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Abstract

The effects of an agreement such as the Kyoto Protocol, which implicitly imposes a ceiling on the stock of pollution, have recently been studied in Hotelling models. We add pollution and a ceiling to the endogenous growth model of Tsur and Zemel (2005) to study the effects of the ceiling on capital accumulation and research investments. The ceiling increases the scarcity of the exhaustible resource in the short run, which boosts backstop utilization. This implies that R&D becomes more beneficial compared with capital accumulation. How the short run development path of an economy is affected depends on its capital endowment or richness, respectively. Only economies which are neither too rich nor too poor may invest more into research. In the long run an economy with a ceiling follows basically the same long run development path as an economy without the ceiling.

Keywords: Endogenous growth, Environmental agreements, Fossil fuels, Nonrenewable resources, Research and Development

1. Introduction

Climate change has been one of the major issues both in public and academic discussion in recent decades. A wide range of nations agreed in the Kyoto Protocol to limit climate change. The best known political goal is the 2°C climate target which allows for long-run global temperature increase of 2°C above pre-industrial level. This target had been a subject to political and scientific discussion and was finally endorsed by the United Nation Conference of the Parties in Cancun in 2010.[1] The supporters of the climate target agree that the consequences of climate change remain manageable as long as the global temperature increase does not exceed 2°C. According to Graßl et al. (2003)

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[1]Cf. Graßl et al. (2003) and UNFCCC (2010).

the 2°C climate target translates into a maximum CO_2 concentration of 400 - 450ppm. Hansen et al. (2008) advocate a maximum CO_2 concentration of 350ppm to preserve the planet as it was during the development of civilization. Regardless of one follows the 2°C climate target or Hansen et al. (2008), both implicitly impose a ceiling on the stock of CO_2. Since other agreements follow a similar approach, e.g. the Montreal Protocol on Substances that Deplete the Ozone Layer, it seems likely that a successor of the Kyoto Protocol will include an implicit or explicit ceiling on the stock of CO_2. One of the main sources of CO_2 emissions are fossil fuels. Therefore, a ceiling on the CO_2 concentration might have a significant impact on the energy generation of the economy.

Chakravorty et al. (2006a) was motivated by the described problem to analyze the effects of a ceiling on the stock of CO_2, or more generally pollution. Further works in this literature strand are Chakravorty et al. (2006b), Chakravorty et al. (2008), Chakravorty et al. (2012) and Lafforgue et al. (2008). This literature analyzes how a ceiling on the stock of pollution changes the optimal resource utilization path. A Hotelling model with polluting exhaustible resources and a renewable non-polluting resource serves as the basic framework, which is augmented in several ways. Abatement activities are considered by Chakravorty et al. (2006a) and Lafforgue et al. (2008). Chakravorty et al. (2008) focus on the consequences of two differently polluting exhaustible resources. Chakravorty et al. (2012) extend the model of Chakravorty et al. (2006a) by technological progress, which is caused by a learning-by-doing effect and decreases the costs of the backstop. It is shown that the optimal resource utilization path depends on the cost structure established by the standard assumption of the Hotelling model and the assumption related to the specific augmentation. Owning to its Hotelling based structure, the literature fails to consider capital or research activities, which are both determinants of economic growth, structural change and changes of the energy mix as shown by Tsur and Zemel (2005). R&D in particular seems to be a non-negligible factor, as it is the driving force behind a steadily positive growth rate in many endogenous growth models, e.g. Rivera-Batiz and Romer (1991).[2] Therefore, this paper strives to analyze the effects of a ceiling on the stock of pollution in an economy incorporating a polluting exhaustible resource and a backstop as well as capital and research driven technological progress. For this purpose we augment

[2] A comprehensive review of the endogenous growth theory is given by Aghion et al. (1998) and Barro and Sala-i Martin (2003).

the suitable endogenous growth model of Tsur and Zemel (2005) with both pollution and a ceiling on the stock of pollution. Utilization of the two resources occasions costs. In contrast to the usual assumption of endogenous growth models, technological progress does not augment the productivity of resources or capital, but reduces the costs associated with the use of the backstop. With regard to energy generation, the chosen modeling constitutes the more realistic approach. For clarification we refer to Stiglitz (1974). By modifying Solow's neoclassical growth model, Stiglitz shows that sustainable economic development is compatible with exhaustible resources or fossil fuels, respectively, if technology, which enhances the resource's productivity, increases sufficiently fast.[3] However, the result rests upon the assumption that a vast amount of goods can be produced by a vanishingly low amount of fossil fuel, and sufficiently advanced technology. Without renewable energies, which are not considered by Stiglitz (1974), fossil fuels are the only energy source. However, thermodynamics require some minimum stake of energy for every production process.[4] In this light, the assumption of Tsur and Zemel (2005) seems more realistic. Other features from the Hotelling models, such as abatement or differently polluting exhaustible resources, are left for further research, in order to keep the analysis as simple as possible.

In the present paper we show that the social optimum consists of three time periods (or phases) which appear in the Hotelling models in a similar manner. As in Chakravorty et al. (2006a), the only possible sequence containing all three time periods starts with a non-binding ceiling which becomes binding later on. After a phase with a binding ceiling, the ceiling becomes non-binding again and will stay it forever. Thus, neither capital nor research can explain other sequences. However, research reduces the costs of the backstop. As long as the backstop is used, the unit costs of the backstop determine the energy price as well as the marginal costs of the last used unit of exhaustible resources. Technological progress implies therefore a reduction of both. Together with changing energy demand, caused by the variable capital stock, and in contrast to Chakravorty et al. (2006a), the model can explain a decreasing scarcity rent of the exhaustible resource endogenously. By analyzing the development during the three phases and taking the only possible sequence into account, we can describe the optimal path of the economy. The development of

[3] See Solow (1956).

[4] Compare Meyer et al. (1998), page 171.

the economy depends on its state described by capital stock and technology. Due to the ceiling the scarcity of the exhaustible resource is increased above its "natural" level, i.e. the level without the ceiling. This "additional" scarcity increases the number of capital-technology combinations with the optimality of research instead of capital accumulation in the first two phases. Since the economy must be described by one of this capital-technology combinations, which change from the optimality of capital accumulation to the one of research, to be affected, we call the effect of the ceiling an increase of R&D incentives. During the phase with a binding ceiling the additional scarcity and therefore also the excess incentives are eliminated. In the long run, i.e. in the last phase, the ceiling will be never reached. Thus, there is no additional scarcity and the economy develops as the unconstrained one of Tsur and Zemel (2005). Hence, the constrained economy will basically follow the same long run development path as the unconstrained economy. To sum up, we show how the ceiling affects capital accumulation and research activities and give an endogenous explanation for decreasing scarcity rents of the exhaustible resource. To complete the discussion, we decentralize the social optimum in a competitive market. The analysis is based upon a neoclassical framework with price-taking composite product manufacturers and individuals, as well as Cournot competition on the resource market between two resource owning companies. Neither the individuals nor the companies take their influence on the emission stock into account. Therefore, the exhaustible resource has to be taxed in the short run. In the long run, the tax is not needed due to the high scarcity of the resource. To adjust for market power effects resulting from the Cournot competition both resources must be subsidized at all times.

The outline of the paper is as follows. Section 2 gives a description of the model. The social optimum is described in section 3. The market economy and government interventions necessary for the social optimum are discussed in section 4. Section 5 concludes the discussion.

2. Model

We augment the endogenous growth model of Tsur and Zemel (2005) with a pollution stock and a ceiling on the stock of pollution. For that purpose we describe the model structure of Tsur and Zemel (2005) briefly.[5] A single composite good Y is produced by using capital K and energy x according to the well behaved production function $Y = F(K,x)$, with $F(0,x) = F(K,0) = 0$, $F_K > 0$, $F_x > 0$, $F_{KK} < 0$, $F_{xx} < 0$, $F_{Kx} = F_{xK} > 0$ and $J = F_{KK}F_{xx} - F_{Kx}^2 > 0$. To avoid a collapse of production, the assumptions $\lim_{K\to 0} F_K = \infty$ and $\lim_{x\to 0} F_x = \infty$ are added. Energy is generated by a one to one transformation of an exhaustible resource R (fossil fuel) or a backstop (e.g. solar energy) b, i.e. $x = R + b$. The costs of supplying resources are $M(R)$ in the case of fossil fuel and $M_b B(A) b$ in the case of the backstop. The fossil fuel extraction cost function is increasing and strictly convex, i.e. $M'(R) > 0$ and $M''(R) > 0$. Furthermore, we assume that no fixed costs exists $M(0) = 0$ and that the marginal costs of the first supplied unit are zero $M'(0) = 0$. The backstop cost function is composed of a fixed cost parameter $M_b > 0$ and a function $B(A)$, with $B(A) > 0 \ \forall A > 0$.[6] The latter reflects the influence of technology A on the backstop unit costs. We assume that unit costs decline with technology but that the effect vanishes for large A, i.e. $B'(A) < 0$, $\lim_{A\to\infty} B(A) = \bar{B} > 0$ and $\lim_{A\to\infty} B'(A) = 0$. Furthermore, the technology endowment A_0 is positive and $B'(A)$ differentiable with $B''(A) > 0$. The net income is given at each point in time by $Y^n = F(K,x) - M(R) - M_b B(A) b$ and can be used for consumption C, physical capital (dis)investment $\dot{K}$ or research I. Capital stock develops according to

$$\dot{K} = F(K,x) - C - M(R) - M_b B(A) b - I. \tag{1}$$

Technology A increases in research investment I in compliance with

$$\dot{A} = I. \tag{2}$$

R&D investments are limited by the net income, i.e. $I \in [0, Y^n]$. Hereafter the upper bound is represented by $\bar{I}$. As long as fossil fuel is used, the resource stock S_R, with the initial value S_{R_0}, decreases according to

$$\dot{S}_R = -R. \tag{3}$$

At every point in time the representative household exhibits a strictly concave utility function $U(C)$ which increases in consumption with $\lim_{C\to 0} U'(C) = \infty$. To avoid the optimality

[5] We refer to Tsur and Zemel (2005) for details. Deviations from Tsur and Zemel (2005) are indicated explicitly. For the sake of simplicity time index t is suppressed. It is only added, if needed for understanding.

[6] Tsur and Zemel (2005) assume $M_b = 1$.

of $C = 0$, we also assume $U(0) = -\infty$.[7] Therefore, the utility is given by

$$U(C) \begin{cases} \geq 0, & \text{for } C > 0, \\ = -\infty, & \text{for } C = 0. \end{cases} \tag{4}$$

Following Chakravorty et al. (2006a) and the other Hotelling models mentioned above, utilization of fossil fuel causes pollution E.[8] By appropriate unit choice we can set $R = E$. Thus, R and E are used synonymously. The stock of pollution is S_E, while its initial value is denoted by S_{E_0}. With γ being the natural regeneration rate, S_E develops according to[9]

$$\dot{S}_E = E - \gamma S_E. \tag{5}$$

The ceiling $\bar{S}_E$ is imposed exogenously, for example by an political decision.[10] Then $\bar{S}_E - S_E \geq 0$ must hold at every point in time. Due to the ceiling, it is possible to divide the complete planning period into three time phases depending on the ceiling's status. Phase 1 is characterized by a non-binding ceiling. In phase 2 the ceiling is binding for a limited time period. In phase 3 the ceiling is non-binding and stays that forever.

3. Social Optimum

In the following section we derive the (constrained) social optimum. Thus, we assume that a constrained social planner maximizes the utility over the complete planning period given the initial state $(K_0, A_0, S_{R_0}, S_{E_0})$ and subject to (1), (2), (3), (5), $\bar{S}_E - S_E \geq 0$, $K \geq 0$, $S_R \geq 0$, $0 \leq I \leq \bar{I}$ and $E, b, C \in [0, \infty[$.[11] The present value of utility is given by

[7] Due to $\lim_{K \to 0} F_K = \infty$ and (15) a decreasing capital stock is accompanied by increasing consumption. Therefore, $K = 0$ and $C = 0$ could be reached in finite time, if the assumption $U(0) = -\infty$ is not made.

[8] While it is reasonable to assume that pollution has negative effects on utility and/or production, we follow Chakravorty et al. (2006a), Chakravorty et al. (2006b), Chakravorty et al. (2008), Chakravorty et al. (2012), Lafforgue et al. (2008) and neglect these effects to concentrate on the effect of a ceiling on pollution and to keep the model as simple as possible. This implies that there are no marginal costs of pollution.

[9] This form is widely used in the literature. E.g. by Guruswamy Babu et al. (1997) and Tsur and Zemel (2009).

[10] The political decision can be both an election outcome or the result of an international agreement. Chakravorty et al. (2006a), Chakravorty et al. (2012) and Chakravorty et al. (2008) refer to the latter. According to Chakravorty et al. (2008) and Lafforgue et al. (2008) the ceiling may also reflect a damage function that impose zero (or negligible) damages below the ceiling but prohibitive high damages above it. The ceiling can also be imposed by some regulatory authority, as stated by Chakravorty et al. (2006a) and Chakravorty et al. (2008). Since the ongoing international climate negotiations refer mainly to the 2°C climate target, Eichner and Pethig (2013) use also a ceiling in their analysis of unilateral climate policy.

[11] Following Chakravorty et al. (2008) and Chakravorty et al. (2012) we refer to the social planer has a constrained one, since utility is maximized subject to the exogenously given ceiling. Thus, we are not going to determine whether the ceiling is optimal or not but to analyze the optimal solution given the ceiling.

$\int_0^\infty U(C)e^{-\rho t}dt$, with ρ as the time preference rate. Thus, with λ, κ, τ and θ representing the current-value costate variables of K, A, S_R and S_E, and μ representing the Lagrange multiplier associated with the ceiling, the current-value Lagrangian is[12]

$$L = U(C) + \lambda\left[F(K, b+R) - C - M(E) - M_b B(A) b - I\right] \\ + \kappa I - \tau E + \theta[E - \gamma S_E] - \mu[E - \gamma S_E]. \quad (6)$$

Analogous to Tsur and Zemel (2005), an interior optimum is given by the following necessary conditions:[13]

$$\frac{\partial L}{\partial C} = U_C - \lambda = 0, \quad (7)$$

$$\frac{\partial L}{\partial E} = \lambda[F_x - M'] - \tau + \theta - \mu = 0, \quad (8)$$

$$\frac{\partial L}{\partial b} = \lambda[F_x - M_b B(A)] = 0. \quad (9)$$

The total energy supply, as well as the energy mix can be determined graphically by means of (8) and (9). In Fig. 1 the marginal productivity of energy is given by $F_x(K, x)$, while $M_b B(A)$ represents the marginal costs of backstop.[14] $M'(E) + \frac{\tau - \theta + \mu}{\lambda}$ denotes the marginal extraction costs of fossil fuel plus the term $m^q := \frac{\tau - \theta + \mu}{\lambda}$ which sets the costate variable related to fossil fuel into relation to the shadow price of capital. Therefore, m^q is called the relative scarcity index in the following. If marginal backstop costs are sufficiently high, i.e. $M_b B(A) > M'(E^{\#}) + m^q$, only fossil fuel are used and energy utilization $x = E^{\#}$ is given by $F_x(K, E^{\#}) = M'(E^{\#}) + m^q$. Energy generation relies only on backstop, if the marginal backstop costs fall short of the sum of the marginal extraction costs of the first fossil fuel unit and the relative scarcity index $M'(0) + m^q$. Energy input is then determined by $M_b B(A) = F_x(K, x)$. If marginal backstop costs lie between the two extremes like illustrated in Fig. 1, i.e. $M'(0) + m^q < M_b B(A) < M'(E^{\#}) + m^q$, both energy sources are used and total energy utilization is given by $F_x = M_b B(A)$. In this case $M_b B(A) = M'(E) + m^q$ determine the share of fossil fuel. The amount of used

[12]Notice that there are two mathematically approaches to solve a dynamic optimization problem with state space constraints. We apply here the approach which Chiang (1992), p. 298 et seqq. calls the "alternative approach" and Feichtinger and Hartl (1986), p. 164 et seqq. the "indirekte Methode".

[13]It can be shown that the sufficient conditions hold as long as $B''(A) \geq \frac{M_b}{b}(B'(A))^2\left[\frac{1}{M''(R)} - \frac{F_{KK}}{J}\right]$. Due to $B''(A) > 0$, $M''(R) > 0$ and $\frac{F_{KK}}{J} < 0$ both sides of the inequality are positive. As long as the backstop is used, which is assumed, the inequality holds if M_b is sufficiently small.

[14]A similar figure with $\theta = \mu = 0$ can be found in Tsur and Zemel (2005), p. 488. Thus, the figure of Tsur and Zemel (2005) is a special case of Fig. 1.

backstop equals the difference $x - E$. Following Tsur and Zemel (2005) we assume that both resources are used.[15]

In the following, the index * denotes optimal values, while unmarked variables denote

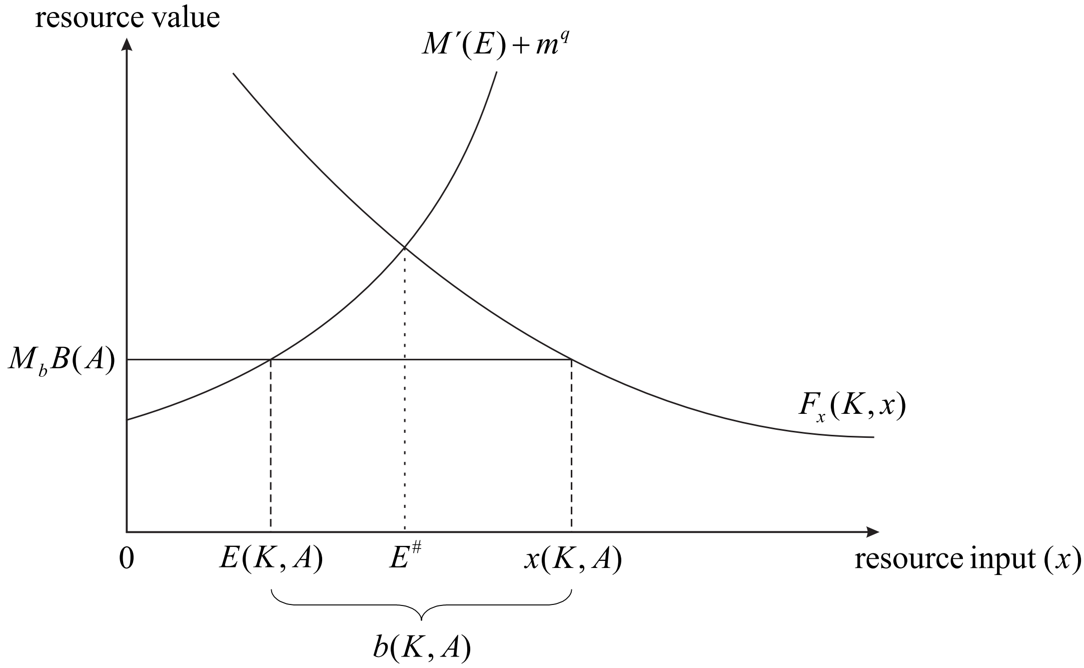

Figure 1: Usage of exhaustible resource and backstop

values of any possible path. The maximization of (6) with respect to the R&D investments I gives

$$\begin{aligned} I^* &= 0, \text{ if } -\lambda + \kappa < 0, \\ 0 \leq I^* &\leq \bar{I}, \text{ if } -\lambda + \kappa = 0, \\ I^* &= \bar{I}, \text{ if } -\lambda + \kappa > 0. \end{aligned} \tag{10}$$

[15]Notice that both resources can be used simultaneously, since the extraction costs of fossil fuel are increasing in fossil fuel utilization. If the extraction costs are independent of R or zero, like in Hoel (2011), a simultaneous use of both resources is not possible.

Depending on the relation of κ to λ, R&D investments are minimal, singular or maximal. The costate variables grow according to

$$\frac{\partial L}{\partial K} = \lambda F_K = \rho\lambda - \dot{\lambda}, \tag{11}$$

$$\frac{\partial L}{\partial S_E} = -\theta\gamma + \mu\gamma = \rho\theta - \dot{\theta}, \tag{12}$$

$$\frac{\partial L}{\partial S_R} = 0 = \rho\tau - \dot{\tau}, \tag{13}$$

$$\frac{\partial L}{\partial A} = -\lambda M_b b B' = \rho\kappa - \dot{\kappa}. \tag{14}$$

Combining (11) with (7) establishes the well-known Ramsey - rule

$$\hat{C} = \frac{F_K - \rho}{\eta}. \tag{15}$$

The rule states that the growth rate of consumption $\hat{C}$ is positive as long as the marginal product of capital is higher than the time preference rate. Consumption reacts the stronger to the difference the smaller the positive elasticity of marginal utility (η) is.

The complementary slackness condition is given by

$$\begin{aligned}
&\frac{\partial L}{\partial \mu} = -E + \gamma S_E \geq 0, \qquad && \mu \geq 0, \qquad && \mu\frac{\partial L}{\partial \mu} = 0, \\
& && \bar{S}_E - S_E \geq 0, && \mu[\bar{S}_E - S_E] = 0, \\
& && \rho\mu - \dot{\mu} \geq 0, && [= 0 \quad \text{if} \quad \bar{S}_E - S_E > 0].
\end{aligned} \tag{16}$$

To complete the equation system the transversality conditions

$$\begin{aligned}
&(a) \lim_{t\to\infty} e^{-\rho t}\lambda(t)\,[K(t) - K^*(t)] \geq 0, \quad && (b) \lim_{t\to\infty} e^{-\rho t}\tau(t)\,[S_R(t) - S_R^*(t)] \geq 0, \\
&(c) \lim_{t\to\infty} e^{-\rho t}\theta(t)\,[S_E(t) - S_E^*(t)] \geq 0, && (d) \lim_{t\to\infty} e^{-\rho t}\kappa(t)\,[A(t) - A^*(t)] \geq 0
\end{aligned} \tag{17}$$

are needed.

Before analyzing the three phases it is useful to determine the possible sequences of phases. In Appendix A.1 it is shown that the only sequence containing all three phases starts with a non-binding ceiling, i.e. in phase 1. At the end of phase 1, at $t = t_1$, the ceiling becomes binding, so that the economy switches into phase 2, and stays binding for a limited time period. At the moment $t = t_2$ the ceiling becomes non-binding again and the economy switches into phase 3. Thus, the ceiling stays non-binding for all following points in time. This sequence of phases was already found by Chakravorty et al. (2006a). Thus, the introduction of capital and R&D cannot explain other sequences. Furthermore, the switch from one phase to the next is smooth, i.e. the development paths of consumption, backstop and fossil fuel utilization, technology and capital do not exhibit discontinuities. The sign of θ during phase 1 can be easily explained by its interpretation as the shadow price of the emission stock. An external marginal increase of the stock narrows the problem of the social planner. Therefore, the increase has a negative value, which implies $\theta < 0$ in phase 1. Phase 3 was characterized by an emission stock that will never reach the ceiling. Since we abstain from direct effects of pollution on utility or production, the emission stock has no relevance for the constrained social planer.[16] $\theta = 0$ follows directly. With a binding ceiling θ cannot be interpreted as a shadow price.[17] However, in Appendix A.1 we show that θ is continuous at $t = t_2$. Together with $\theta = 0$ during phase 3, we get that θ equals zero at the end of phase 2. (12) implies that θ decreases constantly, if $\theta(t) < 0$ holds for point in time during phase 2. Since this contradicts $\theta(t_2) = 0$, a binding ceiling implies $\theta > 0$.

Proposition 1 *The development of consumption, capital, technology, backstop and fossil fuel utilization is continuous. The only sequence containing all three phases begins with phase 1, switches over to phase 2 and ends with phase 3.*

3.1. The phases

In this section we turn to the analysis of the three phases. In phase 3 the ceiling is never reached so that $\mu = \theta = 0$, which implies the identity of phase 3 with an unconstrained economy described by Tsur and Zemel (2005).[18] Since Tsur and Zemel (2005) are also one basis for our model, the analysis starts with phase 3 before we turn to phase 1 and 2.

3.1.1. Phase 3 - the long run

As phase 3 is identical with the economy of Tsur and Zemel (2005), the following remarks are limited to the extent that is necessary for understanding. For proofs, as well as for more detailed explanations, we refer to Tsur and Zemel (2005). As mentioned above, phase 3 is characterized by $\theta = \mu = 0$ so that the relative scarcity index reads $m_3^q := \frac{\tau}{\lambda}$. Due to (11) and (13) the growth rate of m_3^q is given by $\hat{m}_3^q = F_K > 0$. Thus,

[16]Notice that this does not mean that pollution is irrelevant for the economy or that the ceiling is set to low. Environmental concerns, like the damage function mentioned in footnote 10, are reflected by the ceiling. Since the ceiling is exogenously given, it goes beyond the scope of this paper if it is reflecting the environmental concerns correctly.

[17]See Feichtinger and Hartl (1986), p. 175-176. According to Feichtinger and Hartl (1986), p. 171, θ equals the sum of the shadow price of the emission stock and μ during phase 2.

[18]$\mu = 0$ follows directly from (16).

m_3^q steadily increases in time. If, as assumed, both resources are used, (8) and (9) give

$$F_x(K, x(K, A)) = M'(E(A)) + \frac{\tau}{\lambda} = M_b B(A). \tag{18}$$

(18) determines $b(K, A)$, $E(A)$ and $x(K, A)$. The optimal R&D investments are given by (10). Thus, the optimization problem reduces to the task of identifying optimal consumption and capital accumulation for every point in time. Tsur and Zemel (2005) show that this can be done by comparing the position of the economy in the three-dimensional technology-capital-time space with two characteristic manifolds (planes).[19] The time-dimension reflects the development of the scarcity index and therefore of the energy mix. As can be seen in Fig. 1 the higher the scarcity index the higher is the backstop share ceteris paribus.

The first plane describe all points in the A, K, t space which allow the steady state $\dot{C} = \dot{K} = \dot{A} = 0$. Therefore, we refer to it as the steady state plane (SSP). It is given by the steady state, (7) and (11), which imply $F_K(K, x(K, A)) - \rho = 0$. The plane is increasing in A but is independent from time, as $F_K(K, x(K, A))$ depends on total energy but not on the energy mix. Consumption increases (decreases) below (above) the SSP.

The second plane describes all points where singular R&D is optimal and is therefore called the singular plane (SiP). Only above the SiP maximal R&D is optimal, while below it no research can be conducted. The plane is given by the no-arbitrage condition with respect to net production between investments into the capital stock and into technology, i.e. by $\frac{\partial Y^n}{\partial A} = \frac{\partial Y^n}{\partial K}$. The SiP increases in technology but decreases in time (scarcity) as long as $S_R > 0$. If the resource stock is exhausted it is independent from time. The interpretation of the SiP decline in time is straightforward. As the increasing scarcity implies a higher share of backstop, the effect of a technology increase on net production strengthen while the effect of capital accumulation remains unchanged. Thus, R&D (capital accumulation) becomes feasible for more (less) A, K combinations. This process could be interpreted as an increase of R&D advantageousness, and will play a major role in the following discussion of the other two phases and economic development over the whole time. Both planes SiP and SSP are illustrated in Fig. 2, with T_R denoting the point in time S_R becomes exhausted.

Tsur and Zemel (2005) show that the economy approaches the SiP or the SSP with either

[19] Although "manifold" is mathematically correct, we use the term "plane" in the following, as it is more descriptive.

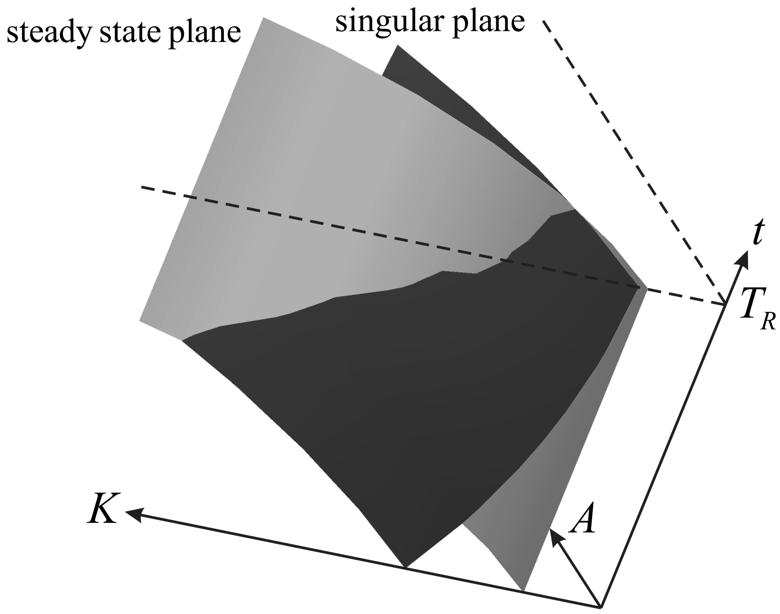

Figure 2: Singular- and steady state plane in the A, K, t space

maximal or minimal R&D, i.e. on a most rapid approach path (MRAP). If located above the SiP, the economy converges against the SiP with maximal R&D or against the SSP (i. exception) with minimal R&D. Below the SiP the economy conducts no research and accumulates capital to reach the SiP or reduces capital to reach the SSP (ii. exception). Once reached, the economy conducts singular R&D on the SiP for ever, or switches into a steady state at the intersection of SSP and SiP for $S_R = 0$, if the SiP lies above the SSP for huge A. Thus, positive R&D investments are only feasible above or on the SiP, while capital can only accumulate on or below the SiP.

3.1.2. Phase 1

Phase 1 is characterized by a non-binding ceiling that becomes binding in the future. Thus, from (16) we get $\mu = 0$. Since the ceiling becomes binding later on, changes of the emission stock are valued by the social planner by $\theta < 0$, as shown before. By using θ_1 to indicate the phase the shadow price belongs to, we get from (8) the variant of phase 1 for (18):

$$F_x(K, x(K, A)) = M'(E(A)) + \frac{\tau - \theta_1}{\lambda} = M_b B(A). \quad (19)$$

The relative scarcity index m_1^q is now given by $\frac{\tau + |\theta_1|}{\lambda}$ and its growth rate reads

$$\hat{m}_1^q = F_K + \gamma \frac{|\theta_1|}{\chi}, \text{ with } \chi := \tau + |\theta_1|. \quad (20)$$

Ceteris paribus, the scarcity index is both higher and increasing faster then in an economy without the ceiling, i.e. with the same A, K combination and the same costate variables but without θ_1. In the following, such an economy is called unbounded. As the scarcity index $\frac{\tau}{\lambda}$ of an unbounded economy reflects the pure relative scarcity of fossil fuel, we refer to it as the natural scarcity. During phase 1 the additional scarcity $\frac{|\theta_1|}{\lambda}$, caused by the ceiling, adds to the natural one. Fig. 1 implies that the additional scarcity boosts backstop utilization and reduces fossil fuel use while leaving total energy input x unchanged. As stated in section 3.1.1 the higher backstop utilization implies a greater effect of an increasing technology on net production. Therefore, R&D instead of capital accumulation becomes feasible for more A, K combinations implying an increase in R&D advantageousness, i.e. a SiP that is both lying below and decreasing faster in time then the SiP of an unbounded economy. As shown in section 3.1.1, the SSP is not affected by scarcity. Therefore, the additional scarcity has no effect on the SSP.

The development program is not affected by the ceiling. However, the economy cannot be in phase 1 for ever. If it were, the emission stock converges to the ceiling for $t \to \infty$. This implies $\lim_{t\to\infty} E(t) = \gamma \bar{S}_E$ and therefore the exhaustion of S_R in finite time. But with $S_R = 0$ the emission stock decreases to zero, contradicting a forever binding ceiling. Thus, the economy cannot reach the steady state at the intersection of SSP and SiP for $S_R = 0$ during phase 1. Only the two exceptions form section 3.1.1 remain for reaching a steady state.

Proposition 2 *During phase 1 the prospectively binding ceiling adds an additional scarcity to the natural one and is therefore increasing the R&D advantageousness. Compared with an unbounded economy the R&D advantageouness is both higher and increasing faster.*

3.1.3. Phase 2

During phase 2 the ceiling is binding. (16) implies $\mu_2 > 0$. As stated above $\theta > 0$ during phase 2. To indicate the phase we use the notation θ_2 and μ_2. (8) is rewritten to form the variant of phase 2 for (18):

$$F_x(K, x(K, A)) = M'(\bar{E}) + \frac{\tau - \theta_2 + \mu_2}{\lambda} = M_b B(A). \tag{21}$$

The relative scarcity index is given by $m_2^q := \frac{\tau - \theta_2 + \mu_2}{\lambda}$. According to Feichtinger and Hartl (1986), p. 171, θ_2 equals the sum of the shadow price of the emission stock and μ_2. As the shadow price of the emission stock is negative, $\mu_2(t) - \theta_2(t) \geq 0$.[20] Thus, the relative scarcity index m_2^q is higher than in an unbounded economy. Thus, in phase 2 there is

[20] A marginal exogenous decrease of the emission stock has a positive value for the constrained social planer.

still an additional scarcity, which implies that the R&D advantageousness is higher then in an unbounded economy. However, the growth rate $\hat{m}_2^q$ is ambiguous, because no exact information is given about $\hat{\mu}_2$ in (16):

$$\hat{m}_2^q = F_K + \gamma - \frac{\mu_2}{\psi}[\rho - \hat{\mu}_2] - \gamma\frac{\tau}{\psi}, \text{ with } \psi := \tau + |\theta_2| + \mu_2. \tag{22}$$

To obtain more information about the development of m_2^q and therefore also about the development of the R&D advantageousness, we use the binding ceiling. It implies that the fossil fuel utilization is fixed at $E = \bar{E} = \gamma\bar{S}_E$. Fig. 1 shows how the binding ceiling restricts the possible changes of m_2^q. Let be $\bar{E}$ equal $E(K, A)$ from the figure. If the marginal backstop costs remain unchanged, i.e. if no research is conducted, the fixed fossil fuel amount implies a fixed scarcity index. On the other hand, if R&D is conducted and therefore technology A increasing, the marginal backstop costs are lowered. To ensure $E = \bar{E}$, the scarcity index m_2^q needs to decrease, too. Otherwise, fossil fuel utilization decreases violating the assumption of a binding ceiling during phase 2. Thus, the binding ceiling establishes a link between R&D investments and the relative scarcity index. As long as R&D is minimal the relative scarcity index and therefore the R&D advantageousness remain unchanged in time. Generally, the economy needs to be located below the SiP to ensure minimal R&D. On the other hand, positive R&D investments are only possible, if the economy is located above or on the SiP. In this case, the scarcity index decreases implying a decrease of R&D advantageousness. The reason will be discussed as a part of the following section 3.2.

As in the other two phases, neither the SSP nor the development program are affected by the ceiling. The former is independent from the scarcity index. The latter one is not affected, because the economy cannot stay in phase 2 forever.[21] Therefore, the steady state at the intersection of SSP and SiP for $S_R = 0$ is ruled out and only the two exceptions from 3.1.1 are left.

Proposition 3 *Due to the constant resource input $\bar{E}$, the ceiling establishes a link between R&D activities and the R&D advantageousness. R&D advantageousness remains unchanged if R&D investments are minimal and decreases in time if R&D investments are positive. Compared with an unbounded economy R&D advantageousness is higher.*

[21] Otherwise, $E(t) = \gamma\bar{S}_E$ implies the exhaustion of S_R in finite time and therefore the violation of $S_E = \bar{S}_E$. Thus, the economy has to switch from phase 2 to phase 3 at some later point in time.

3.2. Optimal development

To analyze the development over the whole planning period $[0, \infty]$ it is necessary to join the analysis of the three phases. For this purpose we use the relative scarcity indices m_1^q, m_2^q and m_3^q. Furthermore, the smooth transition from one phase to the next must be noticed. Therefore, the SiP and SSP of the three phases can be attached to each other. Since the SSP is independent of time t, it is identical to the one described in section 3.1.1. The SiP decreases in time during phase 1 and during phase 3 as long as $S_R > 0$. In phase 2 it is either constant or increasing in time. Since an increasing SiP requires singular or maximal R&D investments, there are two more, mixed possibilities. The first one appears if the development path approaches the SiP from below, i.e. the economy is accumulating capital to realize the research option. In this case, the SiP remains unchanged until the path reaches it and increases afterwards. In the second case, maximal R&D investments are reduced to minimal investments before the SiP is reached, i.e. the economy converges to a steady state. Then the SiP increases at the beginning of the phase and remains unchanged at the end. Figure 3 illustrates the two border cases of a completely unchanged and a steadily increasing SiP in the A, K, t space. The development of the SiP is closely

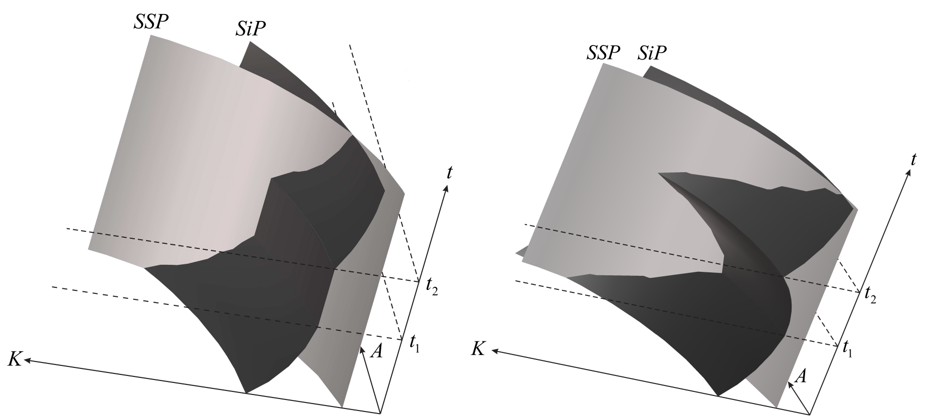

Figure 3: SSP and SiP with no or maximal R&D in Phase 2

related to the relative scarcity indices, since a higher index corresponds with a lower SiP, and therefore with a higher R&D advantageousness. Figure 4 shows how the development of the index in time must look like to generate the both SiP variants of Fig. 3. As stated in section 3.1.2, the relative scarcity index of phase 1 (m_1^q) both lies above and grows faster than its equivalent of an unbounded economy, implying a higher and faster growing R&D advantageousness. The driving force behind the increase of $m_3^q = \frac{\tau}{\lambda}$ is the exhaustibility of R, i.e. the natural scarcity. In phase 1, the relative scarcity index also

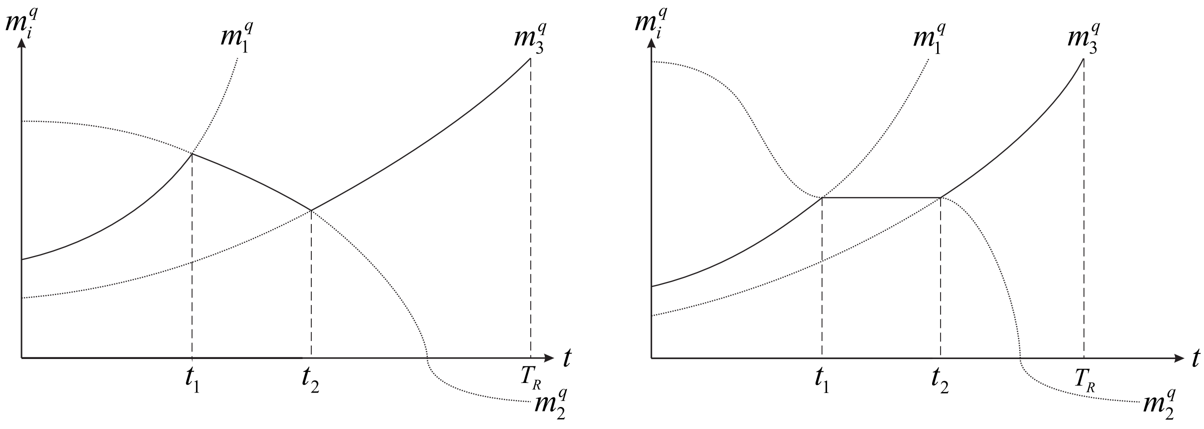

Figure 4: Relative scarcity index

entails the additional scarcity $\frac{|\theta_1|}{\lambda} > 0$, which represents the prospectively binding ceiling. This additional scarcity enhances the scarcity of R and therefore increases the R&D advantageousness, which corresponds with a decline in the number of A, K combinations with feasibility of capital accumulation. In Fig. 4, the additional scarcity equals the gap between m_3^q and m_1^q. It widens, since the economy approaches the ceiling during phase 1 implying a tighter restriction, which is reflected by $\hat{m}_1^q > \hat{m}_3^q$. The gap also indicates the amount of R which would be used, if the ceiling does not exists. The wider the gap the higher is the amount of not used fossil fuel. During phase 2 the additional scarcity still exists but is reduced passively and possibly actively. In the first case, R&D investments are minimal and the relative scarcity index m_2^q remains constant. Since fossil fuel utilization is constant, the natural scarcity increases, reducing the gap between m_2^q and m_3^q, i.e. the additional scarcity. In the second case, R&D investments are positive. Therefore, utilization of the backstop increases, implying a decreasing share of R in the energy mix. Since fossil fuel is less important, its relative scarcity declines. Hence, singular or maximal R&D reduces the additional scarcity actively, establishing a second driving force in addition to the passive reduction of additional scarcity. As backstop utilization declines ceteris paribus with a lower scarcity index, the R&D advantageousness decreases. To switch over to phase 3, the additional scarcity must be eliminated completely. This follows directly from $\theta = \mu = 0$ in phase 3 and the smooth transition from one phase to the next.[22] The phase itself is equivalent to an economy without a ceiling.

Thus, the ceiling causes an increase in the scarcity of fossil fuel and thereby a reduction in

[22]See Appendix A.1 for a detailed proof.

its usage during phase 1 and 2. Since the ceiling would be violated without the additional scarcity, the result is quite intuitive. The additional scarcity increases as the pollution concentration approaches the ceiling, indicating a smaller growth rate of R. At the ceiling, the additional scarcity decreases due to the declining resource stock S_R and possibly increasing utilization of the backstop. The scarcity induced reduction of R causes an increase in backstop utilization and therefore of R&D advantageousness. Consequently, the advantageousness of capital accumulation declines.

The short-run effects of the ceiling of the development path of an economy are ambiguous. The ceiling does not only introduce the additional scarcity but can also alter the initial shadow prices of capital λ_0 and fossil fuel τ_0 and therefore of the natural scarcity.[23] However, if the initial shadow prices are sufficiently similar, the following analysis holds. We distinguish between poor economies with a low capital endowment and rich ones with a high capital endowment. As stated in section 3.1.1 a poor economy which is located below the SiP generally approaches the SiP with minimal R&D by capital accumulation. Thus, its optimal investment choice is only affected by the additional scarcity, if it reaches the SiP during phase 1 or 2. In this case, the economy starts R&D at a lower capital stock than an economy without the ceiling, i.e. earlier in its development process.[24] In other words, an economy is less developed or poorer when it starts R&D. Since the R&D investments are not available for capital accumulation, the economy investments into the capital stock are correspondingly lower. If an poor economy ignores the research option at all (ii. exception from section 3.1.1) or is only accumulating capital during phase 1 and 2, its optimal investment choice is not affected. However, due the increased use of backstop the energy costs are higher with the ceiling in the first two phases. Therefore, less net production is available for consumption and capital or R&D investments, which may stretch the time period until the development process reaches the critical level allowing R&D.

A rich economy which is located above the SiP can be affected by the ceiling in two ways. Generally, the economy approaches the SiP with maximal R&D. Since the ceiling increases the backstop utilization, at least in early periods less net production can be invested into research. Furthermore, the optimal investment decision is affected, if

[23] A short analysis with respect to an altered λ is given below.

[24] Notice that "early" is here not revering to time but to the capital stock level.

the economy would reach the SiP without the ceiling at a point in time that belongs either to phase 1 or 2. As the additional scarcity increases the R&D advantageousness by strengthen the decline of the SiP in time, the economy switches to singular research at a lower capital stock level. This may extend the time period of maximal R&D. If this time period is sufficiently extended, it compensates for the lower net production such that the economy switches to singular R&D at a higher development stage, i.e. with a higher technology level. As above, more R&D investments imply that less investments are made into the capital stock, or in the case of maximal R&D more capital is consumed. If the economy conducts maximal R&D during phase 1 and 2 regardless of whether the ceiling exists or not, i.e. if the economy is sufficiently rich, the ceiling induced additional scarcity lowers only the net production. In the case of minimal R&D above the SiP (i. exception from section 3.1.1) the higher per unit energy costs may shorten the time period until the economy reaches a steady state on the SSP.

In the long run an economy with a ceiling and one without a ceiling will exhibit the same SiP (and SSP) after the resource stock S_R is exhausted, because all energy is then generated by the backstop. Therefore, the R&D advantageousness is the same in both cases. This implies for the two standard cases from section 3.1.1 that the position of the long run development path is not affected by the ceiling. However, the position of a constrained economy on the path may be different from that of an economy without the ceiling at some specific point in time. In case of the ii. exception, an poor economy does not conduct R&D but converges to the SSP. Positive R&D would be only possible if the relative scarcity reaches a level higher than that of an exhausted resource stock. However, in this case fossil fuel is no longer used. Thus, if the additional scarcity had increased the scarcity to this level, it would have been impossible to reach or stay at the ceiling, which contradicts the fact that the economy must be in phase 1 or 2 to justify the additional scarcity. However, the long run development may be affected by the ceiling in case of the i. exception. This rich economy follows a MRAP with minimal R&D investments above the SiP. If the MRAP with minimal R&D is only a part of the development path, and was preceded by a part with maximal R&D, higher (lower) net production in early periods may cause a higher (lower) technology level and capital stock in the steady state.

The analysis of Chakravorty et al. (2006a) is based essentially on the scarcity of fossil fuel, which translates directly into the price of the resource in a Hotelling model. To explain a decreasing scarcity fossil fuel at the ceiling Chakravorty et al. (2006a) needed the assumption of decreasing global energy demand. In the current model a rising (declining) capital stock increases (decreases) energy demand.[25] However, the backstop absorbs all changes in total energy demand caused by a variation of the capital stock at the ceiling.[26] A decreasing scarcity is caused here by R&D, which increases the utilization of the backstop. Consequently, the share of fossil fuel in the energy mix declines.[27] The opposite effect of increasing scarcity at the ceiling cannot be explained, since there is no possibility of increasing the share of fossil fuel in the energy mix. For this purpose, either depreciation with respect to technology or a second technology that is related to fossil fuel must be taken into account.

The development of the relative scarcity index together with Fig. 1 allows a qualitative statement about the fossil fuel extraction path.

Proposition 4 *During phase 1 (3) R decreases monotonically due to the increasing relative scarcity index m_1^q (m_3^q) and the constant or decreasing unit costs of the backstop. On the other hand, phase 2 is characterized by constant utilization of fossil fuel.*

Figure 5 illustrates a corresponding fossil fuel path and a development path of the emission stock which follows directly from the fossil fuel path.

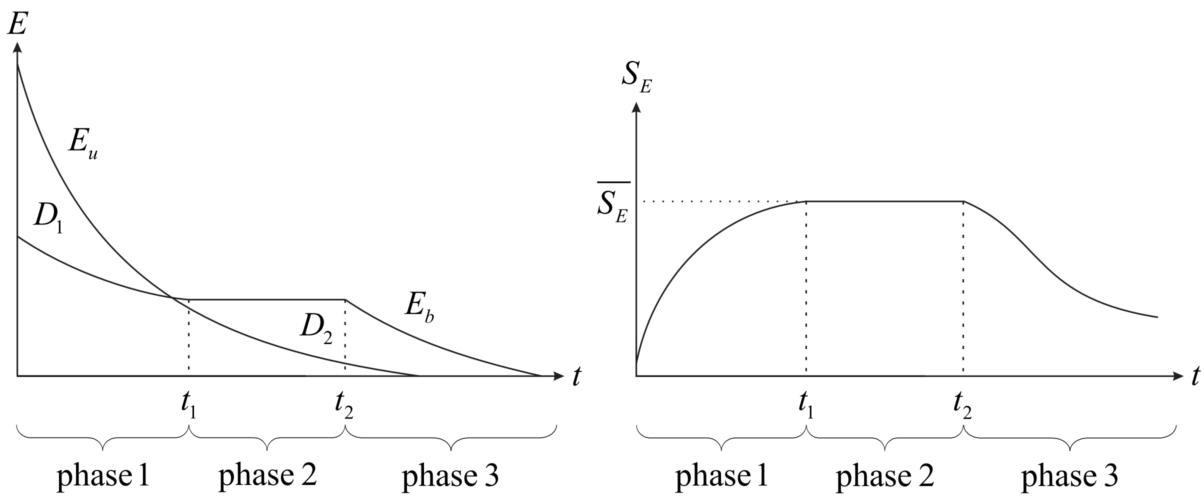

Figure 5: fossil fuel extraction path and emission stock development

The extraction path denoted with E_b shows how the resource is extracted in the

[25]See Fig. 1.
[26]If $b = 0$, a decreasing (increasing) capital stock and therefore a lower (higher) energy demand implies a lower (higher) scarcity to ensure $E = \bar{E}$.
[27]Chakravorty et al. (2012) get a similar result due to a learning - by - doing effect which decreases the costs of the backstop.

constrained economy. The other path, E_u, illustrates the path of an economy that is identical with the constrained one at $t = 0$ but without the ceiling. Since the complete resource stock must be used, both the area under E_b and under E_u equals S_R. Thus, the area marked with D_1 represents the amount of fossil fuel that is not used at early points of time in the constrained economy. Therefore, this amount must be used later on. The corresponding area is denoted by D_2. Because the areas under both paths are equal, $D_1 = D_2$ must hold. Note that E_b must not lie below E_U at $t = 0$. Since both economies are identical at the starting time, this only happens if the relative scarcity index $m_1^q(0) = \frac{\tau(0)+|\theta_1(0)|}{\lambda(0)}$ is greater than its equivalent of the economy without the ceiling $m_u^q(0) = \frac{\tau_u(0)}{\lambda_u(0)}$. Even if $\tau_u(0) = \tau(0)$ holds, there are two possible effects left. On the one hand $|\theta_1(0)| > 0$ increases the numerator of $m_1^q(0)$, indicating less usage of fossil fuel. On the other hand $\lambda(0)$ can be greater than $\lambda_u(0)$. Due to (7) this implies lower consumption in the constrained economy. If the reduction of consumption is large enough, the second effect outweighs the first one and $E_b(0) > E_u(0)$. In this case, the constrained economy uses more fossil fuel and consumes less. Both imply a higher net income Y^n, which can be used for either capital accumulation or research. Thus, the economy tends to adjust to the ceiling rather with strong measures than by gaining time to implement the necessary measures. The latter happens rather if $E_b(0) < E_u(0)$, as shown in Fig. 5. This solution is the one intuition suggests, since it would be expected that a ceiling on the stock of pollution, as an environmentally friendly measure, should decrease utilization of fossil fuel. Therefore, if $E_u(0) < E_b(0)$ holds, we have a kind of a green paradox.[28] However, in contrast to Sinn (2008a) and Sinn (2008b) the greater usage of fossil fuel is part of an optimal path for the entire economy and does not violate the ceiling. If the intuitive solution holds, the constrained economy will use more backstop, which implies a lower net income. But the lower fossil fuel use extends the time period until the ceiling binds.

4. Market Economy

After having analyzed the (constrained) social optimum we turn to a market economy in this section. We will show how fossil fuel tax needs to develop to ensure the ceiling. The

[28] The concept of the green paradox was introduced by Sinn (2008a) and Sinn (2008b) to describe a situation where a tighter environmental policy on the demand side of the economy induces a higher supply of polluting goods, therefore harming the environment instead of protecting it. The idea can be applied here in a more general sense.

economy consists of a great number of identical individuals and composite good producers as well as two resource owning companies.[29] The individuals own all companies in the Y - and resource sector, as well as the capital stock. They maximize their intertemporal utility with respect to their budget constraint. The companies in the composite good sector rent capital and buy resources to generate energy. Since they do not face an intertemporal problem, they maximize their profit at every point in time. The two resource owners sell the resources and, in the case of the backstop owning company, conduct research. Therefore, they maximize their intertemporal profit with respect to either the resource stock or the technology. We assume a Cournot competition on the resource market and perfect competition on all other markets. The government has the possibility of taxing fossil fuel, with ϕ denoting the corresponding quantity tax rate. Additionally, both resources can be subsidized with s_R or s_b, respectively. To balance the budget, the government can levy a lump-sum tax or grant a lump-sum transfer, both denoted with $T \lesseqgtr 0$.

Given the described market structure, a representative individual faces the intertemporal optimization problem

$$\max_C \int_0^\infty \left[U(C)e^{-\rho t}dt\right],$$

$$\text{subject to } \dot{K} = \frac{r}{p_Y}K + \frac{\pi}{p_Y} + \frac{\pi_b}{p_Y} + \frac{\pi_R}{p_Y} + \frac{T}{p_Y} - C. \tag{23}$$

The interest rate and the price of the composite good are represented by r and p_Y respectively. π, π_b and π_R denote the profits of the composite good producers and the two resource owners. From the necessary conditions $\lambda_H = U'(C)$ and $\hat{\lambda}_H = \rho - \frac{r}{p_Y}$ we get[30]

$$\hat{C} = \frac{\frac{r}{p_Y} - \rho}{\eta}. \tag{24}$$

λ_H represents the costate variable associated with the capital stock and η the (positive) elasticity of $U'(C)$. The Ramsey rule (24) states that consumption will increase as long as the real interest rate is greater than the time preference rate. According to $\lambda_H = U'(C)$, the marginal utility equals λ_H, i.e. the price the individual would pay for an increase of

[29] The assumption of two resource owners and a Cournot competition is a simplification of the well known patent assumption in the endogenous growth theory. See for example Romer (1990). A more complex model with several research firms would not alter the results.

[30] The current value Lagrangian as well as the necessary conditions are presented in Appendix A.2

his capital stock. For an equilibrium on the composite good market $\lambda_H = p_Y$ is necessary. Otherwise, the individual would buy more (less) Y for investing in the capital stock and consumption, if $\lambda_H < p_Y$ ($\lambda_H > p_Y$).

As mentioned above, the composite good producers do not have to solve an intertemporal optimization problem. Instead they maximize their profit at every point in time. Omitting a firm index and with p_b and p_R denoting the resource prices, the representative producer's profit is given by $\pi = p_Y F(K, b+R) - rK - p_b b - (p_R + \phi)R$. The first order conditions $\frac{\partial F}{\partial K} = \frac{r}{p_Y}$ and $\frac{\partial F}{\partial x} = \frac{p_b}{p_Y} = \frac{p_R+\phi}{p_Y}$ state that the marginal product of any input has to equal its real price. Since the composite good producers have no market power, these conditions hold, if the capital and resource market are cleared. By substituting $\frac{r}{p_Y}$ into (24) we get the socially optimal Ramsey rule (15).

The resource owners know the profit maximization problem of the composite good producers and therefore the price-demand functions for both resources $p_Y F_x(K, b+R) = p_b = p_R + \phi$.[31] The profits of the resource owners for each point in time are then given by

$$\pi_R = [p_Y F_x(K, b+R) - \phi]R - p_Y M(R) + s_R R, \tag{25}$$

$$\pi_b = p_Y F_x(K, b+R)b - M_b B(A)b + s_b b - p_Y I. \tag{26}$$

The fossil fuel owner maximizes its discounted flow of profits with respect to $S_R \geq 0$ and $\dot{S}_R = -R \leq 0$. Appendix A.3 shows the current value Lagrangian as well as the derivation of the first order conditions. As long as fossil fuel is used, the first order condition with respect to R can be written as

$$F_x(K, b+R) = M'(R) - Fxx(K, b+R)R + \frac{\phi}{p_Y} - \frac{s_R}{p_Y} + \frac{\tau_M}{p_Y}. \tag{27}$$

τ_M denotes the costate variable of the resource stock, which grows with the constant rate ρ and is therefore determined by its initial value τ_{0M}. Since the capital stock, the tax, the subsidy and the price p_Y are exogenous to both resource owner and τ_M determined by τ_{0M}, equation (27) defines implicitly the optimal resource supply R^* subject to the amount of supplied backstop. Thus, the reaction function is given by $R^* = R^*(b)$.
Using the same approach for the backstop owner we get for $b > 0$:[32]

$$F_x(K, b+R) = M_b B(A) - F_{xx}(K, b+R)b - \frac{s_b}{p_Y}. \tag{28}$$

[31] For the resource owners the capital stock is a known but exogenous factor.
[32] The Lagrangian and the derivation of the first order conditions can be found in Appendix A.3.

(28) defines implicitly the optimal backstop supply subject to R and the technology level A, which increases with the resource owner's R&D investments. We get $b^* = b^*(R, A)$. The optimal R&D investments are given by the maximization of the Lagrangian with respect to I, with κ_M denoting the costate variable of technology:

$$\begin{aligned} I^* = 0, \text{ if } & -p_Y + \kappa_M < 0, \\ 0 \leq I^* \leq \bar{I}, \text{ if } & -p_Y + \kappa_M = 0, \\ I^* = \bar{I}, \text{ if } & -p_Y + \kappa_M > 0. \end{aligned} \tag{29}$$

κ_M evolves according to $\hat{\kappa}_M = \rho + \frac{p_Y}{\kappa_M} M_b B'(A) b$. By substituting $R^*(b)$ and $b^*(R, A)$ in (27) and (28) respectively, the Nash - Cournot equilibrium is implicitly given:

$$\begin{aligned} F_x(K, b^*(R^*, A) + R^*) =& M'(R^*) - F_{xx}(K, b^*(R^*, A) + R^*) R^* + \frac{\phi}{p_Y} - \\ & \frac{s_R}{p_Y} + \frac{\tau_M}{p_Y}, \end{aligned} \tag{30}$$

$$F_x(K, b^* + R^*(b^*)) = M_b B(A) - F_{xx}(K, b^* + R^*(b^*)) b^* - \frac{s_b}{p_Y}. \tag{31}$$

Table 1 summarizes and compares the results of the market economy with the socially optimal solution. The Ramsey rule and the capital accumulation equation of the social optimum are identical with the market equilibrium equations. To reveal the latter, we substitute the profit of the composite good producer, (25), (26) and the government's budget constraint $T = \phi R - s_b b - s_R R$ into $\dot{K} = \frac{r}{p_Y} K + \frac{\pi}{p_Y} + \frac{\pi_b}{p_Y} + \frac{\pi_R}{p_Y} + \frac{T}{p_Y} - C$. For the further analysis we assume that the constrained social planner values capital, the resource stock S_R, and knowledge A in the same way as the subjects of the economy, which implies $\lambda = \lambda_H = p_Y$, $\tau = \tau_M$ and $\kappa = \kappa_M$. In this case the equations related to R&D are identical. The ceiling has no effect on the economy in phase 3, which suggests $\phi = 0$. Therefore, the optimal subsidies are determined by comparing the marginal products of R and b of this phase. We get

$$s_b = -p_Y F_{xx} b > 0, \tag{32}$$

$$s_R = -p_Y F_{xx} R > 0. \tag{33}$$

Using (33) we get for the optimal tax

$$\phi(t) = \begin{cases} |\theta_1(t)|, & \text{if } t \in [0, t_1), \\ \mu_2(t) - \theta_2(t), & \text{if } t \in [t_1, t_2), \\ 0, & \text{if } t \geq t_2. \end{cases} \tag{34}$$

	Phase One	Phase Two	Phase Three	Market Equilibrium
Ramsey - rule	$\hat{C} = \frac{F_K - \rho}{\eta}$			$\hat{C} = \frac{F_K - \rho}{\eta}$
marginal product of R	$F_x = M'(R) + \frac{\tau - \theta}{\lambda}$	$F_x = M'(R) + \frac{\tau - \theta + \mu}{\lambda}$	$F_x = M'(R) + \frac{\tau}{\lambda}$	$F_x = M'(R) - F_{xx}R + \frac{\phi}{p_Y} - \frac{s_R}{p_Y} + \frac{\tau_M}{p_Y}$
marginal product of b	$F_x = M_b B(A)$			$F_x = M_b B(A) - F_{xx}b - \frac{s_b}{p_Y}$
capital accumulation	$\dot{K} = F(K, x) - M_b B(A)b - M(R) - I - C$			$\dot{K} = \frac{r}{p_Y}K + \frac{\pi}{p_Y} + \frac{\pi_b}{p_Y} + \frac{\pi_R}{p_Y} + \frac{T}{p_Y} - C$
R&D	$\hat{\kappa} = \rho + \frac{\lambda}{\kappa} M_b B'(A)b$ $I^* = 0, \text{ if } -\lambda + \kappa < 0$ $0 \leq I^* \leq \bar{I}, \text{ if } -\lambda + \kappa = 0$ $I^* = \bar{I}, \text{ if } -\lambda + \kappa > 0$			$\hat{\kappa}_M = \rho + \frac{p_Y}{\kappa_M} M_b B'(A)b$ $I^* = 0, \text{ if } -p_Y + \kappa_M < 0$ $0 \leq I^* \leq \bar{I}, \text{ if } -p_Y + \kappa_M = 0$ $I^* = \bar{I}, \text{ if } -p_Y + \kappa_M > 0$

Table 1: Comparison of the market equilibrium and the socially optimal solution

Proposition 5 *The market equilibrium replicates the social optimum, if* $\lambda = \lambda_H$, $\tau = \tau_M$ *and* $\kappa = \kappa_M$ *holds, the usage of both resources is subsidized according to* (32) *and* (33), *and fossil fuel is taxed according to* (34).

Due to (12) the tax increases during phase 1 at the rate $\rho+\gamma$, reflecting the increasing emission stock, and therefore the tightening ceiling. In other words, as the emission stock increases, the amount of possible new emissions decreases, implying a higher tax. During phase 2, this amount is fixed at $\bar{E} = \gamma \bar{S}_E$. However, this does not imply a constant tax but, (in the long run) a decreasing one, because the natural scarcity of fossil fuel increases. Thus, the tax at the second junction point at $t = t_2$ equals $\phi(t_2) = 0$. As the growth rate of μ_2 is variable, $\phi_2(t)$ may increase as well as decrease. Nevertheless, Fig. 6 shows that the tax can only increase, if the composite good price p_Y grows fast enough. In this case the growth rate of the natural scarcity $\frac{\tau}{p_Y}$ is small, while the denominator of

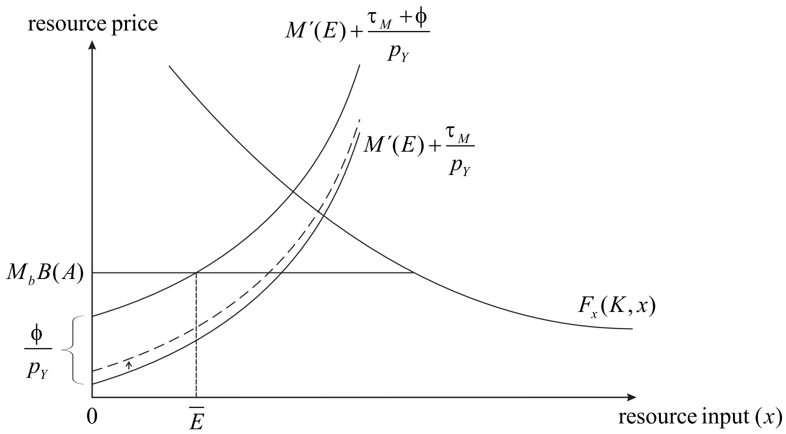

Figure 6: Increasing tax during phase 2

$\frac{\phi}{p_Y}$ increases. To comply with $E(t) = \bar{E}$ the tax needs to increase. Since $\hat{p}_Y = \rho - F_K$, a high capital stock is necessary for a sufficiently high inflation rate. Due to the Ramsey rule (24), a low marginal product of capital implies decreasing consumption. By using $\hat{\phi}_2 = \rho + \gamma - \frac{\mu_2}{\phi_2}[\rho - \hat{\mu}_2]$ we can show that consumption must indeed decline if the tax increases. The tax increases if, and only if, $F_K + \gamma - \frac{\mu_2}{\psi}[\rho - \hat{\mu}_2] - \gamma\frac{\tau}{\psi} > F_K - \rho\frac{\phi_2}{\psi}$ holds. The left hand side equals $\hat{m}_2^q$, which can only decrease or stay unchanged. Therefore, $0 > F_K - \rho\frac{\phi_2}{\psi}$ and because of $\phi_2 < \psi$ it follows $F_K - \rho < 0$. According to the Ramsey rule (24) $F_K - \rho < 0$ implies decreasing consumption, which is only possible if the economy lies above the SSP. Thus, during phase 2 the tax cannot increase in a sufficiently poor economy.

Proposition 6 *The optimal tax increases during phase 1 due to the increasing emission stock. Since the natural scarcity increases monotonically, the tax decreases to zero during phase 2. If inflation is sufficiently high, and consumption declines, the tax increases in the short run of phase 2.*

5. Conclusion

This paper analyzes the effects on R&D and the capital stock of a ceiling on the pollution stock. For this purpose we augment the endogenous growth model of Tsur and Zemel (2005) with a polluting resource and a ceiling on the stock of pollution as known from the literature following Chakravorty et al. (2006a). We show that the ceiling mainly affects the short run development of the economy by imposing an additional scarcity on fossil fuel. Since the costs of the backstop can be reduced by R&D, the additional scarcity increases the R&D advantageousness. If the ceiling does not affect the natural scarcity, the short run development is affected in two ways. On the one hand, higher energy costs decrease at least in early periods the amount of available production for capital or R&D investments. On the other hand, the optimal investment decision is affected, if the economy is neither too poor nor too rich. A poor economy will conduct R&D at an early development stage. In a rich economy the period of maximal R&D may be extended which can lead to a higher technology level. Consequently, both economies invest fewer into the capital stock. As long as the economy conducts some singular R&D, the long run development is hardly affected by the ceiling. In other words, the long run development path remains unchanged, whereas the position of the economy on the path to one specific point in time may be altered. If R&D is omitted, the ceiling has also no effect. The ceiling may alter the steady state of the economy if and only if maximal R&D is canceled in favor of minimal R&D.

As in Chakravorty et al. (2006a), Chakravorty et al. (2006b), Chakravorty et al. (2008), Chakravorty et al. (2012) and Lafforgue et al. (2008), we are able to distinguish three time phases. Analogous to Chakravorty et al. (2006a), the only sequence containing all three phases starts with a non-binding ceiling that will bind later on to become and stay non-binding afterwards. In contrast to the Hotelling models we can explain changes of total energy demand endogenously by the variable capital stock. Similar to Chakravorty et al. (2012) a declining resource scarcity at the ceiling is caused by an increasing technology level. However, the necessary R&D is an explicit decision and R&D can be abandoned, while Chakravorty et al. (2012) assumes a cost reducing learning-by-doing effect. In both

cases, the importance of fossil fuel for the energy mix vanishes as the utilization of the backstop is intensified.
The optimal fossil fuel extraction path is affected by the ceiling, since it exhibits a plateau during phase 2. While intuition suggests a reduction of fossil fuel utilization at the starting time to delay the moment the ceiling becomes binding, the results also permit some kind of a green paradox. In this case, the natural scarcity of fossil fuel is affected by the ceiling and both fossil fuel utilization and non-consumed income are higher. This implies greater investments in the capital stock and/or research to adjust to the ceiling.
We show that the social optimum is implemented by a market economy, if the government subsidizes both resources to counter market power effects resulting from Cournot competition on the resource market. Additionally, fossil fuel has to be taxed during phases 1 and 2 to comply with the ceiling. During phase 1, the tax increases monotonically, reflecting the rising emission stock. During phase 2, the emission stock remains unchanged, while the natural scarcity of fossil fuel increases, resulting in the tax being abolished at the end of phase 2. If inflation is sufficiently high, the tax can increase in phase 2 temporary. In this case, consumption decreases which requires a sufficiently rich economy. It is noteworthy that the model does not support subsidies for the backstop that are granted for pollution control reasons.

The presented model is rather a basic model, as it ignores several augmentations used in the literature following Chakravorty et al. (2006a). These are abatement investments, differently polluting exhaustible resources and limited carbon sinks. Especially the former seems interesting as abatement may substitute for R&D. We have also ignored that the amount of already used fossil fuel may influence the extraction costs. Concerning the backstop one could assume a non-linear cost structure. Another promising research field is the endogenization of the ceiling by means of a specific damage function or, in a model with multiple countries, by a multilateral bargaining process. By introducing a second technology which associated to fossil fuel it would be possible to analyze the influence of the ceiling on the direction of technical change.

A. Appendix

A.1. Junction points

The jump conditions for the costate variables at a junction point j which is characterized by a ceiling that becomes binding (entry point) is[33]

$$\Gamma^{+}(j) = \Gamma^{-}(j) + B\frac{\partial[\bar{S}_E - S_E]}{\partial \Gamma_V}, \ B \geq 0, \tag{A.1}$$

with $\Gamma = \tau, \theta, \lambda$; Γ_V being the associated state variable S_R, S_E, K as well as $^+$ and $^-$ denoting the values just after and just before the junction point, respectively. It shows that τ and λ are continuous while θ may jump. At a junction point where the ceiling becomes non-binding (exit point) all costate variables are continuous.[34] Due to (7), the continuity of λ implies a continuous consumption path. Since the indirect approach is used for (6), the jump condition can be written as[35]

$$\theta^{+}(j) = \theta^{-}(j) + \mu^{+}(j) - \mu^{-}(j) + B_\theta, \ B_\theta \geq 0. \tag{A.2}$$

Due to (16), $\mu = 0$ during phase 1 and 3. In phase 3 the ceiling is non-binding and will never be reached. Since pollution does not affect production or utility directly, it is then irrelevant for the social planner. Thus, θ must be zero. Therefore, the relative scarcity index in phase 1, phase 2 and phase 3 is $\frac{\tau-\theta}{\lambda}$, $\frac{\tau-\theta+\mu}{\lambda}$ and $\frac{\tau}{\lambda}$, respectively. At a junction point the used amount of exhaustible resources can exhibit a jump, because E is a control variable. If the ceiling becomes binding at the junction point, a jump upwards is prevented by the natural regeneration rate. If the ceiling becomes non-binding, the ceiling itself prevents an upward jump. However, jumps downward are possible in both cases. The necessary changes can be derived from Fig. 1. It does not matter whether the backstop is used. As the demand function F_x and the marginal extraction costs function $M'(E)$ are not affected by a junction point, a sudden drop in E is only possible if the relative scarcity index increases. Therefore, the following conditions must hold at junction points between phase 1 and 2 as well as between phase 2 and 3.

- At a junction point t_1 from phase 1 to phase 2:
  $\frac{\tau^{-}(t_1)-\theta^{-}(t_1)}{\lambda^{-}(t_1)} \leq \frac{\tau^{+}(t_1)-\theta^{+}(t_1)+\mu^{+}(t_1)}{\lambda^{+}(t_1)} \Leftrightarrow \theta^{+}(t_1) \leq \theta^{-}(t_1) + \mu^{+}(t_1)$

[33] Cf. Feichtinger and Hartl (1986), p. 166 et seq.
[34] See Feichtinger and Hartl (1986), p.170.
[35] Cf. Chiang (1992), p. 300 et seq. and Feichtinger and Hartl (1986), p. 171-172.

- At a junction point t_2 from phase 2 to phase 3:
  $\frac{\tau^-(t_2)-\theta^-(t_2)+\mu^-(t_2)}{\lambda^-(t_2)} \leq \frac{\tau^+(t_2)}{\lambda^+(t_2)} \Leftrightarrow \mu^-(t_2) \leq \theta^-(t_2)$

- At a junction point t_3 from phase 2 to phase 1:
  $\frac{\tau^-(t_3)-\theta^-(t_3)+\mu^-(t_3)}{\lambda^-(t_3)} \leq \frac{\tau^+(t_3)-\theta^+(t_3)}{\lambda^+(t_3)} \Leftrightarrow \theta^+(t_3) \leq \theta^-(t_3) - \mu^-(t_3)$

Substituting (A.2) shows that all three conditions must hold equally. This implies the continuity of E, since the state variables capital K and technology A have to be continuous, too. The total energy input depends only on K and A, so that its continuity, as well that of $b = x(K, A) - E(K, A)$, follows directly. Thus, both production factors are continuous, which implies the continuity of Y. The one of consumption C results from the continuity of λ and (7). Therefore, the economy switches smoothly from one phase to the next.

If we denote the variables by the corresponding phase, we can rewrite the conditions at the junction points as $\theta_2(t_1) = \theta_1(t_1) + \mu_2(t_1)$, $\theta_2(t_2) = \mu_2(t_2)$ and $\theta_2(t_3) = \theta_1(t_3) + \mu_2(t_3)$, respectively. Obviously, the first and third conditions are identical. Thus, should there be more than one junction point between phase 1 and 2, the conditions must hold for two or more different points in time. However, by solving (12) and (16) for θ_1, θ_2 and μ_2, i.e. $\theta_1(t) = \theta_{01}e^{(\rho+\gamma)t}$, $\theta_2(t) = \theta_{02}e^{(\rho+\gamma)t} - \gamma\mu_{02}e^{(\rho+\gamma)t}\int e^{-(\rho+\gamma)t+\rho\int\xi(t)dt}dt$ and $\mu_2(t) = \mu_{02}e^{\rho\int\xi(t)dt}$, with $\theta_{01} < 0$, θ_{02} and $\mu_{02} > 0$ as constants of integration and $\xi(t) \leq 1$, the conditions can be written as

$$\begin{aligned} &\theta_{02}e^{(\rho+\gamma)t} - \gamma\mu_{02}e^{(\rho+\gamma)t}\int e^{-(\rho+\gamma)t+\rho\int\xi(t)dt}dt = \theta_{01}e^{(\rho+\gamma)t} + \mu_{02}e^{\rho\int\xi(t)dt} \\ \Leftrightarrow\ &\frac{\theta_{02}-\theta_{01}}{\mu_{02}} = e^{-(\rho+\gamma)t+\rho\int\xi(t)dt} + \gamma\int e^{-(\rho+\gamma)t+\rho\int\xi(t)dt}dt, \text{ for } t = t_1, t_3 \end{aligned} \tag{A.3}$$

and

$$\begin{aligned} &\theta_{02}e^{(\rho+\gamma)t} - \gamma\mu_{02}e^{(\rho+\gamma)t}\int e^{-(\rho+\gamma)t+\rho\int\xi(t)dt}dt = \mu_{02}e^{\rho\int\xi(t)dt} \\ \Leftrightarrow\ &\frac{\theta_{02}}{\mu_{02}} = e^{-(\rho+\gamma)t+\rho\int\xi(t)dt} + \gamma\int e^{-(\rho+\gamma)t+\rho\int\xi(t)dt}dt, \text{ for } t = t_2. \end{aligned} \tag{A.4}$$

The right hand side of (A.3) and (A.4) is called $T_f(t)$. It is continuous in time and $\frac{dT_f}{dt} < 0$ for $\xi(t) < 1$. As long as the growth rate of μ is lower than the time preference rate, T_f decreases strictly. In this case, both (A.3) and (A.4) hold only for one point in time, which implies just one junction point between both phase 1 and 2 and between phase 2 and 3. Furthermore, θ_{01} must be negative. Otherwise, the junction point between phase 2 and 3

would be located before the junction point between phase 1 and 2 on the time line, which is impossible due to the definition of phase 3. The only possible sequence containing all phases is 1, 2, 3. In the case of $\xi(t) = 1$ the right hand side of (A.3) and (A.4) reduces to zero. It follows $\theta_{02} = \theta_{01} = 0$. This implies that the emission path will be only tangent to the ceiling. Therefore, this case is negligible.
The condition $\theta_2(t_1) = \theta_1(t_1) + \mu_2(t_1)$ reveals that θ jumps at the entry point, if $\mu(t_1) > 0$. Since all costate variables are continuous at the exit point and $\theta = 0$ in phase 3, the corresponding condition $\theta_2(t_2) = \mu_2(t_2)$ implies $\theta^-(t_2) = \mu^-(t_2) = 0$.

A.2. Individual

The current value Hamiltonian of the representative individual is:

$$H = U(C) + \lambda_H \left[\frac{r}{p_Y} K + \frac{\pi}{p_Y} + \frac{\pi_b}{p_Y} + \frac{\pi_R}{p_Y} + \frac{T}{p_Y} - C \right] \tag{A.5}$$

The first order conditions and the transversality condition are given by:

$$\frac{\partial H}{\partial C} = U'(C) - \lambda_H = 0 \tag{A.6}$$

$$\frac{\partial H}{\partial K} = \lambda_H \frac{r}{p_Y} = \rho \lambda_H - \dot{\lambda}_H \tag{A.7}$$

$$\lim_{t \to \infty} e^{-\rho t} \lambda_H(t) [K(t) - K^*(t)] \geq 0 \tag{A.8}$$

A.3. Resource Owners

The current value Hamiltonian of the firm owning the exhaustible resource is:

$$H = [p_Y F_x(K, b + R) - \phi] R - p_Y M(R) + s_R R - \tau_M R \tag{A.9}$$

The first order condition as well as the transversality condition are given by:

$$\frac{\partial H}{\partial R} = p_Y F_{xx}(K, x) R + p_Y F_x(K, x) - \phi - p_Y M'(R) + s_R - \tau_M = 0 \tag{A.10}$$

$$\frac{\partial H}{\partial S_R} = 0 = \rho \tau_M - \dot{\tau}_M \tag{A.11}$$

$$\tau_M(T_R) = \gamma_{S_R} \tag{A.12}$$

$$\gamma_{S_R} \geq 0, \ \gamma_{S_R} S_R(T_R) = 0 \tag{A.13}$$

$$H(T_R) = \begin{cases} \leq 0, \ \text{if } T_R = 0 \\ = 0, \ \text{if } 0 < T_R < \infty \\ \geq 0, \ \text{if } T_R = \infty \end{cases} \tag{A.14}$$

T_R denotes the point in time the resource stock S_R becomes exhausted.

The current value Hamiltonian of the firm owning the backstop is:

$$H = p_Y F_x(K, b+R)b - p_Y M_b B(A)b + s_b b - p_Y I + \kappa_M I \tag{A.15}$$

The first order condition as well as the transversality condition are given by:

$$\frac{\partial H}{\partial b} = p_Y F_{xx}(K,x)b + p_Y F_x(K,x) - p_Y M_b B(A) + s_b = 0 \tag{A.16}$$

$$\frac{\partial H}{\partial A} = -p_Y M_b B'(A)b = \rho\kappa_M - \dot{\kappa}_M \tag{A.17}$$

$$\lim_{t\to\infty} e^{-\rho t}\kappa_M(t)[A(t) - A^*(t)] \geq 0 \tag{A.18}$$

The maximization of the Hamiltonian with respect to I gives:

$$I^* = 0, \text{ if } -p_Y + \kappa_M < 0 \tag{A.19}$$

$$0 \leq I^* \leq \bar{I}, \text{ if } -p_Y + \kappa_M = 0 \tag{A.20}$$

$$I^* = \bar{I}, \text{ if } -p_Y + \kappa_M > 0 \tag{A.21}$$

The index * marks the optimal value of the variable in question.

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