Relation between lease finance and purchase

Mohajan, Haradhan

International Journal of Economics and Research

18 January 2012

Online at https://mpra.ub.uni-muenchen.de/50681/
MPRA Paper No. 50681, posted 16 Oct 2013 07:12 UTC
RELATION BETWEEN LEASE FINANCE AND PURCHASE

Haradhan Kumar Mohajan
Assistant Professor, Premier University, Chittagong, Bangladesh
E-mail: haradhan_km@yahoo.com

Abstract
This paper discusses the long-term financial lease contracts with lease evaluation. Here a comparatively simple and straightforward solution of neutralizing the risk of lease financing is explained. The lease is a contract between the owner and the user of assets for a certain period during which the second party uses an asset in exchange of making periodic rental payments to the first party without purchasing it. In the long-term lease contract the lessee (the user of assets) is generally given an option to buy or renew the lease. An attempt has been taken here to investigate the buy or lease decision of an asset of a competitive firm.

Keywords: Lessor and lessee, Purchase and lease, Cash flow.

INTRODUCTION
At the first sight it would be appear that the choice between the two alternatives lease and purchase is relatively simple. If the net present value lease, \( NPV(L) \), is greater than the net present value of the purchase option, \( NPV(P) \), the machine should be leased and vice versa. But such a decision without comparing present values may be wrong. If we compare the \( NPV \) of the purchase and lease then we find different cash flows. The lease is like borrowing in that it commits the firm to a series of fixed rental payments. Hence even if the lease alternative has a greater \( NPV \), it may also expose the firm’s shareholders to greater risk. This difference risk can be determined by carefully specifying the cash flows of the two alternatives. We assume that the lease defined over the duration of the asset’s life and that there is no residual value. The lease or purchase decision is a type of capital budgeting problem which requires the application of present value techniques. The other investigations needed are tax implications and the relevant after tax cash flows of the two alternatives. The choice of discount rates needs a decomposition of the cash flows into their risky and riskless components. Yan (2006) examines the relation between lease and debt financing and shows that there exists a large finance literature with mixed results on the empirical relationship between lease and debt. He also examines the variation in the substitutability between leases and debt across different firms.

LEASE FINANCING
Leasing is a contract between the owner of the asset which is called lessor and the business that wants to lease the equipment is called lessee/client. Hence lease is a contract between the owner and the user of assets for a certain time period during which the second party uses an asset in exchange of making periodic rental payments to the first party without purchasing it. In up-fronted lease more rental are charged in the initial period and less in the later years of the contract and the opposite happens in back-ended leases. Under lease financing, the lessee regularly pays the fixed lease rent over a period of time at the beginning or at the end of 1 month, 3 months, 6 months or 1 year and at the end of the lease contract the asset reverts to the real owner. Leasing is a standard way of financing through which the payment being spread over the period of the lease and the installments paid being deductible according to the type of leasing contract. The client agrees to make payments to the leasing company over the life of the agreement and can purchase the equipment, return it to the lessor, or negotiate a lease extension, when the original agreement expires. Most lease contracts run from 3 to 7 years; however,
irrigation and other agricultural equipment leases often run a bit longer, to about 10 years. In the long-term lease contract the lessee is generally given an option to buy or renew the lease. Lease financing is an asset management-based business that requires specialized expertise. It is more profitable in some special sectors where other financing will be less profitable than lease financing. Recently lease financing is the most emphasized topic to any challenging institution or organization to develop their financial resources as well as profit maximization or maximization of owner’s equity. All types of assets such as land, buildings, plant and machinery equipment and transports are related to lease. The three major types of leases are the operating lease, financial/capital lease and the direct financing lease. By lease financing an organization can reach its specific destination. If an organization has effective lease financing efficiency it can survive and develop quickly than others. Leasing contracts usually include the following terms:

- Although the lessor is the legal owner of a leased asset, the lessee bears the risk and enjoys the returns. The lessee benefits if the assets operates ownership and use as two economic activities, and facilitates asset use without ownership (Miller and Upton 1976). Leasing contracts usually include the following terms:
- The leasing company remains the equipment owner. The client acquires the right of temporary possession and use.
- The client must pay one or more lease payments when the lease is signed and the client obtains possession of the equipment; subsequent payments are usually made at periodic intervals.
- The leasing company may or may not recognize a salvage value in calculating leasing payments.
- Often the lease cannot be canceled, and if canceled, a substantial penalty may be imposed.
- Typically, the client is responsible for property taxes, insurance, and repairs not covered by the warranty.
- When the lease period ends, the client has the option to purchase the equipment, renew the lease, or return the equipment to the lessor.

Sometimes the lease contract is divided into primary and secondary lease for the purpose of lease for the purpose of lease rentals. Primary lease provides for the recovery of the cost of the asset and profit through lease rentals during a period of about four to five years. A perpetual secondary lease may follow it on nominal rentals. Leasing can cover anything from the hiring of a power tool for a day or the hiring a car for months to the hiring of a fleet of aircraft for decades or the hire of a building for centuries. Where assets are hired for longer periods they are usually referred to as being leased rather than hired but there is no clear distinction between the two terms.

A lessee can be individual or a firm interested in the use of an asset without owning. Lessors may be equipment manufacturer or leasing who bring together the manufacturer and users. In the USA equipment manufacturers and the largest group of lessors followed by bank (Mohajan 2012).
CLASSIFICATION OF LEASES

A lease can be modified to the lessee’s specific needs and structured in a number of ways. From the perspective of the lessor, there are three basic types of leases as follows:

Capital or Financial Lease

Long-term, non-cancelable lease contracts are known as financial leases. It combines some of the benefits of leasing with those of ownership. Hence a finance lease is structured as a non-cancelable agreement, where the leasing company buys the equipment which the client has chosen and the client uses the equipment for a significant period of its useful life. Financial leases also are called full-payout leases because payments during the lease term amortize the lessor’s total purchase costs with a residual value of up to 5% of the original gaining price. Sometimes the present value of the minimum lease payment equals or exceeds 90% of the fair value of the leased property. Most financial leases are direct leases. The lessor buys the asset identified by the lessee from the manufacturer and signs a contract to lease it out to the lessee. Office building, multipurpose industrial building and even complete shopping centers are frequently financed with this method. Most lease backs are on a net-net basis, which means that the lessee pay all maintenance expense, property taxes, insurance and lease payment (Hamilton 1992, John 1964, Khanam 1995, Islam 1999, Bass and Henderson 2000). A financial lease agreement may provide for renewal of contract or purchase the asset by the lessee after the contract expires.

Operating Lease

An operational lease involves the lessee only renting an asset over a time period which is substantially less than the asset’s economic life. In such cases operating lease may run for 3 to 5 years. The lease is usually responsible for maintenance and insurance. It is cancelable by the lessee prior to its expiration, the lessor provides service, maintenance and insurance, and the sum of all lease payments by the lessee does not necessary fully provide for the recovery of the asset cost. The leasing agency retains ownership of the equipment during the lease and recovers its capital costs through multiple rentals and the asset’s final sale (Jones 1992, Islam 1999, Bass and Henderson 2000). In economic matter a finance lease is a loan of money with the asset as security. The economic ownership of the asset, the risks and rewards of ownership lies with the lessee. Hence the finance lessee buys the asset with a loan from the finance lessor. A finance lease may be viewed as an arrangement under which the lessor provides the money to buy an asset which is used by the lessee in return for an interest charge and the lessor has security because he owns the asset.

Sale Leaseback

The sale leaseback is a transaction in which the owner of the property sells the property to another which is simultaneously leases it back from the new owner. The use of the property is generally continued without disruption. The advantages of a sale lease back from the seller’s perspective as follows: If the purchase of asset has already been financed then a sale leaseback can allow the seller to refinance at lower rates, if rates of asset have dropped and provide another source of working capital, if liquidity is tight.
When a company purchases equipment then it may not realize that it was going to be a minimum tax and that ownership might increase its minimum tax liability. By selling the property, the seller lessee may deduct the entire lease payment that is may not be a minimum tax (Islam 1999). In 1989, Shipping Credit and Investment Corporation of India purchased Great Eastern Shipping Company’s bulk carrier Jag Lata, for Rs.12.5 crore (1 crore = 10,000,000) and then leased it back to Great Eastern Shipping Company on a 5-year lease, the rental being Rs.28.13 lakh (1 lakh = 100,000) per month and the ship’s written down book value was Rs.2.5 crore.

**Hire-purchase Lease**

A hire-purchase lease is an alternative to a lending transaction for the asset purchase and usually employed for retail or individual financing of smaller ticket items, such as motorcycles, sewing machines, refrigerators etc. The client assumes a higher down payment up to 30% of the purchase price, and with each lease payment retains a higher percentage of equipment ownership to build equity. In this type of purchase the asset price and risk involved in the financial transaction are spread over the lease term (Bass and Henderson 2000).

**LEASE CASH FLOW**

Let a firm leases a machine for \( n \) years and pays a rent of \( l_t \) in year \( t \), and the firm earns revenue from the sale of the machine is \( R \) in year \( t \). The production cost from labors, raw materials, electricity, and transports etc., associated with this output is \( C \) at time \( t \). Let \( T \) be the appropriate tax rate of the firm. Therefore, the net cash flow produced by the lease can be written as follows:

\[
(1 - T)(R - C - r) = (1 - T)(R - C) - (1 - T)l_t.
\]

Now if we assume that the riskness of the net receipt from this investment, \( (R - C) \), does not change from the firm’s standard risk. The appropriate after tax discount rate is equal to \( k \) which should be used to calculate the first term \( (1 - T)(R - C) \) of right side of \( (1) \).

The second term \( (1 - T)l_t \) of the right side of \( (1) \) is as like the payments on a bond, which is a fixed charge and should be discounted using the interest rate \( r \). We shall assume that the interest rate \( r \) is riskless and tantamount to assuming the absence of bankruptcy risk.

Now the net present value of the lease, \( NPV(L) \) being as follows (Levy and Sarnat 1979):

\[
NPV(L) = \sum_{t=1}^{n} \frac{(1 - T)(R - C)}{(1 + k)^t} \times \sum_{t=1}^{n} \frac{(1 - T)l_t}{(1 + k)^t}.
\]
PURCHASE CASH FLOW
Now let the firm decides to buy the machine instead of leasing. Suppose the purchase price be $P$ and depreciation be $d_t$ per annum (p. a.). Let $M$ be the additional maintenance, insurance, or other costs engendered by the decision to buy instead of leasing the machine. Hence the relevant cash flow of the purchase in year $t$ is given by:

$$(1 - T)(R - C - M - d_t) + d_t. \quad (3)$$

In (3) we subtract $d_t$ in first term from $R$ to calculate the corporate tax liability and then add as second term because depreciation is not a cash outflow. We assume that $(M + d_t)$ tends to zero, so that the net cash flow of the purchase option in year $t$ reduces to

$$(1 - T)(R - C) + Td_t. \quad (4)$$

The net present value of the purchase option $NPV(P)$, is written as;

$$NPV(P) = \sum_{r=1}^{n} \frac{(1 - T)(R - C)}{(1 + k)^t} + \sum_{r=1}^{n} \frac{Td_t}{(1 + r)^t} + \frac{R'}{(1 + r)^n} - I, \quad (5)$$

where $I$ denotes the initial investment outlay, $R'$ is the estimated after tax salvage value of the equipment, and $r$ denotes the appropriate discount factor for the tax shield and the salvage value. The tax shelter $Td_t$ is for all practical purposes almost completely certain, since it can be obtained against the income of other projects should the project in question fail to generate any taxable income. Even in the case of firm suffers an overall loss, the tax contract continues on the taxable income. For the specificity of the greater equipment, the discount rate will be higher. Abstracting from the tax shield riskness of the tax shield and the existence of any terminal salvage value we can write (Levy and Sarnat 1979),

$$NPV(P) = \sum_{r=1}^{n} \frac{(1 - T)(R - C)}{(1 + k)^t} + \sum_{r=1}^{n} \frac{Td_t}{(1 + r)^t} - I. \quad (6)$$

Here the tax shield from the depreciation is considered to be certain which is discounted using the interest rate $r$ rather than discount rate.
COMPARISON BETWEEN PURCHASE AND LEASE OPTIONS

Subtracting (2) from (6) we get:

\[
NPV(P) - NPV(L) = \sum_{t=1}^{n} \left(1 - \frac{T}{1 + r}\right)l_t + \sum_{t=1}^{n} \frac{Td_t}{(1 + r)^t} - I. 
\] (7)

From (7) we see that the positive difference indicates that the purchase option is preferable and a negative result indicates that lease option is preferable.

The difference of cash flow can be derived by subtracting the annual lease cash flow from the annual purchase cash flow as follows:

\[
CF(P) - CF(L) = (1 - T)l_t + Td_t, 
\] (8)

where \(CF(P)\) and \(CP(L)\) denote the cash flow of the purchase and lease options respectively. The purchase option involves an initial investment outlay of \(I\) but adds with certainty the annual tax shield from depreciation, \(Td\), and the lease option indicates the firm to a series of annual fixed after-tax rentals \((1 - T)l_t\). In critical maximum lease payment \(l_t^*\), the firm is indifferent between \(NPV(P)\) and \(NPV(L)\), hence (7) becomes;

\[
I = \sum_{t=1}^{n} \frac{(1 - T)l_t^* + Td_t}{(1 + r)^t}. 
\] (9)

In practical riskness of the \(NPV(P)\) and \(NPV(L)\) can not be identical, since the lease option commutes the firm to a stream of rental payments fixed in advance. Equations (6) and (8) indicate that obligating the firm under the lease, and giving up the depreciation shelter, engenders no sacrifice in the firms overall borrowing power. To neutralize the difference of risks the purchase option must be made on the explicit assumption that the purchase is partially financed by a loan which commits the firm to a stream of fixed payments. When the firm borrows incurs an interest payment, say \(X\) in year \(t\), then the resulting tax shield \(TX\) must be taken into account to the difference of riskness of the lease and purchase alternatives is to be neutralized. To neutralize the difference of risks between the purchase lease options we get from (7) the sum of pre-tax interest and principal can be expressed as follows:

\[
Td_t + (1 - T)l_t + TX 
\] (10)

where \(TX\) = the interest tax shield in year \(t\). If the interest tax shield in year \(t\) be negligible then from (9) we get the after-tax payment as follows:

\[
Td_t + (1 - T)l_t, 
\] (11)
Let $B_t$ be the balance of the loan outstanding at the end of period $t$ and $B_{t-1}$ be balance at the end of the period ($t-1$) then the principal repaid in period $t$ be as follows:

$$Td_t + (1-T)l_t = (B_{t-1} - B_t) + (1-T)r B_{t-1}. \quad (12)$$

Again from the definition of interest payment we get;

$$X = rB_{t-1}. \quad (13)$$

Hence (12) can be written as;

$$Td_t + (1-T)l_t = (B_{t-1} - B_t) + (1-T)X, \quad (14)$$

$$Td_t + (1-T)l_t + TX = (B_{t-1} - B_t) + X. \quad (15)$$

If the corporate tax rate $T$ is uncertain or systematically varies with the economy or if income can not always be found to exploit the tax shelter risk is not neutralized them the different amounts of shelter flow is,

$$X = Td_t + (1-T)l_t + TX, \quad (16)$$

Since $B_t$ be the balance of a loan at the end of the period $t$, so that for any $n$ period $B_n = 0$, and $B_0$ is the total amount borrowed. To equate the riskness of the lease and buy options the firm should borrow a sum of money which must be a total payments stream. The repayment of principle be with after-tax interest (Gordon 1976). From (12) we see that the debt repayment $(B_{t-1} - B_t)$ plus after-tax interest payment $(1-T)r B_{t-1}$ should equal to the after-tax lease payment plus depreciation shelter. From (12) we get;

$$B_{t-1} + r B_{t-1} = Td_t + (1-T)l_t + rTB_{t-1} + B_t, \quad (17)$$

For $t = n$ we have $B_n = 0$ then (17) becomes;

$$B_{n-1} = \frac{Td_n + (1-T)l_n + rTB_{n-1}}{1 + r}. \quad (18)$$

Again for $t = n-1$ from (17) we get;
\[ B_{n-2} = \frac{Td_{n-1} + (1-T)l_{n-1} + rTB_{n-2} + B_{n-1}}{1+r} \]

Using (18) in (19) we can write;

\[ B_{n-2} = \frac{Td_{n-1} + (1-T)l_{n-1} + rTB_{n-2}}{1+r} + \frac{Td_{n} + (1-T)l_{n} + rTB_{n-1}}{(1+r)^2} \]

\[ B_{n-2} = \sum_{t=n-1}^{n} \frac{Td_{t} + (1-T)l_{t} + rTB_{t-1}}{(1+r)^{t-(n-2)}}. \]  \hspace{1cm} (20)

Continuing such a way we obtain for \( t = 1 \) to \( n \);

\[ B_{0} = \sum_{t=1}^{n} \frac{Td_{t} + (1-T)l_{t} + rTB_{t-1}}{(1+r)^{t}}. \]  \hspace{1cm} (21)

For the obtaining the critical lease payment \( l^* \) which leases the firm indifferent between the buy and lease options, let us substitute \( B_{0} = I \) we get,

\[ I = \sum_{t=1}^{n} \frac{Td_{t} + (1-T)l^{*} + rTB_{t-1}}{(1+r)^{t}}. \]  \hspace{1cm} (22)

Using (13) in (22) we get;

\[ I = \sum_{t=1}^{n} \frac{Td_{t} + (1-T)l^{*} + XT}{(1+r)^{t}}. \]  \hspace{1cm} (23)

Again from (12) we get;

\[ (1+r)B_{t-1} - TrB_{t-1} = Td_{t} + (1-T)l_{t} + B_{t}, \]

\[ B_{t-1} = \frac{Td_{t} + (1-T)l_{t} + B_{t} + TrB_{t-1}}{1 + (1-T)r}. \]  \hspace{1cm} (24)

Again for \( t = n \) we get \( B_{n} = 0 \) then (24) becomes,

\[ B_{n-1} = \frac{Td_{n} + (1-T)l_{n} + TrB_{n-1}}{1 + (1-T)r}. \]  \hspace{1cm} (25)

For \( t = n-1 \) (24) becomes;
\[ B_{n-2} = \frac{Td_{n-1} + (1-T)l_{n-1} + TrB_{n-2} + B_{n-1}}{1 + (1-T)r} \]

\[ = \frac{Td_{n-1} + (1-T)l_{n-1} + TrB_{n-2}}{1 + (1-T)r} + \frac{B_{n-1}}{1 + (1-T)r}. \]  \hspace{1cm} (26)

Using (25) in (26) we get;

\[ B_{n-2} = \frac{Td_{n-1} + (1-T)l_{n-1} + TrB_{n-2}}{1 + (1-T)r} + \frac{Td_n + (1-T)l_n + TrB_{n-1}}{(1 + (1-T)r)^2} \]

\[ = \sum_{i=n-1}^{n} \frac{Td_i + (1-T)l_i + TrB_{i-1}}{(1 + (1-T)r)^{i-(n-2)}}. \]  \hspace{1cm} (27)

Proceeding such way we obtain for \( t = 1 \) to \( n \);

\[ B_0 = \sum_{i=1}^{n} \frac{Td_i + (1-T)l_i}{(1 + (1-T)r)^i}. \]  \hspace{1cm} (28)

Hence neutralizing the risk difference of the lease and purchase alternative leads either to equation (22) or (28) which for the purpose of evaluation are fully equivalent. Because both formulations give the same estimate of the maximum critical lease payment \( l' \) and lead to the same decision regarding the relative desirability of the lease and purchase alternatives. If the proposed lease payments are less than \( l' \), then the machine should be leased and if lease payments are greater than \( l' \), then the purchase will be profitable (Myers et al. 1976). Now solving (28) we get;

\[ I = \sum_{i=1}^{n} \frac{(1-T)l_i''}{(1 + (1-T)r)^i} + \sum_{i=1}^{n} \frac{Td_i}{(1 + (1-T)r)^i}, \]

\[ \sum_{i=1}^{n} \frac{(1-T)l_i'}{(1 + (1-T)r)^i} = I - \sum_{i=1}^{n} \frac{Td_i}{(1 + (1-T)r)^i}, \]

\[ l'' = \frac{I - \sum_{i=1}^{n} \frac{Td_i}{(1 + (1-T)r)^i}}{\sum_{i=1}^{n} \frac{(1-T)}{(1 + (1-T)r)^i}}. \]  \hspace{1cm} (29)
NUMERICAL CALCULATIONS

Let a firm decide to acquire a machine which is available in buying or leasing. The owner of the firm is confused which would give him profit in leasing or buying. Let the price of the machine is \( I = \$1,000,000 \) and its economic life is \( n = 10 \) years, interest rate is \( r = 10\% \) p.a., the corporate tax rate is \( T = 50\% \), and the accelerated depreciation is calculated using the sum of the years digits method (see Appendix-I). The buy or lease the machine depends on the magnitude and timing of the lease payments facing the firm. From (1) we see that the firm leases the machine the annual cash flow will be:

\[
(1-T)(R-C)-(1-T)l = 0.5(R-C)-0.5 \times \$161,676.70.
\]

Also the firm can purchase the machine in cash at a cost \( I = \$1,000,000 \). From Appendix-I we have depreciation of the machine in year 1, \( d_1 = \$181,818.18 \). If the firm purchases the machine then first year’s cash flow will be;

\[
(1-T)(R-C)-Td_1 = 0.5(R-C)+0.5 \times \$181,818.18 = 0.5(R-C)+\$90909.09.
\]

From (11) we have the after-tax payment needed to neutralize the risk in the year 1 as follows:

\[
Td_1 + (1-T)l' = 0.5 \times \$181,818.18+0.5 \times \$161,676.70 = \$171,747.44.
\]

We see that if the machine is purchased, it should be financed by a loan which provides a payment in the year 1 is \$171,747. Now for the first year;

\[
(1-T)(R-C)-Td_1 - \$161,676.70 = 0.5(R-C)+90,909.09-61,676.70 = 0.5(R-C)-\$70,767.61.
\]

The adjustment annual purchase cash flow of all the other years will be identical to the cash flow of the lease option, obviously have the same risk. The purchase or lease decision depends strictly on the amount borrowing \( B \) and in return for the annual payments of principal and interest to neutralize the leverage \( Td_i + (1-T)l_i \) and the size of the initial investment \( I \) need to purchase the machine. If \( B > I \), purchasing will be better than leasing, because we can borrow the amount \( B \) to purchase the machine and invest the amount \( I \) in bank. If \( B < I \), then for the firm leasing will be better than borrowing. If the firm deposit \( SI \) of its own capital in the bank at a post-tax interest rate \( (1-T)r \), then each year the firm reduces its deposit an amount \( Td_i + (1-T)l_i \) and adds this amount to the annual cash flow to the lease which gives identical \( Td_i + (1-T)l_i \) in the year \( t \), then purchase of the machine will be profitable and if the deposit is less than \( SI \) then leasing will be profitable. From Appendix-II we see that if we assume \( l = \$100,000 \) then \( B_0 = 3.861 \times 100,000+375,766.26 = \$764,866 \). In this case loss from buying rather than leasing will be \$235,134. Again if we assume \( l = \$160,899.7 \) then \( B_0 = \$1,000,000 \). In this case leasing and buying will be identical. In this situation the firm finds same benefit with buying or leasing the machine. If we assume \( l = \$200,000 \) then \( B_0 = \$1,150,966 \).
Therefore profit from buying rather than leasing will be $150,966. Hence for \( l > 160,899.70 \) we have obtained that buying is better financing than leasing, and the firm should purchase the machine. Again for \( l < 160,899.70 \) we have obtained that leasing is better financing than buying, and the firm should lease the machine.

Hence we have found from the Appendix-II that if the value of \( l \) increases then profit from leasing the machine rather than buying increases gradually. If we increase \( l \) such a way then only one value of \( l = 160,899.70 \) gives leasing and buying indifferent. If we further increase the value of \( l \) we have found that profit from buying the machine rather than leasing increases gradually.

**CONCLUDING REMARKS**

In this paper we have analyzed the long-term lease of assets to the economic development of a firm. At the first sight the concept of lease or purchase seems very simple. But we have shown that to take correct decision which one is better for the business requires an evaluation of almost all the features of financial decision making. It is also investigated riskness of the lease and purchase. By the mathematical calculations we have discussed when leasing is profitable than purchasing for the firm. We also estimate of the maximum critical lease payment which leads to the same decision regarding the relative desirability of the lease and purchase alternatives. We have tried our best to present the mathematical calculations in some details.

**REFERENCES**


APPENDIX-I

For \( n = 10 \) years, sum of the years in digits, \( S = \frac{n(n+1)}{2} = \frac{10 \times 11}{2} = 55 \), and \( I = $1,000,000 \).

Depreciation at the year 1, \( d_1 = \frac{10}{55} \times 1,000,000 = $181,818.18 \),
Depreciation at the year 2, \( d_2 = \frac{9}{55} \times 1,000,000 = $163,636.36 \),
Depreciation at the year 3, \( d_3 = \frac{8}{55} \times 1,000,000 = $145,454.55 \),
Depreciation at the year 4, \( d_4 = \frac{7}{55} \times 1,000,000 = $127,272.73 \),
Depreciation at the year 5, \( d_5 = \frac{6}{55} \times 1,000,000 = $109,090.91 \),
Depreciation at the year 6, \( d_6 = \frac{5}{55} \times 1,000,000 = $90,909.09 \),
Depreciation at the year 7, \( d_7 = \frac{4}{55} \times 1,000,000 = $72,727.27 \),
Depreciation at the year 8, \( d_8 = \frac{3}{55} \times 1,000,000 = $54,545.45 \),
Depreciation at the year 9, \( d_9 = \frac{2}{55} \times 1,000,000 = $36,363.64 \),
Depreciation at the year 10, \( d_{10} = \frac{1}{55} \times 1000000 = $18,181.82 \).

Now we have, \( T = 50\% = 0.5 \), \( r = 10\% = 0.1 \), and \( I = 1,000,000 \). Also we have;

\[
(1 + (1 - T)r)' = (1 + (1 - 50\%) \times 10\%)' = 1.05'.
\]

We can write equation (29) for \( t = 1 \) to 10 as follows:

\[
l' = \frac{1,000,000 - 0.5 \times \sum_{t=1}^{10} \frac{d_t}{(1.05)^t}}{0.5 \times \sum_{t=1}^{10} \frac{1}{(1.05)^t}}. \tag{AI-1}
\]

Now the term,

\[
\sum_{t=1}^{10} \frac{d_t}{1.05^t} = \frac{d_1}{1.05} + \frac{d_2}{1.05^2} + \frac{d_3}{1.05^3} + \frac{d_4}{1.05^4} + \frac{d_5}{1.05^5} + \frac{d_6}{1.05^6} + \frac{d_7}{1.05^7} + \frac{d_8}{1.05^8} + \frac{d_9}{1.05^9} + \frac{d_{10}}{1.05^{10}}
\]

\[
= \frac{181,818.18}{1.05} + \frac{163,636.36}{1.05^2} + \frac{145,454.55}{1.05^3} + \frac{127,272.73}{1.05^4} + \frac{109,090.91}{1.05^5}
\]
$90,909.09 + \frac{72,727.27}{1.05} + \frac{54,545.45}{1.05^2} + \frac{36,363.64}{1.05^3} + \frac{18,181.82}{1.05^4} = \$751,532.51.$

Again the term,

\[ \sum_{t=1}^{10} \left( \frac{1}{1.05} \right)^t = \frac{1}{1.05} + \frac{1}{1.05^2} + \frac{1}{1.05^3} + \frac{1}{1.05^4} + \frac{1}{1.05^5} + \frac{1}{1.05^6} + \frac{1}{1.05^7} + \frac{1}{1.05^8} + \frac{1}{1.05^9} + \frac{1}{1.05^{10}} \]

\[ = \frac{1}{1.05} \times \frac{1 - \left( \frac{1}{1.05} \right)^{10}}{1 - \frac{1}{1.05}} = 7.722. \]

Hence (AI-1) can be written as;

\[ t^* = \frac{1,000,000 - 0.5 \times 751,532.51}{0.5 \times 7.722} = \$161,676.70. \]  

Therefore the critical value of annual lease payment is, \( l^* = \$161,676.70. \)

**APPENDIX-II**

From (28) for \( n = 1 \) to 10 we get;

\[ B_0 = \sum_{t=1}^{10} Td_t + (1 - T)l \]

\[ = 0.5l \sum_{t=1}^{10} \frac{1}{1.05^t} + 0.5 \sum_{t=1}^{10} d_t \]

\[ = 0.5 \times 7.722 + 0.5 \times 751,532.51 \]

\[ = 3.861 + 375,766.26. \]

If we assume \( l = \$100,000 \) then \( B_0 = 3.861 \times 100,000 + 375,766.26 = \$764,866. \)

Hence loss from buying rather than leasing = 1,000,000-764,866 = \$235,134.

If we assume \( l = \$150,000 \) then \( B_0 = 3.861 \times 150,000 + 375,766.26 = \$957,916. \)

In this case loss from buying rather than leasing = 1,500,000-957,916= \$42,039.

If we assume \( l = \$160,899.7 \) then \( B_0 = 3.861 \times 160,899.7 + 375,766.26 = \$1,000,000. \)

In this case leasing and buying will be identical.

If we assume \( l = \$200,000 \) then \( B_0 = 3.861 \times 200,000 + 375,766.26 = \$1,150,966. \)

Therefore profit from buying rather than leasing = 1,150,966-1,000,000 = \$150,966.

If we assume \( l = \$300,000 \) then \( B_0 = 3.861 \times 300,000 + 375,766.26 = \$1,537,066. \)

Hence profit from buying rather than leasing = 1,537,066-1,000,000 = \$537,066.