Self-Attribution Bias and Consumption

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Abstract

In this paper I examine the implications of self-attribution bias on consumption and savings decisions. When self-attributive learning replaces rational expectations in a model of intertemporal choice, two departures from the permanent-income hypotheses manifest. One is that consumers tend to under-save early in life. Another is a relatively high degree of covariance between changes in consumption and changes in income. No other factor on its own has been able to explain both of these empirical anomalies that the permanent-income hypothesis has faced.

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1 Introduction

Self-attribution bias refers to the tendency to credit one's self for desirable outcomes while blaming undesirable outcomes on external factors.[1] For example, a

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[1]See Taylor and Brown (1988) or Campbell and Sedikides (1999) for surveys on the subject of self-attribution bis.

professional athlete who exhibits the bias may blame his coaches, teammates, the referees, or "bad luck" for poor performance, rather than his own lack of ability. There has been some work in economics and finance that considers self-attribution. Daniel, Hirshleifer, and Subrahmanyam (1998) theorize that self-attribution bias and over-confidence account for excess volatility in securities markets relative to that implied by models with fully rational decision makers. Gervais and Odean (2001) present a model wherein traders are initially ignorant of their ability but tend to become overconfident over time due to self-attribution. Choi and Lou (2010) find that self-attribution bias affects the decisions of mutual fund managers, and that it leads to poor performance. Interestingly, they find biased self-attribution amongst younger managers but little evidence for it among more experienced mutual fund managers.

The present study considers the effects of self-attributive learning on decisions involving consumption spending and savings. Self-attribution may be relevant to consumption/savings decisions because it is thought to be a mechanism by which individuals bolster their self-esteem (Shepperd, Malone, and Sweeny, 2008). If consumers buoy their self-esteem by believing they will earn relatively high incomes over their lifetime (as a result of self-attribution bias) then this should affect their consumption/savings decisions as well, since such decisions are thought to be based upon expectations regarding future income.

This study details two implications of self-attribution bias on the dynamics of consumption spending. One implication is that consumption tends to be unsustainably high early in life, leading to a probable decrease in consumption later in life. Hence, self-attribution bias can be used to explain the fact that many individuals and households over-consume and under-save. Another implication of self-attributive learning is that changes in consumption covary to a higher degree with changes in income under self-attribution bias than for rational expectations. So, self-attributive learning explains the "excessive sensitivity" (Flavin, 1981) of consumption to income found in empirical analyses.

The intuition behind over-consumption early in life is that self-attribution bias tends to entail overly-optimistic expectations for future earnings and an overly-optimistic individual will consume more early in life because he or she expects to finance such spending with greater earnings in future periods. Relative to a consumer with rational expectations, the greater degree of covariation between changes in consumption and changes in income for a consumer with self-attribution bias stems from the fact that income in any period provides a signal about the likelihoods of future incomes for the self-attributor but not for the consumer with rational expectations, the latter of whom inherently knows the probabilities of any level of income in any future period.

It is well-known that there is no single factor that has been able to explain both of these phenomena.[2] For example, hyperbolic discounting on its own generates over-consumption, but it requires at least one other non-standard assumption (such as credit constraints) in order to account for greater covariation between changes in consumption and changes in income (Angeletos, Laibson, Repetto, Tobacman, and Weinberg, 2001). Thus, self-attribution bias represents a more parsimonious theory of consumption that diverges from the permanent-income hypothesis in these ways.

There is some empirical evidence that may corroborate with the theory presented in this paper. Kooreman, Prast, and Vellekoop (2009) find significantly different propensities to save for wages that were labelled differently on the paychecks of employees at a Bank and an insurance company in The Netherlands. In

[2]See, for example, the discussion on pages 397-398 of Romer (2012).

particular, people saved a lower proportion of a "performance bonus" than they did a "vacation allowance" or a "13th month". They explain this finding using the mental accounting framework, that there could be "a mental accounting relationship between the label of an income component and how the component is put to use".[3] Self-attribution bias could be another explanation for the findings regarding performance bonuses. If employees treat performance bonuses as a signal of their own high ability then they will expect to earn more bonuses in the future, and they will consume more and save less of this extra income than they would have otherwise.

The remainder of the paper is organized as follows: Section 2 discusses other theories of consumption, and how the present work aims to fill a gap in the literature. Section 3 presents and analyzes an intertemporal consumption and savings model with self-attributive learning. Section 4 concludes.

2 Theories of Consumption

This section offers a discussion of the various theories of consumption. The purpose of the section is to identify the gap in our understanding of consumption and savings decisions that this work fills.

The theory of aggregate consumption has a history dating back at least to Keynes (1936), who conjectured that consumption in any period depends primarily on income for that period and that other variables have only negligible effects on consumption. A controversial property of Keynes's consumption function is that the average propensity to consume (i.e. the ratio of consumption to income) decreases as income grows. Although empirical analyses of cross-sections sup-

[3] Thaler (1980) initiated the mental accounting literature. See Thaler (2004) for fairly recent review of this literature.

ported the idea of decreasing average propensity to consume, time-series analyses suggested that the ratio is fairly constant despite substantially growing incomes over long periods of times.[4] These empirical failures of the Keynesian consumption function begat a search for new theories of consumption. The most influential substitutes were a pair of related, neoclassical hypotheses: the life-cycle hypothesis of Modigliani and Brumberg (1954) and the permanent-income hypothesis of Friedman (1957). These theories emphasized real wealth, including discounted future real income, as the determinant of consumption. When coupled with the rational expectations hypothesis these theories imply that consumption fluctuates relatively little compared to contemporaneous income, that consumption is *smooth*.[5]

However empirical evidence suggests that consumption is not as smooth as these hypotheses imply. For example, Campbell and Mankiw (1990) find that about half of consumption spending is determined by contemporaneous income while the rest is determined by other variables. Wilcox (1989), Shea (1995), Banks, Blundell, and Tanner (1998), Parker (1999), Souleles (1999), and Bernheim, Skinner, and Weinberg (2001) find evidence that consumption depends more on contemporaneous income than implied by the permanent-income hypothesis. In a similar vein of research, Carroll (1994) provides evidence that consumption is a poor indicator of future income and Startz (2008) finds that lagged income is a much better predictor of future income than present consumption.

There have been a number of theories proposed to explain the apparent lack of smoothness in consumption implied by rational expectations and the life-cycle

[4]Perhaps the most noteworthy example of this empirical work is Kuznets (1946).

[5]Hall (1978) shows that the permanent-income hypothesis coupled with rational expectations imply that the marginal utility of consumption is a martingale. As such, if the rate of interest offsets the discount factor each period then consumption is also a martingale.

and permanent-income hypotheses. Neoclassical theory typically assumes that consumers have preferences only over their consumption. Some studies have relaxed this assumption, and considered preferences over sociological phenomena. The concepts of *conspicuous consumption* and *pecuniary emulation* from Veblen (1899) involve preferences to signal wealth through consumption. In the exposition of the relative-income hypothesis, Duesenberry (1949) reasons that these phenomena lead to lower levels of saving than would occur if consumers only had preferences over consumption. Another sociological theory is proposed by Akerlof (2007), who argues that there is a social *norm* whereby individuals feel entitled to spend their current income (and under-save), a norm that could be modelled by a *direct* preference to save less. A behavioral approach involves time-inconsistent preferences in the form of *hyperbolic discounting*, wherein, at a given point in time, consumption in the present is more valued than the next period by a greater factor than consumption in future proximate periods.[6]

The present study adds to this list of theories that attempt to explain consumption patterns. A notable difference between the theory presented herein and previous alternatives to the fully rational model is that it focuses on irrationality with respect to the beliefs agents hold, whereas previous theories dealt with preferences that were in, in one sense or another, non-standard. An implication of this is that previous theories typically utilize the rational expectations hypothesis, whereas the present study does not (except as a basis for comparison).

To summarize, the theories of consumption that have been offered as an alternative to the permanent income hypothesis have heretofore dealt with non-standard preferences drawn from the fields of psychology and sociology. This

[6]See, for example, Laibson (1997) or Frederick, Loewenstein, and O'Donoghue (2002). Also see the discussion in Akerlof (2002).

work differs from these theories in that it focuses on non-standard beliefs, which tend to be biased in a systematic manner. A complete theory of consumption would utilize realistic preferences *and* beliefs, so this work complements previous theories by moving our understanding closer to that goal.

From a policy perspective, it is important to consider the multitude of explanations for a given phenomenon so that efforts to improve economic outcomes are not doomed to failure simply because our understanding is incomplete. For example, if self-attribution bias is a major reason for why some people under-save, then a policy designed to induce time-consistent discounting of future consumption may not be cost effective for attaining the underlying goal of getting people to not under-save. Therefore, this work has value to policy-makers by giving them a more complete understanding of consumption spending, and how it might be influenced.

3 Self-Attribution and Intertemporal Choice

This section presents a model designed to obtain implications of embedding self-attributive learning in an otherwise conventional consumption and savings model.

Consider an economy in which individuals earn income in each of T periods, $1, \ldots, T$ and decide how much to consume in each of $T+R$ periods $1, \ldots, T+R$, where R is the number of periods spent in retirement. Denote the random variable representing consumer i's income in period t with y_{it}.

Let c_{it} denote consumer i's consumption spending in period t. Assume that each consumer's preferences over $(c_{i1}, \ldots, c_{iT+R})$ satisfy the expected utility hypothesis, and that each consumer makes consumption and savings decisions by maximizing

$$\mathbb{E}\left[\sum_{t=1}^{T+R} c_{it} - \frac{\gamma c_{it}^2}{2}\right], \tag{1}$$

with $\gamma > 0$ and sufficiently small to guarantee that marginal utility is positive over relevant values of c_{it}.

A quadratic period utility function, zero discount rate, and zero interest rate are used in order to focus exclusively on how consumption changes due to changes in beliefs. Generally, consumers choose levels of consumption in order to equalize the present value of discounted marginal utility. Quadratic utility is unique in that it implies that consumption is equal to the expected value of consumption in future dates,[7] which is equal to the average level of income the individual expects to earn over the remaining periods in life. As such, these results generalize, somewhat, to any monotonically increasing utility function since they all imply that consumption is increasing in expected average income. Besides, if this were not the case then consumption would change over time even if beliefs did not change, simply because of one's attitude toward risk and the decrease in the riskiness of total lifetime income as time passes. Similarly, if the discount factor and interest rate were non-zero they would generally interact to affect consumption levels. If these factors were present in the analysis then determining the extent to which fluctuations in consumption were due to changes in expectations would be muddled by attitudes toward risk and the interplay between the discount and the rate of interest, so the confounding factors are eliminated.

[7] To see this, note that a first-order condition for a general utility function u is $u'(c_{it}) = \mathbb{E}_t[u'(c_{it+1})]$. When u is quadratic u' is linear, so the condition becomes $u'(c_{it}) = u'(\mathbb{E}_t[c_{it+1}])$ and when u is monotonically increasing (over the relevant range) it follows that $c_{it} = \mathbb{E}_t[c_{it+1}]$.

Consumer i's lifetime budget constraint is

$$\sum_{t=1}^{T+R} c_{it} = \sum_{t=1}^{T} y_{it}, \tag{2}$$

where equality is ensured by the assumption that the marginal utility of each period's consumption is positive.

Let $\mathbb{E}_t$ denote the general expectation operator given information up to time t.[8] For consumer i choosing how much to consume in period t the optimality conditions for maximization of expression (1) are

$$c_{it} = \mathbb{E}_t[c_{i\tau}], \quad \text{for all } \tau \in \{t+1, \ldots, T\}. \tag{3}$$

Each consumer will expect, at each period t, that lifetime income will eventually be spent, so

$$\sum_{\tau=1}^{t-1} c_{i\tau} + \sum_{\tau=t}^{T+R} \mathbb{E}_t[c_{i\tau}] = \sum_{\tau=1}^{t-1} y_{i\tau} + \sum_{\tau=t}^{T} \mathbb{E}_t[y_{i\tau}]. \tag{4}$$

Here it is important to emphasize that in this model there is no credit constraint on any individual, so it is possible for $c_{it} > y_{it}$ in any period t.

From expression (3), Substitute c_{it} for each $E_t[c_{i\tau}]$ term in expression (4) and solve for c_{it} to obtain

$$c_{it} = \frac{1}{T+R-t+1} \left(\sum_{\tau=1}^{t-1} y_{i\tau} - c_{i\tau} + \sum_{\tau=t}^{T} \mathbb{E}_t[y_{i\tau}] \right), \tag{5}$$

which says that consumption in any period t is equal to accumulated savings ($\sum_{\tau=1}^{t-1} y_{i\tau} - c_{i\tau}$) plus expected income from the current period on ($\sum_{\tau=t}^{T} \mathbb{E}_t[y_{i\tau}]$), divided by the number of remaining periods in which consumption will take place

[8] Information for consumer i up to time t is essentially having observed the realized values of $y_{i1}, \ldots, y_{it}$.

$(T + R - t + 1)$.

3.1 Self-Attribution and Expectations

Suppose income in each of the non-retirement periods take one of two possible values: y' and y, with $y' > y$. Let $\theta_i \in (0,1)$ denote the probability that $y_{it} = y'$ for any i and all t. As θ_i is the probability of earning the higher level of income, we can interpret θ_i as a measure of i's income earning ability.

We will focus on the case in which consumers do not know their values of θ_i (so explicitly assuming that consumers do not hold rational expectations, under which θ_i is known by each i). Consumers will instead infer the value of θ_i by observing their income levels. These inferences will be modelled with the weighted updating model studied in Zinn (2013), which is a generalization of Bayes' rule that allows for biased belief formation. Therefore, beliefs regarding θ_i given the realized values $y_{i1}, \ldots, y_{it}$, for each period t, are summarized by

$$\tilde{\pi}_{it}(\theta_i | y_{i1}, \ldots, y_{it}) = \frac{\pi(\theta_i) \prod_{\tau=1}^{t} f(y_{i\tau}|\theta_i)^{\psi(y_{i\tau})}}{\int_0^1 \pi(\theta_i) \prod_{\tau=1}^{t} f(y_{i\tau}|\theta_i)^{\psi(y_{i\tau})}\, d\theta_i}, \tag{6}$$

where each $f(y_{i\tau}|\theta_i)$ is the likelihood function associated with income $y_{i\tau}$ and $\pi(\theta_i)$ is the prior distribution. The weighting function $\psi : \{y, y'\} \to \mathbb{R}_+$ gives a measure of how informative individual i regards the observed level of income. [9] As Bayesian updating is the case when all weights equal one, the consumer is respectively treating an observation of income $y_{i\tau}$ as less, equally, or more informative compared to a perfect Bayesian if the weight $\psi(y_{i\tau})$ is less than, equal

[9] Results from Zinn (2013) show that larger values of ψ lead to the effective likelihood function proportional to $f(z_i|\theta_i)^{\psi}$ having less information entropy as ψ increases. That ψ is a measure of how informative an individual is treating an observation follows from the interpretation of information entropy as a measure of the average information content of a random variable.

to, or greater than one.

Self-attribution bias involves associating undesirable outcomes with luck while ascribing desirable outcomes to internal, personal factors (such as ability), so assume that consumers blame luck when $y_{it} = y$ and that they attribute $y_{it} = y'$ to their ability. To dismiss an outcome as being due to luck is to consider that outcome as not being very informative, which is modelled with a low weight relative to that of a Bayesian. So, $\psi(y) = \delta \in [0, 1)$. The theory of self-attribution bias posits that those who exhibit the bias ascribe positive outcomes to ability. How this translates to restrictions we ought to place on $\psi(y')$ is unclear, except that it must be the case that $\psi(y') > \psi(y)$. So simply assume that $\psi(y') = 1$, the minimum value of the range suggested by theory. In essence, this assumption stipulates that self-attribution bias involves putting as much weight on the desirable outcome as a perfect Bayesian updater would, implying that the irrational learning is driven entirely by the under-weighting of undesirable outcomes.

Each y_{it} is a Bernoulli trial with parameter θ_i, so the likelihood functions may be expressed as

$$f(y_{it}|\theta_i) = \begin{cases} \theta_i & \text{if } y_{it} = y' \\ (1 - \theta_i) & \text{if } y_{it} = y. \end{cases}$$

Assume that each consumer i's prior distribution $\pi(\theta_i)$ is from the beta family of distributions, with parameters $a_i, b_i \in \mathbb{R}_{++}$. That is

$$\pi(\theta_i) = \frac{\theta_i^{a_i - 1}(1 - \theta_i)^{b_i - 1}}{\int_0^1 \theta_i^{a_i - 1}(1 - \theta_i)^{b_i - 1}\, d\theta_i}$$

This ensures tractability as beta distributions are the conjugate priors of the binomial distribution, ensuring that posterior distributions are in the beta-binomial family.

Let z_{it} denote the number of times consumer i has observed the high income level y' in the first t periods. Then the weighted updating model expressed in (6) can be restated more specifically:

$$\tilde{\pi}(\theta_i|y_{i1},\ldots,y_{it}) = \frac{\theta_i^{z_{it}+a_i-1}(1-\theta_i)^{\delta(t-z_{it})+b_i-1}}{\int_0^1 \theta_i^{z_{it}+a_i-1}(1-\theta_i)^{\delta(t-z_{it})+b_i-1}\,d\theta_i}.$$

Assume consumers use the (subjective) expected value of θ_i as point estimates. Then after observing income levels in the first t periods, consumer i will estimate θ_i to be

$$\begin{aligned}\tilde{\theta}_{it} &\equiv \tilde{\mathbb{E}}(\theta_i|y_{i1},\ldots,y_{it}) \\ &\equiv \int_0^1 \theta_i\tilde{\pi}(\theta_i|y_{i1},\ldots,y_{it})\,d\theta_i \\ &= \frac{a_i+z_{it}}{a_i+b_i+\delta t+(1-\delta)z_{it}}. \qquad (7)\end{aligned}$$

(A full derivation of the formula for $\tilde{\theta}_{it}$ in expression (7) is presented in the appendix.)

A notable aspect of $\tilde{\theta}_{it}$ is how it tends to behave as the number of observations increases without bound. Notice that

$$\begin{aligned}\lim_{t\to\infty}\tilde{\theta}_{it} &= \lim_{t\to\infty}\frac{a_i+z_{it}}{a_i+b_i+\delta t+(1-\delta)z_{it}} \\ &= \lim_{t\to\infty}\frac{a_i+t\theta_i}{a_i+b_i+\delta t+(1-\delta)t\theta_i} \\ &= \lim_{t\to\infty}\frac{\frac{a_i}{t}+\theta_i}{\frac{a_i+b_i}{t}+\delta+(1-\delta)\theta_i} \\ &= \frac{\theta_i}{\delta+\theta_i-\delta\theta_i} \qquad (8) \\ &> \theta_i,\end{aligned}$$

where the inequality in the final line is a consequence of $\delta, \theta_i \in (0,1)$ implying that $\delta + \theta_i - \delta\theta_i - 1 = (1-\theta)(\delta - 1) < 0$, from which it follows that $\delta + \theta_i - \delta\theta_i < 1$. That $\lim_{t\to\infty} \tilde{\theta}_{it} > \theta_i$ suggests that with self-attribution bias any consumer, given enough observations, will (i.e. with *sure convergence*) eventually become more optimistic than counterparts with rational expectations. Thus, one would expect that $\tilde{\theta}_{it}$ will grow over time.

It is not true that $\tilde{\theta}_{it+1} > \tilde{\theta}_{it}$ for every consumer i and in every period t. That depends on the actual observations; if a consumer repeatedly observes the low income level y then he or she will not grow more optimistic. Whether or not $\tilde{\theta}_{it}$ increases also depends on the prior distribution. For example, if the prior distribution is overly optimistic (particularly when $\frac{a_i}{a_i+b_i} > \lim_{t\to\infty} \tilde{\theta}_{it}$) then $\tilde{\theta}_{it}$ will tend to decrease as it converges to $\frac{\theta_i}{\delta+\theta_i-\delta\theta_i}$.

The estimator $\tilde{\theta}_{it}$ will generally be biased by the prior distribution $\pi(\theta_i)$, even in the case where $\delta = 1$, and the consumer updates according to Bayes' rule. In order to study only the bias due to self-attributive learning, this analysis will focus exclusively on cases where the prior distribution does not generate bias by imposing that $\frac{a_i}{b_i} = \frac{\theta_i}{1-\theta_i}$, so the prior distribution is accurate in the sense that $\mathbb{E}(\tilde{\theta}_{i0}) = \theta_i$. Then any remaining bias in the estimate $\tilde{\theta}_{it}$ will be due to self-attribution. To achieve this, do the following: for any $k > 0$, substitute $k\theta_i$ for a_i and $k(1-\theta_i)$ for b_i, so that $\frac{a_i}{b_i} = \frac{\theta_i}{1-\theta_i}$. Now, impose that consumer i experiences an approximately *typical* history, by substituting $t\theta_i = \mathbb{E}(z_{it}|\theta_i)$ for z_{it} in expression (7). Define the beliefs from such an approximately typical experience as

$$\begin{aligned} \bar{\theta}_{it} &\equiv \frac{k\theta_i + t\theta_i}{k\theta_i + k(1-\theta_i) + t\theta_i + \delta(t - t\theta_i)} \\ &= \frac{\theta_i(k+t)}{k + \delta t + t\theta_i(1-\delta)}. \end{aligned} \tag{9}$$

To understand how these beliefs tend to change over time, take the time derivative of expression (9):

$$\frac{\partial \bar{\theta}_{it}}{\partial t} = \frac{k\theta_i(1-\theta_i)(1-\delta)}{[k+\delta t + t\theta_i(1-\delta)]^2} > 0. \tag{10}$$

That this derivative is positive shows that self-attribution bias will tend to induce increasingly optimistic beliefs (up to a limiting value) as time passes.[10] To reiterate, this result suggests that when the bias introduced by the prior distribution is eliminated, beliefs regarding the value of θ_i will typically rise over time.

3.2 Self-Attribution and Consumption

The previous subsection established that beliefs generated with self-attribution tend to become increasingly optimistic over time, increasing surely (in the technical sense) as the number of observations increases without bound. The present subsection analyzes how these beliefs affect consumption over time.

Substituting $\tilde{\theta}_{it}y' + (1-\tilde{\theta}_{it})y$ for $\mathbb{E}_t[y_{i\tau}]$ for all $\tau \geq t$ in expression (5) yields[11]

$$\begin{aligned} c_{it} &= \frac{1}{T+R-t+1}\left(\sum_{\tau=1}^{t} y_{i\tau} - \sum_{\tau=1}^{t-1} c_{i\tau} + \sum_{\tau=t}^{T} \tilde{\theta}_{it}y' + (1-\tilde{\theta}_{it})y\right) \\ &= \frac{\sum_{\tau=1}^{t} y_{i\tau} - \sum_{\tau=1}^{t-1} c_{i\tau} + (T-t+1)[\tilde{\theta}_{it}y' + (1-\tilde{\theta}_{it})y]}{T+R-t+1} \end{aligned}$$

To illustrate this pattern of consumption the analysis will utilize a numerical example, in which the parameter values are as follows: $T = 40$ periods of time over which income is earned and there are $R = 10$ periods of retirement where consumption takes place and income is not earned. Low income $y = 10$, high income $y' = 15$, and each are equally likely in each period, so $\theta_i = 1/2$. Prior

[10] Note that the time derivative is zero under Bayesian updating (when $\delta = 1$), suggesting that beliefs will tend to stay constant over time for such a consumer.

[11] Note also the substitution $y_{it} = \mathbb{E}_t[y_{it}]$.

distribution parameters are $a_i = b_i = 1$, so the prior is a uniform distribution over $[0, 1]$. Importantly, in light of the discussion at the end of the last subsection, the expected value of θ_i given this uniform prior is $1/2$, so the prior estimate is accurate and bias due to the prior distribution's affect on subsequent estimates is eliminated. The combination of $\theta_i = 1/2$ and $a_i = b_i = 1$ implies that $k = 1/2$. The weight consumers put on the likelihood functions associated with the low income level is $\delta = 1/2$, which, since it is less than one, is what drives self-attribution and optimism bias that occurs in this example.

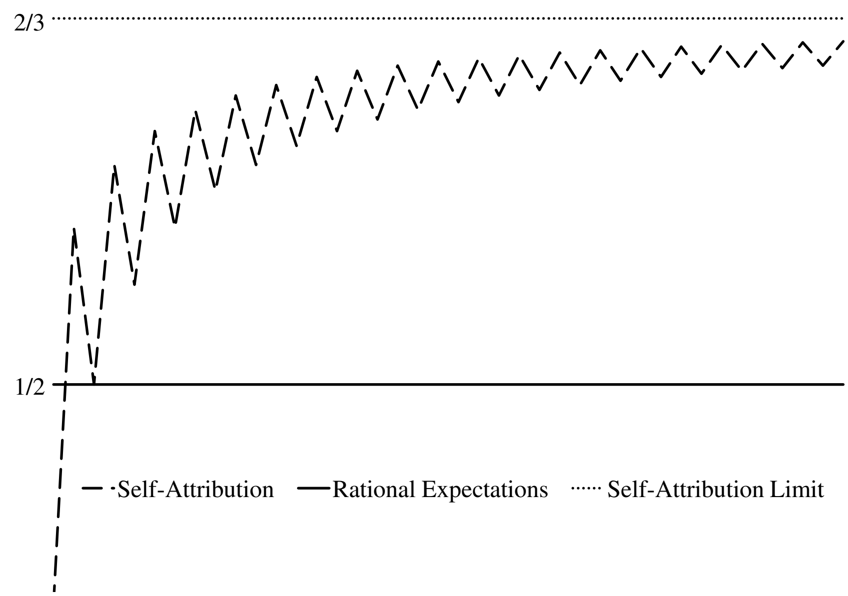
Because it offers clearer insight than a random sequence, consider the case where the income levels $y = 10$ and $y' = 15$ alternate from period to period, starting with $y_{i1} = y$. As the initial income level is low and the weight on that observation is positive, the first-period estimate $\tilde{\theta}_{i1} = 2/5 < 1/2 = \theta_i$ and one could say that the consumer with self-attribution bias is actually *pessimistic* in the first period.[12] This pessimism quickly subsides as the estimate rises substantially in the next period to $\tilde{\theta}_{i2} = 4/7$, falls to $\tilde{\theta}_{i3} = 1/2$, then rises again, remaining greater than the true value $\theta_i = 1/2$, and tending toward the limiting value $\lim_{t\to\infty} \tilde{\theta}_{it} = 2/3$.[13] As such, after the initial period the consumer with self-attribution bias who witnesses this sequence will never believe that the high level of income is less likely than the low level of income, despite the fact that the number of periods during which income is high never outnumbers the number of periods in which income was low.

Figure 1 shows this sequence of beliefs graphically, with time going increasing from left to right, alongside the analogous beliefs of a consumer with rational expectations (i.e. one who knows that $\theta_i = 1/2$). The local maxima (corresponding

[12] Though, compared to a perfect Bayesian with identical priors (who estimates θ_i to be $1/3$), this consumer with self-attribution is not as pessimistic as he or she "should" be.

[13] From expression (8), $\lim_{t\to\infty} \tilde{\theta}_{it} = \frac{\theta_i}{\delta+\theta_i-\delta\theta_i}$. Substituting the numerical values yields $\frac{0.5}{0.5+0.5-0.25} = 2/3$.

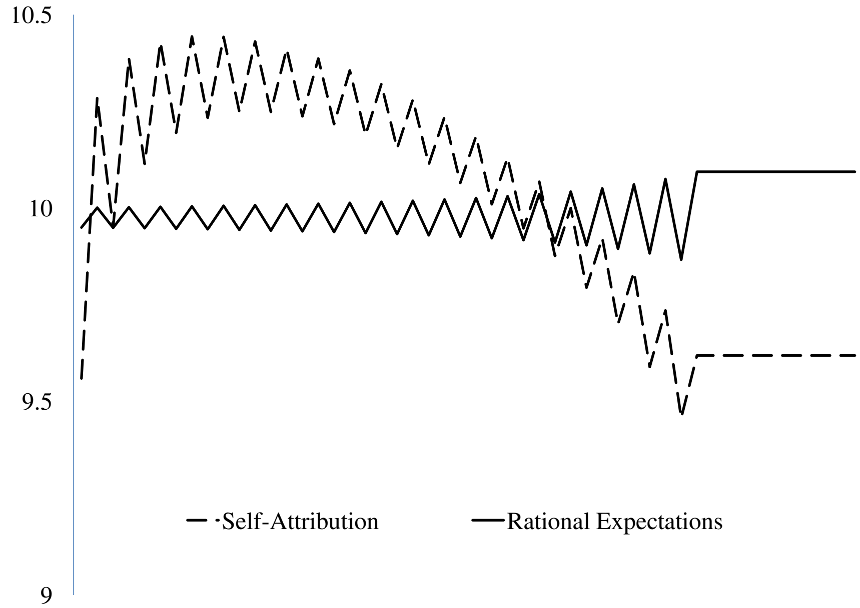
Figure 1: Estimates of θ_i over the First Forty Periods of Life



to when the time index is even-numbered) on the graph for beliefs formed under self-attribution coincide with the approximately typical set of beliefs $\bar{\theta}_{it}$ because at those points it is true that $z_{it} = \theta_i t$. Notice that these local maxima increase monotonically, in agreement with the findings of expression (10).

Figure 2 depicts the sequence of consumption levels corresponding to this alternating sequence of income levels for both a consumer with self-attribution bias and a consumer with rational expectations. As the consumers in this example both earn each of the income levels in 20 periods each during the 40 years of pre-retirement, lifetime income and consumption for these consumers is 500 units. This consumption is spread over 50 periods during which consumption can take place, so consumption will average 10 units per period. As these consumers both at least intend to smooth consumption over these periods, consumption fluctuates

Figure 2: Lifetime Consumption Profiles



around this level of 10 units of consumption.

A glaring disparity between the consumption sequences depicted in Figure 2 is that the consumer with rational expectation has consumption levels that fluctuate consistently about the average level of 10 while the self-attributor has consumption that fluctuates above 10 for low t (other than the first period, because income in that period is low) but then fluctuates below 10 for high t. There is another disparity that is related to the one just mentioned: the rational consumer enjoys a higher level of consumption in retirement than the self-attributor.

These disparities occur because self-attribution bias tends to lead to overly-optimistic expected income in the future, inducing consumers to consume at levels that are not likely to be sustainable. These levels of consumption require the consumer to have lower savings than what would likely be required to have relatively smooth income throughout life, possibly driving the consumer to incur debt which must be paid back. Such low savings early in life will likely necessitate that spending levels fall later on. Taking another look at expression (5), for the consumer with self-attribution bias the expression for savings $\sum_{\tau=1}^{t-1} y_{i\tau} - c_{i\tau}$ will tend to be lower than for the rational consumer, likely causing a drag on consumption and causing it to decrease when after enough time passes and the reality of less-than-anticipated lifetime income sets in. As such, self-attribution bias offers a novel explanation for under-saving and low levels of consumption in retirement.

3.3 Period-to-Period Variation in Consumption

As depicted in Figure 2, these consumption sequences both vary with income levels from period to period. For the consumer with rational expectations, this covariation occurs through one channel, the "direct wealth effect." The direct wealth effect is the change in consumption due to actual income in any given period differing from its expected level. Holding expectations constant, expected lifetime earnings changes by this exact amount. Therefore, for an individual with rational expectations who intends to perfectly smooth consumption over a lifetime, consumption from one period to the next will change by the difference between actual income and expected income in that period divided by the number of periods left in which to consume. This is readily apparent by subtracting the period t version of expression (5) from the period $t+1$ version[14]

$$c_{it+1} - c_{it} = \frac{1}{T+R-t}\left(\sum_{\tau=t+1}^{T} \mathbb{E}_{t+1}[y_{i\tau}] - \mathbb{E}_t[y_{i\tau}]\right), \tag{11}$$

[14]See the appendix for the algebraic details of this.

then substituting the newly-observed value y_{it+1} for $\mathbb{E}_{t+1}[y_{it+1}]$, and imposing that expectations for the observations of future income levels do not change (as is the case with rational expectations), so that $E_{t+1}[y_{i\tau}] = \mathbb{E}_t[y_{i\tau}]$ for $\tau > t+1$, to conclude

$$\text{Rational Expectations} \quad \Longrightarrow \quad c_{it+1} - c_{it} = \frac{y_{it+1} - \mathbb{E}_t[y_{it+1}]}{T + R - t}. \tag{12}$$

For the consumer with rational expectations depicted in Figure 2, taking the absolute value of expression (12) while substituting the numerical values for the relevant terms yields $|c_{it+1} - c_{it}| = \frac{2.5}{50-t}$, from which it is clear that consumption fluctuates with greater magnitude as time passes (as t increases, going from left to right in Figure 2). The interpretation of this is that the difference between the actual income and expected income each period is divided into less remaining periods as time passes.

In contrast, the consumer who forms belief through self-attribution has consumption levels that vary with income through an additional channel: the "updated beliefs effect". This accounts for the effect of the newly-observed level of income on beliefs. For example, if this consumer earns high income (in any period $t < T = 40$) then lifetime expected wealth will increase by that amount, minus the expected level of income for that period, plus the increase in expected future income due to the change in beliefs due to this observation. To see this mathematically, take expression (11) and, to emphasize that these are expectations formed through self-attribution bias, substitute

$$\tilde{\mathbb{E}}_t[y_{i\tau}] \equiv \tilde{\theta}_{it} y' + (1 - \tilde{\theta}_{it}) y, \qquad \text{for all } t \text{ and } \tau > t$$

for $\mathbb{E}_t[y_{i\tau}]$ and then substitute y_{it+1} for $\tilde{\mathbb{E}}_{t+1}[y_{it+1}]$. These yield

$$c_{it+1} - c_{it} = \frac{y_{it+1} - \tilde{\mathbb{E}}_t[y_{it+1}]}{T + R - t} + \frac{\sum_{\tau=t+2}^{T} \tilde{\mathbb{E}}_{t+1}[y_{i\tau}] - \tilde{\mathbb{E}}_t[y_{i\tau}]}{T + R - t}. \tag{13}$$

The term

$$\frac{y_{it+1} - \tilde{\mathbb{E}}_t[y_{it+1}]}{T + R - t} \tag{14}$$

in expression (13) is the direct wealth effect and the term

$$\frac{\sum_{\tau=t+2}^{T} \tilde{\mathbb{E}}_{t+1}[y_{i\tau}] - \tilde{\mathbb{E}}_t[y_{i\tau}]}{T + R - t} \tag{15}$$

represents the updated beliefs effect.

Because income affects consumption through an additional channel (the updated beliefs effect) for the consumer with self-attribution bias, one might expect that changes in consumption and changes in income have a higher degree of covariance for this consumer relative to the rational consumer. Indeed this is the case, as the covariance between changes in income and changes in consumption for the self-attributor is 0.812 and for the rational consumer it is 0.396.

Consider the direct wealth effect for the consumer with self-attribution bias in expression (14). From expression (10), $\tilde{\mathbb{E}}_t[y_{it+1}]$ tends to increase as the consumer with self-attribution bias becomes increasingly optimistic with additional observations. This tends to make the direct wealth effect weaker when the income level is high than the decrease in consumption when income is low. And this phenomenon gets stronger as time proceeds, both because of the fact that $\tilde{\mathbb{E}}_t[y_{it+1}]$ tends to increase and the fact that there are fewer time periods over which to spread the discrepancies between observed income and expected levels of income. One can see how this effect gets larger on the right side of Figure 2, where t is relatively large, as consumption tends to decrease overall.

The updated beliefs effect works somewhat differently than the direct wealth effect. For additional clarity, utilize the fact that this effect can be rewritten

$$\frac{\sum_{\tau=t+2}^{T} \tilde{\mathbb{E}}_{t+1}[y_{i\tau}] - \tilde{\mathbb{E}}_t[y_{i\tau}]}{T+R-t} = \frac{T+R-t-1}{T+R-t}(\tilde{\theta}_{it+1} - \tilde{\theta}_{it})(y' - y)$$

As t increases: $\frac{T+R-t-1}{T+R-t}$ decreases and, since $\tilde{\theta}_{it}$ is bounded and converges to $2/3$ for the parameter values used in this example, $\tilde{\theta}_{it+1} - \tilde{\theta}_{it}$ tends to decrease as the consumer becomes more optimistic at a slower rate.[15] Thus, the updated beliefs effect is greater for relatively low t. Also, because of the nature of self-attribution bias, $\tilde{\theta}_{it+1} - \tilde{\theta}_{it}$ will increase more in a period when income is high than it decreases when income is low. This fact is what drives θ_{it} to tend to grow over time, and one can see it clearly in Figure 1 where any downward movement in θ_{it} is always smaller than the immediately subsequent upward movement. Together, these facts imply that the updated beliefs effect results in larger magnitudes for the changes in consumption for low t, with increases being larger than decreases.

In combination, the direct wealth effect and the updated beliefs effect imply that consumption for the individual with self-attribution bias will tend to rise early in life (for low t) and decline later in life (for high t). Thus, these phenomena, which themselves are implied by self-attributive learning, tend to cause over-consumption.[16]

[15] One can also argue that $\tilde{\theta}_{it+1} - \tilde{\theta}_{it}$ tends to decrease because $\bar{\theta}_{it}$ is monotonically increasing and strictly concave.

[16] It is interesting to note that parsing of the change of consumption into the direct wealth effect and the updated beliefs effect obscures the phenomenon of under-saving until one considers how the direct wealth effect and the updated beliefs effect play out over the lifetime. The under-saving explanation for the broader consumption pattern was obscured in the analysis of period-to-period changes in consumption by the fact that the changes in savings from one period to the next are relatively small, and one can see this mathematically as the savings terms $\sum_{\tau=1}^{t-1} y_{i\tau} - c_{i\tau}$ and $\sum_{\tau=1}^{t} y_{i\tau} - c_{i\tau}$ largely cancel each other out (i.e. they "telescope" away) in the operation of subtracting c_{it} from c_{it+1}.

4 Concluding Remarks

In this study, I show that self-attribution bias leads to two well-known phenomena that are inconsistent with the permanent-income hypothesis: under-saving and excess sensitivity of consumption to income. Because previous explanations of these two phenomena require multiple factors, it is noteworthy that they can now be explained by a single factor embedded within a standard intertemporal choice model. Thus, not only does self-attributive learning represent a novel theory that is capable of explaining some of the most notable stylized facts about consumption, but it also embodies a theory that is more parsimonious than alternatives.

I believe that these theoretical findings warrant empirical investigation. A clear method of testing the theory would be to look at data on performance bonuses and measuring the propensity to consume such bonuses versus income with other labels, as is done in Kooreman, Prast, and Vellekoop (2009). So that there is plenty of opportunity for employees to mistakenly credit themselves for desirable outcomes, it would be particularly valuable to investigate industries in which it is difficult to determine whether performance is due to an employee's ability or outside factors. An example is the performance bonuses of professional traders, who may seem to do well because the broader stock market increases, or simply by luck.

Appendix

Deriving the Expression for $\tilde{\theta}_{it}$

In the following derivation, Γ denotes the gamma function, B denotes the beta function, and we make use of the properties

$$\Gamma(r+1) = r\Gamma(r) \qquad \text{and} \qquad B(r,s) \equiv \frac{\Gamma(r)\Gamma(s)}{\Gamma(r+s)}$$

for all $r, s \in \mathbb{R}_{++}$. Now,

$$\begin{aligned}
\tilde{\theta}_{it} &\equiv \tilde{\mathbb{E}}(\theta_i|y_{i1},\ldots,y_{it}) \\
&\equiv \int_0^1 \theta_i \tilde{\pi}(\theta_i|y_{i1},\ldots,y_{it})\,d\theta_i \\
&= \int_0^1 \frac{\theta_i^{z_{it}+a_i}(1-\theta_i)^{\delta(t-z_{it})+b_i-1}}{\int_0^1 \theta_i^{z_{it}+a_i-1}(1-\theta_i)^{\delta(t-z_{it})+b_i-1}\,d\theta_i}\,d\theta_i \\
&= \frac{B(z_{it}+a+1, \delta(t-z_{it})+b)}{B(z_{it}+a, \delta(t-z_{it})+b)} \\
&= \frac{\Gamma(z_{it}+a+1)\Gamma(\delta(t-z_{it})+b)}{\Gamma(z_{it}+a+1+\delta(t-z_{it})+b)} * \frac{\Gamma(z_{it}+a+\delta(t-z_{it})+b)}{\Gamma(z_{it}+a)\Gamma(\delta(t-z_{it})+b)} \\
&= \frac{\Gamma(z_{it}+a+1)}{\Gamma(z_{it}+a)} * \frac{\Gamma(z_{it}+a+\delta(t-z_{it})+b))}{\Gamma(z_{it}+a+1+\delta(t-z_{it})+b)} \\
&= \frac{a_i+z_{it}}{a_i+b_i+z_{it}+\delta(t-z_{it})}.
\end{aligned}$$

Deriving the Expression for $\frac{\partial \bar{\theta}_{it}}{\partial t}$

$$
\begin{aligned}
\frac{\partial \bar{\theta}_{it}}{\partial t} &= \frac{\partial \frac{\theta_i(k+t)}{k+\delta t+t\theta_i(1-\delta)}}{\partial t} \\
&= \frac{\theta_i[k+\delta t+t\theta_i(1-\delta)] - \theta_i(k+t)[\delta+\theta_i(1-\delta)]}{[k+\delta t+t\theta_i(1-\delta)]^2} \\
&= \frac{\theta_i[k+\delta t+t\theta_i(1-\delta) - (k+t)(\delta+\theta_i(1-\delta))]}{[k+\delta t+t\theta_i(1-\delta)]^2} \\
&= \frac{\theta_i[k-k\delta-k\theta_i(1-\delta)]}{[k+\delta t+t\theta_i(1-\delta)]^2} \\
&= \frac{k\theta_i[1-\delta-\theta_i(1-\delta)]}{[k+\delta t+t\theta_i(1-\delta)]^2} \\
&= \frac{k\theta_i(1-\theta_i)(1-\delta)}{[k+\delta t+t\theta_i(1-\delta)]^2}.
\end{aligned}
$$

Deriving the Expression for $c_{it+1} - c_{it}$

The period $t+1$ version of expression (5) is

$$
c_{it+1} = \frac{1}{T-t}\left(\sum_{\tau=1}^{t} y_{i\tau} - c_{i\tau} + \sum_{\tau=t+1}^{T} \mathbb{E}_{t+1}[y_{i\tau}]\right). \tag{16}
$$

To obtain an expression for $c_{it+1}-c_{it}$, one can add and subtract the sum $\sum_{\tau=t}^{T} \mathbb{E}_t[y_{i\tau}]$ within expression (16) to obtain

$$
c_{it+1} = \frac{1}{T-t}\left(\sum_{\tau=1}^{t} y_{i\tau} - c_{i\tau} + \sum_{\tau=t}^{T} \mathbb{E}_t[y_{i\tau}] + \sum_{\tau=t+1}^{T} \mathbb{E}_{t+1}[y_{i\tau}] - \sum_{\tau=t}^{T} \mathbb{E}_t[y_{i\tau}]\right),
$$

pull $y_{it} - c_{it}$ from $\sum_{\tau=1}^{t} y_{i\tau} - c_{i\tau}$ and y_{it} from $\sum_{\tau=t}^{T} \mathbb{E}_t[y_{i\tau}]$, and then utilize expression (5) to substitute in $(T - t + 1)c_{it}$, yielding

$$\begin{aligned} c_{it+1} &= \frac{1}{T-t}\left((T-t+1)c_{it} + y_{it} - c_{it} - y_{it} + \sum_{\tau=t+1}^{T} \mathbb{E}_{t+1}[y_{i\tau}] - \mathbb{E}_t[y_{i\tau}]\right) \\ &= c_{it} + \frac{1}{T-t}\left(\sum_{\tau=t+1}^{T} \mathbb{E}_{t+1}[y_{i\tau}] - \mathbb{E}_t[y_{i\tau}]\right), \end{aligned}$$

which implies that

$$c_{it+1} - c_{it} = \frac{1}{T-t}\left(\sum_{\tau=t+1}^{T} \mathbb{E}_{t+1}[y_{i\tau}] - \mathbb{E}_t[y_{i\tau}]\right).$$

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