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Abstract

Although the Structural Economic Dynamic approach provides a simultaneous consideration of demand and supply sides of economic growth, it does not fully take into account the possible role played by demand in the generation of technical progress. From a neo-Kaldorian perspective, this paper seeks to establish the concepts of demand and productivity regimes in an open version of the pure labour Pasinettian model. In order to derive the demand regime, a disaggregated version of the Keynesian multiplier is derived for an open economy, while the productivity regime is built in terms of disaggregated Kaldor-Verdoorn laws. The upshot is a multi-sector growth model of structural change and cumulative causation, in which an extended version of the Pasinettian model to foreign trade may be obtained as a particular case. Furthermore, we show that the evolution of demand patterns, while being affected by differential rates of productivity growth in different sectors of the economy, also play an important role in establishing the pace of technical progress.

JEL Classifications: O19, F12.

Keywords: Cumulative causation, structural change, Kaldor-Verdoorn law.

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1. Introduction

While structural change and economic growth register as interrelated processes, the mainstream assigns a key role to issues such as technical progress and capital accumulation, relegating structural change to a secondary position in explaining economic growth. The traditional Neoclassical approach, with its emphasizes on the supply side, and originally built in terms of one or two sector models [see e.g. Solow (1956) and Uzawa (1961)] cannot take into account the possible links between growth and changes in the structure of an economy\(^1\). According to this view, changes in structure are simply a by-product of the growth in per capita gross domestic product (GDP) [see McCombie (2006) and McMillan and Rodrik (2011)].

This perspective can be sharply contrasted with the post-Keynesian view, where structural change is central to economic development. Different approaches have taken into account the connections between growth and change in this tradition, with particular emphasis on the role played by demand, even in the long run [see e.g. Pasinetti (1981, 1983), Setterfield (2010), Thirlwall (2013) and Ocampo et al. (2009)\(^2\)]. Within this tradition, the Structural Economic Dynamic (SED) view is distinguishable by its simultaneous consideration of supply and demand in a multi-sectoral framework; in particular, the interaction between the evolving patterns of demand and technical progress is responsible for particular dynamics of output, prices and structural

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\(^1\) See the introductory chapter of Arena and Porta (2012) for a survey on the state of the art of the literature on structural change after the renewal interest by the mainstream.

\(^2\) According to these views, structural changes register not just a by-product of growth as claimed by the mainstream but rather play a central role in spurring growth. The migration of the labour force from diminishing returns activities to increasing return activities may be one of the outcomes of proper structural changes that allow developing economies to grow faster.[see McMillan and Rodrik (2011)].
transformation of economies in different stages of the development process. In this regard, Pasinetti’s emphasis upon demand composition offers a significant qualitative improvement vis-a-vis traditional, aggregated models, which fail to adequately consider the composition of consumption demand, and thus conceal changes in structure.

Although the SED approach provides a sophisticated treatment of structural change, some authors have pointed to the necessity of a more inclusive treatment of the demand side in order to provide a full characterization or even endogenisation of technical progress and structural change. Gualerzi (2012) for instance notes that the SED is an approach rooted in the theory of demand-led growth insofar as demand matters shape how supply factors and technical change in particular will evolve not only in the short run but also in the long run. But elsewhere the author states that: “the integration of the demand side into the analysis of growth, which is potentially the most fruitful step forward, does not lead to an analysis of the endogenous growth mechanisms because of a fully inadequate theory of demand” [Gualerzi (1996, p. 157)].

According to that view, demand still plays a somewhat passive role in the SED approach since increases in per capita income are motivated by technical change, which is wholly exogenous. Admittedly, being the focus of Pasinetti’s analysis on the effect

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3 Pasinetti (1993, p.69) himself acknowledges the importance of considering a better treatment of the demand side when questing the origins of technological progress. According to him: “[t]his means that any investigation into technical progress, must necessarily imply some hypothesis on the evolution of consumers’ preferences as income increases. Not to make such an hypothesis, and to pretend to discuss technical progress without considering the evolution of demand, would make it impossible to evaluate the very relevance of technical progress and would render the investigation itself meaningless.”

4 This view is also emphasized by Silva and Teixeira (2008, p.286) : “Although Pasinetti relates both factors with the learning principle, learning itself is essentially unexplained and therefore the question of what moves the driving forces of the economy remains unanswered.”
of productivity growth differentials over sectoral dynamics, exogenous technical progress hinders a deeper understanding of the endogenous growth mechanisms. In this vein, if on the one hand the Pasinettian model emphasizes the main channels of interdependence between economic growth and structural change, on the other hand, it overlooks the emphasis of the demand-led-growth theory in which consumption and growth feedback in a cumulative process.

Hence, the SED approach in its original formulation is unable to take into account a deeper conception of endogenous technical change according to which the rate of technical progress is sensitive to the rate of output growth. That view of endogenous growth is emphasized by the Neo-Kaldorian literature, which assigns to demand a central role in generating technical progress through the Kaldor-Verdoorn law. [Roberts and Setterfield (2007)].

In this article we intend to fill that gap by by building a bridge between the SED formulation and the Neo-Kaldorian theory. To accomplish this task, we conceptualize the notion of a demand regime that departs from a multi-sectoral version of the Keynesian multiplier in an extended version of Pasinetti’s pure labour model that takes into account international trade. Trigg and Lee (2005) have shown that it is possible to

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5 There have been some developments of the neo-Kaldorian tradition related to models of balance of payments-constrained growth (BPCG). Araujo and Lima (2007) and Araujo (2013), for instance, have derived versions of the balance of payment constrained growth rates that explain the growth performance by considering that the evolution of patterns of consumption plays a crucial role in the performance of the external sector, and as a consequence for the overall economy.

6 Pasinetti (2005, p. 839-40) assigns to a lack of theoretical cohesion amongst models in the Keynesian tradition as a possible explanation for the difficulties faced in providing a successful alternative paradigm to mainstream economics.
derive a simple multiplier relationship from multisectoral foundations in the original version of the Pasinetti model, meaning that a scalar multiplier can legitimately be applied to a multisector economy.

Here we use the Trigg and Lee (2005) analysis as a crucial step to establish the links with the Neo-Kaldorian literature. But we have to extend their analysis to an economy with foreign trade, since the Neo-Kaldorian view assigns to exports a key role in autonomous aggregate demand. According to that view, the dynamism of the export sector may give rise to virtuous cycles of economic growth not only through its effect on aggregate demand but also due to dynamic economies of scale\(^7\) that accrue from an increase in output.

Hence, the first contribution of this paper is the derivation of the multi-sectoral Keynesian multiplier in the extension of Pasinetti’s model to an open-economy. This derivation allows us to derive a proper demand regime for the model. The sectoral productivity regime departs from Araujo (2013), where sectoral Kaldor-Verdoorn’s law were introduced in Pasinetti’s model. With this analysis, we are able to introduce the concepts of growth regimes [see Blecker (2010)] in the SED approach, which also allows us to endogenise technical progress in this model. Moreover, it affords a connection between many of the arguments that underpin the importance of the endogenous concept of economic growth.

\(^7\) Cornwall and Cornwall (2002, p. 206) highlighted these mechanisms by considering that the contribution of the external sector to productivity growth is twofold: first it allows the larger scale production methods to improve productivity; second it encourages the adoption of the best available productivity-enhancing technologies.
The second contribution of the paper shows that the extended version of the Pasinettian model to foreign trade, derived by Araujo and Teixeira (2004), may be seen as a particular case of the multi-sectoral Keynesian approach derived here. While the Pasinettian solution holds as potential or equilibrium production, the solution derived here registers as effective production, the latter being equal to the former when the condition of full employment of the labour force is satisfied\(^8\). As a consequence, it is shown that the extended version of the Pasinetti model to foreign trade generates different levels of employment, only one of which will be the full employment level.

In order to emphasize this point, we carry out the formulation of a sectoral demand regime both in terms of effective and equilibrium sectoral output. The first analysis is developed under the label of Sectoral Demand Regime – SDR hereafter – while the latter is referred as the Structural Economic Dynamic Regime (SEDR). Notwithstanding the Neo-Kaldorian emphasis on the role of effective demand in interacting with productivity in a cumulative sense, the derivation of the SEDR allows us to take into account the role that potential demand plays in generating technical change, without denying the main role of effective demand.

Besides, it allows us to show that the Neo-Kaldorian analysis may also reap benefits from a disaggregated refinement of its basic framework. Even departing from a somewhat narrower view of cumulative causation based on Adam Smith dictum that “the division of labour is limited by the extent of the market” – which emphasizes the sectoral aspect of dynamic increasing returns of scale – we arrive at a Macroeconomic notion, in which technical change in one sector spurs productivity in other sectors

\(^8\) This registers as a well-known result in the SED framework, and one of the main outcomes of the Pasinettian analysis is that in general it is not fulfilled, meaning that unemployment is the most probable outcome of structural change.
through its effect on per capita income growth [see Young (1928)]. Central to this development is the concept of Engel’s law, according to which an evolving pattern of consumption arises when per capita income grows.

This article is structured as follows: in the next section we present the derivation of the multi-sector multiplier for an open version of the pure labour Pasinettian model. In the third section the demand and productivity regimes are modeled in the Pasinettian framework. Section 4 introduces the notion of a SED regime and section 5 concludes.

2. The Derivation of the Multi-sectoral Multiplier for an open economy

In order to develop a DR in the Pasinettian approach, we depart from Trigg and Lee (2005)\(^9\), who derived a multisectoral version of the Keynesian multiplier. This is a natural step since the DR in the Neo-Kaldorian model is developed in terms of the growth rates of the output given by Keynesian multiplier. But due to the importance of foreign demand in the Neo-Kaldorian literature we go a step further by developing an extended version of the disaggregated Keynesian multiplier\(^10\) that takes into account international trade. Let us consider an extended version of the Pasinettian model to foreign trade as advanced by Araujo and Teixeira (2004). According to these authors the system of physical quantities may be expressed as:

\(^9\)The idea of developing a multi-sectoral version of the Keynesian multiplier dates back to Goodwin (1949) and Miyazawa (1960) who accomplished to develop a disaggregated version of the income multiplier in Leontief’s framework from the relatively simple Keynesian structure. Both authors emphasized that although there are important differences between the Keynes and Leontief approaches, a bridge between them, namely a disaggregated version of the multiplier, is an important development for both views.

\(^10\) The procedure adopted here is similar to the Pasinettian analysis.
where $X_i$ denotes the domestic physical quantity produced of consumption good $i$, $i = 1, 2, \ldots, n - 1$, and $X_n$ represents the quantity of labour in all internal production activities; per capita demand of consumption goods is represented by a set of consumption coefficients: both $a_{in}$ and $a_{i\hat{n}}$ stand for the demand coefficients of final commodity $i$, $i = 1, 2, \ldots, n - 1$. The former refers to domestic and the latter to foreign demand. The production coefficients of consumption goods are represented by $a_{ni}$, $i = 1, 2, \ldots, n - 1$. The family sector in country $A$ is denoted by $\hat{n}$ and the population sizes in both countries are related by the coefficient of proportionality $\xi$. The first $(n - 1)$ equations in the above system refer to the equilibrium in the consumption goods sectors: all production, $X_i$, is either consumed internally, $a_{in}X_n$, or abroad, $\xi a_{i\hat{n}}X_n$. And, the expression in the last line of system (1) refers to the equilibrium in the labour market. The quantity of labour in all internal production activities, $X_n$, should be employed either in one of the consumption goods sectors, $a_{ni}X_i$. This characterization of the equilibrium does not mean that it will hold throughout the period covered by the analysis but Pasinetti (1981) assumes that it holds at time zero. The above system may be written in matrix form as:

$$
\begin{bmatrix}
1 & -(c + \xi c) \\
-a & 1 \\
\end{bmatrix}
\begin{bmatrix}
X \\
X_n \\
\end{bmatrix} =
\begin{bmatrix}
0 \\
0 \\
\end{bmatrix}
$$

(2)
where \( \mathbf{I} \) is an \((n-1)\times(n-1)\) identity matrix, \( \mathbf{X} = \begin{bmatrix} X_1 \\ \vdots \\ X_{n-1} \end{bmatrix} \) is the \((n-1)\) column vector of physical quantities, \( \mathbf{c} = \begin{bmatrix} a_{1,1} \\ \vdots \\ a_{n-1,1} \end{bmatrix} \) is the \((n-1)\) column vector of consumption coefficients, 

\[
\hat{\mathbf{c}} = \begin{bmatrix} a_{1,\hat{1}} \\ \vdots \\ a_{n-1,\hat{1}} \end{bmatrix}
\]

refers to the \((n-1)\) column vector of foreign demand coefficients, and 

\( \mathbf{a} = \begin{bmatrix} a_{1,1} & \cdots & a_{n-1,1} \end{bmatrix} \) is the \((n-1)\) row vector of labour coefficients. System (2) is a homogenous and linear system and, hence a necessary condition to ensure non-trivial solutions of the system for physical quantities is:

\[
\begin{vmatrix} \mathbf{I} & - (\mathbf{c} + \xi \hat{\mathbf{c}}) \\ -\mathbf{a} & 1 \end{vmatrix} = 0
\]

Condition (3) may be equivalently written as:

\[
\mathbf{a}(\mathbf{c} + \xi \hat{\mathbf{c}}) = 1
\]

By using summations it is possible to rewrite expression (4) as [see Araujo and Teixeira (2004)]:

\[
\sum_{i=1}^{n-1} a_{m}(a_{m} + \xi a_{m}) = 1
\]

If condition (5) is fulfilled then there exists solution for the system of physical quantities in terms of a exogenous variable, namely \( \bar{X}_n \). In this case, the solution of the system for physical quantities may be expressed as:

\[
\begin{bmatrix} \mathbf{X} \\ \bar{X}_n \end{bmatrix} = \begin{bmatrix} \mathbf{c} + \xi \hat{\mathbf{c}} \end{bmatrix} \bar{X}_n
\]
In order to particularize the production in one of the countries let us introduce the superscript $U$ do denote the components of vector $X$ in the underdeveloped country, according to:

$$X_i^U = (a_i^U + \xi a_i^U)X_n$$  \hfill (7)

From (7), we conclude that in equilibrium the physical quantity of each tradable commodity to be produced in country $U$, that is $X_i^U$, $i = 1, \ldots, n - 1$, will be determined by the sum of the internal, namely $a_i^U X_n$, and foreign demand, namely $\xi a_i^U X_n$. It is important to emphasize that solution (7) holds only if condition (5) is fulfilled. If (5) does not hold, then the non-trivial solution of physical quantities cannot be given by expression (7). In order to find meaningful solutions, let us rewrite the system of physical quantities as:

$$\begin{bmatrix} \mathbf{I} & -\mathbf{c} \\ -\mathbf{a} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ \mathbf{X}_n \end{bmatrix} = \begin{bmatrix} \mathbf{E} \\ 0 \end{bmatrix}$$ \hfill (8)

Where $\mathbf{E} = \xi \mathbf{X}_n \hat{\mathbf{c}}$ denotes the sectoral exports. We may rewrite system (8) as

$$\begin{cases} \mathbf{X} - \mathbf{cX}_n = \mathbf{E} \\ -\mathbf{aX} + \mathbf{X}_n = 0 \end{cases}$$ \hfill (9)

From the last line of system (9), it follows that:

$$\mathbf{X}_n = \mathbf{aX}$$ \hfill (10)

---

11 Dealing with the original Pasinettian model, Trigg and Lee (2005) had to assume that investment in the current period becomes new capital inputs in the next period and that the rate of depreciation is 100% (that is, all capital is circulating capital) in order to derive the Keynesian multiplier. By considering an economy extended to foreign trade we do not need this hypothesis.
By pre-multiplying throughout the first line of (9) by $a$, one obtains:

$$aX - acX_n = aE$$

(11)

By substituting (10) into expression (11), and isolating $X_n$, we obtain the employment multiplier relationship:

$$X_n = \frac{1}{1-ac}aE$$

(12)

where $1/(1-ac)$ is a scalar employment multiplier [Trigg and Lee (2005)]. Through further decomposition [see Trigg (2006, Appendix 2)], (12) can be substituted into the first line of (9) to yield:

$$X = \left( I + \frac{ca}{1-ac} \right)E$$

(13)

This is a multiplier relationship between the vector of gross outputs, $X$, and the vector representing final demand $E$, where $\left( I + \frac{ca}{1-ac} \right)$ is the output multiplier matrix.

One of the main differences between this multi-sectoral multiplier for an open economy and the one derived by Trigg and Lee is that the latter is a scalar, and the former is a matrix. We can rewrite expression (13) as:

$$\begin{bmatrix} X_1 \\ \vdots \\ X_{n-1} \\ X_n \end{bmatrix} = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n-1,1} & \cdots & a_{n-1,n-1} \\ a_{n,1} & \cdots & a_{n,n-1} \end{bmatrix} \begin{bmatrix} E_1/1-ac \\ \vdots \\ E_{n-1}/1-ac \\ E_n/1-ac \end{bmatrix} + \begin{bmatrix} E_1 \\ \vdots \\ E_{n-1} \\ E_n \end{bmatrix}$$

(14)

The multiplier relationship for the $i$-th sector therefore takes the form:

$$X_i = E_i + \left( \frac{ca}{1-ac} \right)E$$

(15)
Where $e_i = a_{in}$. Expression (15) may be rewritten as:

$$X_i = E_i + \frac{\begin{vmatrix} a_{in} & a_{ni} \\ \vdots & \vdots \\ a_{in} & a_{n,n-1} \end{vmatrix} E}{1 - ac}$$  \hspace{1cm} (15)'

The sectoral physical solution derived from the multi-sectoral Keynesian multiplier corresponds to the effective sectoral production, which contrasts with the equilibrium sectoral production, namely expression (7). Now it follows from (15)', for the underdeveloped country $U$:

$$X_i^U = \xi d_{in}^U X_n + \left\{ \frac{1}{1 - \sum_{j=1}^{n-1} a_{nj}^U a_{jn}^U} \left[ \sum_{j=1}^{n-1} a_{nj}^U a_{jn}^U \xi d_{jn}^U X_n \right] \right\}$$  \hspace{1cm} (16)

After some algebraic manipulation it is possible to rewrite expression (16) as:

$$X_i^U = \left\{ \frac{\sum_{j=1}^{n-1} \xi d_{nj}^U a_{jn}^U}{a_{in}^U \left[ 1 - \sum_{j=1}^{n-1} a_{nj}^U a_{jn}^U \right]} \right\} X_n + \xi d_{in}^U X_n$$  \hspace{1cm} (16)'

Expression (16)' plays a central role in our analysis. It shows that the effective demand for output of the $i$-th sector is due to two components: the domestic demand, conveyed by the domestic consumption coefficient $a_{in}^U$, and external demand, portrayed by the foreign demand coefficient $a_{in}^U$. Due to reasons that will become clearer latter, the domestic coefficient is affected by the structural economic dynamics of the economy as a whole, captured by the quotient: \( \frac{\sum_{j=1}^{n-1} \xi d_{nj}^U a_{jn}^U}{1 - \sum_{j=1}^{n-1} a_{nj}^U a_{jn}^U} \). This quotient particularizes the solution obtained here, given by expression (16)', from the solution (7) derived by
Araujo and Teixeira (2004) for an open version of the Pasinettian model. While the latter refers to sectoral equilibrium production the former registers as effective sectoral production.

We can prove that solution (7) is a particular case of solution (16)’ when condition (5) holds. In other words, the equilibrium solution is a particular case of the effective solution, given by multi-sectoral Keynesian multiplier, when the full employment condition is satisfied. Then we have the following:

**Proposition 1**

Expression (7) is a particular case of expression (16)’ when expression (5) holds.

**Proof.**

The proof is straight. If condition (5) holds then rearranging it we obtain:

$$\sum_{j=1}^{n-1} \xi a_{nj} a_{jn} = 1 - \sum_{j=1}^{n-1} a_{nj} a_{jn}.$$  

By replacing this result into numerator of the first term of the high rand side of expression (16)’, namely

$$X_i^U = \left( \frac{\sum_{j=1}^{n-1} \xi a_{nj} a_{jn}}{1 - \sum_{j=1}^{n-1} a_{nj} a_{jn}} + \xi a_{in}^U \right) X_n ,$$

it yields:

$$X_i^U = (a_{in}^U + \xi a_{in}^U ) \bar{X}_n ,$$

which is expression (7). □

Proposition 1 shows that the solution put forward by Araujo and Teixeira (2004) for an open version of the Pasinetti model is in fact a particular case of the solution obtained here. That result is of key importance. One of the central results of the SED analysis [See Pasinetti (1981, 1993)] is that even departing from an equilibrium position, where full employment prevails, condition (5)’ will not hold in the long run due to the particular dynamics of technical progress and evolution of demand for each
sector. It means that, in general, we should expect that: \( \sum_{i=1}^{n-1} a_{ni}(a_{in} + \xi a_{in}) < 1 \). We may consider a symmetrical case, namely \( \sum_{i=1}^{n-1} a_{ni}(a_{in} + \xi a_{in}) > 1 \), which corresponds to the case of overemployment. Then we have the following proposition:

**Proposition 2**

If \( \sum_{i=1}^{n-1} a_{ni}(a_{in} + \xi a_{in}) < 1 \) then effective production is smaller than equilibrium production. Otherwise, effective production is larger than equilibrium production.

**Proof.**

If \( \sum_{i=1}^{n-1} a_{ni}(a_{in} + \xi a_{in}) < 1 \), then it is possible to show after some algebraic manipulation that: 
\[
1 - \sum_{j=1}^{n-1} a_{nj} a_{jn} > \sum_{j=1}^{n-1} \xi a_{nj} a_{jn}.
\]
As a consequence, \( \frac{\sum_{j=1}^{n-1} \xi a_{nj} a_{jn}}{1 - \sum_{j=1}^{n-1} a_{nj} a_{jn}} < 1 \), and the sectoral output solution (16)’ obtained from the multi-sectoral Keynesian multiplier is smaller than the sectoral production from the SED approach (7). Now if \( \sum_{i=1}^{n-1} a_{ni}(a_{in} + \xi a_{in}) > 1 \)
\[
\sum_{j=1}^{n-1} \xi a_{nj} a_{jn} < 1.
\]
In this case, solution (16)’, namely the sectoral effective production, is larger than the corresponding sectoral equilibrium production. □

In sum, we should expect that the sectoral effective output will gravitate around the equilibrium output. In the Pasinettian analysis the first case, namely
\[ \sum_{i=1}^{n-1} a_{ni}(a_m + \xi a_m) < 1, \] 
receives more attention since one of the probable outcomes of 
structural change is structural unemployment. In the next sections we will make use of 
the results (7) and (10)’ to derive growth regimes in the SED approach.

3. Macroeconomic Regimes in a Structural Economic Dynamic Approach

3.1. The Sectoral Demand Regime

The traditional Neo-Kaldorian growth schema [see McCombie and Thirlwall (1994) and 
Setterfield and Cornwall (2002)] is presented in terms of both demand – DR hereafter – and 
productivity regimes – PR hereafter. The latter is portrayed by a Kaldor-Verdoorn 
function while the former is depicted by the effects of growth rate of exports – and in 
some cases the growth rate of autonomous investment – on the growth rate of output via 
aggregate demand.

In order to determine the sectoral growth rate of output from expression (16)’ we 
have to specify the dynamic path of terms of trade since price competitiveness plays a 
crucial role in the theory of cumulative causation. Following Araujo (2013) let us 
consider that \( p_i^U \) and \( p_i^A \) stand for prices of the \( i \)-th consumption good in countries \( U \) 
and \( A \), respectively. By considering that \( e \) stands for the nominal exchange rate in the \( U \) 
country, let us also consider that per capita export coefficient \( a_{ni}^U \) is given according to:

i) On one hand, if \( ep_i^A < p_i^U \), that is, if country \( U \) has no comparative cost advantage in 
the production of consumption good \( i \), then the per capita foreign demand for good \( i \) is 
assumed to be zero: \( a_{ni}^U = 0 \). If \( ep_i^A \geq p_i^U \), then let us consider that the foreign demand
for the consumption good $i$ is given by an export function à la Thirlwall (1979) [see Araujo and Lima (2007)]:

$$
a_{in}^U = \left( \frac{p_i^U}{e p_i^A} \right)^{\eta_i} y_A^\beta_i X_n^{1-\beta_i} \tag{17}
$$

where $y_A$ denotes the per capita income of country $A$ and $e$ stands for the the population of the $A$ country, denoted by $X_n$. $\eta_i$ designates a price elasticity of demand for exports of good $i$, with $\eta_i < 0$. While $\beta_i$ denotes an income elasticity of demand for exports, and with $\beta_i > 0$. According to this specification, it is not assumed ex-ante full specialization.

ii) On the other hand, if country $A$ has no comparative cost advantage in the production of consumption good $i$, we assume country $U$ does not import it, that is, $a_{in} = 0$, where $a_{in}$ stands for the per capita import coefficient for good $i$. But if $p_i^U \geq e p_i^A$, let us consider that the demand coefficients for imports are given by the following import function:

$$
a_{in} = \left( \frac{e p_i^A}{p_i^U} \right)^{\phi_i} y_U^\phi_i X_n^{\phi_i-1} \tag{18}
$$

where $\phi_i$ is the price elasticity of the demand for imports of good $i$, with $\phi_i < 0$, $\phi_i$ is the income elasticity of the demand for imports of good $i$ and $y_U$ is the per capita income of country $U$. Following Pasinetti (1981), the coefficient of internal demand is assumed to vary according to:

$$
a_{in}^U(t) = a_{in}^U(0) \exp(r_i^U t) \tag{19}
$$

where $r_i^U$ stands for the growth rate of domestic demand of good $i$ in the $U$ country. In what follows let us assume that the evolution of consumption patterns is endogenous.
considering that the growth rate of sectoral demand is a function, not only of technical coefficients, \( a_{ni}^U \), but also of their variations. From expressions (17) and (19) and by adopting the following convention: 
\[
\frac{\dot{p}_{i}^U}{p_{i}^U} = \sigma_{i}^U, \quad \frac{\dot{p}_{i}^A}{p_{i}^A} = \sigma_{i}^U, \quad \varepsilon = \epsilon, \quad \dot{\gamma}_{A} = \sigma_{i}^A, \quad \text{and} \quad \dot{X}_{\hat{\lambda}} = \hat{g} 
\]
we conclude that the growth rate of foreign and home demand for consumption good \( i \) are given respectively by:

\[
\frac{\dot{a}_{ni}^U}{a_{ni}^U} = \eta_i (\sigma_{ni}^U - \sigma_{ni}^A - \epsilon) + \beta_i \sigma_{i}^A + (1 - \beta_i) g 
\]  
(20)

\[
\frac{\dot{a}_{ni}^U}{a_{ni}^U} = r_{i}^U 
\]  
(21)

In what follows, let us consider that the growth rate of foreign demand for the \( i \)-th consumption good is denoted by \( \dot{r}_{i} = \eta_i (\sigma_{ni}^U - \sigma_{ni}^A - \epsilon) + \beta_i \sigma_{i}^A + (1 - \beta_i) g \). Following Araujo and Teixeira (2004) domestic and foreign prices are given by:

\[
p_{i}^U (t) = \frac{\dot{a}_{ni}^U}{a_{ni}^U} (t) w_{ni}^U 
\]  
(22)

\[
p_{i}^A (t) = \frac{\dot{a}_{ni}^U}{a_{ni}^U} (t) w_{ni}^A 
\]  
(23)

Where \( w_{ni}^U \) and \( w_{ni}^A \) stand for the wages in countries \( U \) and \( A \), respectively, and \( \frac{\dot{a}_{ni}^A}{a_{ni}^A} (t) \) stands for the labour coefficient of the \( i \)-th sector in country \( A \). According to this formulation, prices are given by the costs of production. By taking logs, and differentiating these expressions in relation to time, we obtain the dynamics of prices as given by:

\[
\sigma_{i}^U = \sigma_{w}^U - \rho_{i}^U 
\]  
(24)

\[
\sigma_{i}^A = \sigma_{w}^A - \rho_{i}^A 
\]  
(25)
Where $\sigma^U$ and $\sigma^A$ stand for the growth rates of wages in countries $U$ and $A$ respectively, $\rho^U_i$ is the rate of technical progress in $i$-th sector of $U$ country and $\rho^A_i$ represents technical progress in $i$-th sector of country $A$. The dynamics of technical coefficients, namely $a^U_{ni}$ and $a^A_{ni}$, in countries $U$ and $A$ are given respectively as:

$$a^U_{ni}(t) = a^U_{ni}(0)e^{-\rho^U_i t}$$  \hspace{1cm} (26)$$

$$a^A_{ni}(t) = a^A_{ni}(0)e^{-\rho^A_i t}$$  \hspace{1cm} (27)$$

Hence:

$$\frac{\dot{a}^U_{ni}}{a^U_{ni}} = -\rho^U_i$$  \hspace{1cm} (26)'$$

$$\frac{\dot{a}^A_{ni}}{a^A_{ni}} = -\rho^A_i$$  \hspace{1cm} (27)'$$

By taking logs and differentiating expression (16)' and considering expressions (200, (21), (24), (25), (26)' and (27)' it is possible to obtain the growth rate of the production of the $i$-th sector as:

$$\frac{\dot{X}_i^U}{X_i^U} = \Pi_i^U \left( \left( \frac{r^U_i}{\sigma^U} + g \right) + \varepsilon a^U_{ni} X_n \left( 1 - \sum_{j=1}^{n-1} a^U_{nj} a^U_{jn} \left( \sum_{j=1}^{n-1} \frac{r^U_j}{\sigma^U} + r^U_i + g - \rho^U_j \right) d^U_{nj} a^U_{jn} X_n \right) \right) +$$

$$+ \frac{\sum_{j=1}^{n-1} \left( \frac{r^U_j}{\sigma^U} - \rho^U_j \right) a^U_{nj} a^U_{jn}}{1 - \sum_{j=1}^{n-1} a^U_{nj} a^U_{jn}}$$  \hspace{1cm} (28)$$

Where $\Pi_i^U = \frac{\varepsilon a^U_{ni} X_n}{\varepsilon a^U_{ni} X_n + \left( 1 - \sum_{i=1}^{n-1} a^U_{ni} a^U_{ni} \right)^{-1} \left( \sum_{j=1}^{n-1} a^U_{nj} a^U_{jn} a^U_{jn} X_n \right)}$. 

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In order to make a parallel with the Neo-Kaldorian literature, in what follows let us rewrite expression (28), namely $\frac{\dot{X}_i^U}{X_i^U}$ as a linear function of technical progress of the $i$-th sector:

$$\frac{\dot{X}_i^U}{X_i^U} = \Gamma_{SDR} \rho_i^U + \Omega_{SDR} \tag{29}$$

Where:

$$\Gamma_{SDR} = -\Pi \eta_i + \frac{(1 - \Pi_i) a_{in} a_{mi}^U}{\sum_{j=1}^{n-1} a_{nj} a_{mj}^U} \left(1 - \frac{1}{\sum_{j=1}^{n-1} a_{nj} a_{mj}^U} \right) \hat{z}_{in} X_n + \sum_{j=1}^{n-1} a_{nj} a_{mj}^U$$

$$\Omega_{SDR} = \Pi \left[ \eta_i (\rho_i^A - \varepsilon) + \beta \sigma_y^A + g \right] + \frac{1}{1 - \sum_{j=1}^{n-1} a_{nj} a_{mj}^U} \left[ (1 - \Pi_i) \sum_{j=1}^{n-1} (r_{ij} - \rho_j^U) a_{nj} a_{mi}^U - r_i^U a_{ni} a_{mi}^U \right] + \eta_i (\rho_i^A - \varepsilon) \sum_{j=1}^{n-1} \eta_i (\rho_j^A - \rho_j^U - \varepsilon) + \beta_i \sigma_y^A + g \right)$$

Expression (29) is the sectoral counterpart of the DR, derived from a multi-sectoral Keynesian multiplier. We label this solution as the Sectoral Demand Regime (SDR), and it expresses the growth rate of the $i$-th sector as a function of technical progress. In order to fully determine the pace of technical progress and the growth rate of demand for the $i$-th sector, we also have to develop the notion of a productivity regime in a multi-sectoral set-up. We accomplish this task in the next subsection.

### 2.2. The Sectoral Productivity Regime

In order to establish the sectoral counterpart of the PR, namely a sectoral productivity regime – SPR hereafter – let us assume following Araujo (2013) that the
sectoral growth rate of productivity is given by sectoral Kaldor-Verdoorn laws. According to this view, the dynamic economies of scale result from the increasing specialization of labor provided by sectoral market growth, and from the productivity gains that accrues from the learning by doing. Hence:

$$\rho_i^U = \frac{q_i^U}{q_i^V} = \gamma_i^U + \alpha_i^U \frac{\dot{X}_i^U}{X_i^U}$$  \hspace{1cm} (31)$$

Where $\rho_i^U$ is the rate of technical progress in $i$-th sector of $U$ country, $\gamma_i^U$ is the intercept of the Verdoorn relation, and $\alpha_i^U$ poses itself as the Verdoorn coefficient. According to this view, it does not matter if the production increases occur at the firm level – that is, if they are restricted to one of the firms in a sector – or if they are widespread amongst firms. Both the individual firm and the aggregated sectoral production play an important role in the generation of sectoral productivity gains.

Expression (25) may be rewritten as:

$$\frac{\dot{X}_i^U}{X_i^U} = \frac{\gamma_i^U}{\alpha_i^U} + \frac{1}{\alpha_i^U \rho_i^U}$$  \hspace{1cm} (31)'$$

Expression (31)' plays the role of a PR in our formulation. By equalizing expression (31)' to (29), namely the SDR to SPR, it is possible to obtain after some algebraic manipulation the rate of technical progress in the $i$-th sector as:

$$\left(\rho_i^U\right)_{SDR} = \frac{\gamma_i^U + \alpha_i^U \Omega_{SDR}}{1 - \alpha_i^U \Gamma_{SDR}}$$  \hspace{1cm} (32)$$

Expression (32) conveys one of the important outcomes of this analysis, namely the endogenisation of technical progress in the SED model. This analysis has been suggested by Araujo (2013) who introduced sectoral Kaldor-Verdoorn’s laws in the
SED approach. But his analysis departs from sectoral equilibrium conditions in the Pasinettian model and not from a notion of aggregate demand. As a result the pace of technical progress is established but not in terms of the effective production as in the Neo-Kaldorian set up. Here, the pace of technical progress is determined according to the sectoral effective demand, which makes our analysis closer to the cumulative model. [See Dixon and Thirlwall (1975)].

One important and key property of expression (32) is that technical progress in the $i$-th sector, that is $\rho_i$, is affected by technical progress in other sectors, namely $\rho_j$. This raises an important property of the model: when demand is fully taken into account in Pasinetti’s model, it highlights the role of productivity spillovers emphasized by the Neo-Kaldorian literature. The straight effect of an increase in $\rho_j$ is to increase $\rho_i$, meaning that positive effects of technical progress in the $j$-th sector will not be restricted to that sector, but will affect the generation of technical progress in other sectors$^{12}$.

The rationale behind this interaction may be grasped by considering that technical progress in the $j$-th sector has a negative effect on the price of good $j$. A smaller price for good $j$ is translated in terms of higher purchasing power, which may be unevenly spent on consumption of other goods, let us say $i$. A higher level of consumption for good $i$ means, through the Kaldor-Verdoorn relation, a higher level of technical progress for the $i$-th sector$^{13}$. By substituting expression (32) into expression (31) we obtain the growth rate of production of the $i$-th sector in the $U$ country:

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$^{12}$ This can be grasped from expression (30).

$^{13}$ Note that this property was not evinced in the SED version of the endogenised technical progress derived by Araujo (2013).
\[
\frac{\dot{X}_i^U}{X_i^U} = \frac{\Gamma_{SDR} \dot{Y}_i^U + \Omega_{SDR}}{1 - \alpha_i^U \Gamma_{SDR}}
\]  

(33)

The analysis here is similar to the aggregated model. Since we are focusing on a sectoral aspect of the dynamics, let us consider as a device the case in which \( r_j = \rho_j^U = 0, \forall j \neq i \). By following this approach we obtain \( \Omega_{SDR} \) and \( \Gamma_{SDR} \) as constants and a graphical approach may be adopted. In this case, \( \Omega_{SDR} \) may be rewritten as:

\[
\Omega_{SDR} = \prod_i \left[ \eta_i \left( \rho_i^A - \varepsilon \right) + \beta_i \sigma_i^A + g \right] + \\
\left[ 1 - \frac{\sum a_m a_m^U}{\sum a_m a_m^U} \right] \frac{\left( \prod_i - \sum a_m a_m^U \right) + \sum \eta_i \left( \rho_i^A - \varepsilon \right) + \beta_i \sigma_i^A + g}{\left( 1 - \sum a_m a_m^U \right) \left( \sum a_m a_m^U \right) \left( \sum a_m a_m^U \right) + \xi a_m X_n}
\]

Hence we plot the SDR and SPR in a graph as follows:

---

14 Although this case is unrealistic it may evince the properties of our model. Note that Pasinetti (1993) considers in his structural economic dynamics as the first approximation the case in which \( r_i = \rho_i^U, \forall i \).
The interpretation of this graph is similar to the traditional Neo-Kaldorian models. If we start with values of $\rho_i^u$ and $\frac{\dot{X}_i}{X_i}$ below their equilibrium values, then the $i$-th sector experience a rate of output growth that will induce the pace of technical progress, leading to higher price competitiveness that by its turn increase the exports. This will lead to a higher rate of sectoral output growth that will induce more productivity gains and further gains in terms of price competitiveness and export performance.

According to this view, structural changes are triggered by exogenous demand that induce technical progress through increasing returns of scale and learning-by-doing. The consequent increase in per capita income due to the raise in productivity will turn into an increase into per capita demand that may also induce more technical progress. In some moment of this virtuous cycle, structural changes are made endogenous. With this approach we overcome one of the shortcomings of the SED approach as pointed out by Gualerzi (2001, p. 26): “[i]n Pasinetti’s scheme, since the very source of income growth, technical change, is itself fully exogenous, potential demand is identified only with available disposable income; as such it is a passive notion”.

4. A Structural Economic Dynamic Regime

Note then that we have two possibilities for the production of sector $i$. First, we have obtained a production that is derived under the hypothesis of equilibrium or full employment, given by expression (7). The second solution, given by expression (16)’, is derived from a multi-sectoral Keynesian multiplier for an open economy. This stands
for the actual or effective production while expression (16)’ stands for the equilibrium output for the \( i \)-th sector.

With the expression of the equilibrium output in hands, that is expression (7), it is possible to derive the growth rate of potential sectoral output – what we call here as our SEDR in contrast to the SDR. By taking logs and differentiating expression (7) it is possible to obtain the growth rate of the production of the \( i \)-th sector as:

\[
\frac{\dot{X}_i^U}{X_i^U} = \theta_i^U \frac{\dot{a}_m^U}{a_m^U} + (1 - \theta_i^U) \frac{\dot{a}_{ji}^U}{a_{ji}^U} + \frac{\dot{X}_n}{X_n}
\]

(34)

Where \( \theta_i^U = \frac{a_m^U}{a_m^U + \xi a_{ji}^U} \) stands for the share of internal demand in total demand of good \( i \), \( 0 \leq \theta_i^U \leq 1 \). By inserting (20) and (21) into expression (34), we obtain after some algebraic manipulation the growth rate of potential output for the \( i \)-th sector as:

\[
\frac{\dot{X}_i^U}{X_i^U} = \theta_i^U r_i^U + (1 - \theta_i^U) \left[ \eta_i (\sigma_i^U - \sigma_i^A - \varepsilon) + \beta_i \sigma_{y_i}^A + (\beta_i - 1) g \right] + g
\]

(34’)

By adopting the same procedure of the previous section, from expression (34’), we can write the growth rate of output in the \( i \)-th sector as a function of technical progress in that sector. Hence expression (34’) may be rewritten as:

\[
\frac{\dot{X}_i^U}{X_i^U} = \Gamma_{SED} \rho_i^U + \Omega_{SED}
\]

(35)

where: \( \Gamma_{SED} = -(1 - \theta_i^U) \eta_i \) and \( \Omega_{SED} = \theta_i^U r_i^U + (1 - \theta_i^U) \left[ \eta_i (\gamma_i^A + \alpha_i^A \lambda_i^A \sigma_i^A - \varepsilon) + \beta_i \sigma_{y_i}^A \right] \).

By replacing expression (31), which represents the SPR, into expression (35), we obtain after some algebraic manipulation the growth rate of productivity in the \( i \)-th sector:
Expression (36) yields the pace of technical progress by considering the interaction between SPR with SEDR following Araujo (2013). By substituting expression (36) into expression (35) we obtain after some algebraic manipulation, the equilibrium growth rate of output under the SEDR regime:

\[
\left( \rho_i^U \right)_{SED} = \frac{\gamma_i^U + \alpha_i^U \Omega_{SED}}{1 - \alpha_i^U \Gamma_{SED}}
\]  

(36)

It is worth recalling that while the derivation of the SDR is based on actual production, the derivation of SEDR is based on potential or full employment production, we should expect at least gravitation of the production under SDR around production under SEDR in the short run. But in the long run, we should expect that the growth rate of production given by expressions (33) and (37) should be equal, that is

\[
\left( \frac{\dot{X}_i^U}{X_i^U} \right)_{SDR} = \left( \frac{\dot{X}_i^U}{X_i^U} \right)_{SED}.
\]

The graph below illustrates this point. Although the intercepts and slopes of the SPR and SEDR are different, there is a point in which they coincide and this corresponds to the long run solution. Following this rationale, the pace of technical progress under SEDR and SDR should be equal in the long run.
At this point it is important to consider an important difference between expressions (33) and (36). While the parameters that enter expression (36) are wholly exogenous, the technical progress of other sectors, namely $\rho_j^U$, that enter expression (33) are not exogenous. Hence, expression (33) generates a system of $n - 1$ variables and equations. If on one hand, this system is useful to evince the connections amongst technical progress in different sectors, on the other hand, the task of determining technical change for a specific sector from effective demand becomes cumbersome. In order to alleviate this difficulty we can use the pace of technical progress reckoned by SED regime, since its determination is straighter\textsuperscript{15}. This point highlights the importance

\textsuperscript{15} The value of $\left(\rho_i^U\right)_{sdr}$ may be endogenised if we consider that the rate of technical progress is given by $\left(\rho_i^v\right)_{sed}$.  

26
of the concept of potential output to establish the growth path in a multi-sectoral economy, thus emphasizing the importance of Pasinettian contribution.

When demand in a particular sector is fostered, the productivity in that sector is spurred due to the Kaldor-Verdoorn effect. But higher productivity is translated into higher real wages, which may give rise to further increases in demand, but not necessarily in demand for the good that kick started the process. Sectors producing goods with higher income elasticity of demand tend to increase their share in national income insofar as per capita income grows. Hence, those sectors will also enjoy higher rates of technical progress following the cumulative rationale. Finally, the present approach stresses that the triggering point of this virtuous cycle is external demand, but once it is under way, indigenous demand may expand and may also be an important component to spur growth. In this vein a vigorous strategy of export led growth may play an important role to trigger the virtuous cycle motioned by cumulative causation.

5. Concluding Remarks

Notwithstanding Pasinetti’s emphasis on the evolving patterns of demand within a multi-sectoral framework, demand still plays a somewhat passive role in his approach to the extent that its evolution registers as a function of technical progress, which is wholly exogenous. In this vein, although the original SED approach provides a simultaneous approach of demand and supply sides of economic growth, it does not take into account the role played by cumulative causation in the generation of technical progress. The present analysis aims to join these lines of research on structural factors in a more fully specified multi-sectoral framework and, in which demand interacts with technical progress.
With this inquiry we have introduced concepts such as demand and productivity regime in an open version of Pasientti’s model, by showing that indeed it can be treated as a particular case of the multi-sectoral version of the Keynesian multiplier for an open economy. That was proven to be a required step to formulate a proper notion of demand regime in the SED framework. Besides, by considering the interaction between demand and productivity regimes, it was possible not only to endogenise technical progress in the Pasinettian approach but also to highlight the spillover connections between technical progress in different sectors.

If on one hand, endogenous technical progress is required to properly explain the evolving patterns of demand, on the other hand, the evolution of demand is seen as a function of the technical conditions. In this respect, a Neo-Kaldorian approach to the SED is convenient since it allows us to evince the connections between demand and technical change through the use of the cumulative causation concept.

If on the SED front, the gains from considering Neo-Kaldorian concepts are pervasive, also in the Neo-Kaldorian view we may reap some benefits from the cross-fertilization between these two strands. They accrue mostly from the use of a disaggregated model embedded with sectoral Kaldor-Verdoorn’s law, thus emphasising the connections between demand and productivity growth not only in an aggregated but also in a disaggregated level. Following this view, once there is an exogenous increase of demand in a particular sector, the productivity increases gives rise to per capita income gains that will be translated into higher demand. This higher per capita income may be translated into higher demand for goods with higher income elasticity of demand.
One strength of the approach presented here is its emphasis on the role played by demand in the process of economic growth. According to this view, demand cannot be limited to drive structural changes, but it should also be considered as one of the engines of economic growth via its effect on stimulating the creation and diffusion of technical progress.

References


