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Rent as a Share of Product and Sraffa's Price Equations

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Abstract

The classical economists usually regarded rent in their analyses as a share of the gross product obtained from the use of land or a mine, which was indeed the way in which rent was treated in bargaining between landowner and tenant. The paper revives this view of rent, proceeding from its historical basis through Smith's analysis to arrive at Sraffa's equations, and also examines the case of the introduction of a tax conceived as a tithe, to which Sraffa referred very briefly (Sraffa 1960, p. 55).

Keywords: Rent, Classical theory of distribution, Smith, Sraffa

JEL Codes: B12, B51, D33, Q15.

1. Introduction

In addressing rent, the French Physiocrats and the British classical economists usually regarded it as a certain share of the harvest, in the case of land, or the ore extracted, in the case of mines.¹ It was only in connection with some specific points, generally concerning the effects of competition amongst landowners or tenant-farmers, that the rate of rent, understood as the rent per unit of land (acre), was taken into consideration.²

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¹ It can also be pointed out that the rate of rent understood as the rent per acre usually makes little sense in the case of mines. While agricultural production employs the soil, ore extraction usually involves the subsoil, and output therefore depends in this case on the depth bored rather than the surface cultivated.

² See for example the following observation by Marx in his criticism of Rodbertus: '[Rodbertus] commits the mistake of dealing with the ratio of the money rent to a quantitatively limited piece of land, for instance to an acre, as though it had been the general assumption of classic economics in its analysis of the rise or fall of rent. This, again, is wrong.'

The view of rent as a share of product gradually disappeared from economic theory with the passing of time. This was mainly due to the advent of the marginalist theory with its concept of distribution variables as the prices of the factors of production to be determined simultaneously.³

This paper seeks to go back to the classical economists and revive the old way of considering rent. With a view to understanding why rent was treated as a share of product, the following section presents a brief overview of the historical evolution of forms of land tenure in Europe and argues, in a nutshell, that the idea of rent as a share of the harvest is a legacy of feudal times that persisted (and perhaps still does) in the capitalist system as a sort of crystallised custom or practice. Section 3 then discusses some actual cases, taken from Adam Smith's *Wealth of Nations*, with rent seen as a share of the gross product of land or mines, which also provides an opportunity to address some issues possibly connected with this approach to rent.

The last two sections are theoretical in nature. Section 4 focuses in particular on one of the least known and analysed passages of Sraffa's *Production of Commodities*, where he considers the effects of the introduction of a tax conceived as tithe, i.e. as a share of the gross production of a certain commodity. In addition to discussing Sraffa's assertions in this connection, it takes the opportunity to insert a tax or a rent, which is ultimately the same, conceived as a share of output into the framework of Sraffa's equations. This operation is then extended and completed in section 5.

2. Historical basis

Land rent was the prevailing form of surplus value in European pre-capitalistic societies under the feudal system. During that period, as reported amongst others by Adam Smith (1976, vol. 2, p. 200, I.xi.e.17) and Marx (1909, vol. 3, ch. XLVII), rents were paid in kind. According to Marx's reconstruction, there was an initial phase in which land rent was paid in labour by means of a *corvée* system whereby farmhands worked unpaid on the feudal lord's estate for part of the week in return for permission to produce their own subsistence, on other land also belonging to the lord, during the rest of it.

Classic economics always treats the rate of rent, so far as it considers rent in its natural form, with reference to the product.' Marx 1909, vol. 3, p. 904, note.

³ While it is always possible to address rent, analytically, in a form other than the one in which it is actually observed, which means that the theory can focus on rates of rent understood as value rent per acre even though rent is actually set in terms of shares, there should be a precise reason (or indeed necessity) for doing so.

When society reached a higher state of civilisation (Marx 1909, vol. 3, p. 923), the feudal lord left the burden of organising production to the workers and, as a result, rent in labour was transformed into a rent in commodities paid with a share of product.

Subsequently, with the increasing development of trade and hence the possibility of establishing the market value of the crop, rent in kind turned into rent in money. This led to a radical social change. To some rudimentary degree at least, farmers became entrepreneurs, no longer merely producers but also sellers. Moreover, as Marx remarked, this was one of the steps toward the rise of a new social system:

[t]he transformation of rent in kind into money rent is not only necessarily accompanied, but even anticipated by the formation of a class of propertyless day laborers, who hire themselves out for wages. During the period of their rise, when this new class appears but sporadically, the custom necessarily develops among the better situated tributary farmers of exploiting agricultural laborers for their own account, just as the wealthier serfs in feudal times used to employ serfs for their own benefit. In this way they gradually acquire the ability to accumulate a certain amount of wealth and to transform themselves even into future capitalists. [1909, vol. 3, p. 928]

During this phase of transition, sharecropping – also known as the *métayage* system – was the typical form of land tenure, especially in continental European countries.⁴ As is known, this is a still pre-capitalistic form of land tenure in which the sharecropper or *métayer* undertakes the farming, primarily with his own labour and that of his family, and shares the produce with the owner of the soil. While the rent was usually half of the crop, a broad variety of different agreements were actually in use in different countries and periods.⁵

Similarly, there were different agreements as to the responsibility for furnishing the means of production employed together with land and labour. It was, however, common practice for the landowner to provide the initial seed and livestock⁶ and the sharecropper the tools, which were usually made during the winter, when there was less to do in the fields.

Though typically associated with the feudal period, this form of land tenure persisted for a long time in the capitalist system too⁷ side by side with tenancy, the new form of agricultural contract. In

⁴ As John Stuart Mill wrote, '[t]he metayer system has met with no mercy from English authorities' (1909, p. 183). After developing an in-depth analysis of experience in France and Italy, however, he then drew this conclusion: 'the unmeasured vituperation lavished upon the system by English writers, is grounded on an extremely narrow view of the subject' (pp. 191, 2).

⁵ While this point is addressed in greater detail in the following section, where Adam Smith's observations on rent are considered, it can be briefly recalled here that, according to John Stuart Mill (1909, p. 183), the rent was up to two-thirds of the product in many parts of Italy. Moreover, there were different possible arrangements for the payment of taxes and retention of stock.

⁶ As a result of this provision in advance, Marx says that 'the landlord claims his share not exclusively in his capacity as the owner of the land, but also as a lender of capital' (1909, vol. 3, p. 933).

⁷ The dissolution of the feudal system was, however, an extremely gradual process and did not take place simultaneously in every country. For example, Adam Smith wrote in the *Wealth of Nations* that sharecropping had been

France for example, according to Turgot, sharecropping and tenancy were the most widespread forms of land tenure during the last decades of the *ancien régime*,⁸ being more common respectively in poorer and richer areas (Turgot, [1770], p.25).

A similar view can be found in Quesnay's entry 'Fermiers' for the *Encyclopédie* (1756,7), where he tried to foster the spread of tenancy by proclaiming its superiority with respect to the sharecropping system that still predominated in France at the time. The capitalist tenant-farmer, he claimed, can obtain more produce from land and, in so doing, benefits not only himself but also the landowner and the kingdom as a whole. This greater productivity of land is due, in his view, to the more appropriate means of production that tenant farmers can afford to employ because of their capital. Quesnay dwells in particular upon the advantage arising from the use of draught horses, which the sharecropper cannot afford, instead of oxen. Moreover, while the sharecropper tills the soil mainly by means of his own labour and that of his family, the tenant farmer employs wage-earning farmhands of an age and strength more suitable to agricultural work.

The coexistence of sharecroppers and tenant farmers is therefore a matter of fact, lasting only a short time in countries like Great Britain, where the capitalist system arrived more quickly, and a long time in others like France, where the dissolution of the feudal system was a very gradual process. In Italy in particular, the signing of new sharecropping contracts under the *mezzadria* system was not prohibited until 1974. It then became possible in 1982 for sharecroppers to transform their contracts into leaseholds even without the landowner's agreement.

It was therefore completely normal and natural in Europe during the 18th and 19th century to view rent as a share of gross product or its value. Landowners certainly understood rent in these terms, both because they were accustomed to this view and because, as Torrens wrote, they 'might (and in point of fact frequently have done so) require to be paid for the use of the soil, not a fixed sum in money per acre, but a fixed proportion of the whole produce' (1827, p. 231). In the case of tenancy contracts too, in setting the terms of the lease, rent was thus conceived as a share of product or as the market value of a share of what was supposed to be the average or normal yield of the land in question.

Nowadays too, however, there appear to be important cases in which rent is conceived as a share of output. As Ravagnani (2008) points out with reference to royalties for oil extraction, 'negotiations over royalties have been regulated throughout the existence of the US oil industry by

in disuse in England for so long that he was forced to use the French term "metayer" because he did not know the English word for it (Smith 1976, vol. 2, p. 389 – III.ii.11).

⁸ Adam Smith wrote that in France in his day, 'five parts out of six of the whole kingdom' were occupied by *metayers* (Smith 1976, vol. 2, p. 391 – III.ii.13).

conventional arrangements entitling landowners to a pre-established share of the oil extracted, or of its value' (pp. 86, 87).

3. Rent as a share of product in classical economics

As seen in the previous section, there was in Europe, and indeed elsewhere too, a long period during which both landowners and tenants treated rent as a share of product in their bargaining. It is therefore not at all surprising that the classical economists, who lived and made their observations at that time, usually considered rent in their analyses as a fraction or share of the gross product of land (or its value).

Even though this view of rent can be found in the writings of almost every economist directly or indirectly linked to the classical tradition,⁹ attention will be focused in this section on some passages from Adam Smith, whose *Wealth of Nations* provides all the cases required for a complete treatment of rent as a share of product.

As stated above, while the share of rent was usually half the produce in the French *métayage* and Italian *mezzadria* systems, a variety of different agreements was possible. In particular, it is evident that areas of lands suited and indeed devoted to different uses will generally differ in terms of their rent shares. The fraction of product obtained by the landowner for the use of agricultural soil is in fact generally different from the share earned as royalties by the owner of a mine, and further differences can arise with the extraction of different kinds of minerals (coal, tin, lead, oil, etc.).

Adam Smith thus observes that a share considered normal for agricultural production would be considered excessive for a coal mine:

[t]he rent of an estate above ground commonly amounts to what is supposed to be a third of the gross produce [...]. In coal-mines a fifth of the gross produce is a very great rent; a tenth the common rent [Smith 1976, vol. 2, p. 184 – I.xi.c20].

⁹ While a whole series of quotations from Malthus, Ricardo, Torrens, J.S. Mill and others could be presented, it would never be complete, no matter how long, because this was the way in which economists thought about rent, explicitly or implicitly. Even those who rigidly adopted the differential theory of rent (and chiefly considered agricultural rents) could not avoid referring to rent as a share of production despite the fact that this was not fully consistent with (or at least required by) their theory. It is, however, our belief that the differential theory of rent could be reformulated in terms of shares, as discussed at the end of section 6.

As regards Ricardo in particular, many passages of his *Principles* refer to rent as a share: sometimes as a share of the harvest of a certain agricultural product – e.g. where it is described as a 'proportion of the produce, obtained with a given capital on any given farm' (1951-73, vol. 1, p. 83) – and sometimes as the share of the whole national product paid to landlords (cf. 1951-73, vol. 1, p. 402; see also Cannan, 1903, pp. 352, 3). He did, however, perceive a possible contradiction with the theory of differential rent, as shown by this deleted passage from his *Notes on Malthus*: '[r]ent is not a proportion of the produce obtained [...] depending as it does on the difference between the quantity of produce obtained by two equal capitals' (1951-73, vol. 2, 196, footnote). For this passage, see also Gehrke (2013).

Moreover, there is generally a difference in the share of product that constitutes the rent in coal mining as against tin mining:

[a] sixth part of the gross produce may be reckoned the average rent of the tin mines of Cornwall, the most fertile that are known in the world, as we are told by the Reverend Mr. Borlace, vice-warden of the stannaries. Some, he says, afford more, and some do not afford so much. A sixth part of the gross produce is the rent, too, of several very fertile lead mines in Scotland [p. 186 – I.xi.c24].

In Smith's day, to sum up, the standard rent was thus a third of product for farming agricultural soil, a tenth for a coal mine and a sixth for tin and lead mines. There is, however, a point still to be clarified. While coal is the only output to be obtained from a coal mine (and tin and lead are respectively the sole physical outputs of tin and lead mines), a whole variety of crops can be grown on agricultural land. How can we deal with this fact?

The answer is once again to be found in *The Wealth of Nations*. According to Smith, a key role is played in determining the conditions of agricultural agreements by 'the principal produce of land', which he takes to be corn:

[i]n Europe corn is the principal produce of land which serves immediately for human food. Except in particular situations, therefore, the rent of corn land regulates in Europe that of all other cultivated land [p. 174 – I.xi.b35].

Once the level of the rent paid on land used for the 'principal produce' (corn) is set, competition will make the rent per unit of land under every other form of cultivation equal to that in the corn sector:

[i]n all great countries the greater part of the cultivated lands are employed in producing either food for men or food for cattle. The rent and profit of these regulate the rent and profit of all other cultivated land. If any particular produce afforded less, the land would soon be turned into corn or pasture; and if any afforded more, some part of the lands in corn or pasture would soon be turned to that produce [p. 168 – I.xi.b23].

The mechanism described by Smith is thus as follows. There is a principal agricultural sector, which is (usually, but not necessarily) the sector producing the fundamental element of the human diet. It can in fact be assumed, following Smith, that most of the soil in the country is devoted to this crop, at least in the case of an almost closed economy. The share of product that forms the rent of agricultural soil is established in this sector by bargaining between landowners and tenant farmers. Once this is determined, e.g. as one third of the corn harvest according to Smith,¹⁰ as a

¹⁰ Differences in the fertility of land are ignored here for the sake of simplicity.

result of competition, every piece of land must then pay a rent equal to the value of one third of the corn that can be grown upon it regardless of its actual use.

It is within this levelling mechanism that the rate of rent, conceived as the value rent per unit of land, has relevance in Smith's analysis.

While this point is taken up again in section 5, let us now consider the way in which a tithe, i.e. a tax or a rent conceived as a share of output, can be included within Sraffa's analysis of prices and distribution.

4. Sraffa and the tithe

In chapter VIII of *Production of Commodities* (1960), with the aim of elucidating the different roles played by basic and non-basic commodities in determining the rate of profit, Sraffa considers the effects of a tax levied on the production of a particular commodity 'as a tithe, which can be defined independently of prices' (Sraffa, 1960, p. 55),¹¹ i.e. a certain share (usually a tenth) of the gross output. According to Sraffa:

[a] tax on a basic product then will affect all prices and cause a fall in the rate of profits that corresponds to a given wage, while if imposed on a non-basic it will have no effect beyond the price of the taxed commodity and those of such other non-basics as may be linked with it [Sraffa, 1960, pp. 55].

Being placed at the end of a complex chapter devoted to cases with joint production, this point has received little attention.¹² We shall therefore present some simple examples here in which a given share σ , with $0 < \sigma < 1$, of the gross production of a commodity is paid either to the state as a tax, as Sraffa assumed, or to landowners as a rent, as we can also assume.¹³

¹¹ As Sraffa wrote, the different role played by basic and non-basic industries is easy to perceive in the single-products system, where a technical improvement in a basic industry would bring about a change in the rate of profits and the prices of all commodities, whereas the same improvement in a non-basic sector would not affect the rate of profits but only some prices (and perhaps its price alone). This sort of distinction cannot, however, be extended directly to the case with joint production because, as Sraffa points out, basic and non-basic commodities may be outputs of the same process. He therefore suggests consideration of the effects of a tax on a particular commodity rather than a technical improvement.

¹² An in-depth analysis of how Sraffa arrived at using the tithes in order to distinguish basic and non-basic commodities in the case of joint production is presented by Kurz and Salvadori (2007). Besides Kurz and Salvadori's essay, to the best of our knowledge, there are just a few brief references to this passage of *Production of Commodities* (e.g. Schefold 1989, p. 68; Bidard 2004, p. 36) and one attempt at formal representation (Ballesteros et al., 1979, pp. 30-4).

¹³ In this respect, Torrens wrote: 'Tithes have generally been considered as a direct tax upon agriculture. But this is not the correct way of viewing them. Rent is correctly defined to be, that portion of the produce which is given to the proprietor for the use of the soil. The church, by a title antecedent to any other which can now be shewn, is, to a certain extent, a proprietor in common of the lands of the country; and that portion of the produce of land which the cultivator pays to the church, for the use of the soil, comes, in strictness, under the definition of rent.' Torrens, 1827, p. 230, 1.

In each example there are just two commodities, one basic and one non-basic. We shall consider first a very simple case without joint production and with the tax (or rent) levied on the non-basic commodity, then the case with the tax imposed on the basic commodity, and finally the case with joint production and different rates of taxation levied on the two commodities.

4.1 Example one

Let us begin our analysis with a simple case involving two commodities, one basic, commodity [a], and one non-basic, commodity [b]. Let σ_b be the share of the gross output of commodity [b] paid to the state as tax or the landowner as rent (as the reader prefers).

Commodity [a] is the numéraire¹⁴ and Sraffa's assumptions and symbols are adopted for all the rest.

The price equations for the case considered here are:

$$1 = a_a(1+r) + \ell_a w \quad [1]$$

$$p_b = (a_b + b_b p_b)(1+r) + \ell_b w + \sigma_b p_b \quad [2]$$

The wage-profit relation emerges directly from equation [1] and is therefore not affected by the share σ_b . It follows instead from equation [2] that the price of commodity [b] in terms of [a] depends on the tax (or rent) share:

$$p_b = \frac{a_b(1+r) + \ell_b w}{1 - b_b(1+r) - \sigma_b} \quad [3]$$

In particular, as clearly emerges from equation [3], p_b increases monotonically as σ_b increases and tends to ∞ as σ_b approaches $1 - b_b(1+r)$.

4.2 Example two

In this example, the share σ_a is paid from the gross output of the basic commodity [a] and no tax or rent is paid for the production of the non-basic commodity [b].

The price equations for this case are:

$$1 = a_a(1+r) + \ell_a w + \sigma_a \quad [4]$$

¹⁴ In the three examples, given that commodity [a] is the only basic commodity, it is also the standard commodity.

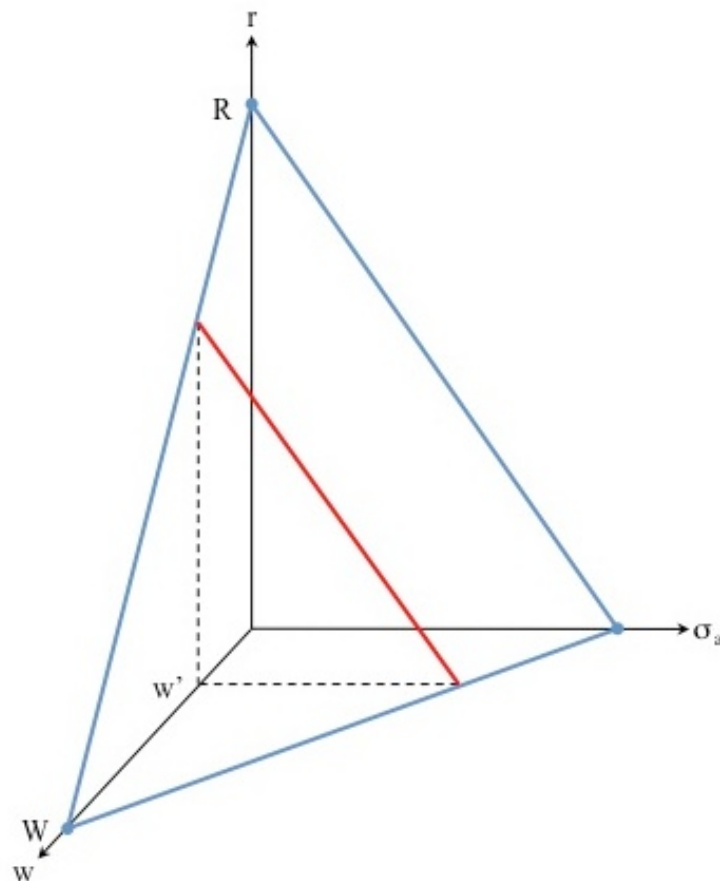
$$p_b = (a_b + b_b p_b)(1+r) + \ell_b w \quad [5]$$

In the case considered here, as well the one above, the wage-profit relation emerges directly from the first equation, equation [4]. It is, however, now affected by the share σ_a , as we obtain the following from equation [4]:

$$r = \frac{1 - a_a - \ell_a w - \sigma_a}{a_a} \quad [6]$$

It can now be observed that since there are three distribution variables, namely w , r and the share σ_a , two of them will change in the same direction when the third goes in the opposite. At the same time, however, when one of the three variables is kept the same, the other two are inversely related. In particular, for this case without joint production, we find confirmation of Sraffa's assertion in the passage quoted above: a rise in the share σ_a brings about a fall in the rate of profits associated with a given wage rate. Given a wage rate w' , there is in fact an inverse relationship between the rate of profits and the share σ_a , as emerges clearly from equation [6] and fig. 1.

Fig. 1: The relation between r and σ_a for a given wage rate



4.3 Example three

The third case considered – in which the two commodities, one basic and one non-basic, are produced jointly – is the one closest to what Sraffa had in mind in introducing the idea of a tithe.

Two processes are in use. For a process i , with $i = 1, 2$, $a_{(i)}$ denotes the amount of commodity [a] jointly produced with each unit of [b], and a_i and ℓ_i denote the inputs of commodity [a] and labour employed per unit of commodity [b] produced. As before, σ_a and σ_b are the shares of tax (or rent) of gross output of commodities [a] and [b] respectively.

The price equations for the case under consideration are thus:

$$(1 - \sigma_a)a_{(1)} + (1 - \sigma_b)p_b = a_1(1 + r) + \ell_1 w \quad [7]$$

$$(1 - \sigma_a)a_{(2)} + (1 - \sigma_b)p_b = a_2(1 + r) + \ell_2 w \quad [8]$$

By subtracting equation [8] from equation [7] we obtain:

$$(1 - \sigma_a)(a_{(1)} - a_{(2)}) = (a_1 - a_2)(1 + r) + (\ell_1 - \ell_2)w \quad [9]$$

and equation [9], properly reorganised, becomes:

$$r = \frac{(1 - \sigma_a)(a_{(1)} - a_{(2)}) - (a_1 - a_2) - (\ell_1 - \ell_2)w}{(a_1 - a_2)}. \quad [10]$$

Therefore, as Sraffa wrote, the tax share σ_b – the one imposed on the non-basic commodity – does not affect the wage-profit relation but only the price p_b . The wage-profit relation is instead influenced by the share σ_a . The rest of Sraffa's claim, namely that a rise in the share σ_a will 'cause a fall in the rate of profits that corresponds to a given wage', does not, however, appear to be generally valid,¹⁵ since we shall show at least a case in which this does not happen.

If the processes are ordered in such a way that $(a_{(1)} - a_{(2)}) > 0$, three cases are possible: i) $(a_1 - a_2) > 0$ and $(\ell_1 - \ell_2) > 0$; ii) $(a_1 - a_2) > 0$ and $(\ell_1 - \ell_2) < 0$ and iii) $(a_1 - a_2) < 0$ and $(\ell_1 - \ell_2) > 0$. There is an inverse relationship between w and r , for a given share σ_a , in the first case, but the relationship is direct in the other two.

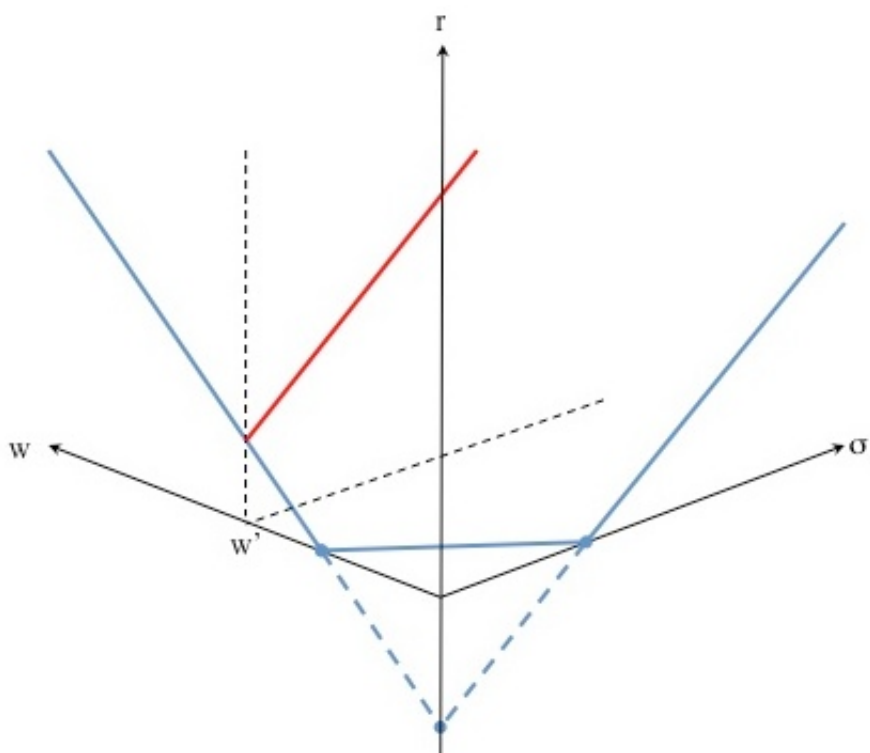
¹⁵ This point is not noticed in the two papers dealing with Sraffa's tithe analitically. Ballesteros et al. (1979), in particular, focussed their attention on the effects of tax on prices only. Kurz and Salvadori's analysis (2007) is, instead, essentially aimed at verifying the most important part of Sraffa's claim, i.e. that the tax share imposed on a non-basic commodity does not affect the wage-profit relation.

While Sraffa's idea of an inverse relationship between r and σ_a holds in cases i) and ii), in the third, contrary to what he wrote, the rate of profits corresponding to a fixed wage rate increases with the rise in the share σ_a . In fact, given a wage rate w' , equation [10] implies:

$$r = \frac{(a_{(1)} - a_{(2)}) - (a_1 - a_2) - (\ell_1 - \ell_2)w'}{(a_1 - a_2)} - \frac{(a_{(1)} - a_{(2)})}{(a_1 - a_2)} \sigma_a. \quad [11]$$

Therefore, since $(a_{(1)} - a_{(2)}) > 0$, $(a_1 - a_2) < 0$ entails a direct relationship between r and σ_a , as is shown in fig. 2 and in the appendix by means of a numerical example.

Fig. 2. The relation between r and σ_a , for a give wage rate in the case with joint production



In the case considered, given the shares σ_a and σ_b and the wage rate w , the rate of profits – together with the price p_b – must allow the two processes to coexist. Therefore, since a rise in the share σ_a is more disadvantageous to process (1) than to process (2) – because process (1) produces a greater output of commodity [a] per unit of [b] – the change in the rate of profit must compensate this disequilibrium. Given that process (1) employs less capital per unit of output of commodity [b] than process (2), i.e. $a_1 < a_2$, the rate of profit associated with the same wage rate must increase. An increase in p_b can also be expected in this case.

5. Rent shares in Sraffa's price equations

While cases in which rent shares appear within Sraffa's equations have already been studied in the previous section, there are a few further points to be considered. This will be done here by means of a simple example.

Let us consider a system in which it is assumed for simplicity that there are two natural means of production, namely agricultural land and mines. The former can be used for two different types of produce, say apples and barley, and the latter produce coal. As is usual in the study of rent, the presence is also assumed of a 'pure industrial product', namely a commodity, say dishes, whose production requires neither land nor mines.

In accordance with the notation introduced in the previous section, σ_a , σ_b and σ_c are the fractions of the gross production of apples, barley and coal paid as rent. Adopting the usual notation for the other magnitudes and assuming that wages and rents are paid post-factum, we can write the following equations:

$$p_a = (a_a p_a + b_a p_b + c_a p_c + d_a p_d)(1+r) + \ell_a w + \sigma_a p_a \quad [12]$$

$$p_b = (a_b p_a + b_b p_b + c_b p_c + d_b p_d)(1+r) + \ell_b w + \sigma_b p_b \quad [13]$$

$$p_c = (a_c p_a + b_c p_b + c_c p_c + d_c p_d)(1+r) + \ell_c w + \sigma_c p_c \quad [14]$$

$$p_d = (a_d p_a + b_d p_b + c_d p_c + d_d p_d)(1+r) + \ell_d w. \quad [15]$$

If the three shares σ_a , σ_b and σ_c can be regarded as given, then no new unknown has been added to Sraffa's usual system of equations and therefore no new analytical problem arises. Once a numéraire is adopted, say dishes, and the wage rate is known, the system [12]–[15] can determine the prices p_a , p_b and p_c in terms of dishes together with the rate of profits r .

The share σ_c is, however, unquestionably independent of σ_a and σ_b , as we cannot grow apples or barley in a coal mine or, normally, extract coal from agricultural soil (while coal could be produced from wood, this possibility is overlooked here), but σ_a and σ_b could instead be linked to one another.

If λ_a and λ_b are the areas of agricultural land needed to obtain one unit of apples and one unit of barley respectively, and if σ_a and σ_b are arbitrarily given, then the land employed in the apple sector receives a rent per acre of $\sigma_a p_a$, which is generally different from the rent for land devoted to the production of barley $\sigma_b p_b / \lambda_b$. Therefore, if there is no qualitative difference between the land employed in the two sectors, it is in the landowners' interest, as Adam Smith pointed out, to lease

their land only in the sector that pays the higher rate of rent. And this fact, together with the classical mechanism of competition among producers, would bring the rates of rent to equality in both sectors, that is:

$$\frac{\sigma_a p_a}{\lambda_a} = \frac{\sigma_b p_b}{\lambda_b}. \quad [16]$$

When this equation is added to the system [12]–[15],¹⁶ one of the two shares σ_a and σ_b becomes an unknown to be determined.¹⁷ Following Smith's analysis as presented in section 3, we can assume that one of the two crops, say apples, is the principal agricultural product of the economy. In this case, the rent of land devoted to apple growing regulates the rent of all the other cultivated land and σ_b is therefore a variable to be determined simultaneously with relative prices and the rate of profits by the system [12]–[16], in which we take as given the technical coefficients, the wage rate and the shares σ_a and σ_c .

6. Conclusions

In many parts of their works, the classical economists refer to rent as a share of the gross output of land or mines. This was indeed, as argued in section 2, the way in which rent was conceived in the world that they observed (and that can still be observed in many cases today).

In section 3, where Adam Smith's *Wealth of Nations* is taken as representative of this approach to rent analysis, it is shown that different rent shares can be paid for the use of different natural resources: agricultural soil, coal mines, tin mines, etc. It is also shown that in the case of agricultural soil, as different crops can be grown on it, the relevant rent share, according to Smith, must be referred to the principal agricultural product, which is assumed in his analysis to be corn. Given the fraction of gross production of corn paid as rent, every piece of land, regardless of its

¹⁶ It is also possible to use equation [16] directly within the system, substituting σ_b with $\sigma_a p_a \lambda_b / \lambda_a$ in equation [13].

¹⁷ The point can be put in terms even closer to the Ricardian (and neo-Ricardian) approach to rent. The cultivation of apples and barley can be viewed as two different production activities simultaneously under way on land of the same quality. Now, if land of this quality is fully employed, the rate of rent must be such as to allow the two processes to coexist, which means in our framework that not only the price equations but also condition [16] must be satisfied. Therefore, as the coexistence of two processes side by side on the same kind of land determines the intensive rent in the case considered by Sraffa (1960, pp. 75, 76), so here one of the two rent shares is endogenously determined.

actual use, must provide the landowner with rent equal in value to that it would fetch in the corn sector.

The ideas found in Adam Smith's *Wealth of Nations* are used in section 5 to include the rent shares for the use of natural resources in Sraffa's price equations. In particular, it is seen that once the rent shares are taken as given, the resulting system is essentially analogous to the one considered by Sraffa. The only possible further complication is represented by the endogenous determination of the rent shares for crops other than the principal one.

The compatibility of this way of conceiving rent with the theory presented in *Production of Commodities* is also demonstrated by the case of a tax in the form of a tithe – i.e. as a share of gross output – that Sraffa addressed very briefly in his book. Since little analysis exists on this case, some possible examples are put forward and discussed in section 4, where it is discovered that Sraffa's claim that 'a tax on a basic product then will [...] cause a fall in the rate of profits that correspond to a given wage' (p. 55) holds in almost all the cases considered even though, as we have shown, it is also possible – and precisely in the case with joint production that Sraffa refers to in the said passage – that the rate of profit increases when the tax share rises for a given wage rate.

In conclusion, we believe there are still many aspects and issues connected with the conception of rent as a share of the gross output of the commodities obtained by the use of natural resources that are not taken into consideration here in order to avoid the introduction of too many arguments all at once. Possibilities include reformulating the differential rent theory – especially with reference to mining rents – in terms of shares and using the conception of rent as a given share in order to include rents other than the differential ones¹⁸ within Sraffa's price equations.

Appendix: a numerical example

The following numerical example is presented here with the aim of clarifying the result showed in sec. 4.3 as regards the possibility, in the case of joint production, of a direct relation between the rent share on a basic output and the rate of profits associated with a given wage rate.

Let us assume the following system of production:

¹⁸ On this point, see also Fratini (2008) and, in particular, (2012).

Tab. 1 – *Technical coefficients*

	INPUTS			OUTPUTS	
	commodity [a]	labour		commodity [a]	commodity [b]
Activity 1	2	20	→	10	1
Activity 2	5	10	→	9	1

In this case, with commodity [a] as the numéraire, the price equations are:

$$(1 - \sigma_a)10 + (1 - \sigma_b)p_b = 2(1 + r) + 20w \quad [\text{a.1}]$$

$$(1 - \sigma_a)9 + (1 - \sigma_b)p_b = 5(1 + r) + 10w \quad [\text{a.2}]$$

Therefore, by subtracting equation [a.2] from equation [a.1], we obtain:

$$1 - \sigma_a = -3(1 + r) + 10w. \quad [\text{a.3}]$$

Two remarks can be made on equation [a.3]. First, as is well known, a direct rather than inverse relationship between w and r can emerge when commodities are jointly produced, and this is exactly what happens in the case we are considering. From equation [a.3], assuming $\sigma_a = 0$, we have:

$$w = \frac{4 + 3r}{10}. \quad [\text{a.4}]$$

and $w = 0.4$ is therefore the minimum wage rate compatible with the coexistence of the two production activities if $\sigma_a = 0$.

Second, given a wage rate, say $w = 0.41$, equation [a.3] brings about a direct relation between r and σ_a :

$$r = \frac{0.1 + \sigma_a}{3} \quad [\text{a.5}]$$

which corresponds to the red straight line in fig. 2.

The levels of the rate of profits and the price of commodity [b] in terms of [a] associated to different rent shares σ_a – with $w = 0.41$ and $\sigma_b = 0$ – are shown in tab. 2.

Tab. 2 – Rate of profits and price p_b associated to different rent shares σ_a
(with $w = 0.41$ and $\sigma_b = 0$)

σ_a	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0
r	0,03	0,07	0,10	0,13	0,17	0,20	0,23	0,27	0,30	0,33	0,37
p_b	0,27	1,33	2,40	3,47	4,53	5,60	6,67	7,73	8,80	9,87	10,93

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