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Abstract

This study analyzes the effects of inflation on the long-run nexus between unemployment and economic growth. We introduce money demand via a cash-in-advance (CIA) constraint on R&D investment into a scale-invariant Schumpeterian growth model with matching frictions in the labor market. Given the CIA constraint on R&D, a higher inflation that raises the opportunity cost of cash holdings leads to a decrease in innovation and economic growth, which in turn decreases labor-market tightness and increases unemployment. In summary, the model predicts a positive relationship between inflation and unemployment, a negative relationship between inflation and R&D, and a negative relationship between inflation and economic growth. These theoretical predictions are consistent with recent empirical evidence. Therefore, when inflation is a fundamental variable that affects the economy, unemployment and economic growth exhibit a negative relationship.

JEL classification: E24, E41, O30, O40
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1 Introduction

The relationship between unemployment and economic growth has long been a fundamental question in economics. This study provides a growth-theoretic analysis on this important relationship in a Schumpeterian model. Creative destruction refers to the process through which new technologies destroy existing firms. On the one hand, the destructive part of this process leads to job losses. On the other hand, new technologies also create new firms with new employment opportunities. In a frictionless labor market, these job destructions and job creations could offset each other leaving the labor market with full employment. However, given the presence of matching frictions between firms and workers, this continuous turnover in the labor market as a result of creative destruction leads to what Joseph Schumpeter (1939) referred to as "technological unemployment". At the first glance, it may seem that technological unemployment is a very specific kind of unemployment; however, as Schumpeter [1911] (2003, p.89) wrote, "[i]t doubtlessly explains a good deal of the phenomenon of unemployment, in my opinion its better half."

To analyze the effects of inflation on the long-run nexus between unemployment and economic growth, we introduce money demand into a scale-invariant Schumpeterian growth model with equilibrium unemployment via a cash-in-advance (CIA) constraint on R&D investment. Early empirical studies such as Hall (1992) and Opler *et al.* (1999) find a positive and significant relationship between R&D and cash flows in US firms. Bates *et al.* (2009) document that the average cash-to-assets ratio in US firms increased substantially from 1980 to 2006 and argue that this is partly driven by their rising R&D expenditures. Brown and Petersen (2011) provide evidence that firms smooth R&D expenditures by maintaining a buffer stock of liquidity in the form of cash reserves. A recent study by Berentsen *et al.* (2012) argues that information frictions and limited collateral value of intangible R&D capital prevent firms from financing R&D investment through debt or equity forcing them to fund R&D projects with cash reserves. We capture these cash requirements on R&D investment using a CIA constraint.

Given the CIA constraint on R&D, an increase in inflation that determines the opportunity cost of cash holdings raises the cost of R&D investment. Consequently, a higher inflation decreases R&D. Given that we remove scale effects[1] by considering a semi-endogenous-growth Schumpeterian model in which the long-run rate of creative destruction is determined by exogenous parameters, a decrease in R&D leads to a decrease in the growth rate of technology only in the short run but decreases the level of technology in the long run. Although the rate of creative destruction decreases temporarily, the decrease in innovation decreases the number of labor-market vacancies in the long run causing a *positive* effect on long-run unemployment. In other words, due to the decrease in labor market tightness, a higher inflation increases unemployment. This positive long-run relationship between inflation and unemployment is consistent with the empirical relationship based on US data documented in Ireland (1999), Beyer and Farmer (2007) and Berentsen *et al.* (2011). As for the negative relationship between inflation and R&D, it is consistent with the cross-country evidence in Chu and Lai (2013). Finally, the negative relationship between inflation and economic growth is supported by the cross-country evidence in Fischer (1993), Guerrero (2006), Vaona (2012) and Chu *et al.* (2013). Therefore, when inflation is a fundamental variable that affects the economy, unemployment and economic growth (measured by the level of technology and output) exhibit a negative relationship.

This study relates to the literature on Schumpeterian growth; see Segerstrom *et al.* (1990),

[1]See Jones (1999) for a discussion of scale effects in the R&D-based growth model.

Grossman and Helpman (1991) and Aghion and Howitt (1992) for seminal studies. However, these studies feature full employment rendering them unsuitable for the purpose of analyzing unemployment. Early contributions in the Schumpeterian theory of unemployment are Aghion and Howitt (1994, 1998), Cerisier and Postel-Vinay (1998), Mortensen and Pissarides (1998), Pissarides (2000), Şener (2000, 2001) and Postel-Vinay (2002). The present study complements these seminal studies by introducing money demand into the Schumpeterian model with unemployment and analyzing the effects of inflation on unemployment and economic growth. To our knowledge, this combination of Schumpeterian growth, money demand and equilibrium unemployment is novel to the literature.

This study also relates to the literature on inflation and economic growth. In this literature, Stockman (1981) and Abel (1985) analyze the effects of inflation via a CIA constraint on capital investment in a monetary version of the Neoclassical growth model. Subsequent studies in this literature explore the effects of inflation in variants of the capital-based growth model. This study instead relates more closely to the literature on inflation and *innovation-driven* growth. In this literature, Marquis and Reffett (1994) analyze the effects of inflation via a CIA constraint on consumption in a variety-expanding growth model based on Romer (1990). In contrast, we analyze the effects of inflation in a Schumpeterian quality-ladder model. Chu and Cozzi (2013) and Chu and Lai (2013) also analyze the relationship between inflation and economic growth in the Schumpeterian model. However, all these studies exhibit full employment due to the absence of matching frictions in the labor market. The present study provides a novel contribution to the literature by introducing equilibrium unemployment driven by matching frictions to the monetary Schumpeterian growth model. A recent study by Wang and Xie (2013) also analyzes the effects of inflation on economic growth and unemployment driven by matching frictions in the labor market. Their model generates money demand via CIA constraints on consumption and wage payment to production workers. In contrast, we model money demand via a CIA constraint on R&D. More importantly, they consider capital accumulation as the engine of economic growth whereas our analysis complements their interesting study by exploring a different growth engine that is R&D and innovation.

The rest of this study is organized as follows. Section 2 describes the Schumpeterian model. Section 3 analyzes the effects of inflation on unemployment and economic growth. The final section concludes.

2 A monetary Schumpeterian model with unemployment

The simple Schumpeterian model with equilibrium unemployment in the present study is based on Mortensen (2005). In this model, economic growth is driven by quality improvement. R&D entrepreneurs invent higher-quality products in order to dominate the market and earn monopolistic profits. When R&D entrepreneurs create new inventions, they open up vacancies to recruit workers from the labor market, in which the number of job separations is determined by creative destruction and the number of job matches is determined by an aggregate matching function and labor market tightness. Due to matching frictions between workers and firms with new technologies, the economy features equilibrium unemployment in the long run. We modify the model in Mortensen (2005) by (a) allowing for population growth and removing scale effects via increasing R&D difficulty as in Segerstrom (1998), (b) introducing money demand via CIA constraints on consumption and R&D investment as in Chu and Cozzi (2013), and (c) considering elastic labor supply.

2.1 Households

In the economy, the population size is L_t that increases at an exogenous rate $g > 0$, and there is a representative household who has the following lifetime utility function:

$$U = \int_0^{\infty} e^{-(\rho-g)t} \left[\ln c_t + \gamma \ln(1 - l_t)\right] dt, \tag{1}$$

where c_t denotes per capita consumption (numeraire) and l_t denotes per capita labor supply at time t. The parameter $\rho > g$ determines subjective discounting, and $\gamma \geq 0$ determines leisure preference. The asset-accumulation equation expressed in real terms is given by

$$\dot{a}_t + \dot{m}_t = (r_t - g)a_t - (\pi_t + g)m_t + i_t d_t + I_t - \tau_t - c_t. \tag{2}$$

a_t is the real value of financial assets (in the form of equity shares in monopolistic intermediate goods firms) owned by each member of the household. r_t is the real interest rate. π_t is the inflation rate. m_t is the real money balance accumulated by each member of the household. d_t is the amount of money lent to R&D entrepreneurs subject to the following constraint: $d_t + \xi c_t \leq m_t$, where $\xi \in [0, 1]$ parameterizes the strength of the CIA constraint on consumption. The interest rate on money lending d_t to R&D entrepreneurs is the nominal interest rate,[2] which is equal to $i_t = r_t + \pi_t$ from the Fisher identity. τ_t is a lump-sum tax levied on each member of the household. I_t is the per capita amount of labor income given by $I_t \equiv (w_t x_t + \omega_t R_t + b_t u_t)/L_t$, where w_t is the wage rate of production labor x_t, ω_t is the wage rate of R&D labor R_t, and b_t is unemployment benefits provided to unemployed labor u_t. To ensure balanced growth, we assume that $b_t = \bar{b} y_t / L_t$ is proportional to total output per capita, where $\bar{b} \in (0, 1)$ is an unemployment-benefit parameter. The size of the labor force at time t is $l_t L_t$, and the resource constraint on labor is

$$x_t + R_t + u_t = l_t L_t. \tag{3}$$

Following the standard treatment in the literature, we assume that workers are mobile across R&D and manufacturing. Given this substitutability between manufacturing labor and R&D labor, it is better not to think of R&D labor as high-skilled R&D entrepreneurs but instead as low- to medium-skilled research assistants, administrative and support staff employed by the R&D entrepreneurs.

The household maximizes (1) subject to (2) and (3). The resulting optimality condition for labor supply is

$$l_t = 1 - \frac{\gamma(1 + \xi i_t)c_t}{\omega_t}, \tag{4}$$

where the opportunity cost of leisure is the R&D wage rate ω_t because workers can freely choose between employment in the R&D sector and job search.[3] The intertemporal optimality condition is

$$-\frac{\dot{\zeta}_t}{\zeta_t} = r_t - \rho, \tag{5}$$

[2] It can be easily shown as a no-arbitrage condition that the interest rate on d_t must be equal to i_t.

[3] Although this formulation appears to be unrealistic, Mortensen (2005) argues that "in the real world R&D requires "skill," which is either an endowment that is distributed over the population or is acquired through education. Still these two alternative specifications require either that the marginal worker is indifferent between unemployed search and R&D activity or that the value of acquiring the needed education is equal to the value of unemployment respectively. My conjecture is that the comparative static implications of either specification are not materially different from those implied by the simpler model studied here."

where ζ_t is the Hamiltonian co-state variable on (2) and determined by $\zeta_t = [(1+\xi i_t)c_t]^{-1}$. In the case of a constant nominal interest rate i, (5) becomes the familiar Euler equation $\dot{c}_t/c_t = r_t - \rho$.

2.2 Final goods

Final goods y_t are produced by perfectly competitive firms that aggregate a unit continuum of intermediate goods using the following Cobb-Douglas aggregator:

$$y_t = \exp\left\{\int_0^1 \ln\left[A_t(j)x_t(j)\right] dj\right\}, \tag{6}$$

where $A_t(j) \equiv q^{n_t(j)}$ is the productivity or quality level of intermediate good $x_t(j)$. The parameter $q > 1$ is the exogenous step size of each quality improvement, and $n_t(j)$ is the number of innovations that have been invented *and implemented* in industry j as of time t. The price of final goods is normalized to $P_t = 1$. From profit maximization, the conditional demand function for $x_t(j)$ is

$$x_t(j) = y_t/p_t(j), \tag{7}$$

where $p_t(j)$ is the price of $x_t(j)$ for $j \in [0,1]$.

2.3 Intermediate goods

There is a unit continuum of industries producing differentiated intermediate goods. Each industry is temporarily dominated by a quality leader until the arrival and implementation of the next higher-quality product. The owner of the new innovation becomes the next quality leader.[4] The current quality leader in industry j uses one unit of labor to produce one unit of intermediate good $x_t(j)$. We follow Mortensen (2005) to assume that the employer has no outside option and the workers' outside option is unemployment benefit b_t. In this case, the generalized Nash bargaining game is[5]

$$\{x_t(j), w_t(j)\} = \arg\max\{[w_t(j) - b_t]x_t(j)\}^{\beta}\{[p_t(j) - w_t(j)]x_t(j)\}^{1-\beta}, \tag{8}$$

where the parameter $\beta \in (0,1)$ measures the bargaining power of workers. The bargaining outcome on wage is[6]

$$w_t(j) = \beta p_t(j) + (1-\beta)b_t, \tag{9}$$

which is an average between the marginal revenue product $p_t(j)$ of each worker and the value of unemployment benefit b_t weighted by the bargaining power of workers. The employer and workers commit to this wage schedule over the lifetime of the firm. Substituting (9) into (8) shows that the $x_t(j)$ that maximizes (8) is the same as the $x_t(j)$ that maximizes the following profit function:

$$\Pi_t(j) = [p_t(j) - w_t(j)]x_t(j) = (1-\beta)[p_t(j) - b_t]x_t(j) = (1-\beta)[y_t - b_t x_t(j)], \tag{10}$$

[4] This is known as the Arrow replacement effect; see Cozzi (2007a) for a discussion of the Arrow effect.

[5] Mortensen (2005) argues that using a more general bargaining condition with the value functions of employment and unemployment would complicate the model without providing new insight; see footnote 3 in his paper.

[6] This bargaining outcome can also be obtained from $w_t(j) = \arg\max\{[w_t(j) - b_t]^{\beta}[p_t(j) - w_t(j)]^{1-\beta}\}$ (i.e., individual wage bargaining).

where the second equality uses (9) and the third equality uses (7).

We follow Howitt (1999) and Dinopoulos and Segerstrom (2010) to consider the realistic case in which new quality leaders do not engage in limit pricing with previous quality leaders because after the implementation of the newest innovations, previous quality leaders immediately exit the market. Given the Cobb-Douglas aggregator in (6), the unconstrained monopolistic price would be infinity (i.e., $x_t(j) \to 0$). We follow Evans *et al.* (2003) to consider price regulation under which the regulated markup ratio cannot be greater than $z > 1$ such that

$$p_t(j) = zw_t(j) = z\frac{1-\beta}{1-\beta z}b_t, \tag{11}$$

where the second equality uses (9), and we impose an additional parameter restriction given by $\beta z < 1$. Substituting (11) into (7) yields

$$x_t(j) = x_t = \frac{1-\beta z}{(1-\beta)z}\frac{y_t}{b_t} = \frac{1-\beta z}{(1-\beta)z\bar{b}}L_t, \tag{12}$$

where the last equality uses $b_t = \bar{b}y_t/L_t$. Finally, the amount of monopolistic profit is

$$\Pi_t(j) = \Pi_t = (p_t - w_t)x_t = \frac{z-1}{z}y_t. \tag{13}$$

Given that the amount of monopolistic profit is the same across industries, we will follow the standard treatment in the literature to focus on the symmetric equilibrium, in which the arrival rate of innovations is equal across industries.[7]

2.4 R&D

R&D is performed by a continuum of competitive entrepreneurs. If an R&D entrepreneur employs $\widetilde{R}_t$ units of labor to engage in innovation in an industry, then she is successful in inventing the next higher-quality product in the industry with an instantaneous probability given by

$$\widetilde{\delta}_t = \frac{h\widetilde{R}_t}{A_t}, \tag{14}$$

where $h > 0$ is an innovation-productivity parameter that captures the abilities of the R&D entrepreneur. We assume that innovation productivity h/A_t decreases in aggregate quality $A_t \equiv \exp\left(\int_0^1 \ln A_t(j)dj\right)$ in order to capture increasing difficulty of R&D in the economy,[8] and this specification removes the scale effects in the innovation process of the quality-ladder model as in Segerstrom (1998).[9] The expected benefit from investing in R&D is $V_t\widetilde{\delta}_t dt$, where V_t is the value

[7] See Cozzi (2007b) for a discussion of multiple equilibria in the Schumpeterian model. Cozzi *et al.* (2007) provide a theoretical justification for the symmetric equilibrium to be the unique rational-expectation equilibrium in the Schumpeterian model.

[8] See Venturini (2012) for empirical evidence based on US manufacturing data that supports the semi-endogenous growth model with increasing difficulty of R&D.

[9] Segerstrom (1998) considers an industry-specific index of R&D difficulty. Here we consider an aggregate index of R&D difficulty to simplify notation without altering the aggregate results of our analysis.

of the expected discounted profits generated by a new innovation and $\widetilde{\delta}_t dt$ is the entrepreneur's probability of having a successful innovation during the infinitesimal time interval dt. To facilitate the wage payment to R&D labor, the entrepreneur borrows money from the household, and the cost of borrowing is determined by the nominal interest rate i_t. To parameterize the strength of this CIA constraint on R&D, we assume that a fraction $\sigma \in [0,1]$ of R&D expenditure requires the borrowing of money from households. Therefore, the total cost of R&D is $(1+\sigma i_t)\,\omega_t \widetilde{R}_t dt$. Free entry implies

$$V_t \widetilde{\delta}_t dt = (1+\sigma i_t)\,\omega_t \widetilde{R}_t dt \Leftrightarrow V_t = (1+\sigma i_t)\,\omega_t A_t/h, \tag{15}$$

where the second equality uses (14).

2.5 Matching and unemployment

When an R&D entrepreneur has a new innovation, she is not able to immediately launch the new product to the market due to matching frictions in the initial recruitment of workers.[10] Instead, she has to open up x_t vacancies to recruit x_t workers for producing and launching her products to the market. For simplicity, Mortensen (2005) assumes that the cost of a vacancy is zero. We follow the standard treatment in the search-and-matching literature to consider an aggregate matching function $F(v_t, u_t)$, where v_t is the number of vacancies in the labor market and u_t is the number of unemployed workers. $F(v_t, u_t)$ has the usual properties of being increasing, concave and homogeneous of degree one in v_t and u_t. In the economy, the number of successful matches at time t is given by $F(v_t, u_t)$; in other words, the number of workers who find jobs is $F(v_t, u_t)$. Therefore, the job-finding rate is

$$\lambda_t = F(v_t, u_t)/u_t = F(v_t/u_t, 1) \equiv M(\theta_t), \tag{16}$$

where $\theta_t \equiv v_t/u_t$ denotes labor market tightness, and $\lambda_t = M(\theta_t)$ is increasing in θ_t. Similarly, the number of vacancies filled is also $F(v_t, u_t)$, so the vacancy-filling rate is

$$\eta_t = F(v_t, u_t)/v_t = M(\theta_t)/\theta_t, \tag{17}$$

where $\eta_t = M(\theta_t)/\theta_t$ is decreasing in θ_t. Following the usual treatment in the literature,[11] we assume that when matching occurs to a firm at time t, the firm matches with x_t workers simultaneously. In other words, the number of successful matches at time t is first determined by the matching function $F(v_t, u_t)$, and then, these matches are randomly assigned to $F(v_t, u_t)/x_t$ firms. Therefore, the probability for a firm with opened vacancies to match with x_t workers at time t is also η_t. After an entrepreneur sets up her firm by having successful matches with x_t workers at time t, we assume for tractability that she can instantly recruit additional workers at the same wage schedule in (9) as demand x_t increases overtime.[12]

[10] Dinopoulos *et al.* (2013) consider an interesting setting, aimed at studying the importance of rent-seeking activities on unemployment, in which new firms are able to immediately recruit a fraction $\phi \in (0,1)$ of the desired number of workers x_t.

[11] See for example Mortensen (2005) and Dinopoulos *et al.* (2013).

[12] Equation (12) implies that x_t increases at the rate g. We assume g is sufficiently small such that it has negligible effects on the labor market.

2.6 Asset values

Let U_t denote the value of being unemployed. The familiar asset-pricing equation of U_t is

$$r_t = \frac{b_t + \dot{U}_t + \lambda_t(W_t - U_t)}{U_t}, \tag{18}$$

where λ_t is the rate at which an unemployed worker becomes employed and W_t denotes the value of being employed in an industry in which the subsequent innovation has not been invented. The asset-pricing equation of W_t is

$$r_t = \frac{w_t + \dot{W}_t + \widetilde{\delta}_t(S_t - W_t)}{W_t}, \tag{19a}$$

where $\widetilde{\delta}_t$ is the rate at which the subsequent innovation is invented and S_t denotes the value of being employed in an industry in which the subsequent innovation has been invented but not yet been launched to the market.[13] The asset-pricing equation of S_t is

$$r_t = \frac{w_t + \dot{S}_t + \eta_t(U_t - S_t)}{S_t}, \tag{19b}$$

where η_t is the rate at which the subsequent innovation is launched to the market and the worker becomes unemployed. Given that a worker must be indifferent between being employed by an R&D entrepreneur and engaging in job search, the wage of R&D workers is equal to

$$\omega_t = r_t U_t - \dot{U}_t. \tag{20}$$

We follow Mortensen (2005) to assume that the subsequent innovation cannot be invented until the current innovation has been launched to the market.[14] The asset-pricing equation of V_t, which is the value of a new innovation for which its vacancies have not been filled, is given by

$$r_t = \frac{\dot{V}_t + \eta_t\left(Z_t - V_t\right)}{V_t}, \tag{21a}$$

where η_t is the rate at which the product is launched to the market. The asset-pricing equation of Z_t, which is the value of the innovation when its vacancies have been filled, is given by

$$r_t = \frac{\Pi_t + \dot{Z}_t + \widetilde{\delta}_t\left(X_t - Z_t\right)}{Z_t}, \tag{21b}$$

where $\widetilde{\delta}_t$ is the rate at which the subsequent innovation is invented. The asset-pricing equation of X_t, which is the value of the current innovation when the subsequent innovation has been invented but not yet been launched to the market, is given by

$$r_t = \frac{\Pi_t + \dot{X}_t - \eta_t X_t}{X_t}, \tag{21c}$$

where η_t is the rate at which the subsequent innovation is launched to the market.

[13] For simplicity, Mortensen (2005) assumes that the current quality leader stops its operation as soon as the next innovation is *invented*. Here we relax this assumption by allowing the current quality leader to continue its operation until the next innovation is *implemented*. This generalization is rational for the current quality leader, who continues to earn profits, and also for the workers because $S_t > U_t$.

[14] Mortensen (2005) writes, "the assumption that a product cannot be replaced by a better one until after it appears on the market [...] is plausible when the technology of each innovation builds on the last one because reverse engineering is not possible until examples of the product or services are available for inspection."

2.7 Government

The monetary policy instrument that we consider is the inflation rate π_t, which is exogenously set by the monetary authority. Given π_t, the nominal interest rate is endogenously determined according to the Fisher identity such that $i_t = \pi_t + r_t$, where r_t is the real interest rate. The growth rate of the nominal money supply (per capita) is $\mu_t = \pi_t + \dot{m}_t/m_t$.[15] Finally, the government balances the fiscal budget subject to the following per capita version of the balanced-budget condition: $\tau_t = b_t u_t/L_t - (\mu_t + g)m_t$.

2.8 Steady-state equilibrium

We will define the aggregate innovation-arrival rate as $\delta_t = (1 - f_t)\widetilde{\delta}_t$, where f_t is the measure of industries with unlaunched innovations. The outflow from the pool of firms searching for workers is given by $\eta_t f_t$, and the inflow into this pool is given by δ_t. Therefore, in the steady state, we must have $\eta_t f_t = \delta_t$. The aggregate production function of final goods is given by $y_t = A_t x_t$, where (the log of) aggregate technology A_t is defined as

$$\ln A_t \equiv \int_0^1 \ln A_t(j)dj = \int_0^1 n_t(j)dj \ln q = \int_0^t \eta_v f_v dv \ln q, \tag{22}$$

where we have normalized $A_0 = 1$ (i.e., $\ln A_0 = 0$). Differentiating (22) with respect to t yields $\dot{A}_t/A_t = \eta_t f_t \ln q$, where $\eta_t f_t$ is the measure of industries with newly launched innovations at time t. The steady-state growth rate of A_t is

$$\frac{\dot{A}_t}{A_t} = \eta f \ln q = \delta \ln q = g, \tag{23}$$

where the third equality holds because $\delta = hR_t/A_t$ must be constant on the balanced growth path implying that R_t and A_t must both grow at the exogenous rate g in the long run. Therefore, the steady-state rate of creative destruction is determined by exogenous parameters such that $\delta = g/\ln q$.

On the balanced growth path, (20) becomes

$$\omega_t = \rho U_t. \tag{24}$$

Solving (18) and (19) yields the balanced-growth value of ρU_t given by

$$\rho U_t = \rho \frac{(\rho+\eta)\,(\rho+\widetilde{\delta})b_t + (\rho+\eta+\widetilde{\delta})\lambda w_t}{(\rho+\eta)\,(\rho+\widetilde{\delta})\,(\rho+\lambda) - \widetilde{\delta}\lambda\eta}, \tag{25}$$

where $\widetilde{\delta} = \delta/(1-f) = \delta/(1-\delta/\eta)$. From (21), the balanced-growth value of V_t is

$$V_t = \frac{\eta(\Pi_t + \widetilde{\delta}X_t)}{(\rho-g+\eta)(\rho-g+\widetilde{\delta})} = \frac{\rho-g+\eta+\widetilde{\delta}}{(\rho-g+\eta)^2(\rho-g+\widetilde{\delta})}\eta\Pi_t. \tag{26}$$

[15] It is useful to note that in this model, it is the *growth rate* of the money supply that affects the real economy in the long run, and a one-time change in the *level* of money supply has no long-run effect on the real economy. This is the well-known distinction between the neutrality and superneutrality of money. Empirical evidence generally favors neutrality and rejects superneutrality, consistent with our model; see Fisher and Seater (1993) for a discussion on the neutrality and superneutrality of money.

Substituting (24)-(26) into (15) yields

$$\frac{\rho - g + \eta + \widetilde{\delta}}{(\rho - g + \eta)^2(\rho - g + \widetilde{\delta})}\eta\Pi_t = \frac{A_t}{h}\left(1 + \sigma i\right)\rho\frac{(\rho + \eta)\left(\rho + \widetilde{\delta}\right)b_t + (\rho + \eta + \widetilde{\delta})\lambda w_t}{(\rho + \eta)\left(\rho + \widetilde{\delta}\right)(\rho + \lambda) - \widetilde{\delta}\lambda\eta}. \tag{27}$$

For convenience, we define a transformed variable $\alpha_t \equiv A_t/L_t$, which is the per capita level of aggregate technology. Substituting (11), (13) and $b_t = \bar{b}y_t/L_t$ into (27) and then rearranging terms yield

$$\alpha = \frac{z-1}{(1+\sigma i)z\bar{b}}\frac{h}{\rho - g + \widetilde{\delta}}\frac{(\rho + \eta)\left(\rho + \widetilde{\delta}\right)(\rho + \lambda) - \widetilde{\delta}\lambda\eta}{(\rho + \eta)\left(\rho + \widetilde{\delta}\right) + \lambda(\rho + \eta + \widetilde{\delta})(1 - \beta)/(1 - \beta z)}\Theta(\theta), \tag{28}$$

where $\widetilde{\delta} = \delta/(1 - \delta/\eta)$, $\lambda = M(\theta)$, $\eta = M(\theta)/\theta$ and

$$\Theta(\theta) \equiv \frac{\eta}{\rho - g + \eta}\left(1 + \frac{\widetilde{\delta}}{\rho - g + \eta}\right).$$

We refer to (28) as the R&D free-entry (FE) condition, which contains two endogenous variables $\{\alpha, \theta\}$.[16] It is useful to note that the FE condition depends on the nominal interest rate i via the CIA constraint on R&D (i.e., $\sigma > 0$).

Lemma 1 *The FE curve describes a negative relationship between α and θ if ρ is sufficiently large.*

Proof. See Appendix A. ■

To close the model, we use the following steady-state condition that equates the inflow δ into the pool of firms searching for workers to its outflow ηf:

$$\delta = \eta f = \eta v/x = M(\theta)u/x, \tag{29}$$

where the second equality follows from $fx = v$, where f is the number of firms with opened vacancies and x is the number of vacancies per firm. The third equality in (29) follows from (17) and uses the definition of $\theta \equiv v/u$. Furthermore, we need to derive the equilibrium supply of labor l. Substituting (11), (24), (25), $b_t = \bar{b}y_t/L_t$ and $c_t = y_t/L_t$ into (4) yields

$$l(\underset{-}{i}, \underset{+}{\theta}) = 1 - \frac{\gamma(1 + \xi i)}{\rho\bar{b}}\frac{(\rho + \eta)\left(\rho + \widetilde{\delta}\right)(\rho + \lambda) - \widetilde{\delta}\lambda\eta}{(\rho + \eta)\left(\rho + \widetilde{\delta}\right) + (\rho + \eta + \widetilde{\delta})\lambda(1 - \beta)/(1 - \beta z)}, \tag{30}$$

which is increasing in θ as we will show in the proof of Lemma 2. Substituting (3), (12), (14) and (30) into (29) and applying the definition of $\alpha \equiv A/L$ yield

$$\alpha = \frac{h}{\delta}\left\{l(\underset{-}{i}, \underset{+}{\theta}) - \left[1 + \frac{\delta}{M(\theta)}\right]\frac{1 - \beta z}{(1 - \beta)z\bar{b}}\right\}. \tag{31}$$

We refer to (31) as the labor-market (LM) condition, which also contains two endogenous variables $\{\alpha, \theta\}$. It is useful to note that the LM condition depends on the nominal interest rate i via the CIA constraint on consumption (i.e., $\xi > 0$). Finally, (28) and (31) can be used to solve for the steady-state equilibrium values of $\{\theta, \alpha\}$; see Figure 1 for an illustration.

[16] Recall that $\delta = g/\ln q$ is determined by exogenous parameters in the steady state.

Lemma 2 *The LM curve describes a positive relationship between α and θ if ρ is sufficiently large.*

Proof. See Appendix A. ■

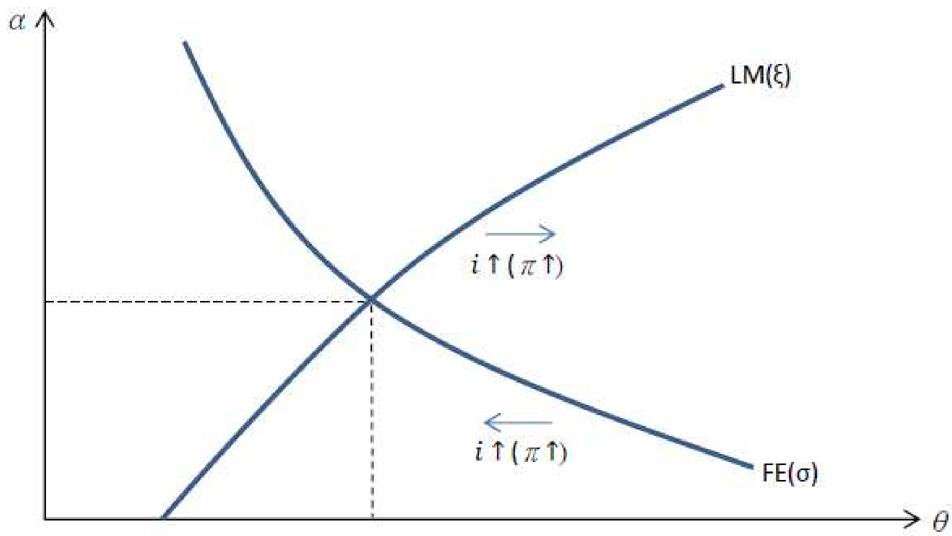

Figure 1: Steady-state equilibrium

3 Inflation, unemployment and economic growth

In this section, we explore the relationship between inflation, unemployment and economic growth. Section 3.1 considers the effects of inflation via the CIA constraint on R&D (i.e., $\sigma > 0$ and $\xi = 0$). Section 3.2 considers the effects of inflation via the CIA constraint on consumption (i.e., $\sigma = 0$ and $\xi > 0$). In Section 3.3, we explore an extension of the model with an additional CIA constraint on manufacturing.

3.1 Inflation via the CIA constraint on R&D

In this subsection, we explore the effects of inflation on unemployment and economic growth under the CIA constraint on R&D. From the Fisher identity, we have $i = \pi + r = \pi + g + \rho$, where the second equality uses the Euler equation and $\dot{c}_t/c_t = \dot{A}_t/A_t = g$. Therefore, a one-unit increase in the inflation rate leads to a one-unit increase in the nominal interest rate in the long run.[17] In Figure 1, we see that an increase in the nominal interest rate i (caused by an increase in inflation π) shifts the FE curve to the left reducing labor market tightness θ and the (per capita) level of technology α. As for the resulting effect on unemployment u, we see from (29) that unemployment $u = \delta x / M(\theta)$, where δ and x are determined by exogenous parameters and independent of i, is

[17] For example, Mishkin (1992) and Booth and Ciner (2001) provide empirical evidence for a positive relationship between inflation and the nominal interest rate in the long run.

decreasing in the job-finding rate $M(\theta)$. Therefore, the increase in inflation π raises unemployment u by reducing labor market tightness θ and the job-finding rate $M(\theta)$. From (14), aggregate R&D is given by $R = \alpha L\delta/h$; therefore, the higher inflation π (that decreases the level of technology α) also reduces R&D. Now we consider the effect of inflation on economic growth. The dynamics of per capita technology $\alpha_t \equiv A_t/L_t$ is given by $\dot{\alpha}_t/\alpha_t = \dot{A}_t/A_t - g$. Therefore, given that a higher inflation π decreases the steady-state value of α, it must also decrease the growth rate of A_t temporarily such that $\dot{A}_t/A_t < g$ before α_t reaches the new steady state. We summarize all these results in Proposition 1.

Proposition 1 *Under the CIA constraint on R&D, a higher inflation has (a) a positive effect on unemployment, (b) a negative effect on R&D, (c) a negative effect on the growth rate of technology in the short run, and (d) a negative effect on the level of technology in the long run.*

Proof. Proven in text. ■

The intuition of Proposition 1 can be explained as follows. A higher inflation leads to an increase in the opportunity cost of cash holdings, which in turn increases the cost of R&D investment via the CIA constraint on R&D. As a result, R&D decreases resulting into a lower growth rate of technology in the short run and a lower level of technology in the long run. Although the rate of creative destruction decreases temporarily, the decrease in innovation also reduces the number of labor-market vacancies in the long run. Consequently, this reduction in labor-market tightness increases long-run unemployment. These theoretical predictions are consistent with the empirical evidence discussed in the introduction. Therefore, when inflation is a fundamental variable that affects the economy, unemployment and economic growth (measured by the level of technology and output) exhibit a negative relationship.

3.2 Inflation via the CIA constraint on consumption

In this subsection, we explore the effects of inflation on unemployment and economic growth under the CIA constraint on consumption. In this case, Figure 1 shows that an increase in inflation π shifts the LM curve to the right increasing labor market tightness θ and decreasing the level of technology α. As for the resulting effect on unemployment u, we see from (29) that unemployment $u = \delta x/M(\theta)$ is decreasing in the job-finding rate $M(\theta)$. Therefore, the increase in inflation π surprisingly reduces unemployment u. From (14), aggregate R&D is given by $R = \alpha L\delta/h$; therefore, the higher inflation π also reduces R&D. As for the effect of inflation on economic growth, given that inflation π decreases the steady-state value of α, it must decrease the growth rate of A_t temporarily before α_t reaches the new steady state. We summarize these results in Proposition 2.

Proposition 2 *Under the CIA constraint on consumption, a higher inflation has (a) a negative effect on unemployment, (b) a negative effect on R&D, (c) a negative effect on the growth rate of technology in the short run, and (d) a negative effect on the level of technology in the long run.*

Proof. Proven in text. ■

The intuition behind the negative relationship between inflation and unemployment can be explained as follows. A higher inflation leads to an increase in the opportunity cost of cash holdings, which in turn increases the cost of consumption relative to leisure. As a result, the household consumes more leisure and reduces labor supply. The decrease in labor supply reduces the number of workers searching for employment. The resulting increase in labor-market tightness decreases unemployment. In other words, our model has opposite implications on the long-run relationship between inflation and unemployment under the two CIA constraints. Specifically, under the CIA constraint on consumption (R&D), the model predicts a negative (positive) relationship between inflation and unemployment.

Berentsen *et al.* (2011) document a positive long-run relationship between inflation and unemployment in the US. Berentsen *et al.* (2011) consider the unemployment rate in the data, but the comparative statics from our model have implications on the share of unemployed workers in the population.[18] Therefore, we follow the approach in Berentsen *et al.* (2011) to document the long-run relationship between inflation and the share of unemployed workers in the population. We consider annual data from 1953 to 2007 to extend the data series in Berentsen *et al.* (2011) to the year before the financial crises in 2008. Figure 2 plots the raw data, 5-year average data, and the HP-filtered trend data on inflation and unemployment in the US. These figures show that inflation and the share of unemployed workers in the population exhibit a positive long-run relationship. Therefore, the theoretical prediction of a negative relationship between inflation and unemployment contradicts empirical evidence suggesting that the CIA constraint on consumption is perhaps a less relevant channel than the CIA constraint on R&D through which inflation affects unemployment. To be more precise, we are not suggesting that the CIA constraint on consumption is irrelevant; instead, we simply find that the consumption-leisure tradeoff via the CIA constraint on consumption is not the empirically relevant channel through which inflation affects unemployment given its counterfactual implication.

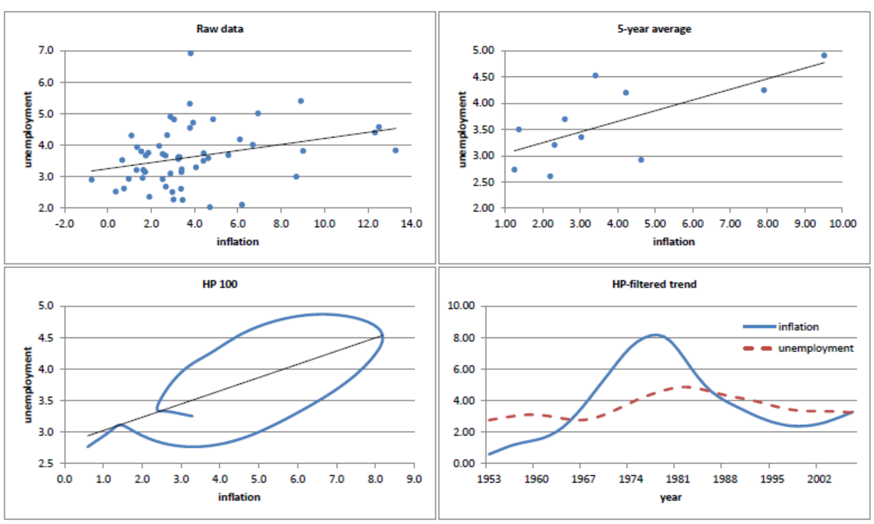

Figure 2: Inflation and unemployment

[18] In our model, the unemployment rate generally moves in the same direction as the share of unemployed workers in the population, but we are unable to derive an absolutely unambiguous result in the case of the CIA constraint on consumption.

3.3 Extension: Inflation via the CIA constraint on manufacturing

In this subsection, we explore an extension of the model under which manufacturing employment x is also affected by inflation. Specifically, we introduce an additional CIA constraint on the wage payment to manufacturing workers.[19] For tractability, we consider inelastic labor supply (i.e., $\gamma = 0$ and $l = 1$) under which the CIA constraint on consumption has no real effect. Under the CIA constraint on manufacturing, the bargaining condition in (8) becomes

$$\{x_t(j), w_t(j)\} = \arg\max\{[w_t(j) - b_t]x_t(j)\}^{\beta}\{[p_t(j) - (1 + \varepsilon i_t)\, w_t(j)]x_t(j)\}^{1-\beta}, \tag{8a}$$

where $\varepsilon \in [0, 1]$ parameterizes the strength of this CIA constraint. The bargaining outcome on wage becomes

$$w_t(j) = \frac{\beta}{1 + \varepsilon i_t} p_t(j) + (1 - \beta)\, b_t, \tag{9a}$$

and the monopolistic price becomes

$$p_t\,(j) = z(1 + \varepsilon i_t)w_t(j) = z(1 + \varepsilon i_t)\frac{1 - \beta}{1 - \beta z} b_t. \tag{11a}$$

Due to the higher price, the demand for $x_t(j)$ decreases to

$$x_t(j) = x_t = \frac{1}{1 + \varepsilon i_t} \frac{1 - \beta z}{(1 - \beta)\, z\bar{b}} L_t. \tag{12a}$$

The rest of the model remains unchanged. Because the nominal interest rate affects x_t via the CIA constraint on manufacturing, its effect εi_t appears in the LM curve in (31), which becomes

$$\alpha = \frac{h}{\delta}\left\{1 - \frac{1}{1 + \varepsilon i}\left[1 + \frac{\delta}{M(\theta)}\right]\frac{1 - \beta z}{(1 - \beta)\, z\bar{b}}\right\}. \tag{31a}$$

Figure 3 shows that an increase in inflation π shifts the LM curve to the left, which by itself leads to a decrease in labor-market tightness θ and an increase in technology α. Together with the shift in the FE curve via the CIA constraint on R&D, the overall effect on θ is negative, but the overall effect on α is ambiguous. However, if we assume that σ (i.e., the strength of the CIA constraint on R&D) is sufficiently large, then a higher inflation π would decrease technology α implying a negative relationship between inflation and R&D and a negative relationship between inflation and economic growth in accordance with the empirical evidence discussed in the introduction. From (29), we have $u = \delta x/M(\theta)$, where x and θ are both decreasing in π. It may seem that the overall effect on unemployment u has become ambiguous; however, if we use the resource constraint on labor in (3), we see that $u = L - (x + R)$ must be increasing in π because both x and $R = \alpha L\delta/h$ are decreasing in π (assuming that σ is sufficiently large). In summary, we find that under the two CIA constraints on R&D and manufacturing, a higher inflation decreases employment in both the R&D and manufacturing sectors and increases unemployment. In other words, the presence of a CIA constraint on manufacturing strengthens the inflation-induced negative relationship between unemployment and economic growth (assuming that σ is sufficiently large).

[19] See also Wang and Xie (2013) who analyze a CIA constraint on manufacturing.

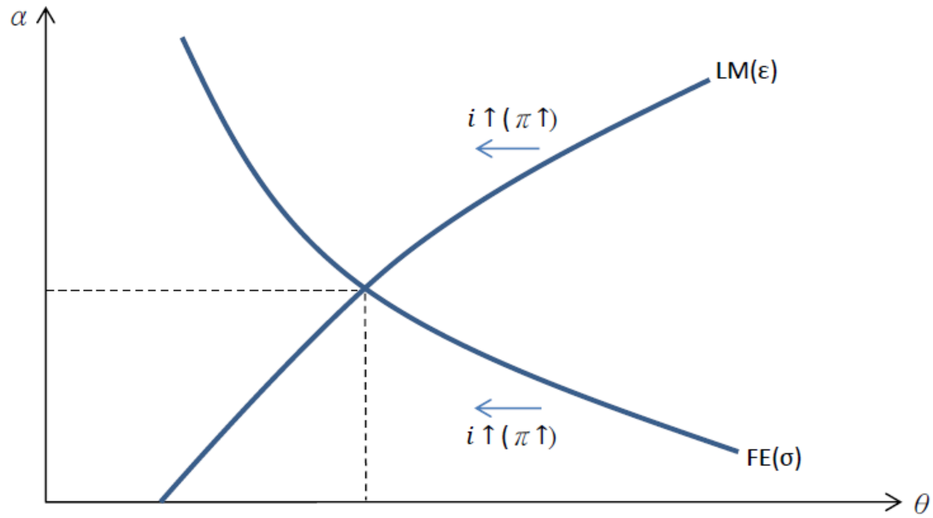

Figure 3: CIA on manufacturing and R&D

4 Conclusion

In this study, we have explored a fundamental question in economics that is the long-run relationship between inflation, unemployment and economic growth. We consider a standard Schumpeterian growth model with the additions of money demand via CIA constraints and equilibrium unemployment driven by matching frictions in the labor market. In this monetary growth-theoretic framework, we discover a positive relationship between inflation and unemployment (under the CIA constraint on R&D), a negative relationship between inflation and R&D, and a negative relationship between inflation and economic growth. These theoretical predictions are consistent with empirical evidence.

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Appendix A

Proof of Lemma 1. First, we restrict the range of values for θ to ensure that (a) $\lambda = M(\theta) < 1$ (i.e., the number of workers who find jobs at a given point in time must be less than the number of workers searching for jobs at that time), (b) $\eta = M(\theta)/\theta < 1$ (i.e., the number of vacancies filled at a given point in time must be less than the number of vacancies on the market at that time), and (c) $\eta > \delta$ so that $f = \delta/\eta < 1$ (i.e., the number of industries with unlaunched innovations must be less than the total number of industries, which is normalized to unity). Then, we examine each term on the right-hand side of (28) separately. The first term in (28) is independent of θ, whereas the second term in (28) is decreasing in θ given that $\tilde{\delta}$ increases with θ. The third term in (28) can be reexpressed as

$$\vartheta(\theta) \equiv \rho \frac{\rho + \tilde{\delta} + \lambda(1 + \frac{\tilde{\delta}}{\rho+\eta})}{\rho + \tilde{\delta} + \lambda(1 + \frac{\tilde{\delta}}{\rho+\eta})(1-\beta)/(1-\beta z)}. \tag{A1}$$

Given $(1-\beta)/(1-\beta z) > 1$, we can show that $\vartheta'(\theta) < 0$ holds if[20]

$$\{[1 - \delta\theta/M(\theta)]^2 \rho + 1\}/\delta > 1/M'(\theta), \tag{A2}$$

which holds if ρ is sufficiently large because $1 - \delta\theta/M(\theta) = 1 - \delta/\eta > 0$. As for the fourth term in (28), noting $\eta = M(\theta)/\theta$ and $\tilde{\delta} = \delta/[1 - \delta\theta/M(\theta)]$, we can show that $\Theta'(\theta) < 0$ holds if and only if[21]

$$\frac{\rho - g}{\delta} \underbrace{\left([1 - \delta\theta/M(\theta)]^2 + \frac{2\delta\,[1 - \delta\theta/M(\theta)]\,\theta/M(\theta)}{(\rho - g)\,\theta/M(\theta) + 1} \right)}_{\equiv \chi(\theta)} > 1. \tag{A3}$$

Note that $\chi(\theta) > 0$ because $\rho > g$ and $1 - \delta\theta/M(\theta) = 1 - \delta/\eta > 0$. Therefore, we can conclude this proof by saying that a large value of ρ is a sufficient (but not necessary) condition for the FE curve in (28) to be downward sloping in θ. ■

Proof of Lemma 2. By (30) and (A1), $l(i, \theta) = 1 - \vartheta(\theta)\gamma(1 + \xi i)/\rho\bar{b}$. From the proof of Lemma 1, $\vartheta(\theta)$ is decreasing in θ if ρ is sufficiently large. Together with (31), one can easily show that the LM curve is upward sloping in θ. ■

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[20] Note that $\vartheta'(\theta) < 0$ holds if and only if

$$(\rho + \tilde{\delta}) \left[\lambda \left(1 + \frac{\tilde{\delta}}{\rho + \eta} \right) \right]' > (\rho + \tilde{\delta}')\lambda \left(1 + \frac{\tilde{\delta}}{\rho + \eta} \right),$$

which can be expressed as

$$\lambda(\rho + \tilde{\delta}) \left[\frac{\tilde{\delta}'(\rho + \eta) - \eta'\tilde{\delta}}{(\rho + \eta)^2} \right] > \left[\tilde{\delta}'\lambda - (\rho + \tilde{\delta})\lambda' \right] \left(1 + \frac{\tilde{\delta}}{\rho + \eta} \right).$$

Given $\lambda' > 0$, $\eta' < 0$, and $\tilde{\delta}' > 0$, this holds if $\tilde{\delta}'\lambda - (\rho + \tilde{\delta})\lambda' < 0$, which is equivalent to (A2) by $\lambda = M(\theta)$, $\tilde{\delta} = \delta/\left[1 - \delta\theta/M(\theta)\right]$, and $\tilde{\delta}' = \tilde{\delta}^2 \left[M(\theta) - \theta M'(\theta)\right]/M(\theta)^2$. Note that $M(\theta) > \theta M'(\theta)$ by the properties of $M(\theta)$.

[21] Note that

$$\Theta'(\theta) = \frac{\varkappa'(\theta)}{[1 - \delta\varkappa(\theta)]^2 \left[(\rho - g)\,\varkappa(\theta) + 1\right]^2} \left[\delta - (\rho - g) \left([1 - \delta\varkappa(\theta)]^2 + \frac{2\delta\varkappa(\theta)\,[1 - \delta\varkappa(\theta)]}{(\rho - g)\,\varkappa(\theta) + 1} \right) \right],$$

where $\varkappa(\theta) \equiv \theta/M(\theta)$ and thus $\varkappa'(\theta) > 0$.