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Abstract

This paper analyzes provision of a differentiated public good within an organization. A moderate principal assigns a public good production to one of two extreme agents. A contributing agent then gets the opportunity to choose a public good variety he prefers but has to carry a cost of production. If a production cost is lower than a benefit from having their preferred public good variety implemented then the agents seek assignment. I show that in this case the principal makes the agents compete by committing to public good varieties they would provide if selected. The agents want to make themselves an attractive choice and so announce moderate (still divergent) varieties if production is costly, and the principal’s preferred variety if production is not costly. However, if the production cost exceeds the benefit from having their preferred public good variety implemented then the agents want to avoid assignment. My results suggest that in this case the principal just assigns an unpopular public good production to a less extreme agent.

JEL classification: H41.

Keywords: Differentiated Public Goods; Public Good Provision; Spatial Competition.

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1. Introduction

This paper addresses the question of provision of a differentiated public good within an organization. Think of this as departmental service in academia, for example. That might include committee work, seminar organizing, chair assignments, etc. All members of the organization prefer the public good to be provided rather than not, but might disagree about certain dimension or variety of the public good. In the context of the academia example, this might reflect research field or methodological specialization which affects the focus of seminar series or hiring priorities. A principal (department chair or median faculty member) assigns the public good provision to one of the agents (say, junior faculty members). A contributing agent then gets the opportunity to provide a public good variety he prefers. However, the public good provision involves certain costs for a contributing agent. At the same time, there might be certain benefits too. In the context of the previous example, running department seminar series requires time but also implies greater academic visibility (useful at earlier career stages) and probably teaching load reduction. Moreover, being a "good citizen" is always appreciated in academic departments and might be taken into account during tenure evaluation. Therefore, provision of some differentiated public goods implies net costs for a contributing agent while provision of others implies net benefits. The agents then tend to avoid providing some public goods but value (and therefore compete for) the opportunity to provide others. What variety of the differentiated public good will be provided in such an institutional environment?

To address this question, I develop a spatial model in which a principal assigns provision of a differentiated public good to one of two agents. The principal prefers a moderate variety of the public good while the agents have more extreme opposite preferences. The public good provision implies either net cost or net benefit for a contributing agent. Whether it is costly or not, as well as the principal’s and the agents’ preferences, is common knowledge.

The principal can adopt different selection procedures to choose an agent for the public good provision. Here, I analyze two selection processes commonly used within organizations. Under the first, referred as appointment, the principal simply evaluates the agents given their preferences, and selects a contributor on the basis of this. Intuitively, under appointment, a selected agent would implement his preferred public good variety. The principal therefore appoints an agent with more moderate preferences over the differentiated public good. Under the second selection procedure, referred as competition, the agents commit to public good varieties they would provide if selected. The principal thus selects an agent who announced a more moderate variety of the public good.
Consider now the agents’ incentives in the competition stage. I show that if the net cost of public good provision exceeds the distance between the agents’ most preferred varieties of the public good then the agents want to avoid being selected. Intuitively, in this case each agent wants his counterpart to provide her preferred public good variety rather than to incur a high cost of providing his own. Therefore, the agents will make themselves an unattractive choice by announcing extreme varieties of the public good. As a result, the principal will prefer appointment to competition.

However, if the net cost of public good provision is lower than the distance between the agents’ most preferred varieties then the agents value the opportunity to provide this public good. Indeed, each agent prefers to incur a relatively low cost of public good provision rather than to let his counterpart implement her preferred option. I show that in this case, there is a unique equilibrium in the competition stage. If one agent is extreme and the other agent is relatively more moderate, this is an equilibrium with asymmetric announcements in which a more moderate agent announces his preferred variety and gets selected, while a more extreme agent announces any variety from a certain equilibrium interval. The principal is then indifferent between competition and appointment. If the agents’ bliss points are extreme, this is an equilibrium with symmetric announcements in which each agent gets selected with probability one half and the announced varieties are more moderate than the bliss points. The principal therefore prefers competition to appointment.

My results emphasize an important feature of competition procedure – announcement divergence in the case of costly public good provision. Indeed, a contributing agent would bear a cost of public good provision only if his gains in terms of a public good variety are large enough, implying that his announced variety is quite different from that of the other agent. Another important characteristic of competition procedure is the existence of equilibrium with asymmetric announcements in which one agent announces his bliss point and gets selected. Intuitively, if he announces a variety different from his bliss point then he can profitably deviate to its direction and still get selected for the public good provision. Thus, in an equilibrium with asymmetric announcements a selected agent necessarily announces his most preferred variety.

Finally, if the public good provision implies net benefit for a contributing agent then the agents compete for the opportunity to provide this public good. Then in the competition stage, the agents will make themselves an attractive choice by announcing a preferred option of the principal. Actually, the present setting then simplifies to a classical spatial model with policy- and office-motivated agents analyzed by Wittman (1990) and Calvert (1985), among many others. In equilibrium, both agents announce a preferred variety of the principal and
each of them gets selected with equal probability. The principal thus prefers competition to appointment.

My results therefore suggest that competition procedure is preferred in the situations in which the costs of public good provision are lower than the agents’ benefits from implementing a variety close to their bliss point. In turn, appointment is preferred in the cases in which the public good provision is relatively expensive and the costs exceed the agents’ benefits from implementing a variety close to their bliss point.

Due to the nature of the public goods under consideration, I consider a somewhat restricted space of instruments available to the principal – she can just set up a contest but cannot offer a contract for provision of the public goods. To this extent, the paper is related to the literature on tournaments and contests, which addresses the issue of contest design (see Konrad 2009 for an introduction to this vast literature). The paper is also related to the literature on spatial political competition going back to the seminal work of Downs (1957), who emphasized platform convergence in a framework with two office-motivated political candidates. A further step was taken by Wittman (1977, 1983, 1990), Calvert (1985) and Roemer (1994), who considered policy- and office-motivated candidates. It has been shown that under full commitment, two policy- and office-motivated candidates announce convergent platforms if the distribution of the voters’ ideal policies is known (Wittman 1977, Calvert 1985, Roemer 1994, Bernhardt et al. 2009, Saporiti 2010). The present paper actually uses these results for the case in which public good provision implies net benefits for a contributing agent. However, the case of spatial competition with net costs which I model here, has not been analyzed in this literature, to the best of my knowledge.

This paper also contributes to the literature on voluntary provision of public goods which goes back to the pioneering work of Samuelson (1954, 1955). More recent classical references on pure public good provision include Bergstrom et al. (1986), Andreoni (1988), Cornes and Sandler (1996), among many others. The net benefit case studied here is also related to the literature on impure public good provision which assumes that agents gain certain private benefits from their own contribution (see Cornes and Sandler 1984, 1994, Glazer and Konrad 1986, Holländer 1990, Harbaugh 1998, among many others). However, the present paper departs from a standard model of public good provision and analyzes a setting with differentiated public good in which agents differ in their preferences over a public good variety to be provided. Differentiated public goods have been studied by Economides and Rose-Ackerman (1993) to model situations in which citizens have varying tastes for public services. They demonstrate that privatization of differentiated public good production is not optimal as it leads to too many producers supplying too much output (as compared to
the socially optimal outcome). In contrast to their research, I disregard privatization issues and concentrate instead on the question of assignment of public good production within organizations.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 proceeds with the formal analysis. Finally, Section 4 concludes the paper.

2. Model

Consider a public good provision within organizations. Suppose moreover that a public good under consideration, denoted by $x$, is differentiated and the set of feasible outcomes is a closed interval $[0, 1]$.

A principal assigns the public good provision to one of the agents. I consider a benchmark case with two agents here. The principal and the agents strictly prefer the public good to be provided rather than not to be provided. However, they differ in their preferences over variety of the public good. In particular, I assume that the principal’s bliss point is $\frac{1}{2}$ while the agents’ bliss points $\alpha_1$ and $\alpha_2$ are such that $\alpha_1 < \frac{1}{2} < \alpha_2$. The agents thus have opposite preferences over the differentiated public good.

The principal and the agents have Euclidean preferences over $x$. Formally, their utility from the differentiated public good $x$ (given bliss point $\chi$) is

$$-|x - \chi|.$$

Thus, they want the public good variety to be close to their bliss point.

A contributing agent incurs a net cost of public good provision, denoted by $C \in \mathbb{R}$. Negative $C$ means that the agent actually benefits from the public good provision. I assume that the cost $C$ and the agents’ bliss points $\alpha_1$ and $\alpha_2$ are common knowledge.

The principal can adopt different selection procedures to choose an agent to provide the public good. Here, I consider two simple and rather common selection processes. Under the first, referred as appointment, the principal simply evaluates the agents’ profiles (i.e., their bliss points) and chooses a public good contributor on the basis of this. Under the second, referred as competition, the principal makes the agents compete by asking about a variety of the differentiated public good they would provide if selected. I assume full commitment here such that once selected, a contributing agent implements the public good variety he has chosen in the competition stage.\(^1\)

\(^1\)One can also assume that if a contributing agent deviates from his announcement he loses credibility and therefore carries a reputational cost, which exceeds potential benefits from deviation.
The timing of events is as follows. First, the principal decides which selection procedure to adopt, appointment or competition. In the case of appointment, she selects one of the two agents and assigns the public good provision to him. The selected agent then provides the public good. In the case of competition, the agents announce to the principal which variety of the public good they would choose. The principal then judges the agents based on their announcements and selects one of them for the public good provision. Finally, the selected agent implements his announcement.

I search for a subgame perfect equilibrium by analyzing the game backwards. I consider the agents’ and the principal’s decisions under appointment procedure first, and under competition procedure second. I turn then to the principal’s decision regarding the selection process. Finally, I discuss robustness of my results.

3. Analysis

3.1. Appointment

Under appointment procedure, the principal simply selects one of the agents to provide the public good. The analysis is straightforward in this case. I study the game backwards and start with a contributing agent’s problem.

Agent’s problem Denote by $x_i$ a differentiated public good provided by agent $i = 1, 2$. If selected for the public good contribution, agent $i$ chooses $x_i$ to maximize his net payoff given by

$$-|x_i - \alpha_i| - C.$$ 

Obviously, the contributing agent then implements his own bliss point, $x_i = \alpha_i$.

Principal’s problem I turn now to the principal’s appointment problem. Given that once selected, an agent sticks to his bliss point, the principal then appoints an agent whose bliss point is closer to hers. Formally, the principal selects agent 1 if $\alpha_1 + \alpha_2 > 1$; agent 2 if $\alpha_1 + \alpha_2 < 1$; and is indifferent between the two agents if $\alpha_1 + \alpha_2 = 1$. The principal’s utility is then

$$-|\alpha_i - \frac{1}{2}|,$$

where $\alpha_i$ is a bliss point of the contributing agent.

I study next the case in which the principal adopts competition procedure to select a contributing agent.
3.2. Competition

Under competition procedure, the principal simply asks the agents which variety of the differentiated public good they would choose if selected. Then the principal selects one of the agents to implement his announcement.

Principal’s problem Consider first the principal’s problem. Intuitively, the principal assigns the public good provision to an agent whose announced variety is closer to her bliss point $\frac{1}{2}$. Now $x_i$ denotes the announcement of agent $i$. Then the probability of agent 1 being selected for the public good provision is

$$p_1(x_1, x_2) = \begin{cases} 
1 & \text{if } x_1 < x_2 \text{ and } x_1 + x_2 > 1, \text{ or } x_1 > x_2 \text{ and } x_1 + x_2 < 1, \\
\frac{1}{2} & \text{if } x_1 = x_2, \text{ or } x_1 \neq x_2 \text{ and } x_1 + x_2 = 1, \\
0 & \text{if } x_1 < x_2 \text{ and } x_1 + x_2 < 1, \text{ or } x_1 > x_2 \text{ and } x_1 + x_2 > 1.
\end{cases}$$

The probability of agent 2 being selected is

$$p_2(x_1, x_2) = 1 - p_1(x_1, x_2).$$

As in the case of appointment, the principal’s decision is rather "mechanical" here. Once the agents announce public good varieties $x_1$ and $x_2$, the outcome of the selection process is decided.

Agents’ problem The agents announce $x_i$ to maximize their expected net payoffs $\Pi_i(\cdot)$ given by

$$\Pi_1(x_1, x_2) = p_1(x_1, x_2)(-|x_1 - \alpha_1| - C) + p_2(x_1, x_2)(-|x_2 - \alpha_1|),$$

$$\Pi_2(x_1, x_2) = p_1(x_1, x_2)(-|x_1 - \alpha_2|) + p_2(x_1, x_2)(-|x_2 - \alpha_2| - C).$$

I search for a pure strategy Nash equilibrium $(x_1^*, x_2^*)$ such that

$$\Pi_1(x_1^*, x_2^*) \geq \Pi_1(x, x_2^*) \quad \forall x \in [0,1],$$

$$\Pi_2(x_1^*, x_2^*) \geq \Pi_2(x_1^*, x) \quad \forall x \in [0,1].$$

Nonpositive cost Consider first the case in which the net cost of public good provision is nonpositive, $C \leq 0$. This actually means that the public good provision either implies net benefit for a contributing agent (if $C < 0$) or at least is not costly for him (if $C = 0$).
Contributing to the public good provision then becomes valuable to the agents. The model is therefore reduced to a standard setting with policy- and office-motivated agents analyzed by Wittman (1990) and Calvert (1985), and predicts convergence to the bliss point of the principal, \( x_1^* = x_2^* = \frac{1}{2} \). Each agent is then selected for the public good provision with probability one half. Intuitively, an agent realizes that in order to get selected, he has to sacrifice his bliss point and to announce a public good variety which the principal would prefer to the other agent’s announced variety. This drives convergence in equilibrium. The principal’s utility is equal to 0 in this case.

Positive cost I turn now to the case in which the net cost is positive, \( C > 0 \). The public good provision then becomes unpopular. The agents value the assignment only as a means of implementing a public good variety close to their bliss points. Therefore, convergence does not occur in equilibrium. Indeed, no agent agrees to carry a cost of provision in exchange for a public good variety that can be implemented by the other agent. The following proposition emphasizes the non-existence of equilibrium with convergence.

**Proposition 1.** When the public good provision implies net costs for a contributing agent, convergence does not occur in equilibrium.

**Proof.** This proposition is easily proved by contradiction. Suppose that in equilibrium the agents announce the same public good variety \( x \in (0, 1) \). Each of them is selected with probability one half and obtains a payoff of \(-|x - \alpha_i| - \frac{C}{2}\), \( i = 1, 2 \). Each agent, however, has an incentive to deviate in order not to get selected and to obtain a payoff of \(-|x - \alpha_i|\), saving the expected net cost \( \frac{C}{2} \). Therefore, \((x, x)\) is not an equilibrium. If the agents announce the same extreme varieties \((0, 1)\) then an agent deviating from that extreme variety will be selected with probability 1 or with probability \( \frac{1}{2} \). Suppose that in equilibrium the agents announce the same variety 0. The payoff of agent \( i \) in this case is equal to \(-\alpha_i - \frac{C}{2}\). Agent 2, however, is better off deviating to \( x_2 = 1 \). This gives agent 2 a payoff of \(-\frac{1}{2} - \frac{C}{2}\), which is strictly greater than \(-\alpha_2 - \frac{C}{2}\) (since \( \alpha_2 > \frac{1}{2} \)). Thus, \((0, 0)\) is not an equilibrium. By analogy, \((1, 1)\) is not an equilibrium as agent 1 is better off deviating to \( x_1 = 0 \); this yields a payoff of \(-\frac{1}{2} - \frac{C}{2}\), which is strictly greater than \( \alpha_1 - 1 - \frac{C}{2} \) (since \( \alpha_1 < \frac{1}{2} \)). There is therefore no equilibrium with convergence of announced public good varieties.

Proposition 1 stresses an important feature of the positive cost case – a lack of announcement convergence. It implies therefore that in equilibrium the agents announce divergent varieties. In what follows, to prevent bizarre outcomes (such as agent 1’s announcing agent
2’s bliss point or vice versa), I restrict the set of choices available to the agents in the following way: \( x_1 \in [0, \frac{1}{2}] \), \( x_2 \in [\frac{1}{2}, 1] \).

Consider now the case in which the net cost of public good provision is larger than the length of the set of feasible outcomes, i.e., \( C > 1 \). For such large \( C \), each agent prefers any variety provided by the other agent to the cost of providing public good himself. He thus has an incentive to announce the most extreme variety from his set of available outcomes. It is easy to show that for \( C > 1 \), there is a unique equilibrium in which the agents announce extreme symmetric varieties \((0, 1)\) and each gets selected with probability one half. The principal’s expected utility is equal to \(-\frac{1}{2}\).

Consider next the non-trivial case in which the net cost of public good provision does not exceed the length of the set of feasible outcomes, \( C \leq 1 \). The following proposition characterizes an equilibrium in which the agents announce symmetric (around \( \frac{1}{2} \)) varieties and each gets selected with probability one half. (The proof of this and other propositions can be found in the Appendix.)

**Proposition 2.** There is a unique equilibrium with symmetric varieties \((\frac{1-C}{2}, \frac{1+C}{2})\) when \( C \in (0, 1) \), \( \alpha_1 \in [0, \frac{1-C}{2}] \), \( \alpha_2 \in [\frac{1+C}{2}, 1] \). There is a unique equilibrium with symmetric varieties \((0, 1)\) when \( C = 1 \), \( \alpha_1 \in [0, \frac{1}{2}] \), \( \alpha_2 \in (\frac{1}{2}, 1] \).

Therefore, in this equilibrium, when \( C \in (0, 1) \) the agents announce more moderate varieties than their bliss points \( \alpha_1 \) and \( \alpha_2 \). The announcements are symmetric around \( \frac{1}{2} \) and at a distance of \( C \) from each other. Each agent gets selected with probability one half. The expected utility of the principal is \(-\frac{C}{2}\). The expected payoff for agent \( i \) is equal to \(-\left| \frac{1}{2} - \alpha_i \right| - \frac{C}{2}\). No agent wants to deviate by announcing a more moderate variety and getting selected for the public good provision. The reason is that the gains in terms of implemented variety (which are less than \( \frac{C}{2} \)) do not compensate the losses in terms of net cost \( \frac{C}{2} \). Neither agent gains by announcing a more extreme variety and not being selected. This is because the gains in terms of net cost \( \frac{C}{2} \) are equal to the losses in terms of implemented variety of the public good. When \( C = 1 \), the agents announce the most extreme varieties \((0, 1)\). In this case, no agent wants to deviate by announcing a less extreme variety and getting selected, as the losses in terms of net cost \( \frac{C}{2} = \frac{1}{2} \) would exceed the gains in terms of the public good variety. The principal’s expected utility is given by \(-\frac{1}{2}\) in this case.

I turn now to the characterization of equilibria in which the agents announce asymmetric varieties and one agent gets selected with probability one. The following lemma establishes an important property of equilibria with asymmetric announcements.
**Lemma 1.** In an equilibrium with asymmetric announcements, the selected agent necessarily announces his most preferred variety of the public good.

**Proof.** This lemma is easily proved by contradiction. Suppose that there is an equilibrium in which the selected agent, say \(i\), announces a variety \(x_i^* \neq \alpha_i\). Agent \(i\), however, can always profitably deviate to the direction of his bliss point \(\alpha_i\) by a small positive number \(\varepsilon\) and still get selected. It follows that in an equilibrium with asymmetric varieties, the selected agent announces his bliss point. ■

An equilibrium \((x_1^*, x_2^*)\) in which the agents announce asymmetric varieties must therefore have one of the two following structures:

1. \((\alpha_1, x_2^*)\) such that \(x_2^* > 1 - \alpha_1\). Agent 1 gets selected, i.e., \(p_1(\alpha_1, x_2^*) = 1\). The principal’s utility is \(\alpha_1 - \frac{1}{2}\).

2. \((x_1^*, \alpha_2)\) such that \(x_1^* < 1 - \alpha_2\). Agent 2 gets selected, i.e., \(p_2(x_1^*, \alpha_2) = 1\). The principal’s utility is \(\frac{1}{2} - \alpha_2\).

The following proposition characterizes equilibria with asymmetric announcements.

**Proposition 3.** For the following values of \(C, \alpha_1, \alpha_2\) and \(x_i^*\), there is an equilibrium with asymmetric announcements \((\alpha_1, x_2^*)\) in which agent 1 gets selected for the public good provision:

- \(C \in (0, 1), \quad \alpha_1 \in (\max \{0, \frac{1-2C}{2}\}, \frac{1-C}{2})\), \(\alpha_2 \in (\frac{1}{2}, \alpha_1 + C]\), \(x_2^* \in (1 - \alpha_1, 1]\);
- \(C \in (0, 1), \quad \alpha_1 = \frac{1-C}{2}\), \(\alpha_2 \in (\frac{1}{2}, 1]\), \(x_2^* \in (1 - \alpha_1, 1]\);
- \(C \in (0, 1), \quad \alpha_1 \in (\frac{1-C}{2}, \min \{\frac{1}{2}, 1 - C\})\), \(\alpha_2 \in (\frac{1}{2}, 1]\), \(x_2^* \in [\alpha_1 + C, 1]\);
- \(C \in (0, 1), \quad \alpha_1 \in \left[\min \left\{\frac{1}{2}, 1 - C\right\}, \frac{1}{2}\right], \quad \alpha_2 \in (\frac{1}{2}, 1]\), \(x_2^* = 1\);
- \(C = 1, \quad \alpha_1 \in (0, \frac{1}{2})\), \(\alpha_2 \in (\frac{1}{2}, 1]\), \(x_2^* = 1\).

For the following values of \(C, \alpha_1, \alpha_2\) and \(x_i^*\), there is an equilibrium with asymmetric announcements \((x_1^*, \alpha_2)\) in which agent 2 gets selected:

- \(C \in (0, 1), \quad \alpha_2 \in \left(\frac{1}{2}, \max \left\{\frac{1}{2}, C\right\}\right]\), \(\alpha_1 \in [0, \frac{1}{2})\), \(x_1^* = 0\);
- \(C \in (0, 1), \quad \alpha_2 \in \left(\max \left\{\frac{1}{2}, C\right\}, \frac{1+C}{2}\right]\), \(\alpha_1 \in [0, \frac{1}{2})\), \(x_1^* \in [0, \alpha_2 - C]\);
- \(C \in (0, 1), \quad \alpha_2 = \frac{1+C}{2}\), \(\alpha_1 \in [0, \frac{1}{2})\), \(x_1^* \in [0, 1 - \alpha_2]\);
- \(C \in (0, 1), \quad \alpha_2 \in \left(\frac{1+C}{2}, \min \{1, \frac{1+2C}{2}\}\right]\), \(\alpha_1 \in [\alpha_2 - C, \frac{1}{2})\), \(x_1^* \in [0, 1 - \alpha_2]\);
- \(C = 1, \quad \alpha_2 \in \left(\frac{1}{2}, 1\right]\), \(\alpha_1 \in [0, \frac{1}{2})\), \(x_1^* = 0\).
It is important to stress here that in some cases there is a continuum of payoff-equivalent equilibria with asymmetric announcements in which one agent, say $i$, announces his bliss point $\alpha_i$, gets selected for the public good provision, and obtains a payoff of $-C$, while the other agent, $j$, announces any variety from an equilibrium interval and obtains a payoff of $-|\alpha_j - \alpha_i|$, $i, j = 1, 2, i \neq j$. I refer to such a continuum of payoff-equivalent equilibria as one equilibrium, specifying that agent $j$ can choose any platform from an equilibrium interval.

Note that the agents’ problem is symmetric and therefore equilibria with asymmetric announcements are symmetric around $\frac{1}{2}$. In other words, if there is an equilibrium in which agent $1$ gets selected for the pair of bliss points $\alpha_1$ and $\alpha_2$, then there is an equilibrium in which agent $2$ gets selected for the pair of bliss points $1 - \alpha_2$ and $1 - \alpha_1$.

Consider an equilibrium with asymmetric announcements in which, say, agent $1$ gets selected, $(\alpha_1, x_2^*)$. (The intuition for an equilibrium in which agent $2$ gets selected is analogous.) Agent $2$’s announced variety is more extreme than agent $1$’s, i.e., $x_2^* > 1 - \alpha_1$. Agent $1$ implements his bliss point and therefore obtains $-C$. He has no incentive to deviate by announcing a more extreme variety and getting selected only with probability $\frac{1}{2}$ or not being selected at all. In this case, the gains in terms of net cost of public good provision ($\frac{C}{2}$ if selected with probability $\frac{1}{2}$ or $C$ if not selected) do not compensate the losses in terms of implemented variety of the public good ($x_2^* - \frac{1}{2}$ if selected with probability $\frac{1}{2}$ or $x_2^* - \alpha_1$ if not selected) for $x_2^*$ specified in Proposition 3. Agent $2$ is not selected and obtains a payoff of $\alpha_1 - \alpha_2$. He would not deviate by announcing a less extreme variety and getting selected with probability $\frac{1}{2}$ or $1$. Indeed, in this case, agent $2$ would carry a net cost ($\frac{C}{2}$ if selected with probability $\frac{1}{2}$ or $C$ if selected with probability $1$) that exceeds the gains in terms of implemented variety ($\min \{\alpha_2, 1 - \alpha_1\} - \frac{1}{2}$ if selected with probability $\frac{1}{2}$ or $\alpha_2 - \alpha_1$ if selected with probability $1$) for the parameter values specified in Proposition 3.

I summarize the results of Propositions 1, 2, and 3 for $C > 0$ as follows.

i) When the net cost of public good provision is larger than the length of the set of feasible outcomes ($C > 1$), there is a unique equilibrium in which the agents announce extreme symmetric varieties $(0, 1)$.

ii) When the net cost of public good provision equals the length of the set of feasible outcomes ($C = 1$), there is an equilibrium with extreme symmetric announcements $(0, 1)$ for any $\alpha_1 \in [0, \frac{1}{2})$ and $\alpha_2 \in (\frac{1}{2}, 1]$. Moreover, there are two equilibria with asymmetric announcements $(\alpha_1, 1)$ and $(0, \alpha_2)$ when the agents’ bliss points are not extreme, i.e., when $\alpha_1 \in (0, \frac{1}{2})$ and $\alpha_2 \in (\frac{1}{2}, 1)$. If agent 1 is extreme ($\alpha_1 = 0$), there is an equilibrium $(0, \alpha_2)$. If agent 2 is extreme ($\alpha_2 = 1$), there is an equilibrium $(\alpha_1, 1)$.
iii) When the net cost of public good provision is lower than the length of the set of feasible outcomes \((C < 1)\), depending on the agents’ bliss points there are either one or two equilibria. If the distance between the agents’ bliss points does not exceed the net cost \((\alpha_2 - \alpha_1 \leq C)\) and the agents are not extreme \((\alpha_1 \neq 0 \text{ and } \alpha_2 \neq 1)\), then there are two equilibria with asymmetric announcements whereby the selected agent chooses his bliss point and the other agent chooses any variety from a certain equilibrium interval. Otherwise, there is a unique equilibrium: if \(\alpha_1 \in \left[0, \frac{1-C}{2}\right]\) and \(\alpha_2 \in \left[\frac{1+C}{2}, 1\right]\) this is an equilibrium with symmetric announcements \((\frac{1-C}{2}, \frac{1+C}{2})\); otherwise, this is an equilibrium with asymmetric announcements in which a less extreme agent announces his bliss point and gets selected and the other agent announces any variety from a certain equilibrium interval.

Table 1 in the Appendix describes equilibria for \(C = 1\). When \(C = 1\), there is an equilibrium with extreme symmetric announcements \((0, 1)\). Moreover, when \(C = 1\) and the agents are not extreme, i.e., \(\alpha_1 \neq 0 \text{ and } \alpha_2 \neq 1\), there are two more equilibria with asymmetric varieties \((\alpha_1, 1)\) and \((0, \alpha_2)\). If one of the agents is extreme, only one equilibrium with asymmetric varieties arises for \(C = 1\): \((0, \alpha_2)\) when \(\alpha_1 = 0\) or \((\alpha_1, 1)\) when \(\alpha_2 = 1\).

Equilibria for \(C \in (0, 1)\) are formally described in Table 2 in the Appendix. Furthermore, Figures 1 and 2 in the Appendix represent equilibria for \(C \in (0, \frac{1}{2})\) and \(C \in (\frac{1}{2}, 1)\), respectively. The horizontal axis depicts the bliss point of agent 1, \(\alpha_1 \in \left[0, \frac{1}{2}\right]\), and the vertical axis depicts that of agent 2, \(\alpha_2 \in \left(\frac{1}{2}, 1\right]\). The dashed lines represent the boundaries of open sets. Figures 1 and 2 specify how many and what equilibria there are for each pair of agents’ bliss points \((\alpha_1, \alpha_2) \in \left[0, \frac{1}{2}\right] \times \left(\frac{1}{2}, 1\right]\).

Note that if the distance between the agents’ bliss points is greater than the net cost \(C\), i.e., \(\alpha_2 - \alpha_1 > C\), or if one of the agents has an extreme bliss point, i.e., \(\alpha_1 = 0\) or \(\alpha_2 = 1\), then there is just one equilibrium for \(C \in (0, 1)\). Otherwise, there are two equilibria. The reason is that when \(\alpha_2 - \alpha_1 > C\), only the agent with a less extreme bliss point gets selected in an equilibrium with asymmetric announcements. If the agent with a more extreme bliss point gets selected, this cannot be equilibrium with asymmetric varieties since the agent with a less extreme bliss point would like to deviate to get selected. Indeed, the losses in terms of net cost \(C\) if selected are less than the gains in terms of implemented public good variety \(\alpha_2 - \alpha_1\). However, when \(\alpha_2 - \alpha_1 \leq C\), there are two equilibria with asymmetric announcements, since both the agent with a less extreme bliss point and the agent with a more extreme bliss point can be selected for the public good provision.

Consider first an equilibrium in which the agent with a less extreme bliss point gets
selected. He does not have incentive to deviate in order to get selected with probability $\frac{1}{2}$ or not to get selected at all. Indeed, by deviating he might avoid the cost of public good provision but incurs even larger losses in terms of implemented public good variety. The other agent does not have incentive to deviate either. Intuitively, since the agents’ bliss points are not very distinct then in equilibrium, he suffers just a modest loss in term of implemented public good variety. By deviating he somewhat reduces this loss but carries even larger costs of public good provision.

The other equilibrium in which a less moderate agent announces his bliss point and gets selected is apparently more counterintuitive. Indeed, why wouldn’t a more moderate agent deviate and announce his bliss point? He could then get selected and implement his preferred variety of the public good. But the same intuition works here. Since the agents’ preferred varieties are rather moderate and not very distinct, the more moderate agent gets a rather small utility loss from the equilibrium variety of the public good. By deviating to his bliss point he would get selected, implement his bliss point, and therefore slightly increase his utility from the differentiated public good. However, he would also incur the cost of public good provision $C$, which exceeds his gains from implementing his preferred variety $\alpha_2 - \alpha_1$.

I must emphasize again that an equilibrium in which a less moderate agent announces his bliss point and gets selected doesn’t arise when the distance between the agents’ bliss points exceeds the net cost of public good provision. In this case, a more moderate agent could profitably deviate to his bliss point as his gains in terms of implemented public good variety would exceed the net cost of public good provision.

3.3. Principal’s Decision regarding Selection Process

I turn next to the principal’s decision regarding the selection procedure. Given the agents’ bliss points $\alpha_1$ and $\alpha_2$, and the net cost of public good provision $C$, the principal chooses between appointment and competition. In what follows, $\hat{\alpha}$ denotes a more moderate bliss point out of $\alpha_1$ and $\alpha_2$.

Under appointment, a more moderate agent ends up providing his preferred public good variety. The principal’s utility is then equal to $-|\hat{\alpha} - \frac{1}{2}|$ for any tuple of $\alpha_1$, $\alpha_2$ and $C$.

Under competition, a selected agent provides a public good variety he has announced in the competition stage. The following table summarizes the principal’s expected utility in this case:
<table>
<thead>
<tr>
<th>Parameter values:</th>
<th>Principal’s expected utility under competition:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C &gt; 1$</td>
<td>$-\frac{1}{2}$;</td>
</tr>
<tr>
<td>$C = 1$</td>
<td>$-\frac{1}{2}$ and/or $-</td>
</tr>
<tr>
<td>$\alpha_2 - \alpha_1 \leq C &lt; 1$</td>
<td>$-</td>
</tr>
<tr>
<td>$0 &lt; C &lt; \alpha_2 - \alpha_1$</td>
<td>$-\frac{C}{2}$ if $\alpha_1 \leq \frac{1-C}{2}$, $\alpha_2 \geq \frac{1+C}{2}$ and $-</td>
</tr>
<tr>
<td>$C \leq 0$</td>
<td>$0$.</td>
</tr>
</tbody>
</table>

The first line corresponds to the case in which the net cost of public good provision is larger than the set of feasible outcomes, $C > 1$. In this case, under competition the agents make themselves an unattractive choice by choosing extreme varieties of the public good. The principal therefore prefers appointment to competition.

The second line reflects the case in which the net cost of public good provision equals the set of feasible outcomes, $C = 1$. Under competition, for any pair of the agents’ bliss points $\alpha_1$ and $\alpha_2$, there is an equilibrium with extreme announcements as in the previous case. Moreover, there might be other equilibria for some $\alpha_1$ and $\alpha_2$. But in none of those the principal’s payoff exceeds $-|\hat{\alpha} - \frac{1}{2}|$. She therefore prefers appointment to competition in this case.

The third line of the table deals with the case in which the net cost of public good provision is lower than the length of the set of feasible outcomes but exceeds the distance between the agents’ bliss points ($\alpha_2 - \alpha_1 \leq C < 1$). Under competition, there is an asymmetric equilibrium in which the principal’s utility is exactly $-|\hat{\alpha} - \frac{1}{2}|$ as under appointment procedure. However, for most pairs of the agents’ bliss points $\alpha_1$ and $\alpha_2$, there is another asymmetric equilibrium in which the principal’s utility is strictly lower than $-|\hat{\alpha} - \frac{1}{2}|$. The principal thus adopts appointment procedure in this case.

Consider now the fourth line which corresponds to the case in which the net cost of public good provision is strictly positive but does not exceed the distance between the agents’ bliss points ($0 < C < \alpha_2 - \alpha_1$). In this case, under competition there is a unique equilibrium with divergent announcements (either symmetric or asymmetric depending on the parameter values) in which the principal’s utility exceeds or equals $-|\hat{\alpha} - \frac{1}{2}|$. The principal therefore
prefers competition to appointment in this case.

Finally, the last line of the table deals with the case of nonpositive production costs, i.e., benefits \((C \leq 0)\). Here, under competition the agents want to make themselves an attractive choice by announcing the principal’s preferred variety. As a result, the principal adopts competition procedure. The following proposition summarizes the results.

**Proposition 4.** The principal uses appointment when the cost of public good provision is greater than or equal to the distance between the agents’ bliss points \((C \geq \alpha_2 - \alpha_1)\).

She uses competition when the cost of public good provision is lower than the distance between the agents’ bliss points \((C < \alpha_2 - \alpha_1)\).

Intuitively, under competition procedure, the agents face a standard cost-benefit trade-off. Providing public good implies certain costs for a contributing agent but at the same time allows him to choose a public good variety closer to his bliss point. Obviously, when the cost exceeds the distance between the agents’ bliss points \((C \geq \alpha_2 - \alpha_1)\) each agent prefers his counterpart to be selected and thus public good provision becomes unpopular. The agents (for some parameter values just one of them) tend to make themselves an unattractive choice and announce extreme varieties of the public good. The principal therefore picks appointment procedure to avoid extreme outcomes. However, when the cost is lower than the distance between the agents’ bliss points \((C < \alpha_2 - \alpha_1)\) neither agent would let his counterpart implement his preferred public good variety. In this case, the agents value the opportunity to provide the public good which makes them to announce more moderate varieties than their bliss points. The principal then picks competition procedure and ends up with a moderate variety of the public good.

### 3.4. Robustness

In this section, I relax some of the important assumptions of the model and discuss robustness of my results.

**Exit of the agents** Assume now that the agents are allowed to exit the competition stage. Intuitively, the agents have incentive to exit only when production of the public good is an unpopular job. However, when the agents value the opportunity to implement a public good variety close to their bliss point, they don’t want to exit the competition stage. In Appendix C, I formally show that for \(C < \alpha_2 - \alpha_1\) neither agent has incentive to exit, while for \(C \geq \alpha_2 - \alpha_1\) at least one of the agents prefers to exit. Therefore, allowing exit affects outcomes of the competition stage only when \(C \geq \alpha_2 - \alpha_1\). Note however that the principal’s
decision regarding the selection procedure stays unaffected in this case. Indeed, even if exit
is allowed in the competition stage, the principal still adopts appointment procedure for
\( C \geq \alpha_2 - \alpha_1 \).

To see this, consider first the case of large costs \((C > 1)\) in which there is a unique
equilibrium \((0, 1)\) of the competition stage and both agents have incentive to exit. The
agents strictly prefer the public good to be provided rather than not. Therefore, in the
simultaneous exit game, only one of the agents ends up exiting.\(^2\) If a more extreme agent
exits then a more moderate agent announces his preferred variety in the competition stage and
gets selected for public good provision. The principal is then indifferent between appointment
and competition. However, if a more moderate agent exits then a more extreme agent gets
selected. The principal then strictly prefers appointment to competition (as when exit is not
allowed).

Consider next the case of \( C = 1 \) in which there are up to 3 equilibria depending on the
parameter values. A symmetric equilibrium \((0, 1)\) is discussed in the previous paragraph. In
equilibrium \((0, \alpha_2)\) agent 2 has incentive to exit. If agent 2 is a more extreme agent then his
exit implies that a more moderate agent 1 will be selected in the competition stage. Therefore,
the principal will be indifferent between appointment and competition. However, if agent 2
is a more moderate agent then his exit leads to a more extreme variety being implemented
by agent 1. The principal then strictly prefers appointment to competition. The similar
intuition works in the case of equilibrium \((\alpha_1, 1)\) in which agent 1 has incentive to exit. It
follows therefore that for \( C = 1 \) the principal prefers appointment to competition (as if exit
is not allowed).

Finally, I turn to the case of \( \alpha_2 - \alpha_1 \leq C < 1 \). Here there are either one or two asymmetric
equilibrria in which one of the agents announces his bliss point, gets selected, but would readily
exit the competition stage if allowed. If he is a more extreme agent then his exit would lead
to a more moderate variety being implemented. This would make the principal indifferent
between appointment and competition. However, if he is a more moderate agent then his
exit would result in a more extreme variety being chosen. The principal would then prefer
appointment to competition.

I can conclude therefore that my results hold when exit is allowed in the competition
stage.

*Preferences of the principal* In the model, I assume that the principal’s bliss point is \( \frac{1}{2} \).
As a result, the principal is indifferent between extreme varieties 0 and 1. Relaxing this

\(^2\)For simplicity, I disregard coordination issues here.
assumption affects some results of the competition stage. In particular, Proposition 1 about the lack of convergence in the competition stage no longer holds. Indeed, if the principal strictly prefers one extreme variety, say 0, to the other, 1, then for sufficiently large $C$ it is easy to construct a convergent equilibrium, $(1, 1)$, in the competition stage. To see this, suppose that the agents announce the same varieties $(1, 1)$. Agent $i$’s payoff is then equal to $- (1 - \alpha_i) - \frac{C}{2}$. If agent $i$ deviates to $\alpha_i$ then he gets a payoff of $-C$. He has no incentive to deviate if $- (1 - \alpha_i) - \frac{C}{2} \geq -C$, which holds if $1 < C \leq 2$ and $\alpha_1 \geq 1 - \frac{C}{2}$ and if $C > 2$. By analogy, if the principal prefers variety 1 to 0 then for $1 < C \leq 2$ and $\alpha_2 \leq \frac{C}{2}$ and for $C > 2$, there is a convergent equilibrium $(0, 0)$. However, this convergence result does not affect the principal’s decision regarding the selection procedure. Indeed, in this case the principal still prefers appointment to competition in order to avoid extreme outcomes.

Note moreover that relaxing the assumption about symmetry of the principal’s preferences does not change the results for $C \leq 1$ either. Indeed, the equilibrium structure of the competition stage stays unaffected. (However, particular quantitative characteristics of the equilibria might change.) Still, the principal will prefer competition when the agents value the opportunity to provide a public good (i.e., when $C < \alpha_2 - \alpha_1$), and appointment when the agents want to avoid it (i.e., when $C \geq \alpha_2 - \alpha_1$). It follows therefore that my results hold when the assumption about symmetry of the principal’s preferences is relaxed.

4. Conclusion

This paper builds a simple model of provision of a differentiated public good within an organization. A principal can adopt different procedures (appointment or competition) to select one of two agents for the public good production. The agents have extreme opposite preferences over the differentiated public good while the principal prefers a moderate variety. Under appointment, the principal just observes the agents’ preferences and selects a contributor on the basis of this. Obviously, an agent with the preferences closer to those of the principal will be selected in this case. In turn, under competition, the agents announce public good varieties they commit to provide if selected. If the public good provision is quite costly then the agents want to avoid being selected and so make themselves an unattractive choice by announcing extreme varieties. The principal then prefers appointment to competition. However, if the public good provision is not very costly then each agent values (and therefore compete for) the chance to choose a public good variety closer to that he prefers the most. The agents thus want to make themselves an attractive choice and announce moderate varieties. The principal prefers competition to appointment in this case.
Even though the model is very stylized, it yields an empirically testable prediction. My results suggest that appointment is preferred in the cases in which provision of a differentiated public good implies considerable costs for a contributing agent and doesn’t pay off in terms of a public good variety. In turn, competition is preferred when provision of a differentiated public good is not so costly and pays off in terms of a public good variety. Therefore, a simple testable hypothesis might be as follows. Within organizations, production of unpopular public goods is simply assigned while production of popular public goods is contested.

Appendix

A. Proof of Proposition 2

Consider a pair of varieties \((x_1, x_2)\) such that \(x_1 = 1 - x_2\), \(x_1 \in (0, \frac{1}{2})\), \(x_2 \in \left(\frac{1}{2}, 1\right)\). Given these \(x_1\) and \(x_2\), each agent gets selected with probability one-half. Agent 1’s payoff is equal to \(\Pi_1(x_1, x_2) = \frac{1}{2} (-|x_1 - \alpha_1| - C) + \frac{1}{2} (-|x_2 - \alpha_1|)\). Agent 2’s payoff is equal to \(\Pi_2(x_1, x_2) = \frac{1}{2} (-|x_1 - \alpha_2|) + \frac{1}{2} (-|x_2 - \alpha_2| - C)\).

1. If agent 1 deviates and announces a variety \(x_1' \in [0, x_1)\), then he is not selected and gets the payoff \(\Pi_1(x_1', x_2) = -|x_2 - \alpha_1|\). Such a deviation is not profitable only if \(\Pi_1(x_1, x_2) \geq \Pi_1(x_1', x_2)\), which yields \(-|x_1 - \alpha_1| - C \geq -(1 - x_1 - \alpha_1)\). If agent 1 deviates and announces a variety \(x_1'' \in (x_1, \frac{1}{2}]\), then he is selected and gets the payoff \(\Pi_1(x_1'', x_2) = -|x_1'' - \alpha_1| - C\). Such a deviation is not profitable only if \(\Pi_1(x_1, x_2) \geq \Pi_1(x_1'', x_2)\), which implies \(\frac{1}{2} (-|x_1 - \alpha_1| - C) + \frac{1}{2} (-1 + x_1 + \alpha_1) \geq -|x_1'' - \alpha_1| - C\).

a) Consider the case where \(x_1 < \alpha_1\). The conditions \(\Pi_1(x_1, x_2) \geq \Pi_1(x_1', x_2)\) and \(\Pi_1(x_1, x_2) \geq \Pi_1(x_1'', x_2)\) become \(\alpha_1 \leq \frac{1-C}{2}\) and \(x_1 \geq \frac{1-C}{2} - |x_1'' - \alpha_1|\), respectively. To guarantee that the latter inequality holds for each \(x_1'' \in (x_1, \frac{1}{2}]\), it is required that \(x_1 \geq \frac{1-C}{2}\). It follows then that \(\alpha_1 \leq \frac{1-C}{2} \leq x_1\), which does not hold for \(x_1 < \alpha_1\). Therefore, when \(x_1 < \alpha_1\), agent 1 can deviate profitably.

b) Consider the case where \(x_1 \geq \alpha_1\). The conditions \(\Pi_1(x_1, x_2) \geq \Pi_1(x_1', x_2)\) and \(\Pi_1(x_1, x_2) \geq \Pi_1(x_1'', x_2)\) become \(x_1 \leq \frac{1-C}{2}\) and \(x_1' \geq \frac{1-C}{2}\), respectively. The latter inequality holds for each \(x_1' \in (x_1, \frac{1}{2}]\) only if \(x_1 = \frac{1-C}{2}\) (where \(C < 1\)). Indeed, if \(x_1 < \frac{1-C}{2}\) then there is \(x_1'' \in (x_1, \frac{1-C}{2})\) that implies that agent 1 has a profitable deviation. Therefore, agent 1 will not deviate only if \(\alpha_1 \leq \frac{1-C}{2}\) and \(x_1 = \frac{1-C}{2}\), where \(C < 1\).
2. If agent 2 deviates and announces a variety $x'_2 \in (x_2, 1]$, then he is not selected and gets the payoff $\Pi_2(x_1, x'_2) = -|x_1 - \alpha_2|$. Such a deviation is not profitable only if $\Pi_2(x_1, x_2) \geq \Pi_2(x_1, x'_2)$, which implies $-|x_2 - \alpha_2| - C \geq 1 - x_2 - \alpha_2$. If agent 2 deviates and announces a variety $x''_2 \in [\frac{1}{2}, x_2)$, then he is selected and his payoff becomes $\Pi_2(x_1, x''_2) = -|x''_2 - \alpha_2| - C$. This deviation is not profitable only if $\Pi_2(x_1, x_2) \geq \Pi_2(x_1, x''_2)$, which yields $\frac{1}{2} (1 - x_2 - \alpha_2) + \frac{1}{2} (1 - x_2 - \alpha_2 - C) \geq -|x''_2 - \alpha_2| - C$.

a) Consider the case where $x_2 > \alpha_2$. The conditions $\Pi_2(x_1, x_2) \geq \Pi_2(x_1, x'_2)$ and $\Pi_2(x_1, x_2) \geq \Pi_2(x_1, x''_2)$ become $\alpha_2 \geq \frac{1+C}{2}$ and $x_2 \leq \frac{1+C}{2} + |x''_2 - \alpha_2|$, respectively. To guarantee that the latter inequality holds for each $x''_2 \in [\frac{1}{2}, x_2)$, it is necessary that $x_2 \leq \frac{1+C}{2}$. Therefore, $x_2 \leq \frac{1+C}{2} \leq \alpha_2$, which is not possible for $x_2 > \alpha_2$. Therefore, agent 2 has profitable deviations when $x_2 > \alpha_2$.

b) Consider the case where $x_2 \leq \alpha_2$. The conditions $\Pi_2(x_1, x_2) \geq \Pi_2(x_1, x'_2)$ and $\Pi_2(x_1, x_2) \geq \Pi_2(x_1, x''_2)$ become $x_2 \geq \frac{1+C}{2}$ and $x''_2 \leq \frac{1+C}{2}$, respectively. The latter inequality holds for each $x''_2 \in [\frac{1}{2}, x_2)$ only if $x_2 = \frac{1+C}{2}$ (where $C < 1$). Indeed, if $x_2 > \frac{1+C}{2}$ then there is $x''_2 \in (\frac{1+C}{2}, x_2)$ that means that agent 2 has a profitable deviation. Thus, agent 2 will not deviate only if $\alpha_2 \geq \frac{1+C}{2}$ and $x_2 = \frac{1+C}{2}$, where $C < 1$.

Therefore, both agents do not deviate from $(x_1, x_2)$ such that $x_1 = 1 - x_2$, $x_1 \in (0, \frac{1}{2})$, $x_2 \in (\frac{1}{2}, 1)$ only if $\alpha_1 \leq \frac{1-C}{2}$, $\alpha_2 \geq \frac{1+C}{2}$, and $x_1 = \frac{1-C}{2}$, $x_2 = \frac{1+C}{2}$, where $C < 1$. In other words, there is a unique equilibrium with symmetric announcements $(\frac{1-C}{2}, \frac{1+C}{2})$ when $\alpha_1 \leq \frac{1-C}{2}$, $\alpha_2 \geq \frac{1+C}{2}$, $C < 1$.

Consider now a pair of announcements $(0, 1)$. Each agent gets selected with probability one-half. The agents’ payoffs are equal to $\Pi_i(0, 1) = -\frac{1}{2} - \frac{C}{2}$, $i = 1, 2$. If agent $i$ deviates and announces a less extreme variety $x'_i$, he is selected and gets the payoff $\Pi_i(x'_i, \cdot) = -|x'_i - \alpha_i| - C$. Note that $\Pi_i(x'_i, \cdot)$ takes its maximum value $-C$ when $x'_i = \alpha_i$. To guarantee that agent $i$ has no profitable deviations it is required $\Pi_i(0, 1) \geq \max \Pi_i(x'_i, \cdot)$, which amounts to $C \geq 1$. Therefore, $(0, 1)$ is an equilibrium for $C \geq 1$. ■

B. Proof of Proposition 3

Characterization of an equilibrium in which the agents announce asymmetric varieties and agent 1 gets selected.

Consider a pair of announcements $(\alpha_1, x^*_2)$ such that $\alpha_1 \in (0, \frac{1}{2})$ and $x^*_2 \in (1 - \alpha_1, 1]$. Given those, agent 1 is selected for the public good provision and gets the payoff $\Pi_1(\alpha_1, x^*_2) =$
The payoff of agent 2 is equal to \( \Pi_2(\alpha_1, x_2') = -|\alpha_1 - \alpha_2| \).

1. If agent 1 deviates and announces a variety \( x'_1 \in (1 - x_2^*, \frac{1}{2}] \), then he still gets selected and his payoff becomes \( \Pi_1(x'_1, x_2^*) = -|x'_1 - \alpha_1| - C \). However, \( \Pi_1(x'_1, x_2^*) < \Pi_1(\alpha_1, x_2^*) \) and so such a deviation is not profitable. If agent 1 deviates and announces a variety \( 1 - x_2^* \), then each agent gets selected with probability one-half. Agent 1’s payoff becomes \( \Pi_1(1 - x_2^*, x_2^*) = \frac{1}{2} (-|1 - x_2^* - \alpha_1| - C) + \frac{1}{2} (-|x_2^* - \alpha_1|) \). Such a deviation is not profitable only if \( \Pi_1(\alpha_1, x_2^*) \geq \Pi_1(1 - x_2^*, x_2^*) \), which implies \( x_2^* \geq \frac{1+\alpha_1}{2} \), where \( C \leq 1 \). Consider the case where \( x_2^* \neq 1 \). If agent 1 deviates and announces \( x'_2 \in [0, 1 - x_2^*] \), then he is not selected and gets the payoff \( \Pi_1(x'_1, x_2^*) = -|x_2^* - \alpha_1| \). Such a deviation is not profitable only if \( \Pi_1(\alpha_1, x_2^*) \geq \Pi_1(x'_1, x_2^*) \), which implies \( x_2^* \geq \alpha_1 + C \). Therefore, when the agents announce \((\alpha_1, x_2^*)\) such that \( \alpha_1 \in (0, \frac{1}{2}) \) and \( x_2^* \in (1 - \alpha_1, 1) \), agent 1 has no profitable deviations only if \( x_2^* \geq \max \{ \frac{1+\alpha_1}{2}, \alpha_1 + C \} \), where \( C < 1 \). When the agents announce \((\alpha_1, 1)\) such that \( \alpha_1 \in (0, \frac{1}{2}) \), agent 1 has no profitable deviations only if \( C \leq 1 \).

2. If agent 2 deviates and announces \( 1 - \alpha_1 \), then each agent gets selected with probability one-half. The payoff of agent 2 becomes \( \Pi_2(\alpha_1, 1 - \alpha_1) = \frac{1}{2} (-|\alpha_1 - \alpha_2|) + \frac{1}{2} (-|1 - \alpha_1 - \alpha_2| - C) \). Such a deviation is not profitable only if \( \Pi_2(\alpha_1, x_2^*) \geq \Pi_2(\alpha_1, 1 - \alpha_1) \), which amounts to \( \alpha_1 - \alpha_2 \geq -|1 - \alpha_1 - \alpha_2| - C \). If agent 2 deviates and announces \( x'_2 \in [\frac{1}{2}, 1 - \alpha_1] \), then he gets selected and his payoff is \( \Pi_2(\alpha_1, x'_2) = -|x'_2 - \alpha_2| - C \). This deviation is not profitable only if \( \Pi_2(\alpha_1, x'_2) \geq \Pi_2(\alpha_1, x_2^*) \), which implies \( \alpha_1 - \alpha_2 \geq -|x'_2 - \alpha_2| - C \).

a) Consider the case where \( \alpha_2 < 1 - \alpha_1 \). The conditions \( \Pi_2(\alpha_1, x_2^*) \geq \Pi_2(\alpha_1, 1 - \alpha_1) \) and \( \Pi_2(\alpha_1, x_2^*) \geq \Pi_2(\alpha_1, x'_2) \) become \( \alpha_2 \leq \frac{1+\alpha_1}{2} \) and \( \alpha_2 \leq \alpha_1 + C \), respectively. It implies therefore that in case \( \alpha_2 < 1 - \alpha_1 \), agent 2 will not deviate only if \( \alpha_2 \leq \min \{ \frac{1+\alpha_1}{2}, \alpha_1 + C \} \).

b) Consider the case where \( \alpha_2 = 1 - \alpha_1 \). The conditions \( \Pi_2(\alpha_1, x_2^*) \geq \Pi_2(\alpha_1, 1 - \alpha_1) \) and \( \Pi_2(\alpha_1, x_2^*) \geq \Pi_2(\alpha_1, x'_2) \) become \( \alpha_2 = \alpha_1 + C \) and \( x'_2 \leq \alpha_1 + C \), respectively. Once the former inequality holds, the latter inequality will hold too since \( x'_2 < 1 - \alpha_1 = \alpha_2 \leq \alpha_1 + C \). It means that in case \( \alpha_2 = 1 - \alpha_1 \), agent 2 will not deviate only if \( \alpha_2 \leq \alpha_1 + C \), which amounts to \( \alpha_1 \geq \frac{1-C}{2} \).

c) Consider the case where \( \alpha_2 > 1 - \alpha_1 \). The conditions \( \Pi_2(\alpha_1, x_2^*) \geq \Pi_2(\alpha_1, 1 - \alpha_1) \) and \( \Pi_2(\alpha_1, x_2^*) \geq \Pi_2(\alpha_1, x'_2) \) become \( \alpha_1 \geq \frac{1-C}{2} \) and \( x'_2 \leq \alpha_1 + C \), respectively. Once the former inequality holds, the latter inequality will hold too since the
former inequality implies $1 - \alpha_1 \leq \alpha_1 + C$, and therefore $x'_2 < 1 - \alpha_1 \leq \alpha_1 + C$.

It follows then that in case $\alpha_2 > 1 - \alpha_1$, agent 2 has no profitable deviations only if $\alpha_1 \geq \frac{1 - C}{2}$.

Therefore, when the agents announce $(\alpha_1, x'_2)$ such that $\alpha_1 \in (0, \frac{1}{2})$ and $x'_2 \in (1 - \alpha_1, 1]$, agent 2 has no profitable deviations only in the following cases: either $\alpha_2 < 1 - \alpha_1$ and $\alpha_2 \leq \min \{\frac{1+C}{2}, \alpha_1 + C\}$ or $\alpha_2 \geq 1 - \alpha_1$ and $\alpha_1 \geq \frac{1 - C}{2}$.

Combining the conditions that guarantee that neither agent has profitable deviations yields the set of the parameters for which there is an equilibrium with asymmetric announcements $(\alpha_1, x'_2)$ with agent 1 selected for the public good provision:

\[
\begin{align*}
C \in (0, 1), & \quad \alpha_1 \in \left(\max \left\{0, \frac{1 - 2C}{2}\right\}, \frac{1 - C}{2}\right), \quad \alpha_2 \in \left(\frac{1}{2}, \alpha_1 + C\right], \quad x'_2 \in (1 - \alpha_1, 1]; \\
C \in (0, 1), & \quad \alpha_1 = \frac{1 - C}{2}, \quad \alpha_2 \in \left(\frac{1}{2}, 1\right], \quad x'_2 \in (1 - \alpha_1, 1]; \\
C \in (0, 1), & \quad \alpha_1 \in \left(\frac{1 - C}{2}, \min \{\frac{1}{2}, 1 - C\}\right), \quad \alpha_2 = \left(\frac{1}{2}, 1\right], \quad x'_2 \in [\alpha_1 + C, 1]; \\
C \in (0, 1), & \quad \alpha_1 \in \left[\min \left\{\frac{1}{2}, 1 - C\right\}, \frac{1}{2}\right], \quad \alpha_2 \in \left(\frac{1}{2}, 1\right], \quad x'_2 = 1; \\
C = 1, & \quad \alpha_1 \in (0, \frac{1}{2})], \quad \alpha_2 \in \left(\frac{1}{2}, 1\right], \quad x'_2 = 1.
\end{align*}
\]

Characterization of an equilibrium in which the agents announce asymmetric varieties and agent 2 gets selected.

Consider a pair of announcements $(x'_1, \alpha_2)$ such that $\alpha_2 \in (\frac{1}{2}, 1)$ and $x'_1 \in [0, 1 - \alpha_2)$. Given those, agent 2 is selected and gets the payoff $\Pi_2(x'_1, \alpha_2) = -C$. The payoff of agent 1 is equal to $\Pi_1(x'_1, \alpha_2) = -|\alpha_2 - \alpha_1|$.  

1. If agent 2 deviates and announces a variety $x''_2 \in \left[\frac{1}{2}, 1 - x'_1\right]$, then he still gets selected and his payoff becomes $\Pi_2(x'_1, x''_2) = -|x''_2 - \alpha_2| - C$. However, $\Pi_2(x'_1, x'_2) < \Pi_2(x'_1, \alpha_2)$ and so such a deviation is not profitable. If agent 2 deviates and announces a variety $1 - x'_1$, then each agent gets selected with probability one-half. Agent 2’s payoff becomes $\Pi_2(x'_1, 1 - x'_1) = \frac{1}{2} (-|x'_1 - \alpha_2|) + \frac{1}{2} (-|1 - x'_1 - \alpha_2| - C)$. Such a deviation is not profitable only if $\Pi_2(x'_1, \alpha_2) \geq \Pi_2(x'_1, 1 - x'_1)$, which implies $x'_1 \leq \frac{1 - C}{2}$, where $C \leq 1$.

Consider the case where $x'_1 \neq 0$. If agent 2 deviates and announces $x''_2 \in (1 - x'_1, 1]$, then he is not selected and his payoff becomes $\Pi_2(x'_1, x''_2) = -|x''_2 - \alpha_2|$. Such a deviation is not profitable only if $\Pi_2(x'_1, \alpha_2) \geq \Pi_2(x'_1, x''_2)$, which yields $x'_1 \leq \alpha_2 - C$. Therefore, when the agents announce $(x'_1, \alpha_2)$ such that $\alpha_2 \in (\frac{1}{2}, 1)$ and $x'_1 \in (0, 1 - \alpha_2)$, agent 2 has no profitable deviations only if $x'_1 \leq \min \left\{\frac{1 - C}{2}, \alpha_2 - C\right\}$, where $C \leq 1$. When the agents announce $(0, \alpha_2)$ such that $\alpha_2 \in (\frac{1}{2}, 1)$, agent 2 has no profitable deviations only if $C \leq 1$.  

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2. If agent 1 deviates and announces $1 - \alpha_2$, then each agent gets selected with probability one-half. The payoff of agent 1 becomes $\Pi_1 (1 - \alpha_2, \alpha_2) = \frac{1}{2} (-|1 - \alpha_2 - \alpha_1| - C) + \frac{1}{2} (-|\alpha_2 - \alpha_1|)$. Such a deviation is not profitable only if $\Pi_1 (x_1^*, \alpha_2) \geq \Pi_1 (1 - \alpha_2, \alpha_2)$, which amounts to $\alpha_1 - \alpha_2 \geq -|1 - \alpha_2 - \alpha_1| - C$. If agent 1 deviates and announces $x_1' \in (1 - \alpha_2, \frac{1}{2}]$, then he is selected and gets the payoff $\Pi_1 (x_1', \alpha_2) = -|x_1' - \alpha_1| - C$. This deviation is not profitable only if $\Pi_1 (x_1^*, \alpha_2) \geq \Pi_1 (x_1', \alpha_2)$, which implies $\alpha_1 - \alpha_2 \geq -|x_1' - \alpha_1| - C$.

a) Consider the case where $\alpha_1 > 1 - \alpha_2$. The conditions $\Pi_1 (x_1^*, \alpha_2) \geq \Pi_1 (1 - \alpha_2, \alpha_2)$ and $\Pi_1 (x_1^*, \alpha_2) \geq \Pi_1 (x_1', \alpha_2)$ become $\alpha_1 \geq \frac{1-C}{2}$ and $\alpha_1 \geq \alpha_2 - C$, respectively.
It implies therefore that in case $\alpha_1 > 1 - \alpha_2$, agent 1 will not deviate only if $\alpha_1 \geq \max \{\frac{1-C}{2}, \alpha_2 - C\}$.

b) Consider the case where $\alpha_1 = 1 - \alpha_2$. The conditions $\Pi_1 (x_1^*, \alpha_2) \geq \Pi_1 (1 - \alpha_2, \alpha_2)$ and $\Pi_1 (x_1^*, \alpha_2) \geq \Pi_1 (x_1', \alpha_2)$ become $\alpha_1 \geq \alpha_2 - C$ and $x_1' \geq \alpha_2 - C$, respectively.
Once the former inequality holds, the latter inequality will hold too since $x_1' > 1 - \alpha_2 = \alpha_1 \geq \alpha_2 - C$. It means that in case $\alpha_1 = 1 - \alpha_2$, agent 1 will not deviate only if $\alpha_1 \geq \alpha_2 - C$, which amounts to $\alpha_2 \leq \frac{1+C}{2}$.

c) Consider the case where $\alpha_1 < 1 - \alpha_2$. The conditions $\Pi_1 (x_1^*, \alpha_2) \geq \Pi_1 (1 - \alpha_2, \alpha_2)$ and $\Pi_1 (x_1^*, \alpha_2) \geq \Pi_1 (x_1', \alpha_2)$ become $\alpha_2 \leq \frac{1+C}{2}$ and $x_1' \geq \alpha_2 - C$, respectively.
Once the former inequality holds, the latter inequality will hold too since the former inequality implies $1 - \alpha_2 \geq \alpha_2 - C$, and therefore $x_1' > 1 - \alpha_2 \geq \alpha_2 - C$.

It follows then that in case $\alpha_1 < 1 - \alpha_2$, agent 1 has no profitable deviations only if $\alpha_2 \leq \frac{1+C}{2}$.

Therefore, when the agents announce $(x_1^*, \alpha_2)$ such that $\alpha_2 \in (\frac{1}{2}, 1)$ and $x_1^* \in [0, 1 - \alpha_2)$, agent 1 has no profitable deviations only in the following cases: either $\alpha_1 > 1 - \alpha_2$ and $\alpha_1 \geq \max \{\frac{1-C}{2}, \alpha_2 - C\}$ or $\alpha_1 \leq 1 - \alpha_2$ and $\alpha_2 \leq \frac{1+C}{2}$.

Combining the conditions that guarantee that neither agent has profitable deviations yields the set of the parameters for which there is an equilibrium with asymmetric announcements $(x_1^*, \alpha_2)$ with agent 2 selected for the public good provision:

\[
\begin{align*}
C \in (0, 1), & \quad \alpha_2 \in \left(\frac{1}{2}, \max \left\{\frac{1}{2}, C\right\}\right), \quad \alpha_1 \in \left[0, \frac{1}{2}\right), \quad x_1^* = 0; \\
C \in (0, 1), & \quad \alpha_2 \in \left(\max \left\{\frac{1}{2}, C\right\}, \frac{1+C}{2}\right), \quad \alpha_1 \in \left[0, \frac{1}{2}\right), \quad x_1^* \in [0, \alpha_2 - C]; \\
C \in (0, 1), & \quad \alpha_2 = \frac{1+C}{2}, \quad \alpha_1 \in \left[0, \frac{1}{2}\right), \quad x_1^* \in [0, 1 - \alpha_2); \\
C \in (0, 1), & \quad \alpha_2 \in \left(\frac{1-C}{2}, \min \left\{1, \frac{1+C}{2}\right\}\right), \quad \alpha_1 \in \left(\alpha_2 - C, \frac{1}{2}\right), \quad x_1^* \in [0, 1 - \alpha_2); \\
C = 1, & \quad \alpha_2 \in (\frac{1}{2}, 1), \quad \alpha_1 \in \left[0, \frac{1}{2}\right), \quad x_1^* = 0. \quad \blacksquare
\end{align*}
\]
C. Exit of the Agents in the Competition Stage

If one of the agents, say agent $i$, exits the competition stage, then agent $j$ can announce his preferred variety and still get selected. Agent $i$’s payoff is then equal to $-|\alpha_j - \alpha_i|$. I calculate next agent $i$’s payoff in the case he does not exit. I consider five different combinations of the parameter values.

1. When $C > 1$ there is a unique symmetric equilibrium $(0, 1)$. Agent $i$’s payoff is $-\frac{C}{2} - \frac{1}{2}$ which is strictly lower than $-|\alpha_j - \alpha_i|$. Agent $i$ therefore prefers to exit the competition stage.

2. When $C = 1$ there is a symmetric equilibrium $(0, 1)$. As discussed in the previous case, agent $i$ will exit competition. For $C = 1$, $\alpha_1 \in (0, \frac{1}{2})$, $\alpha_2 \in (\frac{1}{2}, 1]$ there is also an asymmetric equilibrium $(\alpha_1, 1)$ in which agent 1’s payoff is equal to $-C$ and agent 2’s payoff is $-|\alpha_1 - \alpha_2|$. Agent 1 will exit in this case. For $C = 1$, $\alpha_1 \in [0, \frac{1}{2})$, $\alpha_2 \in (\frac{1}{2}, 1)$ there is also an asymmetric equilibrium $(0, \alpha_2)$ in which agent 1’s payoff is $-|\alpha_2 - \alpha_1|$ and agent 2’s payoff is $-C$. Agent 2 prefers to exit competition.

3. When $\alpha_2 - \alpha_1 \leq C < 1$ there is an asymmetric equilibrium in which a more moderate agent announces his bliss point in the competition stage and gets selected. His payoff is then $-C$ which is lower than $-|\alpha_j - \alpha_i|$. He therefore prefers to exit the competition stage. Moreover, for most pairs of the agents’ bliss points $\alpha_1$ and $\alpha_2$, there is another asymmetric equilibrium in which a more extreme agent announces his bliss point, gets selected and gets payoff of $-C$ which is lower than $-|\alpha_j - \alpha_i|$. He then prefers to exit.

4. When $0 < C < \alpha_2 - \alpha_1$ there is a unique equilibrium in the competition stage. For $\alpha_1 \leq \frac{1-C}{2}$, $\alpha_2 \geq \frac{1+C}{2}$, it is a symmetric equilibrium in which agent $i$’s payoff is equal to $-\frac{C}{2} - \left|\frac{1}{2} - \alpha_i\right|$ which exceeds $-|\alpha_j - \alpha_i|$. Neither agent then wants to exit competition. Otherwise, it is an asymmetric equilibrium in which a more moderate agent gets payoff of $-C$ (which is higher than $-|\alpha_j - \alpha_i|$) while a more extreme agent gets payoff of $-|\alpha_j - \alpha_i|$. Neither agent has incentive to exit.

5. Finally, when $C \leq 0$ there is a unique convergent equilibrium $(\frac{1}{2}, \frac{1}{2})$. Agent $i$’s payoff is $-\frac{C}{2} - \left|\frac{1}{2} - \alpha_i\right|$ which exceeds $-|\alpha_j - \alpha_i|$. Neither agent exits the competition stage in this case.

To summarize, for $C < \alpha_2 - \alpha_1$ agents have no incentive to exit the competition stage. However, for $C \geq \alpha_2 - \alpha_1$ at least one of the agents does prefer to exit.
Figure 1: Equilibria for $C \in \left(0, \frac{1}{2}\right)$. 
Figure 2: Equilibria for $C \in (\frac{1}{2}, 1)$. 
Values of the parameters $C$, $\alpha_1$ and $\alpha_2$:

<table>
<thead>
<tr>
<th>$C$</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>Equilibria</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0, $\alpha_2 \in (\frac{1}{2}, 1)$</td>
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<td>$(0, \alpha_2), (0, 1)$</td>
</tr>
<tr>
<td>1</td>
<td>1, $\alpha_2 \in (0, \frac{1}{2})$</td>
<td></td>
<td>$(\alpha_1, 1), (0, 1)$</td>
</tr>
<tr>
<td>1</td>
<td>$\alpha_1 \in (0, \frac{1}{2}), \alpha_2 \in (\frac{1}{2}, 1)$</td>
<td></td>
<td>$(\alpha_1, 1), (0, \alpha_2), (0, 1)$</td>
</tr>
<tr>
<td>1</td>
<td>0, $\alpha_2 = 1$</td>
<td></td>
<td>$(0, 1)$</td>
</tr>
</tbody>
</table>

Table 1. Equilibria for $C = 1$.

Values of the parameters $C$, $\alpha_1$ and $\alpha_2$:

<table>
<thead>
<tr>
<th>$C$</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>1 equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(0, 1)$</td>
<td>$\alpha_1 \in [0, \frac{1-C}{2}), \alpha_2 \in \left[\frac{1+C}{2}, 1\right]$</td>
<td>$(\frac{1-C}{2}, \frac{1+C}{2});$</td>
<td></td>
</tr>
<tr>
<td>$(0, 1)$</td>
<td>$\alpha_2 \in \left(\max\left{\frac{1}{2}, C\right}, \frac{1+C}{2}\right), \alpha_1 \in [0, \alpha_2 - C)$</td>
<td>$(x_1, \alpha_2), x_1^* \in [0, \alpha_2 - C]$</td>
<td></td>
</tr>
<tr>
<td>$(0, 1)$</td>
<td>$\alpha_1 \in \left[\frac{1-C}{2}, \min\left{\frac{1}{2}, 1 - C\right}\right), \alpha_2 \in (C + \alpha_1, 1]$</td>
<td>$(\alpha_1, x_2^<em>), x_2^</em> \in [\alpha_1 + C, 1]$</td>
<td></td>
</tr>
<tr>
<td>$(\frac{1}{2}, 1)$</td>
<td>$\alpha_1 = 0, \alpha_2 \in \left(\frac{1}{2}, C\right]$</td>
<td>$(0, \alpha_2)$</td>
<td></td>
</tr>
<tr>
<td>$(\frac{1}{2}, 1)$</td>
<td>$\alpha_2 = 1, \alpha_1 \in \left[1 - C, \frac{1}{2}\right]$</td>
<td>$(\alpha_1, 1)$</td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Equilibria for $C \in (0, 1)$.
References


