Approval Voting: A Multi-outcome Election

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ABSTRACT

This paper deals with approval voting and its critical strategy profile. Approval voting is a single winner voting system used for multi-candidate elections. In this method each voter may vote for as many of the candidates as she wishes that is the voter votes for all candidates of whom the voter approves. In Approval voting no ranking is involved, so all the votes have equal weight. Some scientific and engineering societies adopted approval voting but unfortunately yet has not adopted in any public election, despite efforts to institute it, so its success should be judge as mixed. The paper discusses aspects of approval voting and compares with some other voting rules. Approval voting may elect Condorcet winners or Condorcet losers. In addition the paper is enlightened to stability of approval voting outcomes.

JEL. Classification: D72; D73; K41; P16; P48

Key words: Condorcet winner, Strategy profile, AV outcomes.

1 INTRODUCTION

Approval voting (AV) is a single winner voting system used for multi-candidate elections. In this method each voter may vote for as many of the candidates as she wishes that is the voter votes for all candidates of whom the voter approves. This gives them the opportunity to be sovereign by expressing their approval for any set of candidates, which no other voting system permits. Let there is a set of *n* candidates $\{x, y, z, ...\}$. One may cast 0, 1, 2,…, or even *m* votes, where $m \le n$, by assigning a single vote to each candidate he approves and none to each candidate he disapproves, and the candidate with the most votes wins. By setting the upper bound equal to the total number of candidates in the race, approval votes could easily be cast and tallied. Again in AV no ranking is involved, so all the votes have equal weight (Islam, Mohajan and Moolio 2011).

In the state of New York in 1970, a seat in the U.S. Senate was at stake. The Democratic and Liberal-Republican nominees split the liberal vote, and the conservative candidate was elected with only 39% of the vote. The following year, in a mayoral election in Ithaca, New York, the Democratic nominee drew 29.1% of the vote and edged out the Republican nominee, who drew 28.9%; an independent Democratic candidate received 10.2%, and two independent Republican candidates received a combined total of 31.8%. Weber

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(1995), Brams and Fishburn (2005) were supporters of losing candidates in both elections and proposed an alternative to the plurality rule called approval voting.

In the beginning of 1987, several scientific and engineering societies adopted AV; among them the following five are eminent.

- Mathematical Association of America (MAA), with about 32,000 members;
- American Mathematical Society (AMS), with about 30,000 members;
- Institute for Operations Research and Management Sciences (INFORMS), with about 12,000 members;
- American Statistical Association (ASA), with about 15,000 members;
- Institute of Electrical and Electronics Engineers (IEEE), with about 377,000 members.

The selection of the Secretary-General of the UN has involved rounds of approval polling to help discover and build a consensus before a formal vote is held in the Security Council (Brams and Fishburn 1983). AV has also been used in internal elections by the political parties in some states, such as Pennsylvania, where a presidential straw poll using AV was conducted by the Democratic State Committee in 1983 (Nagel 1984). In the late 1980s, AV was used in some competitive elections in countries in Eastern Europe and the Soviet Union, where it was effectively disapproval voting, because voters were permitted to cross off names on ballots but not to vote for candidates (White 1989). Actually this is equivalent to AV as candidates not crossed off are, in effect, approved of.

The idea of AV did not spring forth full-blown only at 1980s; its provenance is much earlier. From 13^{th} - 18^{th} centuries, the Republic of Venice elected the Doge of Venice (Lines 1986) and on papal elections (Colomer and McLean 1998), where multi-stage process were used that featured random selection and voting which allowed approval of multiple candidates and required a super majority. In 19^{th} century approval voting was used in England (Cox 1987).

In a brief essay George Boehm (Boehm 1976) proposed the concept of negative voting (NV) in which voters can either vote for one candidate or against one candidate but can not do both. Robert Weber (Weber 1995) first worked on AV who was apparently coined the term approval voting. NV and AV are equivalent when there are three candidates. Under both systems a voter can vote for just one candidate. In NV, a voter who votes against one candidate has the same effect as a voter who votes for the other two candidates under AV. Also voting for all the three candidates has the same effect as abstaining under both systems. But when there are four or more than four candidates, then AV and NV give different results. In these cases AV gives voters more opportunity to express their views. Endriss (2009) discusses that there are two aspects to strategy-proof voting that may find attractive. First one is anyone can expect to get an outcome which is an honest reflection of the will of the people, because no voter has an incentive to report insincere preferences. Second one is voters do not have to strategize over how to vote, because truthful voting provides them with a dominant strategy. Recently New Hampshire state representatives sponsored a bill which replaces first-past-the-post voting with approval voting. Due to Irrespective of the success approval voting is attracting the attention of political decision-makers (Schlotter, Elkind and Faliszewski 2011). Ahn and Oliveros (2010) compared approval voting with other scoring rules. They showed that for any finite electorate, the best equilibrium under approval voting is more efficient than either plurality rule or other voting systems. Wojtas and Faliszewski (2011) proved that approval voting works for Condorcet voting. P´erez and Vorsatz (2009) discussed approval and disapproval voting in prisoner dilemma game. Brams and Kilgour (2010) proposed a new voting system, satisfaction approval voting for multi-winner elections, in which voters can approve of as many candidates or as many parties as they like. In this type election the winners are not those who receive the most votes, as under approval voting, but those who maximize the sum of the satisfaction scores of all voters, where a voter's satisfaction score is the fraction of his or her approved candidates who are elected.

The paper is organized as follows: In section-2, we discuss that AV is comparatively better than other voting systems. In section-3 we cite that various societies adopted AV successfully. Some illustrative examples and propositions are added in section-4, where we discuss strategy profiles and stability of the AV. In section-5 we introduce briefly the manipulation and non-manipulation of AV. The final section is of concluding remarks.

2 THE ADVANTAGES OF AV

At first blush it seems that AV is less appealing than other voting systems. But if we compare other voting systems with AV, we will find AV is comparatively better than other voting systems. For example, plurality voting (PV) sometimes may be insincere voting where a voter may switch to a second choice if her first choice appears not to be elected. Borda voting (BV) is also vulnerable to insincere, since if a voter moves a second choice down to last place to minimize the candidate's threat to her top choice. Single transferable vote (STV) may eliminate a centrist candidate early on and thereby elect one less acceptable to a majority. But AV encourages sincere voting and all votes are of equal weight.

Assume that there are k candidates, numbered 1, 2,…, k. A scoring rule for an election is a collection of vote-sets, where each vote-set consists of k numbers. A voter selects a vote-set, and assigns the numbers within that set to the candidates. The candidate assigned the greatest total across all voters wins the election. Plurality rule offers to each voter a single vote-set, {1, 0,…, 0}. Borda's rule also offers a single vote-set, {k-1, k-2,…, 0}. Approval voting offers a collection of vote-sets {1, 0,…, 0}, {1, 1, 0,…, 0}, {1 , 1, 1,…, 1, 0}.

The best evidence we have that AV would have elected a different winner is from the 1985 TIMS (The Institute of Management Sciences) experiment, in which ballot data for both the PV official elections and the AV non binding elections are compared (Fishburn and Little 1988; Brams and Fishburn 2005).
In one of the three 1985 elections, the official PV and actual AV ballot totals are shown in table-1 for candidates A, B and C. Also are shown the AV totals extrapolated from the 95% sample of members who returned their AV nonbinding ballots, which is a very high.

Table 1. PV and AV vote totals in 1985 TIMS election.

Candidates	Official PV	Actual AV	Extrapolated AV
A	166	417	486
B	827	1038	1224
C	835	908	1054
Total votes	1828	2363	2764
No. of voters	1828	1563	1828

This extrapolation is justified by the finding that there are no major differences in voting patterns on the official PV ballot between AV respondents and non-respondents. From table-1 we see that candidate C wins the official PV election only by 8 votes (0.4%), but B would have won under AV by a substantial 170 (6.1%). Hence AV gives better result than PV. Brams and Fishburn (2005) proposed the following six propositions in favor of AV which are briefly discussed here.

2.1 AV Gives Voter's Flexible Options

In AV, a voter can vote for a single candidate but if they have no strong preference for one candidate, they can vote for all candidates, they find acceptable. Again, if a voter's most preferred candidate has little chance of winning, then that voter can vote for both a first choice and a more viable candidate without worrying about wasting her vote on the less popular candidate.

2.2 AV Helps Elect the Stronger Candidate

Under PV, the candidate supported by the largest minority often wins but in AV, the candidate with the greatest overall support will generally win. In particular, Condorcet candidates, who can defeat every other candidate in separate pair wise contests, almost always win under AV, whereas under PV they often lose because they split the vote with one or more other centrist candidates.

2.3 AV will Reduce Negative Campaigning

In AV, in the election, a majority of voters' view is reflected, not just cater to minorities whose votes could give them a slight edge in a crowded plurality contest. AV is therefore, likely to cut down on negative campaigning, because candidates will have an incentive to broaden their appeals by reaching out for approval to voters who might have a different first choice.

2.4 AV will Increase Voter Turnout

In AV voters can express their preferences properly so that voters are more likely to vote in the first place. Voters who think they might be wasting their votes, or who cannot decide which of the several candidates best represent their views, will not have to despair about making a choice. As a result the AV encourages greater participation in elections.

2.5 AV will give Minority Candidates their Proper Due

Minority candidates' supporters will not be torn away simply because there is another candidate who, though less appealing to them, is generally considered a stronger contender. Because receive their true level of support under AV, even if they cannot win.

2.6 AV is Eminently Practicable

AV is simple for voters to understand and use but other voting systems are comparatively complicated. Although more votes must be tallied under AV than under PV, AV can readily be implemented on existing voting machines. So far we have praised in favor of AV. No voting system is stainless. AV is also not completely innocuous but it is acceptable than other voting systems. Approval voting usually elects Condorcet winners in practice (Brams and Fishburn 1978).

3 ADOPTION OF AV IN THE SOCIETIES

AV was first adopted in the four societies to elect the president of these societies (will be discussed later). When AV was first proposed as a reform in the four societies that adopted AV in the late 1980s, no candidates or factions, with one major exception, identified AV as a threat either to their candidacies or points of view. The following four societies first adopted AV (Brams and Fishburn 2005):

3.1 Mathematical Association of America (MAA)

In 1985, the president of the MAA, Lynn Arthur Steen, who was familiar with work on AV, asked the Board of Governors of the MAA to consider adoption of AV in its biennial elections for president-elect and other national offices. After heated but not acrimonious debate (Steen 1985), AV was approved by the Board in 1985, passed by the membership in 1986, and used for the first time in the 1987 MAA elections. Both Steen's knowledge and his position as president of the MAA made him a crucial player in the MAA's adoption of AV. So, also, was Steen's successor as president of the MAA, Leonard Gillman, who was a strong advocate of AV and played an active role in its eventual implementation in the l987 elections of the Association (Gillman 1987).

3.2 The Institute of Management Sciences (TIMS)

The use of AV by TIMS in 1988 was preceded by an experiment in which members were sent a nonbinding AV ballot, along with the regular PV ballot, in the 1985 elections. Although the AV ballot did not count, 85% of the members who voted in these elections returned the AV ballot. This permitted Fishburn and Little (1988) to compare the results of voting under the two different systems. On the basis of their empirical analysis, Fishburn and Little (1988) concluded that AV did a better job of electing Condorcet candidates than did PV. Not only was the experiment remarkably successful (Little and Fishburn 1986), but the results also convinced TIMS Council to adopt AV in 1987, leading to its later adoption by INFORMS when it formed in 1995.

3.3 American Statistical Association (ASA)

The former chair of the ASA's Committee on Elections, Richard Potthoff, had read about AV and brought it to the attention of his committee. This committee recommended its adoption first in internal ASA elections; the ASA Board of Directors approved this recommendation. After AV's successful use in 1986 in three elections for Council governors, the election of two editors to serve on the Board, and the election of a Board member to serve on the Executive Committee, the Committee on Elections recommended that AV be used in Association-wide elections, which was approved by the Board and ratified as an amendment in 1987.

3.4 Institute of Electrical and Electronics Engineers (IEEE)

The adoption of AV by the IEEE has a politically charged history (Brams and Nagel 1991). At the start of 1984, AV was considered, along with other voting systems, for possible use in multi-candidate elections. But not until the 1986 elections when, Irwin Feerst, a petition candidate, ran against two candidates for president-elect who were nominated by the Board of Directors, did the issue of election reform take center stage. The reason is that Feerst, with 35% of the vote, defeated one of the two Board-nominated candidates and came within 242 votes (of 52,405 casts) of defeating the other candidate. This result starkly illustrated to the Board how vulnerable their nominees, who together might win a substantial majority in an election, are to a minority candidate if these nominees should split the majority vote more or less evenly.

In 1987 the Board reverted to nominating only one candidate for president-elect, breaking a tradition of nominating two candidates that it had begun in 1982. Feerst was instrumental in bringing the question of how many nominees the board must nominate to a vote of the entire membership in the 1987 election, in which he did not run and there were no other petition candidates. By a 57% majority, members supported a constitutional amendment requiring that the Board nominate at least two candidates, but this fell short of the $\frac{2}{3}$'s majority needed to amend the IEEE's constitution.

In 1987, Brams was invited by the then president of the IEEE, Henry Bachman, to attend an Executive Council meeting to discuss AV. But he was failed and suggested that Jack Nagel of the University of Pennsylvania, who had done extensive research on AV, take his place. Nagel did successively by attending meeting of the full Board of Directors, which adopted AV in November 1987. With its adoption, the board voted to nominate at least two candidates for each office.

4 STRATEGIES UNDER AV

For *n* candidates $\{x, y, z, \ldots\}$ a voter's strict preference relation over candidates will be denoted by *P*, so *xPy* means that a voter strictly prefers *x* to *y*, which we will some times denote by the left-to-right ranking *xy*. We will consider throughout the paper that all voters have strict preferences, so they are not indifferent among two or more candidates. We assume that *P* is connected that is for any *x* and *y*, either *xPy* or *yPx* holds. Moreover, *P* is transitive, so if *xPy* and *yPz*, then *xPz*. The list of preferences of all voters is called a preference profile **P**. For the preference profile **P**, we consider the set of all candidates that can be chosen by AV when voters use sincere strategies. We call this the set AV outcomes at **P**.

An AV strategy *S* is a subset of candidates. Choosing a strategy under AV means that voting for all candidates in the subset and no candidates outside it. The list of strategies of all voters is called a strategy profile **S**. The number of votes that candidate *i* receives at **S** is the number of voters who include i in the strategy *S* that they select. We assume that voters use admissible and sincere strategies. An AV strategy *S* is admissible if it is not dominated in a game-theoretic sense. Admissible strategies under AV involve always voting for a most-preferred candidate and never voting for a least preferred candidate (Brams and Fishburn 1978, 1983). An AV strategy is sincere if, given the lowest-ranked candidate that a voter approves of, she also approves of all candidates ranked higher. Thus, if *S* is sincere, there are no *holes* in a voter's approval set that is everyone ranked above the lowest-ranked candidate that a voter approves of is also approved and everyone ranked below is not approved. A strategy profile **S** is said to be sincere if and only if the strategy *S* that every voter chooses is sincere based on each voter's preference *P*.

In other words, an approval voting is sincere if the outcome is the same as the true preference of the voters (Brams and Fishburn 1978). Let us consider there are four candidates *x*, *y*, *z* and *u*, and a voter's preference profile being as follows:
xPyPzPu.

We can write his possible sincere approval votes as follows:

- vote for *x*, *y*, *z* and *u*,
- vote for *x*, *y* and *z*,
- vote for *x* and *y*,
- vote for *x*, and
- vote for no candidates.

If a voter be indifferent between y and *z* but still *x* is his most preferred candidate, then also above conditions are sincere (in this paper indifferent is not considered). Now we can also include a new combination as a sincere vote which is: vote for *x* and *z*.

Let us consider an example where there are 11 voters, and a set of four candidates {*x*, *y*, *z*, *u* }.

Example 1. Voters are grouped into three different types as follows:

Type1: $xPyPzPu$ by 5 voters,
Type2: $yPzPxPu$ by 3 voters,
Type3: $uPyPzPx$ by 3 voters.

Voters of type-1 have three sincere strategies as follows:

- vote for *x*,
- vote for *x* and *y*,
- vote for *x*, *y* and *z*.

The sincere strategies of other two types of voters are analogous. For simplicity, among the voters of each type choose the same strategy *S*. A typical strategy profile of the 11 voters is,

$$\mathbf{S} = (x, x, x, x, x, yz, yz, yz, uyz, uyz, uyz).$$

The number of votes of each candidate at **S** is 6 votes for *y* and *z*, 5 votes for *x* and 3 votes for *u*. We see that AV selects candidates $\{y, z\}$ as tied winners at **S**.

We can define an *AV* critical strategy profile for candidate *i* at preference profile **P** as follows: Every voter who ranks *i* as her worst candidate votes only for the candidate that she ranks top. The remaining voters vote for *i* and all candidates they prefer to *i*.

Let $C_i(\mathbf{P})$ be the AV critical strategy profile of candidate *i* at **P**. In example-1, the critical strategy profile for candidate *x* is $C_x(\mathbf{P}) = (x, x, x, x, x, yzx, yzx, yzx, u, u, u)$, collecting 8 votes for *x* compared to 3 votes each for *y*, *z* and *u*. The critical strategy profile for candidate *y* is $C_y(\mathbf{P}) = (xy, xy, xy, xy, xy, y, y, y, uy, uy, uy)$, collecting 11 votes for *y* compared to 5 votes for *x* and 3 votes for *u*. The critical strategy profile for candidate *z* is $C_z(\mathbf{P}) = (xyz, xyz, xyz, xyz, xyz, yz, yz, yz, uyz, uyz, uyz)$, collecting 11 votes for each of *y* and *z* compared to 5 votes each for *x* and 3 votes for *u*. The critical strategy profile for candidate *u* is $C_u(\mathbf{P}) = (x, x, x, x, x, y, y, y, u, u, u)$, collecting 5 votes for *x* and each of 3 votes for y and *u*. Here we observe that *x* with 5 votes elects compared to *u* and *y* collecting 3 votes each. Hence in example-1, the set of AV outcome that are possible is $\{x, y, \{y, z\}\}$.

Proposition 1. *At critical strategy profile of any candidate i,* $C_i(\mathbf{P})$*, candidate i gets maximum votes compare with any other strategy profile.*

Proof. From the definition of critical strategy profile **S** and example-1 we see that at $C_x(\mathbf{P})$, *x* gets more votes than other candidates, similar case for candidates *y* and *z*, on the other hand at $C_u(\mathbf{P})$, *u* loses but gets more votes than any other strategy profile. Clearly $C_i(\mathbf{P})$ is the best strategy profile for any candidate *i*. Q.E.D.

A candidate is a Pareto candidate if there is no other candidate that all voters rank higher. In example-1 we see that *x* and *y* are Pareto candidates and AV outcomes; *z* is not a Pareto candidate but is a component of AV outcome as *z* ties with *y* in $C_z(\mathbf{P})$; *u* is a Pareto candidate but is not an AV outcome. The following proposition will give clear idea about Pareto candidate and AV outcomes.

Proposition 2. *At every preference profile* **P** *a Pareto candidate i may be an AV outcome or a tied AV outcome or not an AV outcome at her critical strategy profile* $C_i(\mathbf{P})$.

Proof. At preference profile **P** let *i* be a Pareto candidate. If every voter votes only for her top choice, then if *i* is chosen at top position by most of the voters, at critical strategy profile $C_i(\mathbf{P})$, *i* will be an AV outcome. Consider at the critical strategy profile $C_i(\mathbf{P})$, *i* is a Pareto candidate and also consider another candidate *j* who is not a Pareto candidate then both cast equal but more votes than other candidates. At this situation *i* tied with *j* and both of them are AV outcomes. If more voters place *i* at the last position and few voters place *i* at first position then *i* is a Pareto candidate and at critical strategy profile $C_i(\mathbf{P})$, *i* will not be an AV outcome. Q.E.D.

Proposition 3. *Condorcet winners must be an AV outcome but Condorcet loser sometimes be an AV outcome.*

Proof. Assume *i* is a Condorcet winner. In this case more voters place *i* as their best choice and less voters place her as their worst choice. Again those voters who place *i* among their best and worst choices then more voters prefer *i* to any other candidate *j*. Hence total votes of *i* will be more than *j*. As a result *i* will be always an AV outcome. Again now assume *i* is a Condorcet loser. In this case two situations arise. First, at $C_i(\mathbf{P})$ sometimes candidate *i* collects more votes then any other candidate *j*. Since at $C_i(\mathbf{P})$, those voters who place *i* at the last place their first choices will be render in different candidates *j*, *k*, etc. (say) and those voters who will place *i* as their best choice will collect votes only for *i*. Finally *i* will collect more votes than any other candidate *j*. Consequently *i* will be an AV outcome. Second, at $C_i(\mathbf{P})$, if *i* collects less votes then any other candidate *j*, obviously *i* will not be an AV outcome. Q.E.D.

In voting system if voters vote for a predetermined number of candidates then it is called fixed rule. No fixed rule may elect a unique Condorcet winner but flexibility of AV needed to elect a unique Condorcet winner (Brams and Sanver 2005). The following two examples illustrate the above concept clearly. The first example is due to Moulin (1988).

Example 2. Let us consider 17- voters and 3- candidates election being as follows:

Type 1: $xPyPz$ by 6 voters,
Type 2: $yPxPz$ by 4 voters,
Type 3: $yPzPx$ by 4 voters,
Type 4: $zPxPy$ by 3 voters.

Vote for 1 candidate elects candidate *y* and vote for two candidates also elect candidate *y* defeating candidate *x*. Candidate *x* is a Condorcet winner but by fixed rule *x* is loser. Therefore no fixed rule elects the unique Condorcet winner.

Example 3. Let us consider 6-voters and 3-candidates election is as follows:

Type 1: $xPyPz$ by 3 voters,
Type 2: $yPxPz$ by 2 voters,

Type 3: $zPxPy$ by 1 voter.

Clearly, x is a Condorcet winner. Here different number of voters vote for different number of candidates elect x. Thus, flexibility of AV may be needed to elect a unique Condorcet winner.

Proposition 4. *No fixed rule may elect a unique Condorcet winner but flexibility of AV may be needed to elect a unique Condorcet winner.*

Proof. Assume i is a unique Condorcet winner in a preference profile **P**. According to fixed rule voters may vote for one candidate or they may vote for two candidates and so on. Consider another candidate j which is not a unique Condorcet winner. If more voters' first choice is j, then j will be an AV outcome but not i, the unique Condorcet winner. Similar result arises if voters vote for two candidates and so on. Hence no fixed rule may elect a unique Condorcet winner. If the voters do not follow fixed rule and different voters vote for different number of candidates may elect i that is the flexibility of AV may be needed to elect a unique Condorcet winner. Q.E.D.

Proposition 5. *At a preference profile **P**, a candidate elected by STV is an AV outcome but the converse is not true.*

Proof. Consider two candidates i and j. Let candidate i always collects more votes than candidate j. According to STV j will be eliminated and i is an STV winner. Obviously candidate i is an AV outcome. Therefore an STV outcome is also an AV outcome. Again suppose candidate i is an AV outcome. In critical strategy profile $C_i(\mathbf{P})$, j may be AV outcome but in STV it is impossible. Q.E.D.

There are two kinds of stability as follows: Given a preference profile **P**, a non-tied AV outcome is stable if there exists a strategy profile **S** such that no voters of a single type have an incentive to switch their strategy to another sincere strategy in order to induce a preferred outcome. Given a preference profile **P**, an outcome is strongly stable if there exists a strategy profile **S** such that no types of voters, coordinating their actions, can form a coalition χ, all of whose members would have an incentive to switch their AV strategies to other sincere strategies in order to induce a preferred outcome.

We will cite the following example (Brams and Sanver 2005) to clarify these two definitions as follows:

Example 4. Let us consider 9-voters and 3-candidates election is as follows:

Type 1: $xPzPy$ by 4 voters,
Type 2: $yPzPx$ by 2 voters,
Type 3: $zPyPx$ by 1 voter.

In example-4 neither candidate x nor candidate y is stable AV outcome. The critical strategy profile $C_x(\mathbf{P})$ = (x, x, x, x, x, y, y, z, z, z, z) renders candidate x an AV outcome if 2 voters of type 2 switch to strategy yz, candidate z, whom the type-2 voters prefer to candidate x, wins. At $C_y(\mathbf{P})$ = (x, x, x, x, y, y, zy, zy, zy) that renders candidate y an AV outcome, the type-1 voters have an incentive to switch to strategy xy to induce the selection of candidate z, when they prefer to candidate y. In $C_z(\mathbf{P})$ = (xz, xz, xz, xz, yz, yz, z, z, z); AV outcome z is obviously stable, since candidate z is the unanimous choice of all voters at the critical strategy profile of z. Here it is not possible to switch on the part of the 4 voters of type-1 to x, or 2 voters of type-2 to *y*, or 3 voters of type-3 to *zy*. Here we assume that the coordinating players in χ are allowed to communicate to try to find a set of strategies to induce a preferred outcome for all of them.

In example-1 at $C_x(\mathbf{P})$, 3 voters of type-2 cannot upset the outcome by switching from *yzx* to *yz* or *y*, nor can the 3 voters of type-2 upset the outcome by switching from *u* to *uy* or *uyz*. But if there two types of voters cooperate and form a coalition χ, with 3 voters of type-2 choosing strategy *y* and the 3 voters of type-2 choosing strategy *uy*, they can induce the selection of Condorcet winner *y*, whom both types prefer to candidate *x*. Hence at $C_x(\mathbf{P})$, AV outcome *x* is stable but not strategy stable at $C_y(\mathbf{P})$. If an AV outcome is neither strongly stable nor stable, it is called unstable.

Proposition 6. At critical strategy profile **P** of candidate *i* is $C_i(\mathbf{P})$ then a non-tied AV outcome *i* is stable.

Proof. If a candidate *i* is tied with any other candidate *j* at $C_i(\mathbf{P})$ then *i* is not stable, since *i*'s votes renders and *j* switch to win in the election. If at $C_i(\mathbf{P})$, *i* is non-tied with any other candidate *j* then according to the definition of $C_i(\mathbf{P})$, *i* is non-renders by any other candidate *j*. Q.E.D.

Proposition 7. At critical strategy profile **P** of candidate *i* is $C_i(\mathbf{P})$ then a non-tied AV outcome *i* is strongly stable.

Proof. The proof is similar to proposition-6.

The following discussion results that approval voting can be manipulated and can be non-manipulated.

5. MANIPULATION AND NON-MANIPULATION OF THE AV

To discuss the manipulation of AV first consider an example as follows:

Example 5. There are three electors, where elector 1 is a leader; if there are ties for first place, he breaks them and there are three alternatives *x*, *y*, *z*. The preference profile being as follows:

1 (leader)	2	3
x	*y*	*z*
y	*z*	*x*
z	*x*	*y*.

Each elector may cast 0, 1, 2 or 3 votes. It is foolish to cast votes equally for all or for none. Elector 1(the leader) can vote as follows:

- vote for *x*, *y* and *z*,
- vote for *x* and *y*, and
- vote for *y*,

and so on. Here first and second are sincere but third is insincere. Similarly voter 2's sincere strategies are as follows:

- vote for *y*,
- vote for *y* and *z*.

Voter 3's sincere strategies will be analogous.

Here every voter casts 1 vote for his favorite. So that the results are: 1 for *x*, 1 for *y*, 1 for *z*. Person 1 (the leader) breaks the tie in favor of *x*, so that *x* wins. Voter 2 anticipate that by the leader's favorite *x* will win who is his less favorite, so he could vote falsely as:1 vote for his second favorite *z* instead, but none for *y* or *x*, then the result would be 2 votes for *z* but 1 vote for *x* and none for *y* and finally *z* would be win. Therefore, the approval voting is manipulable. On the other hand person 1 votes 1 for each in a sincere way but both 2 and 3 also vote sincerely in the following ways:

i) person 2 votes 1 for *y*, 1 for *z* but none for *x*
ii) person 3 votes 1 for *z* , 1 for *x* but none for *z*.

In this case the result would be 2 votes for *x*, 2 votes for *y* but 3 votes for *z* and *z* would be winning. For this case the voting is manipulated in a sincere way also.

NON-MANIPULATION OF THE APPROVAL VOTING

Now consider that all the voters except any voter *i* has declared their true strategies, so that voter *i* can not be insincere and the result will be their best output. In this case approval voting is non-manipulated.

6 CONCLUDING REMARKS

In this paper we have discussed aspects of AV. We have introduced some propositions and have proved them. We also include some examples to clarify the concept of AV. The paper deals with sincere AV but we have use insincerity to show the manipulation of AV. Some scientific and engineering societies adopted approval voting but unfortunately yet has not adopted in any public election, despite efforts to institute it, so its success should be judge as mixed. We hope that in future the politicians will find interest to use it in national elections. We have introduced some propositions to incur the quality of the paper and to make the paper easier for the readers. If possible computer counting of the ballots can be used in AV to count votes more accurately and precisely.

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