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Taxes on Cars and Gasoline to Control of Air Pollution: Suggested Models for Bangladesh

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ABSTRACT

The main aim of this paper is to investigate some policies that would influence people to drive fewer miles and to buy smaller cars, use better pollution control equipment, and cleaner fuel. An attempt has been made to quote the vehicle tax rates of Bangladesh. Despite technological advances, the emissions of cars' still can not be measured reliably enough to impose a Pigovian tax. Literature review reveals that the gas tax depends on fuel type, engine size and pollution control equipment. A vehicle tax depends on mileage or a combination of uniform tax rates on gasoline and engine size with a subsidy to pollution control equipment. This study suggested two models, which first considers homogenous consumers and then considers for heterogeneous consumers that differ by income and two taste parameters, one for miles and other for vehicle size. Yet Bangladesh has not imposed emission taxes on vehicles properly; as a result the air pollutions in large cities are increasing dangerously. Dhaka, the capital of Bangladesh, is one of the dangerously polluted cities of the world. The government of Bangladesh should take immediate steps to impose emission taxes on vehicles according to guidelines of this paper to apply the taxes on vehicles.

JEL. Classification: E62; H21; H23; I38; Q51; Q53;

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1. INTRODUCTION

In the last part of the 20th century and in the beginning of the 21st century the area of cities of the world has expanded, and new cities and towns have grown rapidly. As a result in vehicle-miles traveled increases. Again, most of the luxurious people like large vehicles which are increasing externalities from vehicle emissions. Emissions from vehicles pollute air that worsened human health, diminishing visibility and caused global worming (Fullerton and West 2002). The best way is to measure the emission of each vehicle efficiently and accurately but yet no cheaper and accurate measurement technology is invented. Actual vehicle emissions depend not only on vehicle size and age, but also on qualities of the fuel, maintenance of the car's pollution control equipment (*PCE*), frequency of cold start-ups, temperature of the air, speed of the

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vehicle, and aggressive driving (Fullerton and West 2002). More gasoline that is volatile leads to more evaporative emissions. The carburetor setting be unchanged, this may reduce emissions of carbon monoxide (CO) and hydrocarbons (HC), but can also increase emissions of oxides of nitrogen (NO_x). Burmich (1989) finds that for cold start-ups a 5-mile trip has almost three times the emissions per mile trip at the same speed. Sierra Research (1994) finds that a car driven aggressively has a carbon monoxide emissions rate that is almost 20 times higher than when driven normally. As like Fullerton and West (2002) we investigate some policies that would influence people to drive fewer miles and to buy smaller cars, better pollution control equipment, and cleaner fuel.

In our model we first consider homogenous consumers and then consider for heterogeneous consumers that differ by income and two taste parameters, one for miles and one for vehicle size. The motorist can reduce their fees by repairing their vehicles but not by driving less. Sevigny (1998) incorporates the choice of miles with a second-best emissions tax, but this tax requires knowledge of each vehicle's average emission per mile and the accurate measurement of miles traveled. But emissions per mile (*EPM*) cannot be measured perfectly, because it depends on how the car is driven. Miles cannot be measured perfectly, because drivers can roll back the odometer.

The efficiency of the emissions tax can be achieved from the homogeneous agents by a set of uniform tax or subsidy rates on choices such as fuel use, type of fuel, engine size, vehicle age, and *PCE*. Heterogeneous agents maximize different utility functions, so that they have different choices about miles driven, engine size, and vehicle age- three different important determinations of emissions (Fullerton and West 2000, 2010). Here we describe all the three choices using general functional forms, and we find that the first-best requires that each individual a different rate of tax on each such choice following Fullerton and West (2010), and Hoel (1998). Let a car drives m miles, so that total emission is $m \cdot EPM$ and we treat a tax on those emissions as the ideal Pigovian tax. The motorist still has a variety of taxes or subsidies on observable choices such as gasoline, engine size, and vehicle age to induce individuals to drive fewer miles, to buy smaller cars, or to scrap older cars (Fullerton and West 2010). As a result motorists will encourage buying newer cars but the government should take steps to reduce prices of new cars.

2. MODEL FOR HOMOGENEOUS CONSUMERS

We assume perfect information, perfect competition, and no market failure other than a negative externality from emissions for homogeneous consumers (Fullerton and West 2002). Let us consider a simple economy consists of n identical individuals each of which owns one vehicle. Each vehicle is composed of some attributes that affect emissions (such as engine size, fuel efficiency, and *PCE*) and other attributes that do not affect emissions (such as leather seats or a sunroof). Households buy gasoline to drive miles, and they choose among grades of fuel-cleanliness.

They gain utility from driving miles m, the size of the vehicle s, and other goods and services, x. The size s of engine is measured as cubic inches of displacement (*CID*). The consumers may gain or lose utility from pollution-control equipment c, and per gallon fuel cleanliness f. Fuel cleanliness is an attribute of gasoline such as volatility or oxygenation. Again, household utility is affected by aggregate auto emissions, E. Thus the household's utility function is,

$$u = u\,(m, s, c, f, x, E\,) \qquad (1)$$

where u is continuous, differentiable and strictly quasi-concave in its first five arguments.

EPM = *X* discharge by a car depends positively on the size and negative on *PCE* and the clean-fuel characteristic i.e., $X = X(s, c, f)$. Each of the households drives *m* miles, then aggregate emission *E* can be written as;

$$E = n\, m\, X. \tag{2}$$

The fuel efficiency is measured drives in miles per gallon (*MPG*) = *Y* and depends on engine size and the quantity of the clean-car good on the vehicle i.e., $Y = Y(s, c)$. Cars with larger engines get lower gas mileage, so that $Y_s = \frac{\partial Y}{\partial s} < 0$. Consumers do not purchase m directly, but through the combination they choose gasoline (*g*), size (*s*) and the clean car good (*c*), so that;

$$g = \frac{m}{Y(s,c)}. \tag{3}$$

Consumers use (3) when they decide vehicle size, vintage and how much gasoline will maximize utility (1). The individual is taxed or subsidized on consumption of m, s, c, f and *x*.

Let p_g = price per gallon of gasoline without any clean characteristic, p_f = price per unit of the clean-fuel characteristic per gallon. Therefore the total price of a gallon of gasoline is = ($p_g + p_f f$), and the private cost of driving one mile is ($p_g + p_f f$) / $Y(s, c)$. Again p_s = price of *s*, which represents the price of adding a *CID* to an engine, p_c = the price per unit of the clean-air good. For convenience we normalize the price of *x* equals one. The individual problem is to maximize (1) subject to budget constraint;

$$y = \left(\frac{p_g + p_f f}{Y(s,c)}\right) m + p_s s + p_c c + x. \tag{4}$$

Hence the social planner Lagrangean is;

$$L = u(m, s, c, f, x, E) + \delta\left[y - \left(\frac{p_g + p_f f}{Y(s,c)}\right) m - p_s s - p_c c - x\right] \tag{5}$$

where δ is the marginal social value of income. The first-order conditions with homogeneous consumers for maximization are as follows:

$$u_m + u_E n\, X = \delta\left(\frac{p_g + p_f f}{Y(s,c)}\right), \tag{6a}$$

where bracketed term in (6a) is the total implicit price of a mile.

$$u_s + u_E n\, m\, X_s = \delta\left[p_s - \frac{\left(p_g + p_f f\right) m Y_s}{Y^2} \right], \tag{6b}$$

where the bracketed term in (6b) is the overall cost per unit of size, including the extra amount that must be paid for miles due to the lower *MPG* caused by the incremental unit of *s*.

$$u_c + u_E n\, m\, X_c = \delta\left[p_c - \frac{\left(p_g + p_f f\right) m Y_c}{Y^2} \right], \tag{6c}$$

where the bracketed term in (6c) is the overall cost of *PCE*, including the extra amount that must be paid for miles due to the lower *MPG*.

$$u_f + u_E n\, m\, X_f = \delta\left(\frac{m\, p_f}{Y} \right), \tag{6d}$$

where the bracketed term in (6d) is the overall cost per unit of the clean-fuel characteristic.

$$u_x = \delta\,. \tag{6e}$$

These first-order conditions say that the marginal social gain from driving another mile, or from an additional unit of *s*, *c*, *f*, or *x*, is equal to the marginal social cost of each. The term u_E on the left-hand sides of (6a) to (6d) reflects the effect on utility of the increment to aggregate emissions from driving an additional mile, increasing vehicle size, adding *PCE*, or cleaner gas. An individual usually does not know that his own choices affect aggregate emission but he may face taxes or subsides on its consumption of *s*, *c*, *f*, *x* and *g*. The household's budget constraint becomes;

$$y = \left(\frac{p_g + \tau_g + \left(p_f + \tau_f\right) f}{Y(s,c)} \right) m + \left(p_s + \tau_s\right) s + \left(p_c + \tau_c\right) c + \left(1 + \tau_x\right) x + m\,\tau_e\, X(s,c,f) \tag{7}$$

where τ_g is the tax per gallon of gas, τ_f is the tax per unit of clean-fuel characteristic, τ_s is the tax per unit size, τ_c is the tax per unit *PCE*, and τ_e is the tax per unit of emissions. Hence the household's Lagrangean is;

$$L = u\left(m,s,c,f,x,E\right) +$$

$$\lambda\left[y - \left(\frac{p_g + \tau_g + \left(p_f + \tau_f\right) f}{Y(s,c)} \right) m - \left(p_s + \tau_s\right) s - \left(p_c + \tau_c\right) c - \left(1 + \tau_x\right) x - m\,\tau_e\, X(s,c,f) \right]. \tag{8}$$

The first-order conditions for maximization are as follows:

$$u_m = \lambda \left[\left(\frac{p_g + \tau_g + \left(p_f + \tau_f\right) f}{Y(s,c)} \right) m + \tau_e \, X(s,c,f) \right], \tag{9a}$$

$$u_s = \lambda \left[p_s + \tau_s + m \left(\frac{-\left(p_g + \tau_g + \left(p_f + \tau_f\right) f\right) Y_s}{Y^2} \right) + m \tau_e \, X_s \right], \tag{9b}$$

$$u_c = \lambda \left[p_c + \tau_c + m \left(\frac{-\left(p_g + \tau_g + \left(p_f + \tau_f\right) f\right) Y_s}{Y^2} \right) + m \tau_e \, X_c \right], \tag{9c}$$

$$u_f = \lambda \left[\frac{\left(p_f + \tau_f\right) m}{Y} + m \tau_e \, X_f \right], \text{ and} \tag{9d}$$

$$u_x = \lambda \left(1 + \tau_x\right). \tag{9e}$$

Emissions would be calculated to enter the consumer problem implicitly through the pollution tax τ_e. The price per unit mile, and similar emissions tax calculations would be for *s*, *c* and *f*.

2.1 Analytical Calculations for Taxes and Subsides

Now we are interested to calculate Pigovian tax. The tax on emission τ_e, provides the basic efficient policy against which alternatives can be compared. Let all other tax rates set to be equals to zero i.e., $\tau_g = \tau_f = \tau_s = \tau_c = \tau_x = 0$. Then (6e) and (9e) imply $u_x = \lambda = \delta$. Now using $\lambda = \delta$ in (6a) we get;

$$u_m = \lambda \left(\frac{p_g + p_f f}{Y} \right) - u_E n \, X . \tag{10}$$

Using the value of u_m in (9a) we get;

$$\lambda \left(\frac{p_g + p_f f}{Y} \right) - u_E n \, X = \lambda \left(\frac{p_g + p_f f}{Y} + \tau_E X \right),$$

$$\tau_e = -\frac{u_E n}{\lambda} . \tag{11}$$

Now we can define (11) as the marginal environmental damages (*MED*) per unit of emissions; which is the usual Pigovian tax, and it is greater than zero so long as $u_E < 0$. Hence Pigovian tax on emissions by itself induces households to make all the optimal choices about miles, car size, fuel, and *PCE*.

Now we will calculate gas tax τ_g. For the impossible measurement of gas emission, $\tau_e = 0$ and suppose all other tax rates be zero i.e., $\tau_f = \tau_s = \tau_c = \tau_x = 0$, then from (6e) and (9e) we get, $\lambda = \delta$ and (9a) now becomes;

$$u_m = \lambda \left[\frac{p_g + p_f f}{Y} + \frac{\tau_g}{Y} \right]. \tag{12}$$

From (10) and (12) we get;

$$\lambda \frac{\tau_g}{Y} = -u_E n\, X,$$

$$\tau_g = -\frac{u_E n}{\lambda} X(s,c,f)\, Y(s,c), \tag{13}$$

which represents the additional damage caused by an increase of one gallon of gas. From (13) we see that gas tax depends on fuel characteristic *f* and on the characteristic of the vehicle at the pump (*s* and *c*).

Now we will calculate vehicle tax τ_v. If the gas tax cannot depend on characteristics of the vehicle, the efficiency outcome can still be attained by a vehicle tax that depends on mileage i.e., $\tau_v = \tau_v(m)$. As before suppose other tax rates be zero i.e., $\tau_f = \tau_g = \tau_s = \tau_c = \tau_x = 0$, and suppose that the Lagrangean of (8) is modified by subtracting a tax τ_v per vehicle. Suppose that all other tax rates be zero, and the vehicle tax would be as follows:

$$\tau_v = -\frac{u_E n}{\lambda} m\, X(s,c,f). \tag{14}$$

Authorities know the cars' characteristics (*s* and *c*) and mileage (*m*), hence (14) is product of cars' emission $m.\, X(s, c, f)$ and Pigovian tax rate, $\left(-\frac{u_E n}{\lambda} \right)$.

We now solve separate fixed tax rates. This technique applies if none of the above policies are available and government can set separate tax rates on gasoline, engine size, and *PCE*. We assume that the gas tax can be made to depend on characteristic of the fuel but not the characteristic of the car. From (6b) and (9b) we get,

$$\lambda \left[p_s + m \left(\frac{-(p_g + p_f f) Y_s}{Y^2} \right) \right] - u_E n\, m\, X_s = \lambda \left[p_s + \tau_s + m \left(\frac{-(p_g + p_f f) Y_s}{Y^2} \right) + \mathrm{m} \tau_g \frac{Y_s}{Y^2} \right],$$

$$-u_E n\, m\, X_s = \lambda\, \mathrm{m} \tau_{\mathrm{g}} \frac{Y_s}{Y^2} + \lambda \tau_{\mathrm{s}},$$

$$\tau_{\mathrm{s}} = -\frac{u_E n}{\lambda} m \left(X_s + \frac{X Y_s}{Y} \right). \tag{15}$$

Here the first term gives the direct damage cased by an increase of one unit of size, which is positive as long as emissions affect utility, $u_E < 0$ and size affects emissions, $X_s > 0$. The second term is an indirect effect from an additional unit of size through its effect on fuel efficiency. An additional unit of size decreases fuel efficiency, the household knows that an increase in the size of his vehicle engine will cost an additional gas tax. Again observe that the two components of the size tax are opposite in size, so that we cannot predict the sign of τ_{s}. Since $u_E < 0$, so that $\tau_{\mathrm{s}} > 0$ if

$$\frac{X_s}{X} > \frac{-Y_s}{Y}. \tag{16}$$

We now solve *PCE* tax rates. From (6c) and (9c) we get for $\lambda = \delta$;

$$-u_E n\, m\, X_c + \delta \left[p_c - \frac{(p_g + p_f f) m\, Y_c}{Y^2} \right] = \lambda \left[p_c + \tau_c - \frac{(p_g + p_f f) m Y_c}{Y^2} - \frac{m \tau_g\, Y_c}{Y^2} \right],$$

$$\tau_c = -\frac{n\, u_E m\, X_c}{\lambda} - \frac{n\, u_E m Y_c}{\lambda Y} X \tag{17}$$

which is analogous to the τ_{s}. The first term of (17) is negative to reflect the effect on damages of an added unit of *PCE* and the second term is a rebate due to the effect that *PCE* has no fuel efficiency and hence it is negative. So that τ_{c} is always negative, that is it is necessarily a subsidy. Since $\tau_{\mathrm{e}} = 0$ the subsidy to *PCE* (either in τ_{g} or τ_{e}) can only induce consumers to buy any such equipment if it is equal to the entire private cost of *PCE*, including both the direct cost p_{c} and the extra fuel cost incurred due to the negative effect that indeterminate.

3. MODEL FOR HETEROGENEOUS CONSUMERS

In this section, we introduce heterogeneity when the optimal tax rates need to differ among consumers. Let us assume parameter α to represent the household's preference for miles and β to represent the preference for size of the car. Together with income, these parameters are jointly distributed according to the distribution function $h(\alpha, \beta, y)$ with positive support on $[\underline{\alpha}, \overline{\alpha}] \times [\underline{\beta}, \overline{\beta}] \times [\underline{y}, \overline{y}]$. The people who live fur from their work place have a high demand for miles (α), but they may prefer either a large car for comfort and safety or a small car for better gas mileage. For heterogeneity we ignore the clean-car and clean-fuel characteristics. Hence, fuel efficiency and emissions per mile depend only on size, and each household generates $m\, X(s)$ units of emissions. The total pollution is;

$$E = \iiint_{\alpha\,\beta\,y} mX(s)\,h(\alpha,\beta,y)\,d\alpha d\beta dy\,. \tag{18}$$

A household's utility function is;

$$U = u\,(m,s,x;\alpha,\beta) - \mu E \tag{19}$$

where μ is the household's change in welfare from additional pollution ($\partial U / \partial E$). The social planner must maximize a measure of social welfare such as a weighted sum of utilities of *n* households. We divide each household's utility by its own marginal utility of income (λ). If τ_e is available, we want the maximization of our social welfare function to yield the solution of Pigou (1932). Since this solution is based on marginal conditions, such as marginal environmental damages, at the optimum, we use the values for λ that occur at the first-best social optimum (λ^*). When first-best instruments are not available, we want to be able to find second-best uniform tax rates that maximize the same social welfare function. To evaluate λ^* we use the prices at the Pigovian equilibrium and weights are calculated as (1/λ^*). The social welfare function is;

$$W = \iiint_{\alpha\,\beta\,y} \left[\frac{u(m,s,x)}{\lambda^*} - \mu\, E \right] h(\alpha,\beta,y)\,d\alpha d\beta dy\,. \tag{20}$$

Again the social planner's budget constraints;

$$y = \frac{p_g}{Y(s)} m - p_s - x\,. \tag{21}$$

The social planner's problem is to maximize this welfare function subject to a resource constraint, so that the Lagrangean is,

$$L = \iiint_{\alpha\,\beta\,y} \left[\frac{u(m,s,x)}{\lambda^*} - \mu \iiint_{\alpha\,\beta\,y} m\, X(s)\,h(\alpha,\beta,y)\,d\alpha d\beta dy \right] h(\alpha,\beta,y)\,d\alpha d\beta dy$$

$$+\delta \left[\iiint_{\alpha\,\beta\,y} \left(y - \frac{p_g}{Y(s)} m - p_s - x \right) h(\alpha,\beta,y)\,d\alpha d\beta dy \right] \tag{22}$$

with respect to each consumer's m, s and x. Income plus tax rebates is y, and the marginal social value of income is δ. The first-order conditions for household i is as follows:

$$\frac{1}{\lambda^*}\frac{\partial u_i}{\partial m_i} - n\mu\, X(s_i) = \delta\left[\frac{p_g}{Y(s_i)}\right], \tag{23a}$$

where the second term represents the external cost of an additional mile driven by individual *i*.

$$\frac{1}{\lambda^*}\frac{\partial u_i}{\partial s_i} - n\mu\, m_i X_{s_i} = \delta\left[p_s + m_i\left(\frac{-p_g Y_{s_i}}{Y^2}\right)\right], \tag{23b}$$

where the second term represents the external cost of an additional unit of size purchased by individual *i*.

$$\frac{1}{\lambda^*}\frac{\partial u_i}{\partial x_i} = \delta \ , \tag{23c}$$

where each equation represents *n* first-order conditions, one for each individual *i*. Also first term in each equation represents the individual's money value of marginal utility from each good.

Now we discuss household problem. A household does not identify that his own emissions add to aggregate emissions. The household's budge constraint is;

$$y_i = \frac{p_g + \tau_g}{Y(s_i)} m_i + (p_s + \tau_s)s_i + (1+\tau_x)x_i + \tau_e X(s_i) m_i . \tag{24}$$

Therefore household problem is to maximize the Lagrangean;

$$L = u_i(m_i, s_i, x_i) - \mu\, E + \lambda_i\left[y_i - \left(\frac{p_g + \tau_g}{Y(s_i)}\right)m_i - (p_s + \tau_s)s_i - (1+\tau_x)x_i - m_i\tau_e\, X(s_i)\right]. \tag{25}$$

with respect to m_i, s_i and x_i. The first-order conditions for maximization are as follows:

$$\frac{\partial u_i}{\partial m_i} = \lambda^*\left[\frac{p_g + \tau_g}{Y(s_i)} + \tau_e X(s_i)\right], \tag{26a}$$

$$\frac{\partial u_i}{\partial s_i} = \lambda^*\left[p_s + \tau_s - m_i\left(\frac{(p_g + \tau_g)Y_{s_i}}{Y^2}\right) + \tau_e m_i X(s_i)\right], \text{ and} \tag{26b}$$

$$\frac{\partial u_i}{\partial x_i} = \lambda^*(1+\tau_x). \tag{26c}$$

3.1 Analytical Calculations for Taxes

To calculate Pigovian tax, we set all taxes except τ_e equals to zero i.e., $\tau_s = \tau_g = \tau_x = 0$. Again we use $\delta = 1$ in (23a) and (26a) to equal each other. The household specific variables drop out leaving

$$\tau_e = \frac{n\mu}{\lambda} = MED. \qquad (27)$$

This is the first-best uniform Pigovian tax and can be used to identify other first-best gas tax. In the heterogeneous-consumer model, this tax is as follows:

$$\tau_{g_i} = n\mu\, X(s_i)Y(s_i). \qquad (28)$$

This is similar to homogeneous consumers. For homogeneous consumers a tax rate per gallon of gasoline that depends on the individual's own choice of car characteristic (s_i) can optimally influence the determinants of emissions. But for the heterogeneous consumers optimally choose different car sizes and mileage. Hence each pays a different rate per gallon.

Authorities might be able to impose a tax on each vehicle that depends on a direct measure of $X(s_i)$, and multiply by a measure of mileage, then the vehicle tax be as follows:

$$\tau_{v_i} = n\mu\, X(s_i)m_i \qquad (29)$$

which is similar to the consumer model. It indicates first-best, but the tax amount would differ among heterogeneous households.

Assume that the gas tax and size tax can be set at different rates for different consumers, but that they must be fixed for each consumer. From (23b) and (26b) we get;

$$n\mu\, m_i X_{s_i} = \tau_{s_i} - \frac{m_i \tau_{g_i} Y_{s_i}}{Y},$$

$$\tau_{s_i} = n\mu\, m_i X_{s_i} + n\mu\, m_i \frac{X(s_i)Y_{s_i}}{Y(s_i)}. \qquad (30)$$

Suppose that the first three policies (28) to (30) are calculated above are not feasible, and policy is limited to a single uniform rate of tax on gasoline and single uniform rate of tax on engine size or other vehicle characteristic. This policy achieves first-best in the homogeneous-consumer model, but not in the heterogeneous-consumer model. Moreover, a greater degree of heterogeneity means greater divergence from first-best. For these reasons, we now consider how to set the second-best uniform tax rates on gasoline and engine size.

4. SECOND-BEST TAXES ON GASOLINE AND SIZE

In this section, we consider linear second-best size tax rates. So that we must find the single uniform gas tax rate that maximize social welfare, taking as given households' demand for miles, size and other goods and services. We assume that producers' prices are fixed which is equivalent to maximizing this weighted sum of indirect utilities;

$$\iiint_{\alpha\,\beta\,y} \left[\frac{V(\tau_s, \tau_s, \tau_s; y, \alpha, \beta)}{\lambda^*} - \mu\, E \right] h(\alpha, \beta, y)\, d\alpha d\beta dy\,, \tag{31}$$

with respect to τ_s and τ_g. For normalization we set x equals to zero. First-order conditions of (31) are;

$$\iiint_{\alpha\,\beta\,y} \left[\frac{1}{\lambda^*} \frac{\partial V}{\partial t_s} - \mu \iiint_{\alpha\,\beta\,y} \left(A(t_s) \right) h(\alpha, \beta, y)\, d\alpha d\beta dy \right] h(\alpha, \beta, y)\, d\alpha d\beta dy = 0\,, \text{ and} \tag{32a}$$

$$\iiint_{\alpha\,\beta\,y} \left[\frac{1}{\lambda^*} \frac{\partial V}{\partial t_g} - \mu \iiint_{\alpha\,\beta\,y} \left(A(t_g) \right) h(\alpha, \beta, y)\, d\alpha d\beta dy \right] h(\alpha, \beta, y)\, d\alpha d\beta dy = 0\,, \tag{32b}$$

where

$$A(t_i) = g\, Y(s)\, X_s \frac{\partial s}{\partial t_i} + g\, X(s)\, Y_s \frac{\partial s}{\partial t_i} + X(s)\, Y(s) \frac{\partial g}{\partial t_i} \tag{33}$$

for $i = s, g$.

Using Roy's identity $\frac{\partial V}{\partial t_s} = -\lambda s$ (32a) becomes;

$$\iiint_{\alpha\,\beta\,y} \left[\frac{-\lambda s}{\lambda^*} - \mu \iiint_{\alpha\,\beta\,y} \left(A(t_s) \right) h(\alpha, \beta, y)\, d\alpha d\beta dy \right] h(\alpha, \beta, y)\, d\alpha d\beta dy = 0\,, \tag{34a}$$

where the first term in the integral $\left(\frac{-\lambda s}{\lambda^*} \right)$ represents the change in welfare from a change in the size tax, holding aggregate emissions constant and the second term is the change in utility due to the effect that a size tax has on aggregate emissions.

Again using Roy's identity $\frac{\partial V}{\partial t_g} = -\lambda g$ (32b) becomes;

$$\iiint_{\alpha\,\beta\,y}\left[\frac{-\lambda g}{\lambda^*}-\mu\iiint_{\alpha\,\beta\,y}\left(A(t_g)\right)h(\alpha,\beta,y)\,d\alpha d\beta dy\right]h(\alpha,\beta,y)\,d\alpha d\beta dy=0 \qquad \text{(34b)}$$

where the first term is the change in welfare from a change in the gas tax, holding aggregate emissions constant and the second term the change in welfare from the effect that a gas tax has on aggregate emissions.

Hence the tax rate on size and gasoline must be set such a way that the aggregate marginal gain in private welfare equals the aggregate marginal loss from the effect on emissions. Here *X* (*s*) and *Y* (*s*) are the major determinants of the second-best tax rates. To calculate second-best taxes we take average of all different gas tax rates in (28) as follows:

$$\bar{t}_g=\frac{\iiint_{\alpha\,\beta\,y} n\mu X(s_i)Y(s_i)h(\alpha,\beta,y)\,d\alpha d\beta dy}{\iiint_{\alpha\,\beta\,y} h(\alpha,\beta,y)\,d\alpha d\beta dy}$$

$$=\iiint_{\alpha\,\beta\,y}\mu X(s_i)Y(s_i)h(\alpha,\beta,y)\,d\alpha d\beta dy\,. \qquad \text{(35)}$$

Again if we take gas tax rate for the person with average choices then (28) becomes;

$$\tau_g(\bar{s})=n\mu\,X(\bar{s})Y(\bar{s})\,. \qquad \text{(36)}$$

Convexity of $X(s)$ would mean that increases in size increase emissions per mile at an increasing rate which would raise the weighted average using $X(s_i)$ in (35) relative to the tax rate using average size in (36). Similar result is obtained for $Y(s_i)$. Hence if either function or both are sufficiently convex, then the use of average size to calculate the gas tax rate would result in a lower tax rate than the second-best uniform tax rate.

From (30) the average of the size tax rate becomes;

$$\bar{\tau}_s=\iiint_{\alpha\,\beta\,y}\mu X_{s_i}m_i h(\alpha,\beta,y)\,d\alpha d\beta dy+\iiint_{\alpha\,\beta\,y}\frac{\mu X(s_i)Y_{s_i}}{Y(s_i)}h(\alpha,\beta,y)\,d\alpha d\beta dy\,. \qquad \text{(37)}$$

Again the size tax for the per person with average choices (30) becomes;

$$\tau_s(\bar{s},\bar{m})=n\mu\,\bar{m}\,X_{\bar{s}}+n\mu\,\bar{m}\,\frac{X(\bar{s})Y_{\bar{s}}}{Y(\bar{s})}\,. \qquad \text{(38)}$$

Since both *s* and *m* are in both equations, the difference between the average size tax rate in (37) and the size tax rate using average miles and size in (38) depends both on whether preferences are correlated and on whether $X(s)$ or $Y(s)$ is non-linear. For linearity X_s and Y_s be constants, then the first terms do not affect in either (37) or (38) but the second term of (38) must affects, since $Y_s<0$. Again size and miles are negatively correlated, who own larger cars drive proportionately fewer miles, and then the use of the average person's size tax tends to understate the second-best size tax. Finally we can say to maximize social welfare, we need a comprehensive empirical investigation of the technologies *X*(*s*) and *Y*(*s*), the distribution $h(\alpha, \beta, y)$, and behavioral parameters.

5. VEHICLE TAX RATES OF BANGLADESH

Government of Bangladesh and National Board of Revenue (NBR) can provide from time to time for tax collection on certain items or business under presumptive in lump sum in addition to normal tax on relevant transaction. Finance Act 2010 approved some such Statutory Regulatory Orders (SROs) which were issued by NBR relevant to presumptive income and are given in tables-1 and -2. The tax rates are quoted from Mahmud, Purohit and Bhattacharejee. (2010).

Size of the vehicle	Tax rate in Taka (Tk.), (Tk.70 = $1)
Motor car up to 1500 cc	8,000
Motor car up to 2000 cc	10,000
Motor car exceeding 2000 cc	16,000
Jeep up to 2800 cc	14,000
Jeep exceeding 2800 cc	18,000
Micro bus	8,000

Table-1: Tax on private cars/jeeps.

	Registration period up to 10 years, (Tax rate in Tk.)	After 10 years of Registration, (Tax rate in Tk.)
Air conditioned luxury bus	20,000	10,000
Air conditioned mini bus	10,000	6,000
Air conditioned taxi	7,000	3,000

Table-2: Tax on transports used for hire.

Lump sum taxes on vehicles will take advantages those who are polluting the air. The vehicle taxes should be fixed depending on car size; engine size and the type of gasoline are used by the motorists. We hope the government of Bangladesh will take immediate steps to impose taxes on cars and gasoline properly, so that the air pollution can be controlled in future. The old cars, which are polluting the air, must be removed from the road.

6. CONCLUDING REMARKS

By two models, we have tried to form a tax on emissions. In the first model we have considered homogeneous consumers where we have investigated the combination of a tax on gasoline that depends only on the cleanliness of the fuel, a flat rate of tax on engine size, and flat rate of subsidy to *PCE* and this combination of course first-best. In the second model heterogeneous consumers differ by income, tastes for miles, and tastes for engine size. We show that if the engine size and driving miles are negatively correlated and both $X(s)$ and $Y(s)$ are linear then we would achieve second-best. If the taste for miles is negatively correlated with the taste for engine size, then the second-best uniform size tax would exceed the rate using means (size and miles). Yet Bangladesh has not imposed emission taxes on vehicles properly; as a result the air pollutions in large cities are increasing dangerously. Dhaka, the capital of Bangladesh, is one of the dangerously polluted cities of the world. Hence the government of Bangladesh should take immediate steps to impose emission taxes on vehicles. We have tried to give a guideline to apply the taxes on vehicles, so that the paper will be helpful to the government and environment analysts of Bangladesh.

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