Efficient Market Hypothesis in South Africa: Evidence from a threshold autoregressive (TAR) model

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EFFICIENT MARKET HYPOTHESIS IN SOUTH AFRICA: EVIDENCE FROM A THRESHOLD AUTOREGRESSIVE (TAR) MODEL

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ABSTRACT: This study deviates from the conventional use of a linear approach in testing for the efficiency market hypothesis (EMH) for the Johannesburg Stock Exchange (JSE) between the periods 2001:01 to 2013:07. By making use of a threshold autoregressive (TAR) model and corresponding asymmetric unit root tests, our study demonstrates how the stock market indexes evolve as highly persistent, nonlinear process and yet for a majority of the time series under observation, the formal unit root tests reject the hypothesis of stationarity among the variables. These results bridge two opposing contentions obtained from previous studies by concluding that while a number of stock prices under the JSE stock market may not evolve as pure unit root processes, the time series are, however, highly persistent to an extent of being able to be deemed as weak-form efficient.

Keywords: Efficient Market Hypothesis (EMH), Johannesburg stock Exchange (JSE), South Africa, Threshold Autoregressive (TAR) model, Unit Roots.

JEL Classification Code: C01, C13, C2, C22, G10.

1 INTRODUCTION

The South African economy stepped into the global arena and experienced financial liberalization subsequent to gaining independence in 1994. Since then, one of the major challenges in the Republic of South Africa (RSA) has been to regulate financial markets with the ultimate goal of making a positive contribution towards social and economic
development. In advancing towards this goal, academics and policymakers alike, are increasingly realizing the importance of capital markets in contributing towards the economic growth of the country. Policymakers and other observers view the regulation and development of capital markets as a worthy macroeconomic policy objective because of the belief that stabilized capital markets create a solid foundation towards a stable financial system. From an academic perspective, examples of the economic benefits of a developed or stabilized capital market are not difficult to come across in the literature. For instance, Boyd and Smith (1998) ascertain that stable stock markets are indeed compliments to the banking sector in promoting economic growth via the financial sector. Similarly, Greenwood and Smith (1997) suggest that stock market components of the financial system play an important role in the efficient allocation of resources which helps in promoting specialization, reducing the cost of mobilizing savings and ultimately higher economic growth. Overall, the aforementioned authors conclude that success in the development of stable financial markets is fundamentally dependent upon the assumption of an efficient capital/stock market.

The ability of the stock market to perform its role efficiently is highly contingent to the extent on which it can be deemed to be efficient (Ajao, 2012). The hypothesis demonstrating the efficiency of capital markets is grounded upon the realization that competitive behaviour existing among profit-seeking participants will ensure that asset prices continuously adjust to reflect all price-influential information (Jawadi et al, 2009). In this regard, an important attribute of efficient capital markets is that the prices of the securities must reflect all available information and any new information should be rapidly absorbed into the prices (Nisar and Hanif, 2012). The efficient market hypothesis (EMH) suggests that stock prices fully reflect all available information in the market and no investor is able to earn excess return on the basis of some secretly held private, public or historic information. In this sense, an efficient capital market makes it impossible for investors to forecast future price variations since the anticipated events are already integrated in the present stock price (Jawadi et al, 2009). Pragmatically, the EMH can be segregated into three forms depending upon the information set to which stock prices adjust. For instance, in the weak form EMH, prices reflect all past security market information; hence information on past prices and trading volumes cannot be used for profit. Within a semi-strong form efficient market, stock prices fully reflect all publically available information and are concerned with both the speed and accuracy of the market’s reaction to information as it becomes available. Under the strong form efficiency, prices are expected to reflect both public and private information and this hypothesis is concerned with the disclosure efficiency of the information market than the pricing efficiency of the securities market.

A plethora of empirical studies have been conducted to test the efficiency of the stock market in the context of both industrialized and emerging market economies. A vast majority of these studies opt to test the weak-form EMH by assimilating this hypothesis to the random walk of stock returns. While the findings of these studies generally support the weak-form efficiency for developed and mature stock exchanges, the empirical evidence for South Africa and other emerging economies remains inconclusive (Bonga-Bonga and Mukanze, 2010). One credible reason for the observed variation of empirical results obtained from previous studies is that they do not take into consideration possible nonlinear behaviour in the JSE stock indices. As conveniently noted by Lim (2011), the assumption of linearity may be trivializing the entire issue since this assumption implicitly implies that the level of market efficiency remains unchanged throughout the estimation period. Sources of asymmetric behaviour in stock markets are well documented in the literature and are inclusive of the presence of transition costs and market frictions; interaction of heterogeneous agents and
diversity in agents beliefs (Hasanov and Omay, 2007). In view of the growing consensus of possible asymmetric behaviour among stock prices, our study, therefore, considers the threshold autoregressive (TAR) model of Hansen (2000) to investigate possible regime-switching market efficiency behaviour in the Johannesburg Stock Exchange (JSE). We further supplement our empirical analysis by applying formal threshold unit roots, a la Bec et. al. (2004).

We present the remainder of our study as follows. The following section presents a brief review of previous literature in the South African context. Section three of the paper outlines the empirical framework used in the study whereas section four presents the data as well as the empirical results obtained from the study. We then conclude our study in section five by drawing out academic as well as policy implications associated with our study.

2 LITERATURE REVIEW

Weak-form efficiency in capital markets has been widely accepted as being a determining factor in supporting evidence of efficient stock markets across the empirical literature with South Africa bearing no exception to this rule. Earlier studies in the international literature (Fama (1965) and Osborne (1962)) as well as in the South African context (Jammine and Hawkins (1974), and Knight and Affleck-Graves (1983)) ran a variety of formal tests to confirm the existence of weak-form efficiency in various stock markets worldwide. However, more currently, the literature tends to present conflicting evidence pertaining to the subject matter, with such a conflict appearing to be more pronounced for developing or emerging economies. In an extensive review of previous studies conducted on the JSE, Thomas and Ward (1995) conclude that different methodologies applied to various time periods in the literature could account for the observed conflicting evidence in the literature. Take for instance, Smith et al. (2002), Jefferis and Smith (2005) and Magnusson and Wydick (2002), who have all found the JSE to be weak-form efficient using the runs test and random walk tests; whereas, other studies such as Appiah-Kusi and Menyah (2003), have on the contrary, concluded that the JSE is not weak-form efficient for the time period 1990 to 1995 while reverting to being weak-form efficient from the year 2000 onwards. As previously mentioned, such inconclusiveness is not only restricted to South African studies and can be also identified for a host of other emerging economies such as India (Gupta, 2007), Sri Lanka (Wickremasinghe, 2005), Jamaica (Robinson, 2005), South Asian economies (Nisar and Hanif, 2012), Latin American economies (Worthington and Higgs, 2003) and other African economies (Ntim et. al., 2011) just to name a few.

Even more recently, a consensus seems to be building up in the academic literature of a possible asymmetric data generating process for various stock prices or indices worldwide. In this regard, the consolidation of nonlinear time series analysis into the empirical literature presents a milestone development in the academic paradigm in the sense of presenting a more widespread interpretation of the empirical results obtained from empirical studies. Essentially, regime-switching models assume that the data generating process of a time series can be captured within differing regimes segregated by a unique threshold or breakpoint value. One of the earliest works on the subject matter was presented by Li and Lam (1995) who used a threshold autoregressive conditional heteroscedastic (TARCH) to establish that the model structure of Hong Kong stock returns data tends to fluctuate over a horizon of time periods. Another study worth taking note of is that presented in Shivley (2003), who finds evidence of stock prices in international markets being consistent with a regime-reverting random walk process containing a deterministic trend. Other forms of nonlinear time series
analysis which have emerged in the literature concerned with modelling regime-switching behaviour in stock markets include the Markov Switching (MS) models (Schaller and van Norden, 1997), Neural Networks (NN) models (Albano et. al., 2013) and smooth transition regression (STR) models (Bonga-Bonga, 2012). However, it is the use of chaotic nonlinearity that has remained dominant in the nonlinear literature even though a majority of the empirical evidence obtained from these models has altogether been deemed as being inconclusive (see Abyyankar et. al. (1997), Kohers et. al. (1997) and Pandey et. al. (1998)).

Moreover, a separate cluster of academic studies can be identified in the academic paradigm which directly incorporates unit root testing within nonlinear statistical frameworks and this strand of empirical literature appears to have attained more success in establishing weak-form EMH for various stock markets. A popular citation among these works are the studies of Narayan (2005, 2006) who applies the unit root testing procedure of Caner and Hansen (2001) to US stock prices and finds that the data evolves as a nonlinear time series characterized by a unit root process, a finding which is highly consistent with the weak-form EMH. Similarly, Munir and Mansur (2009) apply similar unit root tests to those used by Narayan (2006) and establish a unit root process in the behaviour of the Malaysian stock exchange market. Furthermore, Lee et. al. (2013) apply smooth transition regression (STR) heterogeneous panel unit root tests to OECD, G6, Asian and other European economies and establish that a majority of the countries under observation conform to the weak-form EMH; whereas Hasanov and Omay (2007) employ the STR unit root test of Kapetonois et. al. (2003) to establish weak-form market efficiency for Bulgarian, Czech, Hungarian and Slovakian stock markets. Although still in its infants stages of implementation, Oskooe (2011) used nonlinear Fourier unit root tests for the Iran stock market and was able to validate the weak-form EMH in this particular stock market. Without discarding the positive developments presented in the literature thus far, the empirical literature, never-the-less, remain devoid of bridging the aforementioned two strands of empirical works examining asymmetric behaviour in the stock market prices. Undertaking such a task could prove to bridge the empirical hiatus existing between univariate nonlinear modelling of stock prices, on one hand, and nonlinear unit root tests, on the other hand.

3 EMPIRICAL FRAMEWORK

Given that the phenomenon of random walks is associated with EMH, one way to test the weak-form EMH is to examine whether a historical sequence of stock prices are independent of another or whether they are serially correlated. When the stock prices/indices behave as a random walk or similarly contain a unit root, then the best forecast of the following period’s stock prices is the most recently observed stock price and this ensures that the predictability of the stock prices never tends to an average value. Thus, in our study, we endeavour to test the weak-form of the EMH in the following two interrelated phases. In the first phase of our empirical analysis, we estimate a univariate two-regime threshold autoregressive (TAR) model and the rationale behind the choice of this statistical model can be described as follows. In their seminal paper, Andrews and Chen (2001) propose a “naive” technique for diagnosing the integration properties of a univariate autoregressive function of a time series which entails examining the sum of the autoregressive (AR) coefficients of an observed time series. If the sum of the AR coefficients of an observed time series is greater than or equal to unity then the observed time series is assumed to contain a unit root, a result which is in support of the weak-form EMH. Conversely, if the sum of the AR coefficients is found to be less than unity then the series is stationary thus rejecting the weak-form EMH. In the second phase of our empirical investigation, we formally test the stationary properties of the time
series by applying the nonlinear unit root tests of Bec et al. (2004) to the observed time series. A notable advantage of this nonlinear root testing procedure is that they are directly derived from Hansen’s TAR model. In this sense, the results obtained from the formal unit root tests can be compared to those obtained from the naive technique and be interpreted without spurious conclusions.

3.1 Baseline Threshold Autoregressive (TAR) Model

For analytical purposes, we specify our baseline two-regime TAR model as follows:

\[ p_t = (\alpha_0 + \alpha_1 p_{t-1} + \cdots + \alpha_p p_{t-p}) I(p_{t-d} \leq \tau) + \]
\[ (\phi_0 + \phi_1 p_{t-1} + \cdots + \phi_p p_{t-p}) I(p_{t-d} > \tau) + \varepsilon_t \]  

Where \( p_t \) is the observed time series, \( I(\cdot) \) is an indicator function and \( \tau \) is the unknown threshold parameter which needs to be estimated. The sample observations are split into two regimes and the model coefficients (i.e. \( \alpha_i \) and \( \phi_i \)) are allowed to vary depending on whether the observational data lies below (i.e. \( y_{t-d} \leq \tau \)) or above (i.e. \( y_{t-d} > \tau \)) the threshold parameter estimate. By further defining:

\[ x_t = (1; y_{t-1} \ldots y_{t-p}); \]
\[ x_t(\tau) = (x_t I(y_{t-d} \leq \tau); x_t' I(y_{t-d} > \tau))'; \]
\[ \alpha = (\alpha_0, \alpha_1, \ldots, \alpha_p); \]
\[ \phi = (\phi_0, \phi_1, \ldots, \phi_p); \]
\[ \psi = (\alpha', \phi') \]

We can then re-formulate the TAR model in the following compact matrix format:

\[ y_t = x_t(\tau)' + \varepsilon_t \] (2)

Conditional on threshold parameter estimate \( \gamma \), equation (2) is linear in \( \psi \) so that least square (LS) estimator is appropriate. Heuristically, the least squares estimate of \( \psi \) for a given value of \( \tau \) is given by:

\[ \psi(\tau) = \left( \sum_{t=1}^{n} x_t(\tau)x_t(\tau)' \right)^{-1} \left( \sum_{t=1}^{n} x_t(\tau)y_t \right) \] (3)

In identifying a consistent estimator for the vector \( \psi \), the estimation problem is reduced to finding \( \tau \) that minimizes the sum of squared residuals of the model and recovering the estimates of \( \hat{\tau} \) and \( \hat{\psi} \) through \( \hat{\psi}(\tau) \). Specifically, the consistent estimate of the true threshold value \( \hat{\tau} \) is obtained by solving the following search problem over different possible values of belonging to a set \( \Psi = [\underline{\tau}, \overline{\tau}] \) i.e.

\[ \hat{\tau} = \text{argmin}_{\tau \in \Psi} Q_T(\tau) \] (4)

Where \( \Psi = [\underline{\tau}, \overline{\tau}] \), denotes a set of numbers from which the true estimate is searched over and \( Q_T(\gamma) \) is the generalized distance measured. Once we obtain the estimates of \( \hat{\tau} \), we can then estimate the model’s slope parameters as \( \hat{\psi} = \hat{\psi}(\tau) \). In order to ascertain the significance of the threshold effect, one can test the constraint \( H_0: \alpha = \beta \). Since the threshold is not
identified under the null hypothesis, the classical F-test does not have standard distribution. Asymptotically valid p-values for the hypothesis test are therefore constructed by relying on a bootstrap procedure, as suggested in Hansen (2000), which entails simulating the asymptotic distribution of the following LR test:

$$LR(\tau) = \frac{SSE_1(\tau) - SSE_1(\hat{\tau})}{\sigma^2}$$  \hspace{1cm} (5)

Where $SSE_1(\tau)$ and $SSE_1(\hat{\tau})$ are the residual sum of squares under the null hypothesis and the alternative, respectively and $\sigma^2$ is the residual variance under $H_1$. Since the asymptotic distribution of $LR(\tau)$ is non-standard and strictly dominates the $\chi^2$ distribution, Hansen (2000), tabulated valid asymptotic confidence intervals for the estimated values of the threshold by using a non-rejection region $c(\sigma) = -2 \log (1 - \frac{1}{\sqrt{1 - \sigma^2}})$, where $c(\sigma)$ is the a percent critical value. The LR test of the null hypothesis, $H_0$, is to reject for large values of $LR(\tau)$ at the asymptotic level of $\sigma$ i.e. $LR(\tau) > c(\sigma)$.

### 3.2 Unit Root Tests

In order to completely develop the TAR model, it is important to further investigate the integration properties of the time series under observation. If the weak-form EMH is to be deemed as being statistically valid for the JSE, the univariate integration properties of stock prices must be established to contain a unit root as opposed to being stationary. As eloquently demonstrated by Enders and Granger (1998) as well as by Caner and Hansen (2001), conventional linear unit root tests such as the Dickey-Fuller tests have got considerably low power in testing for unit roots when the underlying data generating process is found to be nonlinear. Hence if evidence of asymmetries in a univariate time series exists, then corresponding asymmetric unit root tests must be implemented inorder to determine the stochastic properties of the time series. In our study, the examination of asymmetric effects in the unit root process of the JSE stock prices is examined through the use of Bec et. al. (2004) nonlinear unit root test which is a generalization of the Dicker-fuller unit root testing procedure implemented under Hansen’s (2000) TAR framework. Specifically, Bec et. al. (2004) propose a unit root testing procedure based on the following compact auxiliary TAR specification:

$$\Delta p_t = x_t(\tau)^\prime \beta + \epsilon_t$$  \hspace{1cm} (6)

Where

$$x_t(\tau)^\prime = \begin{pmatrix} \mathbb{I}(p_{t-1} \epsilon I_1(\tau)) - \mathbb{I}(y_{t-1} \epsilon I_3(\tau)) \\ p_{t-1}(\mathbb{I}(p_{t-1} \epsilon I_1(\tau)) - \mathbb{I}(p_{t-1} \epsilon I_3(\tau))) \\ p_{t-1}(\mathbb{I}(p_{t-1} \epsilon I_2(\tau))) \\ p_{t-1}(\mathbb{I}(p_{t-1} \epsilon I_2(\tau))) \end{pmatrix}$$  \hspace{1cm} (7)

And:

$$\beta = \begin{pmatrix} \mu_1 \\ \rho_1 \\ \mu_2 \\ \rho_2 \end{pmatrix}$$  \hspace{1cm} (8)
Restrictions of $\tau_2 = -\tau_1 = \tau$ and $\rho_i \leq 1$ are imposed on the threshold as well as on the parameter variables of the unit root regression in order to rule out the possibility of explosive behaviour in the unit roots and to simultaneously ensure that the time series remains geometrically ergodic. The unit root test is based upon the statistical significance of the parameters in the matrix $\beta$. Under the null hypothesis, a unit root process (i.e. $\rho_1 \neq 0$ or $\rho_2 \neq 0$) is tested against the alternative of a stationary TAR process (i.e. $\rho_1 = \rho_2 = 0$). In order, to effectively test these hypotheses, there must exist a singular threshold estimate value of $\hat{\tau}$, which is to be plugged into the unit root test regression. Bec et al. (2004) suggest that the threshold value can be selected a prior by the statistician in testing for the unit root hypothesis. The asymptotic distributions of these unit root tests are derived from Supremum based tests on the Wald, Lagrange Multiplier and Likelihood Ratio statistics i.e.

\begin{equation}
W_T(\tau) = \frac{1}{\sigma^2} \beta \left[ R \left( \sum_{t=1}^{T} x_t x_t' \right)^{-1} R' \right]^{-1} \hat{\beta}
\end{equation}

\begin{equation}
LM_T(\tau) = \frac{1}{\sigma^2} \left[ \left( \sum_{t=1}^{T} x_t \hat{\epsilon}_t \right)' \left[ \sum_{t=1}^{T} x_t x_t' \right] \left[ \sum_{t=1}^{T} x_t \hat{\epsilon}_t \right] \right]
\end{equation}

\begin{equation}
LR_T(\tau) = T \ln \left( \frac{\sigma^2}{\hat{\sigma}^2} \right)
\end{equation}

Where $R$ is a $3 \times (3p+6)$ selection matrix such that $R \hat{\beta} = \hat{\beta}$ and $Q^{-1}$ denotes the Moore-Penrose generalized inverse of the matrix $Q$. The hypothesis of a unit root can be rejected if the aforementioned test statistics are larger in absolute value in comparison to their associated critical values.

4 DATA AND EMPIRICAL ANALYSIS

4.1 DATA DESCRIPTION

All data used in our study consists of daily closing indices of the all share index ($ALS$); the JSE top 40 companies index ($top40$); the industrials index ($ind$), the financial index ($fin$), the mining index ($min$) and the gold index ($gold$) and has been collected from the McGregor statistical database. In determining our frequency of data, we take heed of the suggestion presented by Bonga-Bonga and Makakabule (2010), who indicate that high frequency data, such as weekly data, are essential towards capturing the nonlinear relationship that exists in the JSE indices data. We therefore collected data covering a weekly sample period from 31st January 2000 to 16th September 2013. From our summary statistics of the time series data, as reported below in Table 1, we conclude that the data under observations are not normally distributed. We base these conclusions since the Jarque-Bera (jb) statistic exceeds the critical p-values for all significance levels. Furthermore, we detect skewness and kurtosis in the data which may be caused from a pattern of volatility in financial markets were periods of volatility are followed by periods of relative stability. The time series plot of the stock indices used in our study, as shown in Figure 1, verify this explanation for non-normality in the data.
### Table 1: Descriptive Statistics of JSE Share Indices

<table>
<thead>
<tr>
<th></th>
<th>ALSI</th>
<th>Top 40</th>
<th>Ind</th>
<th>Fin</th>
<th>Min</th>
<th>Gold</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>20877.94</td>
<td>18919.31</td>
<td>19510.73</td>
<td>16729.89</td>
<td>23295.43</td>
<td>2174.5</td>
</tr>
<tr>
<td>Median</td>
<td>20875.63</td>
<td>18976.45</td>
<td>20642.36</td>
<td>17002.6</td>
<td>24597.7</td>
<td>2364.78</td>
</tr>
<tr>
<td>Maximum</td>
<td>43132.75</td>
<td>38683.17</td>
<td>42443.24</td>
<td>31566</td>
<td>48258.56</td>
<td>3360.39</td>
</tr>
<tr>
<td>Minimum</td>
<td>7243.08</td>
<td>6780.72</td>
<td>5496.68</td>
<td>7397.84</td>
<td>5681.71</td>
<td>685.29</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>818.77</td>
<td>720.14</td>
<td>10558.83</td>
<td>6779.16</td>
<td>858.44</td>
<td>653.25</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.23</td>
<td>0.21</td>
<td>0.22</td>
<td>0.29</td>
<td>0.06</td>
<td>-0.74</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>-1.28</td>
<td>-1.30</td>
<td>-1.12</td>
<td>-1.01</td>
<td>-1.28</td>
<td>-0.45</td>
</tr>
<tr>
<td>JB</td>
<td>127.39</td>
<td>128.339</td>
<td>118.03</td>
<td>112.86</td>
<td>126.04</td>
<td>96.89</td>
</tr>
<tr>
<td>Probability</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

*Source: Authors own computation*

#### Figure 1: Time Series Plots of JSE Share Indices

![Time Series Plots of JSE Share Indices](image)
4.2 Empirical Results

This section applies Hansen’s (2000) conditional least squares (CLS) threshold modelling technique to the JSE all share index, top 40 companies, industrial sector, financial sector, mining sector and gold stock prices with the estimation results being reported below in Table 2. The estimation of the TAR model requires the prior identification of some parameters: the threshold value ($\tau$) and the autoregressive (AR) order (p). Therefore our modelling approach consists of the following steps:

1. Identify a set of possible values of the threshold parameter and estimate a TAR specification for each predetermined threshold value estimated at different lag lengths with a maximum lag length of 8 periods.

2. Select the threshold valued and the associated lag length which maximizes the SSR in the estimated TAR regressions.

3. At the threshold value identified in step 2, perform linearity tests against the alternative hypothesis of threshold nonlinearity.

We begin our empirical analysis by firstly performing our grid search across the predetermined values of the observations of threshold variable i.e. $\mathcal{Y} = [\underline{\tau}, \bar{\tau}]$. In the spirit of Hansen (2000), we restrict our grid search to values of $\tau$ to specific quantiles by eliminating the smallest and largest 15 percent of the observational data. The remaining values consist of the values of $\tau$ which can be search over for the true estimate $\hat{\tau}$. Our estimates from the TAR model, as reported in Table 2 below, depict threshold values of price indexes of 25784 for the all share index, 26028 for top 40 companies, 22582 for industrials, 41534 for financials, 23948 for mining and 2481 for gold. Interestingly enough, each of these estimated break points for all estimated indexes points to two separate periods, the first being between the months of January and May 2007, whereas the second period corresponds to that of between August and November 2009. Coincidentally, we find that we can attribute these periods to the significant supply shocks caused by the financial crisis of 2007-2008 caused by the closing down of major banks in the USA which affected a majority of financial sectors worldwide.

Subsequent to the estimation of the optimal threshold values for each of the time series, we proceed to performing the LR tests for the threshold estimates and derive the associated bootstrap p-values using Hansen (2000) bootstrap procedure. In particular, we estimate the TAR regression given at the optimal threshold value, $\hat{\tau}$, at lag length (p) and extract the regression residuals to be used as an empirical distribution for the bootstrapping procedure i.e. $\mathcal{E}^* = \{\epsilon_{1}^*, \epsilon_{2}^*, ..., \epsilon_{n}^*\}$. We then draw a sample from the empirical distribution in order to create a bootstrap sample which used to calculate the LR statistic of the estimated TAR model under the null and alternative hypothesis, respectively. By replicating this procedure 1000 times and calculating the percentage in which the simulated statistic exceeds the actual we are able to provide the bootstrap estimate of the asymptotic p-values under the null hypothesis of linearity. Furthermore, we form asymptotic confidence intervals for based upon non-rejection region of confidence level of the LR statistic. The estimated LR test statistics and their asymptotic confidence intervals, as shown at the top of Table 2, confirm that the null hypothesis of linearity can be rejected for all indices at a one percent significance level. In other words, the linear AR model can be strongly rejected in favour of a two-regime TAR model thus warranting estimation of the associated coefficients of the TAR models for each of the time series.
Table 2: TAR Regression Estimates

<table>
<thead>
<tr>
<th></th>
<th>ALSI</th>
<th>Top 40</th>
<th>Ind</th>
<th>Fin</th>
<th>Min</th>
<th>Gold</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau )</td>
<td>(25.784)</td>
<td>(26.028)</td>
<td>(22.582)</td>
<td>(14.534)</td>
<td>(23.948)</td>
<td>(24.81)</td>
</tr>
<tr>
<td>LR((\tau))</td>
<td>(0.00)**</td>
<td>(0.00)**</td>
<td>(0.00)**</td>
<td>(0.00)**</td>
<td>(0.00)**</td>
<td>(0.00)**</td>
</tr>
<tr>
<td>c((\sigma))</td>
<td>(18.42)</td>
<td>(29.39)</td>
<td>(24.79)</td>
<td>(23.45)</td>
<td>(17.16)</td>
<td>(13.94)</td>
</tr>
<tr>
<td>(\alpha_1)</td>
<td>(1.05)</td>
<td>(1.39)</td>
<td>(1.61)</td>
<td>(1.44)</td>
<td>(1.43)</td>
<td>(1.15)</td>
</tr>
<tr>
<td>(\alpha_2)</td>
<td>(-0.15)</td>
<td>(-0.51)</td>
<td>(-1.05)</td>
<td>(-0.72)</td>
<td>(-0.57)</td>
<td>(0.13)</td>
</tr>
<tr>
<td>(\alpha_3)</td>
<td>(0.23)</td>
<td>(0.52)</td>
<td>(0.74)</td>
<td>(0.51)</td>
<td>(0.36)</td>
<td>(-0.27)</td>
</tr>
<tr>
<td>(\alpha_4)</td>
<td>(-0.13)</td>
<td>(-0.39)</td>
<td>(-0.29)</td>
<td>(-0.24)</td>
<td>(-0.21)</td>
<td>(-0.01)</td>
</tr>
<tr>
<td>(\phi_1)</td>
<td>(1.14)</td>
<td>(1.06)</td>
<td>(0.93)</td>
<td>(0.86)</td>
<td>(0.98)</td>
<td>(1.03)</td>
</tr>
<tr>
<td>(\phi_2)</td>
<td>(-0.08)</td>
<td>(-0.01)</td>
<td>(0.22)</td>
<td>(0.25)</td>
<td>(0.01)</td>
<td>(-0.35)</td>
</tr>
<tr>
<td>(\phi_3)</td>
<td>(-0.18)</td>
<td>(-0.12)</td>
<td>(-0.39)</td>
<td>(-0.33)</td>
<td>(0.02)</td>
<td>(0.27)</td>
</tr>
<tr>
<td>(\phi_4)</td>
<td>(0.10)</td>
<td>(0.06)</td>
<td>(0.21)</td>
<td>(0.20)</td>
<td>(-0.02)</td>
<td>(0.04)</td>
</tr>
</tbody>
</table>

\[
\sum \phi_i = 1.00, \quad \sum \alpha_i = 0.98, \quad \text{Observations } l.(y_{t-d} \leq \tau) = 69.57\% \quad \text{Observations } l.(y_{t-d} > \tau) = 30.43\%
\]

\[
\text{MAPE} = 3.17\% \quad 3.34\% \quad 3.08\% \quad 3.11\% \quad 4.67\% \quad 6.40\%
\]

Significance Level Codes: ‘***’, ‘**’ and ‘*’ denote the 1%, 5% and 10% significance levels respectively. The confidence intervals for the threshold estimates are reported in [ ] whereas the associated p-values for the regression coefficients are reported in ().

The coefficient estimates of the TAR model provide some intriguing preliminary evidence on the stationary properties of each of the time series. As can be seen from Table 2, the sum of the autoregressive coefficients (SARC) is very close to unity for each of the regimes under observations such that each of the series displays a high level of persistence. In particular, we find that for alsi, fin, min and gold; the upper regime the SARC is given by unity (i.e. \(\sum \phi_i \geq 1\)) whereas in the lower regime the number is close to unity (i.e. \(\sum \alpha_i < 1\)). Overall, the coefficient estimates from our TAR models, provide preliminary
evidence on the existence of high persistence in both regimes for all the estimated TAR models. This result implies that a shock to any of the observed stock indices is likely to persist for a significant period, and future returns cannot be easily predicted using most recent lagged returns. In view of this evidence, we therefore proceed to apply more formal unit root tests to the time series data.

In view that all time series under observation can be modelled as two-regime TAR processes, we therefore proceed to apply the nonlinear unit roots of Bec et. al. (2004) in order to examine the stationary properties of each of the time series variables under observation. We implement the aforementioned unit root testing procedure as follows. Firstly, we assume that the threshold value does not need to be estimated but is rather based on the estimates obtained in the previous section. Secondly, we take the Supremum of the Wald, Lagrange Multiplier and Likelihood Ratio statistics over an interval of predetermined values of the threshold value, $\hat{\tau}$. We then make a comparison of the derived Supremum statistics with the empirical critical values as reported in Bec et. al. (2006). Since the distribution of this test under the null hypothesis depends on nuisance parameters, the associated p-values for each the test statistics are computed using similar simulations to those performed in the previous section. The results of Bec et. al. (2004) nonlinear unit root tests are reported below in Table 3.

<table>
<thead>
<tr>
<th></th>
<th>$W_T(\tau)$</th>
<th>$LM_T(\tau)$</th>
<th>$LR_T(\tau)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALSI</td>
<td>9.12</td>
<td>8.63</td>
<td>8.87</td>
</tr>
<tr>
<td>Top 40</td>
<td>8.55</td>
<td>8.12</td>
<td>8.33</td>
</tr>
<tr>
<td>Ind</td>
<td>11.12</td>
<td>10.41</td>
<td>10.76</td>
</tr>
<tr>
<td>Fin</td>
<td>6.58</td>
<td>6.32</td>
<td>6.45</td>
</tr>
<tr>
<td>Gold</td>
<td>19.75**</td>
<td>17.60*</td>
<td>18.64**</td>
</tr>
<tr>
<td>Min</td>
<td>16.64*</td>
<td>15.86</td>
<td>15.84**</td>
</tr>
<tr>
<td>10%</td>
<td>16.181</td>
<td>15.87</td>
<td>15.77</td>
</tr>
<tr>
<td>Critical Values</td>
<td>5%</td>
<td>18.4</td>
<td>17.63</td>
</tr>
<tr>
<td></td>
<td>1%</td>
<td>23.01</td>
<td>21.75</td>
</tr>
</tbody>
</table>

Significance Level Codes: ***”, **” and ‘*’ denote the 1%, 5% and 10% significance levels respectively. P-values are reported in ().

As a first step to practically examining the stationary properties of the time series variables, we compute the threshold unit root test statistics (i.e. $W_T(\tau)$, $LM_T(\tau)$ and $LR_T(\tau)$) together with the associated bootstrap critical p-values values at significance values of 1 percent, 5 percent and 10 percent using 1000 bootstrap replications. Our estimation results, as reported below in Table 3, shows that for each of the series, the unit root hypothesis is rejected for all time series with the exception of the mining sector and gold stock prices. These results
obtained from our unit roots tests are generally contrary to the preliminary evidence of a weak-form capital market efficiency as established in the previous section. In evaluating the combined evidence as obtained from the estimation of our univariate TAR models and from the results of our formal unit roots, we conclude that whilst the JSE stock prices exhibit high levels of persistence in their data generating process, they, however do not contain pure unit roots. The inability of unit root test to distinguish, in finite samples, pure unit root from “arbitrarily-close” root processes has been long documented as being problematic in the literature (Diebold and Kilian, 2000). Furthermore, high, “close-to-unity” persistence levels in a time series are as much a generalization of random walks as are unit root processes (Cochrane, 1991). Hence at this juncture, it would be pre-mature to reject the weak-form hypothesis on the strict basis of the applied unit root tests without taking into consideration the high levels of persistence observed in the time series.

5 CONCLUSIONS

Our study sought to evaluate possible asymmetric behaviour in JSE all share index, top 40 companies, industrial sector, financial sector, mining sector and gold stock prices using weekly data collected between the period of 2000 and 2013. Our objective was accomplished through two distinct empirical phases. In our first stage we, estimate conventional TAR models for each of the indexes and our empirical results failed to reject nonlinear effects for each of the indices. In specific we were able to establish that the autoregressive (AR) properties of the various indexes investigated proved to slightly and yet significantly vary between different regimes as segregated by the estimated threshold values. Furthermore, the sum of the autoregression coefficients (SARC) obtained from the TAR estimates provided preliminary evidence of weak-form market efficiency, as each of the observed time series was found to be highly nonlinear and persistent for all TAR models.

In the second phase of our empirical analysis, we extended the TAR model to accommodate unit root testing by implementing the procedure as described in Bec et. al. (2004). We find that the results obtained from our formal unit root tests are directly contrary to the preliminary evidence which were derived from the estimation of the univariate threshold models for all JSE share indices. In particular, we find that the stock indices associated with the primary sectors (i.e. mining sector and gold prices) are market efficient whereas the indices associated with secondary sectors (i.e. all share index, top 40, financial sector, industrial sector) prove to reject the EMH. Generally, our empirical analysis demonstrates how the stock market indexes evolve as highly persistent, nonlinear process and yet for a majority of the time series under observation, the formal unit root tests reject the hypothesis of stationarity among the variables.

The overall results obtained in our study bridge two opposing contentions obtained from previous studies by suggesting that while a number of stock prices under the JSE stock market may not evolve as pure unit root processes, the time series are, however, highly persistent to an extent of being able to be deemed as weak-form efficient. Given the combination of high levels of persistence exhibited in the time series variables as well as the rejection of unit roots for the time series data may be indicative of stock prices being indeed close to a unit root. These empirical results obtained from our study may serve as a convenient guideline for future research. In this sense, possible avenues for future research may focus on investigating the weak-form EMH for JSE stock indices through the use time
series models which can detect close-to-unit root processes i.e. quantile unit root regressions or the local-to-unity autoregressive (AR) model.

REFERENCES


