in the perceived quality of projects submitted for review may come at the expense of the evaluator’s own judgment of the project.

At a formal level, the paper from the literature that is closer to ours is Taylor and Yildirim (2011), which studies a model of project proposals in which an agent chooses the amount of effort to exert in generating a project that is then submitted for review. A blind review system, in which payoff relevant information about the proposer is hidden from the reviewer, is compared with an informed regime in which the proposer’s type is public information. While their results are driven by the moral hazard effects of the potentially available public information, we consider the effort level as sunk, and the agent’s decision to be whether and where to submit a project, as a function of the available public and private information.

The rest of the paper is organized as follows. Section 2 defines the model, while in section 3 we characterize the equilibrium of the game. In section 4 we investigate the effect of submission fees on equilibrium strategies and on the efficiency of the outcome. Section 5 concludes.

2 The model

There are two players, an agent (A) and an evaluator (E). A owns a project and considers proposing it to E. The project is of either high (h) or low (l) quality. The common prior probability of state h is π. There are two tiers of evaluation, A and B, and when A submits a project, he has to select the tier to which to submit it. Upon receiving a project for review into a certain tier, E has the choice of whether to accept it or not. Submitting the project to tier t ∈ {A, B} incurs a fee ct on A. Irrespective of its ex-post observed quality, a project accepted in tier t, yields A a payoff bt. A also has the option to not submit the project; the corresponding payoff is normalized to zero. E’s payoff from accepting a high-quality project in either tier is 1, while the loss incurred by E from
accepting a low-quality project in tier $t$ is $L_t$. $E$’s payoff from rejecting a project is normalized to zero.\(^5\) A project that is rejected once cannot be resubmitted for review to either tier.\(^6\) We make the following assumption on the payoff parameters of the model.

**Assumption 1** (i) $b_A > b_B$; (ii) $\frac{b_A}{c_A} > \frac{b_B}{c_B}$; (iii) $L_A > L_B$.

By (i) and (ii), tier A delivers a better absolute and relative reward to $A$ from an accepted project than tier B. We will refer to A and B as the upper and lower tier, respectively. Part (iii) implies that $E$ is adversely affected more by the acceptance of a low-quality project in the upper tier.

Prior to making their decisions, $A$ and $E$ perform assessments of the project that result in subjective evaluations of its quality. $A$’s assessment yields a private signal $\mu \in [0, 1]$; $E$’s assessment yields a private signal $\sigma \in [0, 1]$. For quality $q \in \{h, l\}$, let $G^q(\mu)$ and $F^q(\sigma)$ denote the cumulative distribution functions of the agent and the evaluators’ signals, respectively. Also, let $g^q(\mu) > 0$ and $f^q(\sigma) > 0$ be the corresponding probability density functions.

**Assumption 2** (i) $f^q$ and $g^q$ are bounded and twice continuously differentiable for $q \in \{h, l\}$; (ii) $\frac{d}{d\sigma} \frac{f^h(\sigma)}{f^l(\sigma)} > 0$, $\frac{d}{d\mu} \frac{g^h(\mu)}{g^l(\mu)} > 0$.

Part (ii) of the assumption is the usual monotone likelihood ratio, essentially implying that a higher signal is more informative of a high-quality project.

### 3 The equilibrium

Consider some arbitrary strategies of $A$ and $E$, respectively, $S^a : [0, 1] \rightarrow \{s_A, s_B, n\}$ and $S^e : \{A, B\} \times [0, 1] \rightarrow \{a, r\}$, with the obvious interpretation of the action labels. Upon observing a

\(^5\)The analysis does not change in a meaningful way if we allow the agent’s payoff to also depend on the quality of the project by having him prefer that an accepted project is of high quality. See section 3 for the discussion. Also, the analysis also does not change if we allow that the evaluator be also concerned with the quality of projects that he rejects. Finally, since the submission fee may often take a non-monetary form, we do not include it in the evaluator’s payoff. This is without too much loss of generality for the ensuing results. These simplifying modelling specifications are also adopted elsewhere in the literature (see for instance, Cotton (2012)).

\(^6\)In line with the motivating example from the introduction, we thus restrict attention to the analysis of those situations when resubmission of a rejected project to a different tier is not possible (according to the principle of no double jeopardy, a defendant acquitted in a criminal trial cannot be prosecuted again for the same offense). The analysis of the case when resubmission is possible requires a different approach to be undertaken in a separate paper.
project submitted to tier $t$, and after acquiring the signal $\sigma$, $E$ accepts the project if and only if

$$\Pr(h|\{s_t\}, \sigma) \cdot 1 + [1 - \Pr(h|\{s_t\}, \sigma)] (-L_t) \geq 0 \iff \Pr(h|\{s_t\}, \sigma) \geq \frac{L_t}{1 + L_t}$$

(1)

where the event $\{s_t\} \equiv \{\mu \in [0, 1] : S^q(\mu) = s_t\}$. Denoting the event $\{a_t\} \equiv \{\sigma \in [0, 1] : S^w(t, \sigma) = a\}$, it follows that $A$ submits a project with quality signal $\mu$ to tier $A$ if

$$b_A \Pr (\{a_A\}|\mu) - c_A \geq \max \{0, b_B \Pr (\{a_B\}|\mu) - c_B\}$$

(2)

to tier $B$ if

$$b_B \Pr (\{a_B\}|\mu) - c_B \geq \max \{0, b_A \Pr (\{a_A\}|\mu) - c_A\}$$

(3)

and does not submit the project in the remaining case.

In appendix A1 we show that in any Bayesian Nash Equilibrium, $E$ adopts a cutoff strategy with respect to his informative signal, according to which he accepts a project if and only if his signal is higher than a threshold specific to each tier. Since $E$’s equilibrium strategy can be defined in terms of the corresponding thresholds, for the rest of the paper, we will use $(\sigma_{As}, \sigma_{Bs})$ to denote a generic cutoff strategy, with thresholds of acceptance for the two tiers $\sigma_{As}$ and $\sigma_{Bs}$, respectively. We also show in appendix A1 that the set of values of $\mu$ for which $A$ submits projects to a particular tier is an interval (possibly empty), and that the set of values of $\mu$ for which $A$ does not submit a project consists of either one (possibly empty) or two disjoint intervals.

The next lemma states that, under assumption 1, if $E$ adopts a cutoff strategy $(\sigma_{As}, \sigma_{Bs})$, with $\sigma_{Bs} \geq \sigma_{As}$, then $A$’s best response is to never submit to tier $B$. Its proof from appendix A2 shows that whenever $A$ has a higher expected payoff from submitting to tier $B$ than to $A$, then that payoff is in fact negative. In the following we thus examine the interesting equilibria where $\sigma_{Bs} < \sigma_{As}$.

**Lemma 3.1** If $\sigma_{Bs} \geq \sigma_{As}$, then $A$ either submits the project to tier $A$ or does not submit it at all.

In appendix A3, we examine the agent’s best response function and provide the necessary and sufficient condition for positive assortative matching between the agent’s signal of quality and the rank of the tier to whom he submits a project. More precisely, we identify the condition on $\sigma_{As}$, $\sigma_{Bs}$, and the payoff parameters of the model under which for a given prior $\pi$, the set of $\mu$ for
which $A$ submits to the upper tier $A$ is above the set of signals for which he submits to $B$.\(^7\) This condition is not always satisfied, and thus negative assortative matching may emerge, where $A$ submits projects with low signals to the upper tier, and projects with high signals to the lower tier.

As shown in the appendix, negative assortative matching emerges when the following conditions are satisfied: (i) $c_A$ is low, (ii) $\sigma_{As}$ is high, (iii) $\sigma_{Bs}$ belongs to a subset of moderate values of $[0,1]$ that allows a high level of identification of the quality of the project when submitting it to $B$.\(^8\) When these conditions are satisfied, if $A$ has a project with a high signal, he prefers submitting it to tier $B$, to have it identified as of high quality and thus accepted, rather than submitting it to $A$, where the probability of acceptance is very low because of the very high standards of acceptance. When $A$’s signal is lower (but not too low), he will submit to $A$ because the low submission cost will allow for a non-negative payoff in spite of the low probability of acceptance. For the lowest signals, $A$ will refrain from submitting the project. On the other hand, given this strategy adopted by $A$, $E$’s best response is precisely to employ very high standards of acceptance to the upper tier and moderate ones to the lower tier. Thus, negative assortative matching may occur in equilibrium.

This is an interesting and surprising insight, as it suggests that when the project evaluation relies insufficiently on the agent’s self-screening mechanism at the upper tier (i.e., when $c_A$ is low), thus requiring the evaluator to rely heavily on his own assessment of the project at that tier, by imposing very high acceptance standards, then negative assortative matching may emerge.

We focus the analysis on the interesting case of interior equilibria with positive assortative matching in which both tiers receive submissions and in which the set of values of $\mu$ for which the agent does not submit the project is an interval.\(^9\) More precisely, we investigate properties of equilibria in which $A$ adopts a cutoff strategy characterized by two thresholds ($\mu_{As}, \mu_{Bs}$), with $\mu_{As} \in (0,1)$, $\mu_{Bs} \in (0,1)$, and $\mu_{As} > \mu_{Bs}$, such that $A$ submits to tier $A$ for $\mu \in [\mu_{As}, 1]$, to tier $B$ for $\mu \in [\mu_{Bs}, \mu_{As})$, and does not submit the project for $\mu \in [0, \mu_{Bs})$. We will assume thus implicitly throughout the rest of the paper that the parameters of the model are such that the corresponding

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\(^7\)This condition is reminiscent of the supermodularity condition, which since Becker (1973) is known to be necessary and sufficient for positive assortative matching in the equilibrium allocation of many applications.

\(^8\)More precisely, when the difference $F^l(\sigma_{Bs}) - F^h(\sigma_{Bs})$ is high, which implies that $E$ will observe a signal $\sigma \geq \sigma_{Bs}$ with high probability, conditional on $h$, and with a low probability, conditional on $l$.

\(^9\)A’s best response function may involve a corner solution. In particular, if $\sigma_{As}$ and $\sigma_{Bs}$ are high enough for some $t \in \{A, B\}$, $A$ never submits to tier $t$. Moreover, when $\frac{\lambda_A}{\lambda_B}$ is much higher than $\frac{\lambda_B}{\lambda_A}$, tier $B$ receives no submissions. To focus our analysis on developing intuition rather than solving for corner solutions, we restrict attention to the case of interior equilibria.
equilibria satisfy this regularity property, without explicitly mentioning this assumption each time.

The next two lemmas present the equations that define implicitly the two players’ best response functions in these Bayesian Nash equilibria. Their proofs are in appendices A4 and A5.\textsuperscript{10}

**Lemma 3.2** Given \( \mathcal{E} \)’s cutoff strategy, \((\sigma_{As}, \sigma_{Bs})\), with \( \sigma_{As} > \sigma_{Bs} \). \( A \)’s best response is characterized by two thresholds \( \bar{\pi}_A (\sigma_{As}, \sigma_{Bs}) \) and \( \bar{\pi}_B (\sigma_{As}, \sigma_{Bs}) \), with \( \bar{\pi}_A (\sigma_{As}, \sigma_{Bs}) > \bar{\pi}_B (\sigma_{As}, \sigma_{Bs}) \), implicitly defined by the equations

\[
\frac{\pi}{1 - \pi} \frac{g^h(\bar{\pi}_A)}{g^l(\bar{\pi}_A)} = \frac{b_B [1 - F^l (\sigma_{Bs})] - b_A [1 - F^l (\sigma_{As})]}{b_A [1 - F^h (\sigma_{As})] - b_B [1 - F^h (\sigma_{Bs})]} + c_A - c_B \tag{4}
\]

\[
\frac{\pi}{1 - \pi} \frac{g^h(\bar{\pi}_B)}{g^l(\bar{\pi}_B)} = \frac{c_B - b_B [1 - F^l (\sigma_{Bs})]}{b_B [1 - F^h (\sigma_{Bs})] - c_B} \tag{5}
\]

such that \( A \) submits to tier \( A \) if \( \mu \geq \bar{\pi}_A (\cdot, \cdot) \), to tier \( B \) if \( \mu \in [\bar{\pi}_B (\cdot, \cdot), \bar{\pi}_A (\cdot, \cdot)] \), and forgoes submitting the project if \( \mu < \bar{\pi}_B (\cdot, \cdot) \).

**Lemma 3.3** Given \( \mathcal{A} \)’s cutoff strategy \((\mu_{As}, \mu_{Bs})\), \( \mathcal{E} \) accepts a project submitted to tier \( A \) if and only if \( \sigma \geq \sigma_A (\mu_{As}, \mu_{Bs}) \), with \( \sigma_A (\mu_{As}, \mu_{Bs}) \) given implicitly by

\[
\frac{\pi}{1 - \pi} \frac{f^h(\sigma_A)}{f^l(\sigma_A)} \frac{1 - G^h(\mu_{As})}{1 - G^l(\mu_{As})} = L_A \tag{6}
\]

and accepts a project submitted to tier \( B \) if and only if \( \sigma \geq \sigma_B (\mu_{As}, \mu_{Bs}) \), with \( \sigma_B (\mu_{As}, \mu_{Bs}) \) given implicitly by

\[
\frac{\pi}{1 - \pi} \frac{f^h(\sigma_B)}{f^l(\sigma_B)} \frac{G^h(\mu_{As}) - G^h(\mu_{Bs})}{G^l(\mu_{As}) - G^l(\mu_{Bs})} = L_B \tag{7}
\]

The best-response functions, as elicited by equations (4), (5), (6) and (7), determine the equilibrium strategies of the two players denoted by \((\sigma_A^*, \sigma_B^*)\) and \((\mu_A^*, \mu_B^*)\). The next lemma, whose proof is in appendix A6, presents the monotonicities of these best-response functions.

\textsuperscript{10}We can model a situation in which \( \mathcal{A} \) also prefers that an accepted project is of high quality, by having \( \mathcal{A} \) receive an additional benefit \( \delta \) under this contingency. In this case, equation (4) becomes \([b_A + \delta_A \Pr (h|\{a_A\}, \mu)] \Pr (\{a_A\}|\mu) - c_A \geq \max \left(\frac{b_A + \delta_B \Pr (h|\{a_B\}, \mu)}{b_A + \delta_B \Pr (h|\{a_B\}, \mu)} \Pr (\{a_B\}|\mu) - c_B, 0\right)\), which after some calculations can be rewritten as \( \frac{\pi}{1 - \pi} \frac{f^h(\sigma_A)}{f^l(\sigma_A)} = \frac{b_A [1 - F^h (\sigma_{As})] - b_A [1 - F^l (\sigma_{As})]}{b_A [1 - F^h (\sigma_{As})] - b_B [1 - F^l (\sigma_{Bs})]} + c_A - c_B \). Equations (3) and (5) are altered in a similar way. The ensuing analysis and results are qualitatively similar to the case when \( \delta_A = \delta_B = 0 \).
Lemma 3.4 (i) \( \overline{\mu}_A(\sigma_{A_s}, \sigma_{B_s}) \) is decreasing in \( \sigma_{B_s} \) and increasing in \( \sigma_{A_s} \); (ii) \( \overline{\mu}_B(\sigma_{A_s}, \sigma_{B_s}) \) is constant in \( \sigma_{A_s} \) and increasing in \( \sigma_{B_s} \); (iii) \( \overline{\sigma}_A(\mu_{A_s}, \mu_{B_s}) \) is decreasing in \( \mu_{A_s} \) and constant in \( \mu_{B_s} \); (iv) \( \overline{\sigma}_B(\mu_{A_s}, \mu_{B_s}) \) is decreasing in \( \mu_{A_s} \) and \( \mu_{B_s} \), when \( \mu_{A_s} > \mu_{B_s} \).

For generic payoff parameters and information structures, the Bayesian Nash equilibrium of the game is not necessarily unique. As in other frameworks, when sunspot equilibria emerge, they do so because there exist different sets of self-fulfilling expectations for the same set of fundamentals of the model. The next proposition identifies a consistency requirement across different equilibria. Its corollary provides a sufficient condition for equilibrium uniqueness.

Proposition 1 If \( \xi \equiv (\sigma^*_A, \sigma^*_B, \mu^*_A, \mu^*_B) \) and \( \xi' \equiv (\sigma''_A, \sigma''_B, \mu''_A, \mu''_B) \) are two Bayesian Nash equilibria with \( \sigma''_B > \sigma''_B \), then it must be that \( \sigma''_A > \sigma^*_A, \mu''_B > \mu^*_B \) and \( \mu''_A < \mu^*_A \).

Corollary 3.1 Consider a Bayesian Nash equilibrium \( \xi \equiv (\sigma^*_A, \sigma^*_B, \mu^*_A, \mu^*_B) \) and assume that for fixed values of \( \sigma^*_A \) and \( \mu^*_B \), the two best-response functions \( \overline{p}_A(\sigma^*_A, \sigma_{B_s}) \) and \( \overline{p}_B(\mu_{A_s}, \mu_{B_s}) \) as defined by (4) and (7) have the unique fixed point \( (\mu^*_A, \sigma^*_B) \). Then, if

\[
\frac{\partial \overline{p}_A}{\partial \sigma_{B_s}} (\sigma^*_A, \sigma^*_B) \cdot \frac{\partial \overline{p}_B}{\partial \mu_{A_s}} (\mu^*_A, \mu^*_B) < 1
\]

(8)

the equilibrium \( \xi \) is unique.

Before presenting the proof of these two results, we introduce the three panels in Figure 1 on which we rely heavily in the rest of the analysis. In each panel, we depict the pairwise best-response functions defined by (4)-(7), when the two variables not considered in the respective panel are kept fixed. A solid curve represents a best-response function when the remaining variables are fixed at the values in \( \xi \). A dashed curve depicts a best-response function when the remaining variables are fixed at the values in \( \xi' \). For instance, in panel (a), the solid curve \( \overline{p}^A_A(\sigma_{A_s}) \) represents the best-response function \( \overline{p}_A(\sigma_{A_s}, \sigma_{B_s}^*) \), while the dashed curve \( \overline{p}''_A(\sigma_{A_s}) \) represents the best-response function \( \overline{p}_A(\sigma_{A_s}, \sigma_{B_s}'') \). When there is no dashed curve, the function is the same in the two equilibria. For instance, in panel (a), \( \overline{\sigma}_A(\mu_{A_s}) \) represents the best-response functions \( \overline{\sigma}_A(\mu_{A_s}, \mu_{B_s}^*) \) and \( \overline{\sigma}_A(\mu_{A_s}, \mu_{B_s}'') \), which by lemma 3.4(iii) are the same.\(^{11}\) All curves are generic and are depicted only

\(^{11}\)Similarly, for instance, in panel (b), \( \overline{p}''_B(\sigma_{B_s}) \) represents the best response functions \( \overline{p}_B(\sigma_{A_s}^*, \sigma_{B_s}^*) \) and \( \overline{p}_B(\sigma_{A_s}'', \sigma_{B_s}'') \), while \( \overline{\sigma}_B(\mu_{B_s}) \) and \( \overline{\sigma}_B(\mu_{B_s}) \) represent the best response functions \( \overline{\sigma}_B(\mu_{A_s}^*, \mu_{B_s}) \) and \( \overline{\sigma}_B(\mu_{A_s}''', \mu_{B_s}) \), respectively.
so as to exhibit the salient monotonicity property. In panel (c), since both curves are decreasing, they are presented as crossing each other twice, so as to allow for either of them crossing from below. To save on notation, we define a partial order on these curves by saying that for instance $\overline{\sigma}_A (\sigma Bs) > \overline{\sigma}_A (\sigma As)$ or $\overline{\sigma}_A (\mu As) > \overline{\sigma}_A (\mu As)$ if, as is the case in panel (c), the first curve is above the second one in a panel with $\sigma$ on the horizontal axis and $\mu$ on the vertical axis.

\[\text{Proof of Proposition 1 and Corollary 3.1.}\]

To prove proposition 1, first note in panel (a), that $\sigma_B' > \sigma_B^*$ implies by lemma 3.4(i) that $\overline{\sigma}_A (\sigma As) > \overline{\sigma}_A (\sigma As)$. Since $\overline{\sigma}_A (\mu As)$ is the same as $\overline{\sigma}_A (\mu As)$, it must be that $\sigma_A' > \sigma_A^*$ and $\mu_A' < \mu_A^*$. Second, in panel (b), $\mu_A' < \mu_A^*$ implies by lemma 3.4(iv) that $\overline{\sigma}_B (\mu Bs) > \overline{\sigma}_B (\mu Bs)$. Since $\overline{\sigma}_B (\sigma Bs)$ is the same as $\overline{\sigma}_B (\sigma Bs)$, it must be that $\mu_B' > \mu_B^*$ and $\sigma_B' > \sigma_B^*$. The second implication is consistent with the initial assumption. Finally, in panel (c), $\mu_B' > \mu_B^*$ implies by lemma 3.4(iv) that $\overline{\sigma}_B (\mu As) > \overline{\sigma}_B (\mu As)$, while $\sigma_A' > \sigma_A^*$ implies by lemma 3.4(i) that $\overline{\sigma}_A (\sigma Bs) > \overline{\sigma}_A (\sigma Bs)$.

To show corollary 3.1, consider an equilibrium $\xi = (\sigma_A^*, \sigma_B^*, \mu_A^*, \mu_B^*)$ such that for fixed values of $\sigma_A^*$ and $\mu_B^*$, the curves $\overline{\sigma}_B (\mu As)$ and $\overline{\sigma}_B (\sigma Bs)$ satisfy a single-crossing property, with $\overline{\sigma}_B (\mu As)$ having a steeper downward slope at the intersection of the two curves. In panel (c), this is the case of the point of intersection that is in the upper left corner. Assume by contradiction that there exists another equilibrium of the game, $\xi' = (\sigma_A'^*, \sigma_B'^*, \mu_A'^*, \mu_B'^*)$. Without loss of generality, we can assume that $\sigma_B'^* > \sigma_B^*$. In this case, by proposition 1, $\sigma_B'^* > \sigma_B^*$ implies $\mu_B'^* > \mu_B^*$ and $\sigma_A'^* > \sigma_A^*$. In turn, these imply $\overline{\sigma}_B (\mu As) > \overline{\sigma}_B (\mu As)$ and $\overline{\sigma}_A (\sigma Bs) > \overline{\sigma}_A (\sigma Bs)$. By inspecting panel (c) of Figure 1 (more precisely, the intersection of the two curves in the upper left corner) it follows that it must
be that $\sigma_B^* < \sigma_B^*$ and $\mu_A^* > \mu_A^*$. This is inconsistent with the initial assumption that $\sigma_B^* > \sigma_B^*$. Thus the initial equilibrium is unique. Now, note that the slope of $\sigma_B^* (\mu)_{A_s}$ at the equilibrium values equals $\left[ \frac{\partial \sigma_B^* (\mu)_{A_s}}{\partial \mu_{A_s}} (\mu_A^*, \mu_B^*) \right]^{-1}$. Thus $\sigma_B^* (\mu)_{A_s}$ is steeper than $\mu_A (\sigma)_{B_s}$ if and only if

$$\frac{\partial \mu_A}{\partial \sigma_B} (\sigma_A^*, \sigma_B^*) > \left[ \frac{\partial \sigma_B}{\partial \mu_{A_s}} (\mu_A^*, \mu_B^*) \right]^{-1}$$

which, since both sides are negative, can be rewritten as in (8). □

Proposition 1 shows that if $E$ is more stringent in his acceptance policy for tier $B$ in equilibrium $\xi'$, (i.e., $\sigma_B^* > \sigma_B^*$), then first, $A$ is more reluctant to submit marginal projects to tier $B$, ($\mu_B^* > \mu_B^*$), and second, $A$ is more inclined to submit marginal projects to tier $A$, ($\mu_A^* < \mu_A^*$), since the alternative is less appealing. Given these, $E$ is also more stringent in his acceptance policy at tier $A$, ($\sigma_A^* > \sigma_A^*$), to make up for the lower expected quality of projects submitted. While these feed-forward effects make the result intuitive, proposition 1 ensures that the feed-back effects, such as the effect of the increase in $\sigma_A^*$ on $\mu_A^*$, or of the decrease in $\mu_A^*$ on $\sigma_B^*$, do not offset them.

To understand corollary 3.1, consider two equilibria, $\xi$ and $\xi'$, with corresponding strategies as in the text of proposition 1. Note then that for a fixed value of $\sigma_B^*$, a higher value of $\sigma_A^*$, ($\sigma_A^* > \sigma_A^*$), would induce $\mu_A^*$ to increase. A higher $\mu_A^*$, together with a higher $\mu_B^*$, ($\mu_B^* > \mu_B^*$), would increase the quality of projects received by tier $B$, and thus induce a decrease in $\sigma_B^*$. To instead have $\sigma_B^*$ increasing and $\mu_A^*$ decreasing (since $\sigma_B^* > \sigma_B^*$ and $\mu_A^* < \mu_A^*$ in $\xi$ and $\xi'$), $\sigma_B^*$ and $\mu_A^*$ should feed off each other. This means that $\sigma_B^*$ should be higher because $\mu_A^*$ is lower, and $\mu_A^*$ should be lower because $\sigma_B^*$ is higher. Thus, to have multiple equilibria, $\sigma_B^*$ has to be very responsive to a decrease in $\mu_A^*$, while $\mu_A^*$ has to be very responsive to an increase in $\sigma_B^*$; these would offset the effects of the increases in $\mu_B^*$ and $\sigma_A^*$. In panel (c), this is the case precisely when $\sigma_B^* (\mu_A)$ and $\mu_A (\sigma_B)$ intersect in the lower right corner where both have a steeper slope. Equation (8) is the mathematical representation of the same condition.

4 Results

We examine the effect of increases in the submission fees at the two tiers of evaluation on the equilibrium strategies, and then employ these comparative statics results to investigate their effect.
on the efficiency of the equilibrium outcome. Note that unlike some of the other papers from the literature, we do not calculate the optimal submission fees, but only elicit the effect of a change in these fees on the quality of projects that are implemented by the evaluator in the two tiers.\footnote{In fact, in a variety of situations, these fees can be adjusted only at the margin. For instance, while laws could be passed to make a prosecutor’s job of collecting evidence less costly, this may be possible only up to a limited extent.} We present first the main result from Barbos (2012) for the model with one tier of evaluation.

### 4.1 The model with one tier of evaluation

Consider a model as in section 2, only that with one tier of evaluation. In this case the agent only has to decide on whether or not to submit the project for review. The next lemma states that in the resulting game, an equilibrium exists, is unique, and must be in cutoff strategies.

**Lemma 4.1 (Barbos (2012))** There exists a unique equilibrium of the game with one tier of evaluation. This equilibrium is completely characterized by two values \((\mu^*, \sigma^*) \in [0, 1) \times (0, 1)\) such that \(A\) submits a project if and only if \(\mu \geq \mu^*\), and \(E\) accepts a project if and only if \(\sigma \geq \sigma^*\).

The expected quality of projects that are implemented in this equilibrium, \(h \Pr (h|\mu \geq \mu^*, \sigma \geq \sigma^*) + l \Pr (l|\mu \geq \mu^*, \sigma \geq \sigma^*)\), is isomorphic to the probability \(\Pr (h|\mu \geq \mu^*, \sigma \geq \sigma^*)\). The next proposition elicits the effect of an increase in the submission fee, \(c\), on this measure.

**Proposition 2 (Barbos (2012))** \(\frac{dc}{d\sigma} \Pr (h|\mu \geq \mu^*, \sigma \geq \sigma^*) > 0\) if and only if

\[
\frac{d}{d\sigma} \ln \frac{f^h(\sigma^*)}{f^l(\sigma^*)} > \frac{d}{d\sigma} \ln \frac{1 - F^h(\sigma^*)}{1 - F^l(\sigma^*)}
\]  

(9)

Note that the term \(\frac{f^h(\sigma^*)}{f^l(\sigma^*)}\) is the likelihood of the state \(h\) as inferred from \(E\)’s equilibrium minimum acceptance quality standard \(\sigma^*\). On the other hand, given \(E\)’s cutoff strategy, \(\frac{1 - F^h(\sigma^*)}{1 - F^l(\sigma^*)}\) is the likelihood of state \(h\) as inferred from the fact that \(E\) accepted a project. Thus, in a model with one tier of evaluation, an increase in the submission fee leads to an increase in the expected quality of projects that are implemented if and only if the elasticity of the likelihood of a high quality project that is inferred from the \(E\)’s minimum quality standard is higher than the elasticity of the likelihood of a high quality project that is inferred from the fact that \(E\) accepted a project.
Since the intuition of this result resembles those of the corresponding results from the model with multiple tiers, we defer presenting it to section 4.3.

4.2 The effects of submission fees on equilibrium strategies

In the model with multiple tiers of evaluation, with generic signal distributions, it is not tractable to obtain a closed-form solution for the equilibrium strategies amenable for direct comparative statics analysis. Instead, we perform this analysis in three steps. First, we identify all correlations between the signs of the changes in the equilibrium strategies that are imposed by (4)-(7) under the assumed change in the underlying parameter. Second, we identify the paths of the equilibrium strategies that are consistent with these correlations. Finally, for each equilibrium path, we verify that the shifts in the pairwise best-response functions that are imposed by the changes in the underlying parameter and in the equilibrium strategies are consistent with the assumed changes in the equilibrium strategies. At this step, we identify the equilibrium paths that are artifacts of the multiplicity of equilibria.

The case of a change in $c_A$  We start with the case of an increase in $c_A$.13 Assuming that $c_A$ increases by $dc_A > 0$, by inspecting (4)-(7), one can infer the following necessary correlations among the possible changes in the equilibrium strategies.

(a) From (4), if $d\sigma_A^* > 0$ and $d\sigma_B^* < 0$, then $d\mu_A^* > 0$.

(b) From (5), if $d\sigma_B^* > (\langle)0$, then $d\mu_B^* > (\langle)0$.

(c) From (6), if $d\mu_A^* > (\langle)0$, then $d\sigma_A^* < (\rangle)0$.

(d) From (7), if $d\mu_A^* > (\langle)0$ and $d\mu_B^* > (\langle)0$, then $d\sigma_B^* < (\rangle)0$.

Using these, we have the following possible equilibrium paths following an increase in $c_A$.

1. Assume $d\mu_A^* > 0$. By (c), it follows that $d\sigma_A^* < 0$. If $d\sigma_B^* > 0$, then by (b) $d\mu_B^* > 0$. But by (d), if $d\mu_A^* > 0$ and $d\mu_B^* > 0$, then it must be that $d\sigma_B^* < 0$. This contradicts the previous assumption. Therefore, it must be that $d\sigma_B^* < 0$, and thus by (b) that $d\mu_B^* < 0$.

13It is straightforward to see that this is qualitatively similar to a decrease in $b_A$. 

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2. Assume $d\mu_A^* < 0$. By (c), this implies that $d\sigma_A^* > 0$. By (a), this implies that $d\sigma_B^* > 0$. By (b), this implies that $d\mu_B^* > 0$.

The first of the two equilibrium paths is intuitive. Upon facing a higher $c_A$, $A$ is less inclined to submit marginal products to tier $A$, and thus $\mu_A^*$ increases. The increase in $\mu_A^*$ leads to an increase in the expected quality of projects received by tier $A$, which allows $E$ to lower the corresponding standards, and thus $\sigma_A^*$ decreases. The increase in $\mu_A^*$ also leads to an increase in the expected quality of projects submitted to tier $B$, which allows $E$ to also lower $\sigma_B^*$. This makes $A$ more willing to submit marginal projects to tier $B$, and thus $\mu_B^*$ decreases. We depict these in Figure 2 below.

The solid curves represent the pairwise best-response functions when the remaining variables are fixed at the values from the initial equilibrium $\xi \equiv (\sigma_A^*, \sigma_B^*, \mu_A^*, \mu_B^*)$, and the submission fee is $c_A$. The partially dashed curves $p_A^{\sigma_A} (\sigma_A)$ and $p_A^{\mu_A} (\mu_B)$ represent the best-response functions when the fee is $c_A' \equiv c_A + dc_A$, but the values of the remaining variables are still fixed at $(\sigma_A^*, \sigma_B^*, \mu_A^*, \mu_B^*)$. For instance, from (4), it follows that for fixed values of $\sigma_B^*$ and $\mu_B^*$, to the same cutoff strategy $\sigma_A^*$, $A$ responds with a higher $p_A$ when $c_A$ increases to $c_A'$. Thus, $p_A^{\sigma_A} (\sigma_A) \succ p_A^{\sigma_A} (\sigma_A)$ in panel (a). Similarly, $p_A^{\mu_A} (\mu_B) \succ p_A^{\mu_A} (\mu_B)$ in panel (c). The dashed curves $p_A^{\sigma_A} (\sigma_A)$, $p_B^{\mu_B} (\mu_B)$, $p_A^{\mu_A} (\mu_B)$ and $p_A^{\sigma_B} (\sigma_B)$ and

\footnote{Thus, for instance, $p_A^{\sigma_A} (\sigma_A)$ represents the best response function $p_A (\sigma_A, \sigma_B^*)$, as defined by (4), when the submission fee to tier $A$ is $c_A$, while $p_A^{\mu_A} (\sigma_A)$ represents $p_A (\sigma_A, \mu_B^*)$ when the submission fee in (4) is $c_A'$. On the other hand, $p_A^{\sigma_A} (\sigma_A)$, which is defined below, represents $p_A (\sigma_A, \sigma_B^*)$ when the submission fee to tier $A$ is $c_A$. Finally, $p_A^{\mu_A} (\mu_A)$ represents $p_A (\mu_A, \mu_B)$ when the submission fee to tier $A$ is $c_A$ or $c_A'$, but also $p_A (\mu_A, \mu_B)$ when to tier $A$ when the fee is $c_A'$. Note also that, for instance in panel (c) the values $\mu_A^*$ and $\sigma_B^*$ from the initial equilibrium $\xi$ are at the intersection of the curves $p_A^{\sigma_A} (\sigma_B^*)$ and $p_B^{\mu_B} (\mu_A)$, while the corresponding values from the equilibrium $\xi'$ are at the intersection of the curves $p_A^{\sigma_A} (\sigma_B^*)$ and $p_B^{\mu_B} (\mu_A)$.
$\bar{\sigma}_B(\mu_{As})$ represent the best-response functions that correspond to $c'_A$, and to values of strategies from the new equilibrium $\xi' \equiv (\sigma^*_A, \sigma^*_B, \mu^*_A, \mu^*_B)$. For instance, since $d\sigma^*_B < 0$, from lemma 3.4(i), it follows that at $c'_A$, A’s best-response $\bar{\mu}_A$ is higher when E’s cutoff for tier B is fixed at $\sigma^*_B$ than at $\sigma^*_B$. This implies that $\bar{\mu}_A^A(\sigma_{As}) > \bar{\mu}_A^A(\sigma_{As})$ in panel (a). Similarly, in panel (b), from lemma 3.4(iv) it follows that $\bar{\sigma}_B^B(\mu_{Bs}) > \bar{\sigma}_B^B(\mu_{Bs})$ because $d\mu^*_A > 0$. Finally, in panel (c), $d\mu^*_B < 0$ implies $\bar{\sigma}_B^B(\mu_{As}) > \bar{\sigma}_B^B(\mu_{As})$, while $d\sigma^*_A < 0$ implies $\bar{\mu}_A^A(\sigma_{Bs}) > \bar{\mu}_A^A(\sigma_{Bs})$. As seen in the figure, the equilibrium path is consistent with either type of initial equilibrium.

As we show next, the second equilibrium path can arise only when the initial equilibrium is not unique. Essentially, the second scenario emerges as a consequence of a coordination of expectations on a different sunspot equilibrium in response to the change in the parameters of the model, rather than being driven by an adjustment of the players’ strategies within the same equilibrium. Thus, note that in panel (c) of Figure 3, $dc_A > 0$ and $d\sigma^*_B < 0$ imply $\bar{\sigma}_B^B(\sigma_{Bs}) > \bar{\sigma}_B^B(\sigma_{Bs}) > \bar{\sigma}_B^B(\sigma_{Bs})$, while $d\mu^*_B > 0$ implies $\bar{\sigma}_B^B(\mu_{As}) > \bar{\sigma}_B^B(\mu_{As})$. Therefore, the only ways to have $d\sigma^*_B > 0$ and $d\mu^*_A < 0$ are either if the initial equilibrium is in the upper left corner and the two curves do not satisfy the single crossing condition, or if the initial equilibrium is in the lower right corner where $\bar{\mu}_A^A(\sigma_{Bs})$ crosses $\bar{\sigma}_B^B(\mu_{As})$ from above. These are precisely the conditions under which the equilibrium is not necessarily unique.

We collect these results in the following proposition.

**Proposition 3** Consider an equilibrium $\xi \equiv (\sigma^*_A, \sigma^*_B, \mu^*_A, \mu^*_B)$ and assume $dc_A > 0$. If $\xi$ is unique,
then \(d\mu_A^* > 0, d\mu_B^* < 0, d\sigma_A^* < 0 \) and \(d\sigma_B^* < 0\). If \(\xi\) is not unique, then it may also happen that \(d\mu_A^* < 0, d\mu_B^* > 0, d\sigma_A^* > 0 \) and \(d\sigma_B^* > 0\).

Focusing on the case when the equilibrium is unique, note that while the increase in \(c_A\) does increase the quality of projects submitted to tier A (as \(d\mu_A^* > 0\)), it also leads to a decrease in the expected quality of projects submitted between the two tiers (as \(d\mu_B^* < 0\)). Therefore, unlike the case of a system of evaluation with one tier, when there are multiple tiers, a submission fee increase is not unequivocally beneficial with respect to the quality of projects submitted for review because increases in submission fees to upper tiers exert negative externalities on the quality of projects submitted to the lower tiers. Note that this insight hinges on the underlying assumption that evaluation is imprecise; if evaluation was precise, an increase in \(c_A\) would have no effect on the agent’s decision at the margin on whether to submit a project to tier B or to forgo submitting it.

**The case of a change in \(c_B\)** Similarly to the previous analysis, assuming \(dc_B > 0\) (or \(dc_B < 0\)), one can infer the following necessary correlations among equilibrium strategies.

(a) From (4), if \(d\sigma_B^* > 0\) and \(d\sigma_A^* < 0\), then \(d\mu_A^* < 0\).

(b) From (5), if \(d\sigma_B^* > 0\), then \(d\mu_B^* > 0\).

(c) From (6), if \(d\mu_A^* > (\langle \rangle)0\), then \(d\sigma_A^* < (\rangle)0\).

(d) From (7), if \(d\mu_A^* > (\langle \rangle)0\) and \(d\mu_B^* > (\langle \rangle)0\), then \(d\sigma_B^* < (\rangle)0\).

Therefore, the equilibrium paths that can emerge when \(c_B\) increases are the following.

1. Assume \(d\mu_B^* > 0\) and \(d\sigma_B^* > 0\). By (d), it follows that \(d\mu_A^* < 0\), and then by (c) that \(d\sigma_A^* > 0\).

2. Assume \(d\mu_B^* > 0\) and \(d\sigma_B^* < 0\). If \(d\mu_A^* < 0\), then by (c) \(d\sigma_A^* > 0\).

3. Assume \(d\mu_B^* > 0\) and \(d\sigma_B^* < 0\). If \(d\mu_A^* > 0\), then by (c) \(d\sigma_A^* < 0\).

4. Assume \(d\mu_B^* < 0\). Then by (b), \(d\sigma_B^* < 0\). By (d) it follows that \(d\mu_A^* > 0\), which then by (c) implies that \(d\sigma_A^* < 0\).
The third step of the analysis is along the lines of the case of an increase in $c_A$ and is thus omitted. On the first three equilibrium paths, when $dc_B > 0$, $A$ is more reluctant to submit low-signal marginal projects to tier $B$, and thus $d\mu^*_B > 0$. On the first two paths, $A$ also abstains from submitting high-signal marginal projects to tier $B$, and thus $d\mu^*_A < 0$. If the net effect on the quality of projects submitted to tier $B$ is negative, $E$ becomes more stringent in his acceptance policy at tier $B$, and so $d\sigma^*_B > 0$, as on the first equilibrium path. If the net effect is positive, $E$ is less stringent, and so $d\sigma^*_B < 0$, as on the second equilibrium path. On both paths $E$ becomes more stringent at tier $A$ since the expected quality of projects that are received at that tier is lower. The third equilibrium path occurs when the quality of projects submitted to tier $B$ increases significantly following the increase in $\mu^*_B$. In this case, $\sigma^*_B$ decreases sufficiently so as to induce an increase in $\mu^*_A$, and a consequent decrease in $\sigma^*_A$. The last equilibrium path emerges again only when the initial equilibrium is not necessarily unique.\textsuperscript{15} We collect these results in the next proposition.

**Proposition 4** Consider an equilibrium $\xi \equiv (\sigma^*_A, \sigma^*_B, \mu^*_A, \mu^*_B)$ and assume $dc_B > 0$. If $\xi$ is unique, then $d\mu^*_B > 0$, and one of the following three equilibrium paths occurs: (i) $d\sigma^*_B > 0$, $d\mu^*_A < 0$ and $d\sigma^*_A > 0$; (ii) $d\sigma^*_B < 0$, $d\mu^*_A < 0$ and $d\sigma^*_A > 0$; (iii) $d\sigma^*_B < 0$, $d\mu^*_A > 0$ and $d\sigma^*_A < 0$. If $\xi$ is not unique, then it may also happen that $d\mu^*_B < 0$, $d\mu^*_A > 0$, $d\sigma^*_A < 0$, and $d\sigma^*_B < 0$.

The equilibrium path selection is a local property, in that for given values of the payoff parameters, it is determined exclusively from the local properties of the signal structures in a neighborhood of the initial equilibrium. In particular, depending on the change in the amount of information extracted with an infinitesimal change in the strategy of each player, the equilibrium may follow at each starting point any of these paths. Therefore, additional regularities on the equilibrium paths can only be obtained only by making additional assumptions on the payoff parameters of the model and the information structure beyond that imposed by assumptions 1 and 2.

\textsuperscript{15}To see this, note the following in a $(\mu_A, \sigma_B)$ panel. First, from (4), $dc_B > 0$ implies $p_A^B(\sigma_B) > p_A^B(\sigma_B)$. Second, also from lemma 3.4(i), $d\sigma^*_A < 0$ implies $p_A^A(\sigma_B) > p_A^A(\sigma_B)$. Third, from lemma 3.4(iv), $d\mu^*_B < 0$ implies $p_B^B(\mu_B) > p_B^B(\mu_B)$. It is then straightforward to see that if $p_B^B(\mu_B)$ is steeper than $p_A^A(\sigma_B)$ at the intersection point, it must be that $d\mu^*_A < 0$. 

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4.3 The effects of submission fees on the equilibrium expected quality of the projects that are implemented

In this section, we examine the effect of a change in the two submission fees on the efficiency of the equilibrium outcome. The measures of efficiency that we employ here are the expected qualities of projects implemented by the evaluator in the two tiers, which are isomorphic with $\Pr(h|\mu \geq \mu_A^*, \sigma \geq \sigma_A^*)$, for tier A, and $\Pr(h|\mu_A^* \geq \mu \geq \mu_B^*, \sigma \geq \sigma_B^*)$, for tier B. The next proposition elicits the effect of an increase in $c_A$ on these two values. We restrict attention to the more interesting case where the initial equilibrium is unique, and thus the comparative statics are driven by the fundamentals of the model rather than equilibrium selection.

**Proposition 5** Assume that the equilibrium $(\sigma_A^*, \sigma_B^*, \mu_A^*, \mu_B^*)$ is unique. Then

(i) \( \frac{d}{dc_A} \Pr(h|\mu \geq \mu_A^*, \sigma \geq \sigma_A^*) > 0 \) if and only if \( \frac{d}{d\sigma} \ln \frac{f^h(\sigma_A^*)}{f(\sigma_A^*)} > \frac{d}{d\sigma} \ln \frac{1-F^h(\sigma_A^*)}{1-F(\sigma_A^*)} \).

(ii) \( \frac{d}{dc_A} \Pr(h|\mu_A^* \geq \mu \geq \mu_B^*, \sigma \geq \sigma_B^*) > 0 \) if and only if \( \frac{d}{d\sigma} \ln \frac{f^h(\sigma_B^*)}{f(\sigma_B^*)} > \frac{d}{d\sigma} \ln \frac{1-F^h(\sigma_B^*)}{1-F(\sigma_B^*)} \).

**Proof.** By Bayes’ Rule, we have

\[
\Pr(h|\mu \geq \mu_A^*, \sigma \geq \sigma_A^*) = \frac{\Pr(\mu \geq \mu_A^*, \sigma \geq \sigma_A^*|h) \Pr(h)}{\Pr(\mu \geq \mu_A^*, \sigma \geq \sigma_A^*)} = \frac{\pi}{\pi + (1-\pi)\Pr(\mu \geq \mu_A^*, \sigma \geq \sigma_A^*)} = \frac{\pi}{\pi + (1-\pi)\Pr(\mu \geq \mu_A^*, \sigma \geq \sigma_A^*)}
\]

where for the third equality we used the conditional independence of the two players’ signals. Therefore, \( \Pr(h|\mu \geq \mu_A^*, \sigma \geq \sigma_A^*) \) increases following an increase in $c_A$ if and only if the sum $\ln \frac{1-G^h(\mu_A^*)}{1-G^h(\mu_A^*)} + \ln \frac{1-F^h(\sigma_A^*)}{1-F^h(\sigma_A^*)}$ increases. From (6), written in equilibrium, we have $\ln \frac{1-G^h(\mu_A^*)}{1-G^h(\mu_A^*)} = \ln L_A - \ln \frac{\pi}{1-\pi} - \ln \frac{1-F^h(\sigma_A^*)}{1-F^h(\sigma_A^*)}$, so $\Pr(h|\mu \geq \mu_A^*, \sigma \geq \sigma_A^*)$ increases if and only if $\ln \frac{1-F^h(\sigma_A^*)}{1-F^h(\sigma_A^*)} - \ln \frac{f^h(\sigma_A^*)}{f(\sigma_A^*)}$ increases. But

\[
\frac{d}{dc_A} \left[ \ln \frac{1-F^h(\sigma_A^*)}{1-F(\sigma_A^*)} - \ln \frac{f^h(\sigma_A^*)}{f(\sigma_A^*)} \right] = \frac{d}{d\sigma} \left[ \ln \frac{1-F^h(\sigma_A^*)}{1-F(\sigma_A^*)} - \ln \frac{f^h(\sigma_A^*)}{f(\sigma_A^*)} \right] \left( \frac{d\sigma}{dc_A} \right)
\]

Since by proposition 3, we have $\frac{d\sigma}{dc_A} < 0$ the proof of part (i) is complete. The proof of part (ii) follows the same steps.\(^{16}\)

\(^{16}\)Note that $\Pr(\mu_A^* \geq \mu \geq \mu_B^*|q) = G^h(\mu_A^*) - G^h(\mu_B^*)$, and that one can use (7) to compute $\frac{G^h(\mu_A^*) - G^h(\mu_B^*)}{G^h(\mu_A^*) - G^h(\mu_B^*)}$. 

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To understand these results, consider the effect of an increase in $c_A$ on $Pr(h|\mu \geq \mu_A^*, \sigma \geq \sigma_A^*)$. Note that by proposition 3, the increase in $c_A$ has a positive effect on the expected quality of projects that are implemented in tier $A$ by increasing the quality of projects that are submitted ($\mu_A^*$ increases), and a negative effect by decreasing $E$’s standards of acceptance ($\sigma_A^*$ decreases). On net, the fee increase has a positive effect if $Pr(h|\mu \geq \mu_A^*, \sigma \geq \sigma_A^*)$ is more responsive to the corresponding increase in $\mu_A^*$ than to the decrease in $\sigma_A^*$. Now, as seen in (10), $Pr(h|\mu \geq \mu_A^*, \sigma \geq \sigma_A^*)$ is a monotone transformation of the product of the likelihoods of state $h$ inferred from the fact that $A$ submitted the project, $\frac{1- G_h(\mu_A^*)}{1-G^*(\mu_A^*)}$, and from the fact that $E$ accepted it, $\frac{1- F_h(\sigma_A^*)}{1-F^*(\sigma_A^*)}$. Therefore, the responsiveness of $Pr(h|\mu \geq \mu_A^*, \sigma \geq \sigma_A^*)$ with respect to $\mu_A^*$ can be elicited from the elasticity of the likelihood $\frac{1- G_h(\mu_A^*)}{1-G^*(\mu_A^*)}$ with respect to $\mu_A^*$, while the responsiveness of $Pr(h|\mu \geq \mu_A^*, \sigma \geq \sigma_A^*)$ with respect to $\sigma_A^*$ can be elicited from the elasticity of the likelihood $\frac{1- F_h(\sigma_A^*)}{1-F^*(\sigma_A^*)}$ with respect to $\sigma_A^*$. In turn, the former elasticity can be elicited from $E$’s decision problem, described by (6), as a function of the elasticity of $\frac{F_h(\sigma_A^*)}{F^*(\sigma_A^*)}$ with respect to $\sigma_A^*$. It follows that the sign of $\frac{d}{dc_A} Pr(h|\mu \geq \mu_A^*, \sigma \geq \sigma_A^*)$ can be elicited by comparing the two elasticities as in the text of the proposition.

An inspection of the results of propositions 2 and 5 reveals that the effects of an increase in the submission fee to the upper tier of a tiered system of evaluation on the expected qualities of projects implemented by both tiers are qualitatively similar to the effect of an increase in the submission fee in the case with one tier of evaluation. In particular, because the increase in $c_A$ leads to unambiguous decreases in both $\sigma_A^*$ and $\sigma_B^*$, the effect of an increase in $c_A$ on the quality of projects implemented by the two tiers can be elicited solely by investigating the elasticities of the two likelihoods at the equilibrium values of $\sigma_A^*$ and $\sigma_B^*$.

Intuitively, the effect of the increase in the submission fee on the expected quality of projects implemented in a certain tier depends on the corresponding effect on the likelihoods of $h$ as inferred from the facts that (i) the agent submitted a project, and (ii) the evaluator accepted it. A higher submission fee to the upper tier (or to the single tier, in the case of a system of evaluation with one tier) increases the former likelihood, but decreases the latter. The key fact is then that the effect on the former likelihood can be elicited from the evaluator’s decision problem, as a function of the elasticity of the likelihood of $h$ as inferred from the evaluator’s minimum quality standard (this is because this minimum standard is determined in the evaluator’s problem precisely by the expected quality of projects that are submitted). Therefore, the desired effect can be elicited from
the evaluator’s information structure in the neighborhood of a particular equilibrium.

The next proposition presents the effect of an increase in $c_B$. Its proof shares the same steps as the proof of proposition 5 up to equation (11) and is thus omitted.

**Proposition 6** Assume that the equilibrium $(\sigma^*_A, \sigma^*_B, \mu^*_A, \mu^*_B)$ is unique. Then

\[
(i) \quad \frac{d}{dc_B} \Pr(h|\mu \geq \mu^*_A, \sigma \geq \sigma^*_A) > 0 \quad \text{if and only if} \quad \frac{d}{dc_B} \ln \left(\frac{1-F(h|\sigma^*_A)}{1-F(h|\sigma^*_A)}\right) \quad \frac{d\sigma^*_A}{dc_B} > 0 \\
(ii) \quad \frac{d}{dc_B} \Pr(h|\mu^*_B \geq \mu \geq \mu^*_B, \sigma \geq \sigma^*_B) > 0 \quad \text{if and only if} \quad \frac{d}{dc_B} \ln \left(\frac{1-F(h|\sigma^*_B)}{1-F(h|\sigma^*_B)}\right) \quad \frac{d\sigma^*_B}{dc_B} > 0.
\]

Note that by proposition 4, the endogenous condition $\frac{d\sigma^*_B}{dc_B} < 0$ occurs when the increase in $c_B$ leads to an increase in the quality of projects submitted to tier $t \in \{A,B\}$ (due to $\mathcal{A}$ shifting some high-signal marginal projects from tier $B$ to tier $A$). Thus, to elicit the effect of an increase in $c_B$ on the quality of projects implemented by the two tiers, one needs to investigate the elasticities of the two likelihoods in the neighborhoods of $\sigma^*_A$ and $\sigma^*_B$, respectively, and the sign of the change in the evaluator’s strategy, as determined by the corresponding effect on the quality of projects submitted for review at each tier. In different words, since unlike the case of an increase in $c_A$, the increase in $c_B$ does not have unambiguous effects on the likelihood of $h$ as inferred from the fact that the agent submitted the project to a certain tier, the effect of a higher $c_B$ on the expected quality of projects implemented into a particular tier depends not only on the elasticities of these likelihoods, but also on the sign of the effect of the increase in $c_B$ on these likelihoods.

### 4.4 Introducing a second tier of evaluation

We close by presenting a proposition that compares the equilibrium of a game with one tier of evaluation, and the equilibrium from the game with both tiers. More precisely, we analyze the impact of introducing an additional upper or lower tier in a system of evaluation in which only one tier had existed.\(^\text{17}\) The proof of the proposition is in appendix A7.

**Proposition 7** Let $(\sigma^1_A, \mu^1_A)$ and $(\sigma^1_B, \mu^1_B)$ be the equilibria of the games with only tier of evaluation $A$ or $B$, respectively. Also, let $(\sigma^2_A, \sigma^2_B, \mu^2_A, \mu^2_B)$ be the equilibrium of the game with both

\(^{17}\)The payoff characteristics of each tier are assumed identical across games. Thus, for instance, $b_A$, $c_A$ and $L_A$ are the same in the game in which only tier $A$ exists and in the game with two tiers of evaluation, $A$ and $B$. 

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tiers. Then, $\mu_A^{2*} > \mu_A^{1*}$, $\sigma_A^{2*} < \sigma_A^{1*}$, $\mu_B^{2*} > \mu_B^{1*}$ and $\sigma_B^{2*} > \sigma_B^{1*}$.

Thus, the introduction of a lower tier $B$ in a system of evaluation in which only tier $A$ had existed induces $\mathcal{A}$ to be more selective in submitting to tier $A$, ($\mu_A^{2*} > \mu_A^{1*}$), which allows $\mathcal{E}$ to be less stringent in his standards of acceptance at that tier, ($\sigma_A^{2*} < \sigma_A^{1*}$). On the other hand, the introduction of an upper tier $A$ in a system in which only tier $B$ had existed lowers the expected quality of projects received by tier $B$, inducing $\mathcal{E}$ to become more stringent, ($\sigma_B^{2*} > \sigma_B^{1*}$). In turn, this makes $\mathcal{A}$ more selective in submitting marginal projects to tier $B$, ($\mu_B^{2*} > \mu_B^{1*}$).

These results have two policy implications. First, $\mu_A^{2*} > \mu_A^{1*}$ suggests an additional intuitive mechanism to induce more self-screening by the agent at tier $A$. Thus, by introducing a new lower benefit tier of evaluation, tier $A$ receives for review projects of higher quality (however, note that since the quality of projects submitted between the two tiers decreases in this case, this would not lower the burden on the evaluator). On the other hand, $\mu_B^{2*} > \mu_B^{1*}$ suggests that by introducing a new higher benefit tier, $\mathcal{A}$ will refrain from submitting low-quality projects to the lower tier $B$, decreasing the burden on the evaluator. Moreover, since $\sigma_B^{2*} > \sigma_B^{1*}$ it also follows that introducing tier $A$ increases the quality of projects implemented between the two tiers. These findings lend additional support for a tiered system of evaluation as an efficient mechanism of project screening.

5 Conclusion

In this paper we investigate the effect of an increase in the submission fees on the efficiency of the equilibrium outcome in a game in which the owner of a project of unknown quality faces a tiered system of evaluation to which he can submit his project for review. By considering a setup in which evaluation is imperfect, and thus the evaluator is a strategic player who adjusts his strategy in response to changes in the fundamentals of the model, we argue that increases in these fees are not always beneficial either for the expected quality of projects that are submitted for review or for the expected quality of projects that are implemented. These findings suggest that in those situations where it is likely the agent’s decision of whether or not to submit a project is informative to the evaluator of the project’s quality, one may need to account for the evaluator’s response to a change in the expected quality of projects submitted for review when examining the optimal submission fee.
Appendix

Appendix A1.

First, for \(E\)'s beliefs, by Bayes' Rule we have

\[
\Pr(h|\{s_t\}, \sigma) = \frac{j(\{s_t\}, \sigma|h) \Pr(h)}{j(\{s_t\}, \sigma|h) \Pr(h) + j(\{s_t\}, \sigma|l) \Pr(l)}
\]

where \(j(\cdot|\cdot)\) denotes the conditional probability density function of the relevant continuous random variable. Since \(A\)'s action and the signal \(\sigma\) are conditionally independent, it follows that

\[
j(\{s_t\}, \sigma|q) = \Pr(\{s_t\}|q) f^q(\sigma),
\]

and thus that

\[
\Pr(h|\{s_t\}, \sigma) = \frac{\Pr(\{s_t\}|h) f^h(\sigma) \pi}{\Pr(\{s_t\}|h) f^h(\sigma) \pi + \Pr(\{s_t\}|l) f^l(\sigma) (1-\pi)} = \frac{\Pr(\{s_t\}|h) f^h(\sigma) \pi}{\Pr(\{s_t\}|h) f^h(\sigma) \pi + \Pr(\{s_t\}|l)}
\]

(12)

Since, the last term is increasing in \(f^h(\sigma)\), the fact that \(\frac{d}{d\sigma} \left[ \frac{f^h(\sigma)}{f^l(\sigma)} \right] > 0\) implies \(\frac{d}{d\sigma} \Pr(h|\{s_t\}, \sigma) > 0\).

Thus, given (1), it follows that for any \(S^{ag}\), \(E\) responds with a cutoff strategy by accepting a project submitted to tier \(t\) if and only if \(\sigma \geq \overline{\sigma}_t(S^{ag})\), with \(\overline{\sigma}_t(S^{ag}) \in [0, 1]\). Thus, in any equilibrium, the evaluator uses a cutoff strategy.

On the other hand, for \(A\)'s belief we have

\[
\Pr(\{a_t\}|\mu) = \Pr(\{a_t\}|\mu, h) \Pr(h|\mu) + \Pr(\{a_t\}|\mu, l) \Pr(l|\mu) = \Pr(\{a_t\}|h) \Pr(h|\mu) + \Pr(\{a_t\}|l) \Pr(l|\mu) = [\Pr(\{a_t\}|h) - \Pr(\{a_t\}|l)] \Pr(h|\mu) + \Pr(\{a_t\}|l)
\]

(13)

where for the second equality we used the fact that \(\mu\) is redundant for \(A\)'s inference about \(E\)'s action when conditioning on the quality of the project. Since in any equilibrium, the evaluator uses a cutoff strategy, we have \(\{a_t\} = \{\sigma : \sigma \geq \sigma_{ts}\}\), and thus \(\Pr(\{a_t\}|h) - \Pr(\{a_t\}|l) = \Pr(\sigma \geq \sigma_{ts}|h) - \Pr(\sigma \geq \sigma_{ts}|l) = F^l(\sigma_{ts}) - F^h(\sigma_{ts})\). The monotone likelihood ratio property implies first order stochastic dominance, and thus \(F^l(\sigma_{ts}) - F^h(\sigma_{ts}) > 0\). On the other hand, by Bayes' Rule
we have
\[
\Pr(h|\mu) = \frac{g^h(\mu)\pi}{g^h(\mu)\pi + g^f(\mu)(1-\pi)} = \frac{\frac{g^h(\mu)}{g^f(\mu)}\pi}{\frac{g^h(\mu)}{g^f(\mu)}\pi + 1}
\]
which is increasing in \(\frac{g^h(\mu)}{g^f(\mu)}\), and thus increasing in \(\mu\) since \(\frac{d}{d\mu} \left[ \frac{g^h(\mu)}{g^f(\mu)} \right] > 0\). Thus, \(\frac{d}{d\mu} \Pr(\{a_l\}|\mu) > 0\).

Now, \(b_A \Pr(\{a_A\}|\mu) - c_A \geq \max \{0, b_B \Pr(\{a_B\}|\mu) - c_B\}\) if and only if \(\Pr(\{a_A\}|\mu) \geq \frac{c_A}{b_A}\) and \(b_A \Pr(\{a_A\}|\mu) - b_B \Pr(\{a_B\}|\mu) \geq c_A - c_B\). Since in any equilibrium, \(\{a_l\} = \{\sigma : \sigma \geq \sigma_{ls}\}\), it follows that

\[
\frac{\partial}{\partial \mu} \left[ b_A \Pr(\{a_A\}|\mu) - b_B \Pr(\{a_B\}|\mu) \right] = \frac{\partial}{\partial \mu} \left[ b_A \Pr(\{a_A\}|\mu) - b_B \Pr(\{a_B\}|\mu) \right] = \frac{\partial}{\partial \mu} \left[ b_A [Pr(\sigma \geq \sigma_{As}|h) - Pr(\sigma \geq \sigma_{As}|l)] Pr(h|\mu) + Pr(\sigma \geq \sigma_{As}|l)] Pr(l|\mu) \right] = \frac{\partial}{\partial \mu} \left[ b_A [Pr(\sigma \geq \sigma_{Bs}|h) - Pr(\sigma \geq \sigma_{Bs}|l)] Pr(h|\mu) - Pr(\sigma \geq \sigma_{Bs}|l)] Pr(l|\mu) \right] = \frac{\partial}{\partial \mu} \left[ b_A [F^l(\sigma_{As}) - F^h(\sigma_{As})] - b_B [F^l(\sigma_{Bs}) - F^h(\sigma_{Bs})] \right] = \frac{\partial}{\partial \mu} \left[ \Pr(h|\mu) \right]
\]
where for the third equality we used again \(\Pr(\{a_l\}|\mu, q) = \Pr(\{a_l\}|q)\). Since \(\frac{\partial}{\partial \mu} \Pr(h|\mu) > 0\), it follows that \(\frac{\partial}{\partial \mu} [b_A \Pr(\{a_A\}|\mu) - b_B \Pr(\{a_B\}|\mu)]\) has the same sign for all values of \(\mu\), i.e., the sign of \(b_A [F^l(\sigma_{As}) - F^h(\sigma_{As})] - b_B [F^l(\sigma_{Bs}) - F^h(\sigma_{Bs})]\).

Let \(\mu'\) be the solution to \(b_A \Pr(\{a_A\}|\mu') - b_B \Pr(\{a_B\}|\mu') = c_A - c_B\), \(\mu''\) be the solution to \(\Pr(\{a_A\}|\mu'') = \frac{c_A}{b_A}\) and \(\mu'''\) be the solution to \(\Pr(\{a_B\}|\mu''') = \frac{c_B}{b_B}\), and assume for the time being that all these solutions are interior in \([0, 1]\). We have two cases to consider. (i) Assume \(b_A [F^l(\sigma_{As}) - F^h(\sigma_{As})] - b_B [F^l(\sigma_{Bs}) - F^h(\sigma_{Bs})] > 0\). Then, \(A\) will submit the project to tier \(A\) for \(\mu \in [\mu', 1] \cap [\mu'', 1]\), to tier \(B\) for \(\mu \in [0, \mu'] \cap [\mu'', 1]\), and will not submit the project for the rest of the values of \(\mu\). (ii) Assume \(b_A [F^l(\sigma_{As}) - F^h(\sigma_{As})] - b_B [F^l(\sigma_{Bs}) - F^h(\sigma_{Bs})] < 0\). Then, \(A\) will submit the project to tier \(B\) for \(\mu \in [\mu', 1] \cap [\mu'', 1]\), to tier \(A\) for \(\mu \in [0, \mu'] \cap [\mu'', 1]\), and will not submit the project for the rest of the values of \(\mu\). In either case, the set of values of \(\mu\) for which \(A\) submits the project to each evaluator is connected. The analysis for the cases when the solutions...
to the equations that define \( \mu', \mu'' \) and \( \mu''' \) are not interior is similar and leads to the same salient conclusions. For instance, if \( b_A \Pr(\{a_A\}|\mu') - b_B \Pr(\{a_B\}|\mu') > c_A - c_B \) for all \( \mu \in [0, 1] \), but \( \mu'' \) is interior, then \( A \) will submit the project to tier \( A \) for \( \mu \in [\mu'', 1] \) and will not submit the project for the rest of the values of \( \mu \). ■

Appendix A2. Proof of Lemma 3.1

We show that if \( \sigma_{Bs} \geq \sigma_{As} \), then condition (3) is never satisfied, i.e., that \( b_B \Pr(\sigma \geq \sigma_{Bs}|\mu) - c_B < \max \{0, b_A \Pr(\sigma \geq \sigma_{As}|\mu) - c_A\} \). To this end, since \( \sigma_{Bs} \geq \sigma_{As} \) implies \( \Pr(\sigma \geq \sigma_{As}|\mu) \geq \Pr(\sigma \geq \sigma_{Bs}|\mu) \), it is enough to show that

\[
b_B \Pr(\sigma \geq \sigma_{As}|\mu) - c_B < \max \{0, b_A \Pr(\sigma \geq \sigma_{As}|\mu) - c_A\}
\]

To this aim, we will argue that whenever \( b_B \Pr(\sigma \geq \sigma_{As}|\mu) - c_B \geq b_A \Pr(\sigma \geq \sigma_{As}|\mu) - c_A \), it must be that \( b_B \Pr(\sigma \geq \sigma_{As}|\mu) - c_B < 0 \), which will complete the argument. Since \( b_A > b_B \), by assumption 1(i), this is equivalent to showing that

\[
\Pr(\sigma \geq \sigma_{As}|\mu) \leq \frac{c_A - c_B}{b_A - b_B} \implies \Pr(\sigma \geq \sigma_{As}|\mu) < \frac{c_B}{b_B}
\]

To show this implication, it is enough to show that \( \frac{c_A - c_B}{b_A - b_B} < \frac{c_B}{b_B} \). Rearranging this last condition, we conclude that it is satisfied whenever assumption 1(ii) is satisfied, so the proof of the lemma is complete. ■

Appendix A3.

From (2), it is straightforward to see that this happens if and only if

\[
\frac{\partial}{\partial \mu} \left[ b_A \Pr(\sigma \geq \sigma_{As}|\mu) - b_B \Pr(\sigma \geq \sigma_{Bs}|\mu) \right] > 0
\]

Lemma 5.1 provides conditions under which this is satisfied.

**Lemma 5.1** Let \( \bar{\sigma}_1 \) be the solution to \( \frac{f'(\bar{\sigma}_1)}{f''(\bar{\sigma}_1)} = 1 \), and let \( \bar{\sigma}_2 \) be the solution to \( \frac{b_A}{b_B} = \frac{F'(\bar{\sigma}_1) - F'(\bar{\sigma}_2)}{F'(\bar{\sigma}_2) - F''(\bar{\sigma}_2)} \) on \([\bar{\sigma}_1, 1]\). (i) If \( \sigma_{As} \leq \bar{\sigma}_2 \), then \( \frac{\partial}{\partial \mu} \left[ b_A \Pr(\sigma \geq \sigma_{As}|\mu) - b_B \Pr(\sigma \geq \sigma_{Bs}|\mu) \right] > 0 \) for any \( \sigma_{Bs} < \sigma_{As} \).
(ii) If \( \sigma_{A_s} > \bar{\sigma}_2 \), then there exists a neighborhood \( N_{\sigma_{A_s}} \) of \( \bar{\sigma}_1 \), such that when \( \sigma_{B_s} \in N_{\sigma_{A_s}} \), we have
\[
\frac{\partial}{\partial \mu} \left[ b_A \Pr(\sigma \geq \sigma_{A_s}|\mu) - b_B \Pr(\sigma \geq \sigma_{B_s}|\mu) \right] < 0.
\]

Proof. From (15), we have that when evaluators employ cutoff strategies \( \sigma_{A_s} \) and \( \sigma_{B_s} \),
\[
\frac{\partial}{\partial \mu} \left[ b_A \Pr(\sigma \geq \sigma_{A_s}|\mu) - b_B \Pr(\sigma \geq \sigma_{B_s}|\mu) \right] = \\
= \left\{ b_A \left[ F^l(\sigma_{A_s}) - F^h(\sigma_{A_s}) \right] - b_B \left[ F^l(\sigma_{B_s}) - F^h(\sigma_{B_s}) \right] \right\} \frac{\partial}{\partial \mu} \left[ \Pr(h|\mu) \right]
\]
with \( \frac{\partial}{\partial \mu} [\Pr(h|\mu)] > 0 \).

Note that \( \frac{d}{d\sigma} [F^l(\sigma) - F^h(\sigma)] = 0 \iff f^l(\sigma) = f^h(\sigma) = 0 \iff \sigma = \bar{\sigma}_1 \). Moreover, we have
\[
\frac{d}{d\sigma} [F^l(\sigma) - F^h(\sigma)] > 0 \iff \frac{f^l(\sigma)}{f^h(\sigma)} > 1
\]
Therefore, since \( \frac{f^l(\bar{\sigma}_2)}{f^l(\bar{\sigma}_1)} = 1 \) (by the definition of \( \bar{\sigma}_1 \)) and \( \frac{d}{d\sigma} [F^l(\sigma) - F^h(\sigma)] < 0 \) (from assumption 2(ii)), we have that
\[
\frac{d}{d\sigma} [F^l(\sigma) - F^h(\sigma)] > 0 \iff \sigma < \bar{\sigma}_1 \tag{16}
\]

Now, if \( \sigma_{A_s} < \bar{\sigma}_2 \), where, by its definition, \( \bar{\sigma}_2 \) is the solution to \( \frac{b_A}{b_B} = \frac{F^l(\bar{\sigma}_1) - F^h(\bar{\sigma}_1)}{F^l(\bar{\sigma}_2) - F^h(\bar{\sigma}_2)} \), then
\[
b_A \left[ F^l(\sigma_{A_s}) - F^h(\sigma_{A_s}) \right] - b_B \left[ F^l(\sigma_{B_s}) - F^h(\sigma_{B_s}) \right] > 0 \text{ for any } \sigma_{B_s} < \sigma_{A_s}.
\]
To see this, assume first that \( \sigma_{A_s} > \bar{\sigma}_1 \). Then, since \( \bar{\sigma}_2 > \sigma_{A_s} > \bar{\sigma}_1 \), by (16) \( F^l(\sigma_{A_s}) - F^h(\sigma_{A_s}) \geq F^l(\bar{\sigma}_1) - F^h(\bar{\sigma}_1) \)

Thus, \( b_A \left[ F^l(\sigma_{A_s}) - F^h(\sigma_{A_s}) \right] - b_B \left[ F^l(\bar{\sigma}_1) - F^h(\bar{\sigma}_1) \right] > 0 \). Since \( F^l(\sigma_{B_s}) - F^h(\sigma_{B_s}) \leq F^l(\bar{\sigma}_1) - F^h(\bar{\sigma}_1) \) by the definition of \( \bar{\sigma}_1 \), it follows that indeed \( b_A \left[ F^l(\sigma_{A_s}) - F^h(\sigma_{A_s}) \right] - b_B \left[ F^l(\sigma_{B_s}) - F^h(\sigma_{B_s}) \right] > 0 \) for any \( \sigma_{B_s} \). On the other hand, if \( \sigma_{A_s} < \bar{\sigma}_1 \) then \( F^l(\sigma_{A_s}) - F^h(\sigma_{A_s}) > F^l(\sigma_{B_s}) - F^h(\sigma_{B_s}) \) by (16) and the fact that \( \sigma_{A_s} > \sigma_{B_s} \). Since \( b_A > b_B \), it follows again that \( b_A \left[ F^l(\sigma_{A_s}) - F^h(\sigma_{A_s}) \right] - b_B \left[ F^l(\sigma_{B_s}) - F^h(\sigma_{B_s}) \right] > 0 \) for any \( \sigma_{B_s} < \sigma_{A_s} \).

On the other hand, if \( \sigma_{A_s} > \bar{\sigma}_2 \) and \( \sigma_{B_s} \) is sufficiently close to \( \bar{\sigma}_1 \), then \( b_A \left[ F^l(\sigma_{A_s}) - F^h(\sigma_{A_s}) \right] - b_B \left[ F^l(\sigma_{B_s}) - F^h(\sigma_{B_s}) \right] < 0 \).

To understand the lemma, note first that \( \bar{\sigma}_1 \) is the point at which the difference \( F^l(\cdot) - F^h(\cdot) \) is maximized, whereas when \( \sigma_{A_s} \) is sufficiently high, the difference \( F^l(\sigma_{A_s}) - F^h(\sigma_{A_s}) \) is small. Therefore, when \( \sigma_{A_s} \) is high and \( \sigma_{B_s} \) is close to \( \bar{\sigma}_1 \), the probability that a high-quality project is identified as such from the evaluator’s signal is higher when submitting it to tier B. The likelihood
of a high-quality project is increasing in the signal $\mu$. Thus, given the low probability of acceptance at tier $A$, when $A$ has a higher signal, he is more likely to submit the project to tier $B$ in order to have it identified as being of high quality and accepted. On the other hand, if $c_A$ is small enough, the expected payoff from submitting the project to tier $A$ may be positive even when $\mu$ is small and $\sigma_{As}$ is high. Therefore, when $A$ has a low signal he prefers submitting the project to tier $A$ rather than not submitting it at all. On the other hand, $A$’s strategy of submitting to tier $B$ for high signals and to tier $A$ for lower signals, $E$’s best response is precisely to adopt a high $\sigma_{As}$ and a moderate $\sigma_{Bs}$. Therefore, negative assortative matching may occur in equilibrium.

**Appendix A4. Proof of Lemma 3.2**

Employing (14) in (13), it follows that

$$Pr(\{a_t\}|\mu) = \frac{g^h(\mu)}{g^l(\mu)} \frac{\pi}{1-\pi} \left[ 1 - F^h(\sigma_{ts}) \right] + \frac{1}{\frac{g^h(\mu)}{g^l(\mu)} \frac{\pi}{1-\pi} + 1} \left[ 1 - F^l(\sigma_{ts}) \right]$$  \hspace{1cm} (17)

From (2), under the equilibrium regularity that we assume throughout, we have then that given $\sigma_{As}$ and $\sigma_{Bs}$, $A$ submits a project to tier $A$ if and only if

$$b_A Pr(\{a_A\}|\mu) - c_A \geq b_B Pr(\{a_B\}|\mu) - c_B \iff$$

$$b_A \left\{ \frac{g^h(\mu)}{g^l(\mu)} \frac{\pi}{1-\pi} \left[ 1 - F^h(\sigma_{As}) \right] + \left[ 1 - F^l(\sigma_{As}) \right] \right\} - b_B \left\{ \frac{g^h(\mu)}{g^l(\mu)} \frac{\pi}{1-\pi} \left[ 1 - F^h(\sigma_{Bs}) \right] + \left[ 1 - F^l(\sigma_{Bs}) \right] \right\} \geq$$

$$\geq (c_A - c_B) \left[ \frac{g^h(\mu)}{g^l(\mu)} \frac{\pi}{1-\pi} + 1 \right] \iff$$

$$\frac{g^h(\mu)}{g^l(\mu)} \frac{\pi}{1-\pi} \left\{ b_A \left[ 1 - F^h(\sigma_{As}) \right] - b_B \left[ 1 - F^h(\sigma_{Bs}) \right] \right\} \geq$$

$$\geq b_B \left[ 1 - F^l(\sigma_{Bs}) \right] - b_A \left[ 1 - F^l(\sigma_{As}) \right] + c_A - c_B$$

The last inequality implies that $A$ employs a cutoff $\mu_{As}$ defined by (4) provided that $b_A \left[ 1 - F^h(\sigma_{As}) \right] - b_B \left[ 1 - F^h(\sigma_{Bs}) \right] + c_B - c_A > 0$ and $b_B \left[ 1 - F^l(\sigma_{Bs}) \right] - b_A \left[ 1 - F^l(\sigma_{As}) \right] + c_A - c_B$. These conditions are not satisfied generically, but they are necessary conditions for the regular equilibrium under consideration. To see this, note first that from the argument in appendix A1, a necessary and sufficient condition for positive assortative matching is that $b_A \left[ F^l(\sigma_{As}) - F^h(\sigma_{As}) \right] -
by straightforward computations using (17), implies to tier

Thus, given the cutoffs

Now, we have three cases to consider. (i) If \( c_A - c_B < b_A [1 - F^h(\sigma_{As})] - b_B [1 - F^h(\sigma_{Bs})] \), then it immediately follows that \( b_B [1 - F^l(\sigma_{Bs})] - b_A [1 - F^l(\sigma_{As})] + c_A - c_B < 0 \), but also that \( b_A [1 - F^h(\sigma_{As})] - b_B [1 - F^h(\sigma_{Bs})] + c_B - c_A > 0 \) by using (18). So \( \mathcal{A} \) will never submit a project to tier \( B \), which is something that we precluded by the regularity assumption. (ii) If \( c_A - c_B > b_A [1 - F^h(\sigma_{As})] - b_B [1 - F^h(\sigma_{Bs})] \), then it immediately follows that \( b_A [1 - F^h(\sigma_{As})] - b_B [1 - F^h(\sigma_{Bs})] + c_B - c_A < 0 \), but also that \( b_B [1 - F^l(\sigma_{Bs})] - b_A [1 - F^l(\sigma_{As})] + c_A - c_B > 0 \) by using (18). So \( \mathcal{A} \) will never submit a project to tier \( A \), which is again something that is precluded by the regularity assumption. (iii) Finally, the case when \( b_A [1 - F^h(\sigma_{As})] - b_B [1 - F^h(\sigma_{Bs})] > c_A - c_B > b_A [1 - F^l(\sigma_{As})] - b_B [1 - F^l(\sigma_{Bs})] \) corresponds to the case where \( \mathcal{A} \) submits a project to tier \( A \) if and only if (4) is satisfied.

When (4) is not satisfied, \( \mathcal{A} \) submits a project to tier \( B \) if and only if \( b_B \Pr(\{a_B\}) - c_B \), which by straightforward computations using (17), implies \( \mathcal{A} \) employs a cutoff \( \mu_{Bs} \) defined by (5).

**Appendix A5. Proof of Lemma 3.3**

For the evaluator, from (1) and (12) it follows that \( \mathcal{E} \) will accept a project submitted to tier \( t \) if and only if

\[
\frac{\Pr(\{s_t\}|h) f^h(\sigma) \frac{\pi}{f^l(\sigma)} 1 - \pi + \Pr(\{s_t\}|l)}{1 + L_t} \geq L_t \iff \frac{f^h(\sigma) \frac{\pi}{f^l(\sigma)} 1 - \pi \Pr(\{s_t\}|h)}{1 - \pi \Pr(\{s_t\}|l)} \geq L_t
\]

Thus, given the cutoffs \( \mu_{As} \) and \( \mu_{Bs} \) employed by \( \mathcal{A} \), \( \mathcal{E} \) will accept a project submitted to tier \( A \) with quality signal \( \sigma \) if and only if

\[
\frac{f^h(\sigma) \frac{\pi}{f^l(\sigma)} 1 - G^h(\mu_{As})}{1 - \pi \left( 1 - G^l(\mu_{As}) \right)} \geq L_A
\]

and \( \mathcal{E} \) will accept a project submitted to tier \( B \) with quality signal \( \sigma \) if and only if

\[
\frac{f^h(\sigma) \frac{\pi}{f^l(\sigma)} G^h(\mu_{As}) - G^h(\mu_{Bs})}{1 - \pi \left( G^l(\mu_{As}) - G^l(\mu_{Bs}) \right)} \geq L_B
\]
Therefore, indeed, the two evaluators employ cutoff strategies with cutoffs defined by (6) and (7). This completes the proof of the lemma. □

Appendix A6. Proof of Lemma 3.4

Parts (i) and (ii) are immediate. The proof of part (iii) is identical to the corresponding proof from the case of a unique evaluator presented in Barbos (2012). For part (iv), since \( \frac{f^h(\sigma)}{f^l(\sigma)} \) is increasing in \( \sigma \), from (7) it follows that \( \bar{\sigma}_B(\cdot) \) is decreasing in \( \mu_{As} \) if and only if

\[
\frac{d}{d\mu_{As}} \left( \frac{G^l(\mu_{As}) - G^l(\mu_{Bs})}{G^h(\mu_{As}) - G^h(\mu_{Bs})} \right) < 0 \iff \frac{g^h(\mu_{As})}{g^l(\mu_{As})} > \frac{G^h(\mu_{As}) - G^h(\mu_{Bs})}{G^l(\mu_{As}) - G^l(\mu_{Bs})} \tag{19}
\]

where we used the fact that \( \mu_{As} > \mu_{Bs} \). Similarly, \( \bar{\sigma}_B(\cdot) \) is decreasing in \( \mu_{Bs} \) if and only if

\[
\frac{d}{d\mu_{Bs}} \left( \frac{G^l(\mu_{As}) - G^l(\mu_{Bs})}{G^h(\mu_{As}) - G^h(\mu_{Bs})} \right) < 0 \iff \frac{G^h(\mu_{As}) - G^h(\mu_{Bs})}{G^l(\mu_{As}) - G^l(\mu_{Bs})} > \frac{g^h(\mu_{Bs})}{g^l(\mu_{Bs})} \tag{20}
\]

We will show that (19) and (20) are satisfied under assumption 2(ii). Since \( \frac{g^h(u)}{g^l(u)} \) is increasing in \( u \), it follows that for \( u \in [\mu_{Bs}, \mu_{As}] \), we have \( \frac{g^h(\mu_{As})}{g^l(\mu_{As})} > \frac{g^h(u)}{g^l(u)} \), and thus \( g^l(\mu_{As}) g^l(u) > g^l(\mu_{As}) g^h(u) \). Integrating this last inequality with respect to \( u \) between \( \mu_{Bs} \) and \( \mu_{As} \), we obtain

\[
g^h(\mu_{As}) \left[ G^l(\mu_{As}) - G^l(\mu_{Bs}) \right] > g^l(\mu_{As}) \left[ G^h(\mu_{As}) - G^h(\mu_{Bs}) \right]
\]

which immediately then implies (19). On the other hand, \( \frac{g^h(\mu_{Bs})}{g^l(\mu_{Bs})} < \frac{g^h(u)}{g^l(u)} \) for \( u \in (\mu_{Bs}, \mu_{As}] \), implies \( g^h(\mu_{Bs}) g^l(u) < g^l(\mu_{Bs}) g^h(u) \), which integrated with respect to \( u \) between \( \mu_{Bs} \) and \( \mu_{As} \), implies

\[
g^h(\mu_{Bs}) \left[ G^l(\mu_{As}) - G^l(\mu_{Bs}) \right] < g^l(\mu_{Bs}) \left[ G^h(\mu_{As}) - G^h(\mu_{Bs}) \right]
\]

which implies (20). This completes the proof of the lemma. □
Appendix A7. Proof of Proposition 7

From Barbos (2012), with only one tier of evaluation, the equilibrium is given by

\[
\frac{\pi}{1-\pi} \frac{g^h(\mu^*_1)}{g^l(\mu^*_1)} = \frac{c_l-b_l}{b_l} \frac{1-F^l(\sigma^*_1)}{1-F^h(\sigma^*_1)} - c_l \quad \text{(21)}
\]

\[
\frac{\pi}{1-\pi} \frac{f^h(\sigma^*_1)}{f^l(\sigma^*_1)} 1-G^h(\mu^*_1) 1-G^l(\mu^*_1) = L_t \quad \text{(22)}
\]

where \((\mu^*_1, \sigma^*_1)\) denotes the equilibrium strategies of the game in which \(\mathcal{E}\) only offers tier \(t \in \{A,B\}\).

Consider first the case when the initial tier is A, and then tier B is introduced. Assume by contradiction that \(\mu^*_A > \mu^*_A\). Then

\[
\frac{\pi}{1-\pi} \frac{f^h(\sigma^*_1)}{f^l(\sigma^*_1)} 1-G^h(\mu^*_1) 1-G^l(\mu^*_1) = L_A \quad \text{where the first equality follows from (22) with } t = A, \text{the second equality follows from (6), and the inequality from } \mu^*_A \leq \mu^*_A \text{ and the fact that } \frac{d}{d\mu} \left[\frac{1-G^h(\mu)}{1-G^l(\mu)}\right] > 0.\]

Thus, \(\frac{f^h(\sigma^*_1)}{f^l(\sigma^*_1)} \leq \frac{f^h(\sigma^*_2)}{f^l(\sigma^*_2)}\), so by assumption 2(ii), we have that \(\sigma^*_A > \sigma^*_A\). Therefore, from (21),

\[
\frac{\pi}{1-\pi} \frac{g^h(\mu^*_1)}{g^l(\mu^*_1)} = \frac{c_A-b_A}{b_A 1-F^h(\sigma^*_1) - c_A} \leq \frac{c_A-b_A}{b_A 1-F^h(\sigma^*_2) - c_A}.
\]

On the other hand, from (4), we have

\[
\frac{\pi}{1-\pi} \frac{g^h(\mu^*_2)}{g^l(\mu^*_2)} = \frac{b_B 1-F^l(\mu^*_2)}{b_B 1-F^h(\sigma^*_2) + c_B - c_A} \leq \frac{b_B 1-F^l(\mu^*_1)}{b_B 1-F^h(\sigma^*_1) + c_B - c_A}. \quad \text{We show next that } \frac{\pi}{1-\pi} \frac{g^h(\mu^*_2)}{g^l(\mu^*_2)} > \frac{c_A-b_A}{b_A 1-F^h(\sigma^*_2) - c_A}, \text{ which would then imply that } \frac{g^h(\mu^*_2)}{g^l(\mu^*_2)} > \frac{g^h(\mu^*_1)}{g^l(\mu^*_1)}, \text{ and thus that } \mu^*_A > \mu^*_A \text{ contradicting the initial assumption. Thus, note that}
\]

\[
\frac{b_B}{b_A} \left[1-F^l(\sigma^*_2)\right] - \frac{b_B}{b_A} \left[1-F^h(\sigma^*_2)\right] - \frac{b_B}{b_A} \left[1-F^l(\sigma^*_1)\right] + c_B - c_A \geq \frac{c_A-b_A}{b_A 1-F^h(\sigma^*_2) - c_A}, \quad \frac{b_B}{b_A} \left[1-F^h(\sigma^*_2)\right] - \frac{b_B}{b_A} \left[1-F^l(\sigma^*_2)\right] - \frac{b_B}{b_A} \left[1-F^h(\sigma^*_2)\right] + c_B - c_A \geq \frac{c_A-b_A}{b_A 1-F^h(\sigma^*_2) - c_A}.
\]

But this last inequality follows from the fact that \(\mu^*_A > \mu^*_B\) implies from (4) and (5) that

\[
\frac{b_B}{b_A} \left[1-F^l(\sigma^*_2)\right] - \frac{b_B}{b_A} \left[1-F^h(\sigma^*_2)\right] - \frac{b_B}{b_A} \left[1-F^l(\sigma^*_1)\right] + c_B - c_A > \frac{c_A-b_A}{b_A 1-F^h(\sigma^*_2) - c_A}, \quad \frac{b_B}{b_A} \left[1-F^h(\sigma^*_2)\right] - \frac{b_B}{b_A} \left[1-F^l(\sigma^*_2)\right] - \frac{b_B}{b_A} \left[1-F^h(\sigma^*_2)\right] + c_B - c_A > \frac{c_A-b_A}{b_A 1-F^h(\sigma^*_2) - c_A}.
\]

Therefore, indeed \(\mu^*_A > \mu^*_A\), which then from (22) and (6) immediately also implies that \(\sigma^*_A > \sigma^*_A\).

For the second part of the proof, consider the case when the initial tier is B and then tier
A is introduced, and assume by contradiction that $\sigma_B^{2*} \leq \sigma_B^{1*}$. From (21) and (5), this implies that $\mu_B^{2*} \leq \mu_B^{1*}$. Therefore, from (22) it follows that

$$\frac{f_b(\sigma_B^{1*})}{f_l(\sigma_B^{1*})} = L_B \frac{1-G^e(\mu_B^{1*})}{1-G^h(\mu_B^{1*})} \leq L_B \frac{1-G^e(\mu_B^{2*})}{1-G^h(\mu_B^{2*})}.$$ 

Therefore, to complete the contradiction argument, it would be enough to show that $\frac{f_b(\sigma_B^{2*})}{f_l(\sigma_B^{2*})} > L_B \frac{1-G^e(\mu_B^{2*})}{1-G^h(\mu_B^{2*})}$, because this would immediately imply $\sigma_B^{2*} > \sigma_B^{1*}$. But from (7) we have

$$\frac{f_b(\sigma_B^{2*})}{f_l(\sigma_B^{2*})} = L_B \frac{G^e(\mu_B^{2*})-G^e(\mu_B^{1*})}{G^h(\mu_B^{2*})-G^h(\mu_B^{1*})},$$

so it suffices to show that

$$\frac{G^e(\mu_B^{2*})-G^e(\mu_B^{1*})}{G^h(\mu_B^{2*})-G^h(\mu_B^{1*})} > \frac{1-G^e(\mu_B^{2*})}{1-G^h(\mu_B^{2*})}.$$

This is true from (19). Therefore, indeed $\sigma_B^{2*} > \sigma_B^{1*}$, which from (21) and (5) also implies that $\mu_B^{2*} > \mu_B^{1*}$. This completes the proof of the proposition. ■

References


