Unions’ Coordination and the Central Banker’s behavior in a Monetary Union

Borda, Patrie and Gaumont, Damien and Manioc, Olivier

Université Des Antilles et de la Guyane, CREDDI-LEAD, Université Paris 2, Université Des Antilles et de la Guyane, CREDDI-LEAD

6 March 2011
Unions’ Coordination and the Central Banker’s behavior in a Monetary Union

Patrice Borda
Université des Antilles et de la Guyane.

Damien Gaumont
Université Panthéon Assas et ERMES, (Paris 2), 12 Place du Panthéon, 75231 Paris Cedex, France.

Olivier Manioc
Université des Antilles et de la Guyane.

March 6, 2011

Abstract

In a 2-country monetary union, this paper studies a Stackelberg game between the Central Banker and two symmetrical countries. The central banker chooses the money supply. In each country, there is a union who acts as a monopoly of labor supply. Firms are wage and price takers. We analyze the effects of internationally coordinated unions versus internationally uncoordinated unions. It is shown that wages are lower when unions are internationally coordinated and the money policy is more accommodating. This result is linked to the degree of conservatism of the Central Banker with respect to inflation.1

1Suggestions and remarks from Christopher Crowe, Shelton Nicholls and Delisle Worrell were very useful when writing this paper. All remaining errors are ours.

JEL Classification: E24, E5, J51, P45.

Key Words: Monetary Union, International Union Coordination, Employment and Wage-Setting
1 Introduction

In a 2-country monetary union where the Central Banker chooses the money supply, we analyze the impact of his choice on employment when unions are internationally coordinated or internationally uncoordinated.

This is an important question to study because national unions are trying to organize a European unions' coordination, but until now, they still do not have any effective bargaining power at the monetary union level. The internationalization of all European Economic Union unions (EEU unions) has not been fully accomplished yet. For example, there haven't been any European strikes so far. The objective of this paper is to analyze what could happen if unions became successful in their internationalization process and got some effective collective bargaining power. This is not a trivial question, especially in a monetary union, such as the EEU, where monetary policy choices are imposed on all members of the union, whatever their specific economic constraints are, Canzoneri and Henderson (1991) and Jensen (1993) and Agiomirgianakis (1998). Countries differ in many aspects, but the monetary policy is unique for each one. Countries' own level of unemployment differ, but in general European unemployment is high and may be linked to monetary decisions.

From a theoretical point of view, in our paper, countries trade national goods for foreign goods so that they differ in their consumer price index. In an international environment, this is sufficient to generate country differences in both prices and wages, consequently unemployment rates differ across countries. Depending on his degree of conservatism in term of inflation, the Central Banker chooses the quantity of money in the monetary union, whereas trade union — as a monopoly of labor supply — imposes on firm a workers’ welfare-maximizing wage. Different wages across countries involve different unemployment rates across countries. Since wages affect price level, and prices affect the monetary mass, the optimal choice of the Central Banker depends on wheather unions are coordinated or not at the international level. To our knowledge, this question has not been addressed.

The literature on the impact of monetary union on the labour market is characterized by unions’ wage-setting at the national level. At the national level, Akhand (1992) demonstrated that trade unions coordination solves partially the inflationary bias problem. In the EMU literature, Soskice and Iversen (1998) pay attention to the interaction between central bank monetary rules and systems of national collective wage bargaining. They analyze coordinated wage-bargaining systems at the national level since wage determination is dominated by collective bargaining in all the EMU member states. They discuss the possibility of government union bargains. Cukierman and Lippi (1999) extend Gruner and Hefeker (1999) to the case of a multi-union country which switches from a national monetary policy to a union monetary policy. Considering decentralized multi-union wage-setting at national level leads to changes in real variables even if unions are not inflation averse.

One notable difference from these authors is that in our framework we do not analyze the switch from a national to a monetary union, because this has already been done in 2002 for EEU. We only consider one union in each of the two coun-
tries and study the impact of their coordination, or not, at the international level. In other words, the centralization takes place at the international level, and not at the national one, as usually done in the literature. This is new. We show that international coordination between unions in a monetary union yields to lower wages and a higher employment level compared with uncoordinated unions. Internationally coordinated unions leads to an accommodating monetary policy as long as the Central Banker is sufficiently conservative in terms of inflation.

The paper is organised as follows. Section 2 presents the model. Section 3 is devoted to the resolution of the model. Section 4 analyzes the effects of unions’ coordination. Section 5 concludes.

2 The model

We consider a competitive world of two equal-sized, interdependent and symmetrical countries under certainty. Each country produces an internationally tradable homogeneous good. In the remainder of the paper, subscript i (j) denote domestic (foreign) variables. The unique Central Banker is the leader of Stackelberg who plays against the two countries. He plays the first by determining the amount of nominal money, \( m \), which minimizes a loss function \( \Psi \). In each country, the union represents a labor supply monopoly which imposes the wage rate \( w_i \) on the firm. This wage maximizes the utility \( U \) of all members of the union. The firm maximizes its profits with respect to the labor demand, \( \ell_i \), taking the union’s wage level as given. The Stackelberg game is the following:

\[
\min_{m(w_i, \ell_i, w_j, \ell_j)} \psi(m(w_i, \ell_i, w_j, \ell_j)),
\]

\[
\left\{ \begin{array}{l}
  w_i^* = \arg\max_{w_i} U(w_i(m)), \\
  \ell_i^* = \arg\max_{\ell_i} \Pi(\ell_i(w_i(m)))
\end{array} \right. \quad \left\{ \begin{array}{l}
  w_j^* = \arg\max_{w_j} U(w_j(m)), \\
  \ell_j^* = \arg\max_{\ell_j} \Pi(\ell_j(w_j(m)))
\end{array} \right.
\]

We solve this static Stackelberg game backwards. Consequently, we first present the firms, then the unions before presenting the central bank’s behavior.

2.1 The Firms’ behavior

In each country, there is a representative firm, which is a price-taker. Capital is already fixed, so that output \( Y \) is produced with labor \( L \) according to the following production functions:

\[
Y_i = L_i^a, \quad Y_j = L_j^a,
\]

where \( 0 < \alpha < 1 \) is the elasticity of output with respect to labour. Firms are profit maximizers for a given wage \( W \) and a given price \( P \). They maximize the profit function \( \Pi = L^\alpha - WL \).

Approximating the production function in a neighborhood of any given value \( Y_0 \) and \( L_0 \), we have:

\[
\frac{Y - Y_0}{Y_0} Y_0 = \alpha L_0^{\alpha - 1} \frac{L - L_0}{L_0} L_0.
\]
Defining \( y = (Y - Y_0)/Y_0 \) and \( \ell = (L - L_0)/L_0 \), we have:

\[
yY_0 = \alpha L_0^{\alpha - 1} \ell L_0,
\]
\[
yY_0 = \alpha L_0^{\alpha} \ell.
\]

Using (1) after simplification by \( Y_0 \), we have:

\[
y_i = \alpha \ell_i, \quad \text{and} \quad y_j = \alpha \ell_j.
\]

(2)

2.2 Unions’ behavior

Two types of unions’ behavior are possible: union are uncoordinated or coordinated.

2.2.1 Uncoordinated unions and wage setting

Unions’ behavior is given by the maximization of a loss function of employment \( \ell_k, \ k = i, j \) and real wages \( \hat{w}_k, \ k = i, j \). In line with Driffill (1986), Jensen (1993), Zervoyianni (1996), Agiomirgianakis (1998) the domestic union (country \( i \)) maximizes with respect to national wages the utility of their members subject to the fact that it does not take into account the impact of his decision on foreign wage, as well as on money supply:

\[
\max_{w_k} U_k = - \left( \ell_k - \ell_{0k} \right)^2 + b\hat{w}_k(m), \quad b > 0,
\]

s.c. \( \frac{\partial m}{\partial w_{-k}} = \frac{\partial m}{\partial w_k} = 0 \),

where \( \ell_{0k} \) is the labor force of country \( k \), and index \( -k \) captures the other country. Unions choose wages taking money and foreign wage as given.

We now present the case of coordinated unions.

2.2.2 Coordinated unions and wage setting

We now turn to the coordinated trade unions. Unions maximize the joined utility of their members without taking into account the impact of their wage decision on the money supply. The maximization program is:

\[
\max_{w_i, w_j} U_i + U_j = \left[ - \left( \ell_i - \ell_{0i} \right)^2 + bw_i \right] + \left[ - \left( \ell_j - \ell_{0j} \right)^2 + bw_j \right],
\]

s.c. \( \frac{\partial m}{\partial w_i} = \frac{\partial m}{\partial w_j} = 0 \).
2.3 The Central banker’s policy

In a monetary union, the central bank maximizes a function of the joined utilities of the two countries. The loss function, which is the sum of two national loss functions, depends on the employment in the two countries as well as the consumer price index in the two countries. The Central Banker chooses the money supply in order to minimize

\[
\min_m \Psi = V_i + V_j = (\ell_i - \ell_{0i})^2 + \lambda q_i^2 + (\ell_j + \ell_{0j})^2 + \lambda q_j^2, \tag{5}
\]

where \( \lambda \) captures the relative weight of price into the loss function of the Central Bank. If the Central Bank is a vigorous inflation fighter, \( \lambda \) takes a high value. On the opposite, if \( \lambda \sim 0 \), the Central Bank pays more attention to employment than to prices.

Before solving this static Stackelberg game, we present some useful preliminary price equations which help determine the solution of the game.

2.4 Prices

As national and foreign productions are internationally tradable, they are consumed in both countries. This implies that consumer price indexes \( q_i \) and \( q_j \) depend on national and foreign prices. The relative price of foreign good measured in national good noted \( z \), is simply the difference between the production prices, where \( p_i, p_j \) are approximated prices (See Appendix A.1 for more details):

\[
z = p_j - p_i. \tag{6}
\]

Then we can derive the following relations:

\[
q_i = p_i + \beta z, \quad q_j = p_j - \beta z, \tag{7}
\]

where \( \beta \) captures the preference for foreign good in the domestic country. We assume that national consumers prefer national good, so that \( 0 < \beta < 0, 5 \). This is a usual assumption in this literature, see for example Lane (2000), Schwarz (2004), Agiomirgianakis (1998).

Real wages, which will be defined below, are increasing in nominal wages, \( w_i \) and \( w_j \), and decreasing in consumer price, \( q_i \) and \( q_j \). We have:

\[
\hat{w}_i = w_i - q_i, \quad \hat{w}_j = w_j - q_j. \tag{8}
\]

We assume that the money supply is a linear convex combination of the national and the foreign prices augmented by the national and foreign outputs. Then, the money supply is given by:

\[
m = [vp_i + (1 - v)p_j] + [y_i + y_j], \tag{9}
\]
where $v$ represents the domestic production’s share. In line with Lane (2000) and Agiomirgianakis (1998), we choose from now up to the end $\alpha = 1 - \alpha = 0, 5$. Unified market is equally shared between domestic and foreign productions. Trade balances are given by:

$$b_i = -b_j = \theta z - \beta (y_i - y_j),$$

(10)

where $\theta$ captures the share of the relative prices in the external equilibrium. External equilibrium is reached if $y_i - y_j = \delta z$:

$$y_i - y_j = \frac{\theta}{\beta} (p_i - p_j) \equiv \delta (p_i - p_j), \quad \delta > 1,$$

(11)

$\delta$ is assumed to be higher than unity because changes in money purchasing power has stronger effects on trade balance than changes in incomes.

3 Resolution of the model

The model is solved backward. We start with the solution of the firms, then the solution of the monopoly unions. Knowing the equilibrium activity in each country (commodity market equilibrium and labor market equilibrium), the Central Banker determines the amount of money for the monetary union.

3.1 Firms

Labour demand is a decreasing function in the real wage $W/P$:

$$L_i = \left[\frac{\alpha P_i}{W_i}\right]^\frac{1}{\alpha}, \quad L_j = \left[\frac{\alpha P_j}{W_j}\right]^\frac{1}{\alpha}.$$

(12)

By an appropriate approximation, see appendix (A.1), these functions can be rewritten as:

$$\ell_i = -\left[\frac{w_i - p_i}{1 - \alpha}\right], \quad \ell_j = -\left[\frac{w_j - p_j}{1 - \alpha}\right].$$

(13)

As shown in Appendix A.1, the difference between the wage increase $w_i$ and the price increase $p_i$ captures the increase in the real purchasing power of consumers. Since, by definition, $\ell_i$ represents a variation of the labor demand, equation (8) means that an increase in the real purchasing power of consumers leads to a decrease in the variation of labor demand.

3.2 Unions’ solution

This subsection is devoted to the unions’solution in both uncoordinated unions and coordinated unions.
3.2.1 Uncoordinated unions

The resolution, of (3) gives:

\[ w_i = m + \frac{\phi}{\sigma} (m - w_j) - \frac{\sigma}{\sigma} \ell_0i + \eta, \quad w_j = m + \frac{\phi}{\sigma} (m - w_i) - \frac{\sigma}{\sigma} \ell_0j + \eta, \] \tag{14}

where
\[ \sigma = \alpha + 2\delta > 0, \quad \eta = \frac{br}{\sigma^2} \left[(1 - \alpha)\sigma + 2\beta(1 + \alpha)\right] > 0, \]
\[ \phi = \alpha(1 - 2\delta) < 0, \quad \tau = 2(1 + \alpha)(\alpha + \delta(1 - \alpha)) > 0. \]

The relations (14) summarize the Nash reaction functions. Both nominal wages are negatively correlated to labour force. As expected, an accommodating monetary policy lead to higher wages. Substituting the first equation of (14) into the second equation of (14) yields:

\[ w_i = m + \frac{\phi\tau}{\sigma^2 - \phi^2} \ell_0j - \frac{\sigma\tau}{\sigma^2 - \phi^2} \ell_0i + \frac{\sigma - \phi}{\sigma} \eta. \] \tag{15}

Consequently

\[ w_j = m + \frac{\phi\tau}{\sigma^2 - \phi^2} \ell_0i - \frac{\sigma\tau}{\sigma^2 - \phi^2} \ell_0j + \frac{\sigma - \phi}{\sigma} \eta. \] \tag{16}

In this case unions set \( w_i \) and \( w_j \) together.

3.2.2 Coordinated unions

Coordinated unions’ solutions are given by:

\[ w_i = m - \frac{\tau (\phi \ell_0i + \sigma \ell_0j)}{(\sigma + \theta)^2} - \frac{\tau (\sigma^2 + \phi^2)}{(\sigma - \phi)(\sigma + \phi)^2} \left( \ell_0i - \ell_0j \right) + \frac{b}{2} (1 - \alpha) (1 + \alpha), \]
\[ w_j = m - \frac{\tau (\phi \ell_0j + \sigma \ell_0i)}{(\sigma + \phi)^2} + \frac{\tau (\sigma^2 + \phi^2)}{(\sigma - \phi)(\sigma + \phi)^2} \left( \ell_0i - \ell_0j \right) + \frac{b}{2} (1 - \alpha) (1 + \alpha). \] \tag{17}

3.2.3 The Central Banker’s solution

The central bank’s reaction function is given by:

\[ m = \frac{1 - 2\lambda(1 - \alpha)\alpha}{2 + 2\lambda(1 - \alpha)^2} (w_i + w_j) + \frac{1 + \alpha}{2 + 2\lambda(1 - \alpha)^2} (\ell_0i + \ell_0j) \] \tag{18}

Note that, if \( \lambda \) is zero, then the policy-maker wants to promote employment whatever the inflation costs, and money supply are:

\[ m = \frac{1}{2} (w_i + w_j) + \frac{1 + \alpha}{2} (\ell_0i + \ell_0j). \] \tag{19}

In this case, nominal wages have a positive effect on money supply. When country unions are very strong, the central banker chooses an accommodating monetary policy to avoid a rise in unemployment and to maintain a high level of labour demand.
### 3.3 Equilibrium

This Subsection is devoted to the computation of all equilibria over all markets, knowing the optimal wages, the optimal labor demands and the optimal money obtained in the previous Section.

Equations (2), (11), and (14) have been used to obtain relation (20):

\[ p_j - p_i = \frac{\alpha}{\alpha + \delta(1 - \alpha)}(w_j - w_i). \]  

(20)

Note that \( p_j - p_i \) is positively related to the wage differential. From (2), (9), (14), and (20), we can derive (21):

\[
m = \frac{1 + \alpha}{1 - \alpha}p_i + \frac{1 + \alpha}{1 - \alpha} \left( \frac{-\alpha}{2(\alpha + \delta (1 - \alpha))} \right) (w_i - w_j) - \frac{\alpha}{1 - \alpha} (w_i + w_j). \]

(21)

The solution for \( p_i \) and \( p_j \) can be written as:

\[
p_i = \frac{1 - \alpha}{1 + \alpha} m + \frac{\alpha}{2(\alpha + \delta (1 - \alpha))} (w_i - w_j) + \frac{\alpha}{1 + \alpha} (w_i + w_j), \]

\[
p_j = \frac{1 - \alpha}{1 + \alpha} m - \frac{\alpha}{2(\alpha + \delta (1 - \alpha))} (w_i - w_j) + \frac{\alpha}{1 + \alpha} (w_i + w_j). \]

(22)

Using (6) (7), (22) we determine the consumer price:

\[
q_i = \frac{1 - \alpha}{1 + \alpha} m + \frac{\alpha}{1 + \alpha} (w_i + w_j) + (1 - 2\beta) \frac{\alpha}{2(\alpha + \delta (1 - \alpha))} (w_i - w_j),
\]

\[
q_j = \frac{1 - \alpha}{1 + \alpha} m + \frac{\alpha}{1 + \alpha} (w_i + w_j) - (1 - 2\beta) \frac{\alpha}{2(\alpha + \delta (1 - \alpha))} (w_i - w_j).
\]

(23)

From (23) and (8), it follows immediately that \( \hat{w}_i \) and \( \hat{w}_j \) are related to \( m, w_i \) and \( w_j \) according to:

\[
\hat{w}_i = w_i - \frac{1 - \alpha}{1 + \alpha} m - \frac{\alpha}{1 + \alpha} (w_i + w_j) - (1 - 2\beta) \frac{\alpha}{2(\alpha + \delta (1 - \alpha))} (w_i - w_j),
\]

\[
\hat{w}_j = w_j - \frac{1 - \alpha}{1 + \alpha} m - \frac{\alpha}{1 + \alpha} (w_i + w_j) + (1 - 2\beta) \frac{\alpha}{2(\alpha + \delta (1 - \alpha))} (w_i - w_j).
\]

(24)

Substituting (22) into (14), we get:

\[
\ell_i = \frac{1}{1 + \alpha} m - \frac{1}{1 - \alpha} \left[ w_i - \frac{\alpha}{2(\alpha + \delta (1 - \alpha))} (w_i - w_j) - \frac{\alpha}{1 + \alpha} (w_i + w_j) \right],
\]

\[
\ell_j = \frac{1}{1 + \alpha} m - \frac{1}{1 - \alpha} \left[ w_j + \frac{\alpha}{2(\alpha + \delta (1 - \alpha))} (w_i - w_j) - \frac{\alpha}{1 + \alpha} (w_i + w_j) \right].
\]

(25)
Using (25, (14), (2) we can express the outputs $y_i, y_j$ as:

$$y_i = \frac{\alpha}{1 + \alpha} m + \frac{\alpha}{1 - \alpha} \left[ w_i - \frac{\alpha}{2(\alpha + \delta (1 - \alpha))} w_j - \frac{\alpha}{1 + \alpha} (w_i + w_j) \right],$$

$$y_j = \frac{\alpha}{1 + \alpha} m - \frac{\alpha}{1 - \alpha} \left[ w_i + \frac{\alpha}{2(\alpha + \delta (1 - \alpha))} w_j - \frac{\alpha}{1 + \alpha} (w_i + w_j) \right].$$

(26)

A few interesting results are obtained from (22). Which can be written:

$$p_i = \frac{1 - \alpha}{1 + \alpha} m + \frac{\alpha}{1 + \alpha} \left[ \left( 1 + \frac{1 + \alpha}{2(\alpha + \delta (1 - \alpha))} \right) w_i + \left( 1 - \frac{1 + \alpha}{2(\alpha + \delta (1 - \alpha))} \right) w_j \right],$$

$$p_j = \frac{1 - \alpha}{1 + \alpha} m + \frac{\alpha}{1 + \alpha} \left[ \left( 1 - \frac{1 + \alpha}{2(\alpha + \delta (1 - \alpha))} \right) w_i + \left( 1 + \frac{1 + \alpha}{2(\alpha + \delta (1 - \alpha))} \right) w_j \right].$$

(27)

Relations (27) shows that an increase in both wages and money supply increases the domestic production price when both money and commodity markets are cleared. The analysis of the foreign production price follows by symmetry. There are two effects playing here. The first is a monetary mechanism effect and the second is a trade balance effect. First, from (26) an increase in $w_i$ increases the production. From (21), the monetary equilibrium is reached only if prices go up. Second, from (23) an increase in $w_i$ involves a trade balance surplus due to a lessening of the domestic production.

### 4 The effects of unions’ coordination

The objective of this Section is to determine which of the two regimes (uncoordinated and coordinated of unions) leads to higher wages in the monetary union. Let us assume that $\ell_{0i} = \ell_{0j} = \hat{\ell}$. Economic values in a coordinated regime are indexed by $c$, while economic values in a uncoordinated regime are indexed by $nc$. Replacing $\ell_{0i}$ and $\ell_{0j}$ by $\hat{\ell}$ in (14) and in (17), we obtain:

$$w_{ci} = w_{cj} = m - (1 + \alpha) \hat{\ell} + \frac{b}{2} (1 + \alpha) (1 - \alpha),$$

$$w_{nci} = w_{ncj} = m - (1 + \alpha) \hat{\ell} + \frac{b}{2} (1 + \alpha) \left[ (1 - \alpha) + \frac{2\beta\alpha(1 + \alpha)}{\sigma} \right].$$

(28)

Externalities of unions’ behavior can be internalized in a coordinated wage setting. Indeed wages and prices are low: $w_{ci} < w_{nci}$ and $w_{cj} < w_{ncj}$. Money supplies are given by:

$$m_c = 2\alpha \hat{\ell} + \frac{b(1 - 2\lambda\alpha (1 - \alpha))}{2\lambda},$$

$9$
\[ m_{nc} = 2 \alpha \tilde{\ell} + \frac{b(1 - 2\lambda \alpha (1 - \alpha))}{2\lambda} \left[ 1 + \frac{2\beta \alpha (1 + \alpha)}{\sigma (1 - \alpha)} \right]. \] (29)

Depending on the degree of conservatism of the central banker, it is possible to compare the two money supplies:

\[ m_c > m_{nc} \text{ if } \lambda > \frac{1}{2(1 - \alpha) \alpha} \text{ and } m_c < m_{nc} \text{ if } \lambda < \frac{1}{2(1 - \alpha) \alpha}. \]

When trade unions are coordinated, wage moderation is not a sufficient condition to ensure a more flexible monetary policy. This is only the case if the central banker is very conservative. Production prices are higher in a uncoordinated situation: \( p_c < p_{nc} \). Labour demand is higher when unions are coordinated, \( \ell_c > \ell_{nc} \), which involves \( y_c > y_{nc} \). Consumer price and real wages are lower in a coordinated regime: \( q_c < q_{nc}, w_c < w_{nc} \).

The unions’ interest is to coordinate their decisions in order to maximize employment in the monetary union. These results are independent of the central banker’s behavior.

5 Conclusion

This paper presented a Stackelberg game between the Central Banker of a monetary union and two countries which differ in their consumer price index. This leads all prices to differ across countries. In each country, the labor market is characterized by the presence of a single union, who acts as a monopoly of labor supply. Firms take the already set wage which maximizes the union workers’ welfare. The paper highlighted the effects of internationally coordinated unions versus internationally uncoordinated unions. It has been shown that international coordination between unions in a monetary union yields to lower wages and a higher employment level compared with uncoordinated unions. The main lesson of this paper is that internationally coordinated unions leads to an accommodating monetary policy. There is more money circulating in the economy as long as the Central Banker is sufficiently conservative in terms of inflation.
References

A Appendix

A.1 Approximation

Since subscripts \( i \) and \( j \) do not play any role here, we omit them in this appendix. Let us recall (2):

\[
L^{\alpha - 1} = \frac{W}{\alpha P}.
\]

Let us use a linear approximation of the previous expression in a neigbourhood of \( L_0 \):

\[
(\alpha - 1)L_0^{\alpha - 2} \left[ \frac{L - L_0}{L_0} \right] L_0 = \frac{1}{P_0\alpha} \left[ \frac{W - W_0}{W_0} \right] W_0 - \frac{W_0}{\alpha P_0^2} \left[ \frac{P - P_0}{P_0} \right] P_0.
\]

Let us denote by small letters the following quantities:

\[
\ell = \frac{L - L_0}{L_0}, \quad w = \frac{W - W_0}{W_0} \quad \text{and} \quad p = \frac{P - P_0}{P_0}.
\]

Rewrite (30) with these notations, developp the left hand side and use (2) into the right hand side to get:

\[
(\alpha - 1)L_0^{\alpha - 1}\ell = L_0^{\alpha - 1}w - L_0^{\alpha - 1}p.
\]

Simplify by \( L_0^{\alpha - 1} \)

\[
(\alpha - 1)\ell = w - p \iff \ell = -\frac{w - p}{1 - \alpha} \iff L = L_0 \left[ 1 - \left( \frac{W}{W_0} - \frac{P}{P_0} \right) \right].
\]

Consequently, \( L > 0 \iff \frac{W}{W_0} - \frac{P}{P_0} < 1 - \alpha \). The difference between the wage increase \( W/W_0 \) and the price increase \( P/P_0 \) captures the increase in the real purchasing power of consumers.

A.2 Minimization of the unions’ loss function

The first program (3) is solved as follow:

\[
\begin{align*}
\max_{w_i} U_i &= -(\ell_i - \ell_{0i})^2 + bw_{ci}, \\
\text{s.c.} \frac{\partial w_i}{\partial w_i} &= \frac{\partial m}{\partial w_i} = 0.
\end{align*}
\]

Using (14), (24) and (25), we derive:

\[
U_i = -\left[ -(1 - \alpha)^{-1} (w_i - p_i) - \ell_{0i} \right]^2 + bw_{ci},
\]

\[
U_i = -\left[ \frac{1}{1+\alpha} m - \frac{1}{1-\alpha} \left[ \frac{\alpha}{2(\alpha+\delta(1-\alpha))} (w_i - w_j) - \frac{\alpha}{1+\alpha} (w_i + w_j) \right] - \ell_{0i} \right]^2
\]

\[+b \left[ w_i - \frac{1}{1+\alpha} m - \frac{\alpha}{1+\alpha} (w_i + w_j) - (1 - 2\beta) \frac{\alpha}{2(\alpha+\delta(1-\alpha))} (w_i - w_j) \right].
\]

The first order condition is:

\[
\frac{\partial U_i}{\partial w_i} = -2 \left[ -\frac{1}{1-\alpha} \left( 1 - \frac{\alpha}{2(\alpha+\delta(1-\alpha))} - \frac{\alpha}{1+\alpha} \right) \right]
\]
The first order condition is:

\[
\left[ \frac{1}{1+\alpha} m - \frac{1}{1-\alpha} \left[ w_i - \frac{\alpha}{2(\alpha+\delta(1-\alpha))} (w_i - w_j) - \frac{\alpha}{1+\alpha} (w_i + w_j) \right] - \ell_{0i} \right] + b \left[ 1 - \frac{\alpha}{1+\alpha} - (1 - 2\beta) \frac{\alpha}{2(\alpha+\delta(1-\alpha))} \right] = 0, \\
\]

\( \sigma, \eta, \theta \) and \( \tau \), are defined as follow:

\[
\sigma = \alpha + 2\delta, \quad \eta = \frac{b r}{2\beta r} [(1 - \alpha) \sigma + 2\beta \alpha (1 + \alpha)], \\
\theta = \alpha (1 - 2\delta), \quad \tau = 2 (1 + \alpha) (\alpha + \delta (1 - \alpha)).
\]

The we have:

\[
w_i = m + \frac{\theta}{\sigma} (m - w_j) - \frac{\tau}{\sigma} \ell_{0i} + \eta, \\
w_j = m + \frac{\theta}{\sigma} (m - w_i) - \frac{\tau}{\sigma} \ell_{0j} + \eta.
\]

Resolution of program (5) is similar except that the maximisation must be done simultaneously in reference with \( w_i \) and \( w_j \).

### A.3 Minimization of the Central Banker’s loss function

Program (22) is also based on the derivation of the utility function:

\[
\max_{\Psi} \Psi = V_i + V_j = \left[ - (\ell_i - \ell_{0i})^2 - \lambda q_i^2 \right] + \left[ - (\ell_j - \ell_{0j})^2 - \lambda q_j^2 \right]
\]

\( \text{sc. } \frac{\partial w_i}{\partial m} = \frac{\partial w_j}{\partial m} = 0, \)

with

\[
\Psi = - \left( \frac{1}{1+\alpha} m - \frac{1}{1-\alpha} \left[ w_i - \frac{\alpha}{2(\alpha+\delta(1-\alpha))} (w_i - w_j) - \frac{\alpha}{1+\alpha} (w_i + w_j) \right] - \ell_{0i} \right)^2 \\
- \lambda \left( \frac{1}{1+\alpha} m + \frac{\alpha}{1+\alpha} (w_i + w_j) + (1 - 2\beta) \frac{\alpha}{2(\alpha+\delta(1-\alpha))} (w_i - w_j) \right)^2 \\
- \left( \frac{1}{1+\alpha} m - \frac{1}{1-\alpha} \left[ w_j + \frac{\alpha}{2(\alpha+\delta(1-\alpha))} (w_i - w_j) - \frac{\alpha}{1+\alpha} (w_i + w_j) \right] - \ell_{0j} \right)^2 \\
- \lambda \left( \frac{1}{1+\alpha} m + \frac{\alpha}{1+\alpha} (w_i + w_j) - (1 - 2\beta) \frac{\alpha}{2(\alpha+\delta(1-\alpha))} (w_i - w_j) \right)^2.
\]

The first order condition is:

\[
\frac{\partial \Psi}{\partial m} = 0
\]

\[
\iff \left[ 2 \left( \frac{1 + 2\lambda (1 - \alpha)}{1 + \alpha} \right) \right] m + \left[ 2\alpha - (1 + \alpha) + (2\lambda \alpha (1 - \alpha)) \right] \left( \ell_i + \ell_{0i} + \ell_{0j} \right) = 0.
\]

The solution is

\[
m = \frac{1 - 2\lambda (1 - \alpha) \alpha}{2 + 2\lambda (1 - \alpha)^2} (w_i + w_j) + \frac{1 + \alpha}{2 + 2\lambda (1 - \alpha)^2} (\ell_{0i} + \ell_{0j}).
\]