International comparison of Environmental performance

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International comparison of Environmental Performance

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Abstract

In order to take into account the effects of production on the environment and, more generally, the urgency of finding a path of sustainable development many attempts have been made to set productivity growth measurements, including the negative impact of pollution, the production of goods and services generates. This paper present programs that have been developed to extend the measurement of total factor productivity and its components; technical progress and technical efficiency, to the consideration of environmental performance minimizing infeasibility problems sometime encountered with usual approaches using simple Malmquist indices. This study shows that the choice of a sequential index has a significant impact on productivity measures and on the comparison of the resulting performance.

Key words: productivity growth; environmental efficiency; DEA; directional distance; technological progress; technical efficiency; beta-convergence; sigma-convergence.
Introduction

Concern about climate change and global cooperative regulations such as "Rio de Janeiro (1992), Kyoto (1997), Johannesburg (2002), The Bali roadmap (2007), etc.", attest to needs for better controls over negative externalities related to production.

Regarding economic performance it suggests taking into account pollutants generated beside production of goods and services. In other words, it means that we must take into consideration undesirable outputs linked to desirable outputs in the measurement of productivity growth. In the context of Data Envelopment Analysis, usual models and indices built using distance functions to measure the total factor productivity must be adopted. In order to avoid feasibility problems that may arise when negative outputs (undesirable) are included in the optimization problem. [19], [7] and [2] have adapted efficiency and productivity measurements to the introduction of negative externalities in the technology of the Decision Making Units namely the 15 European countries and US included in this analysis.

This paper is organized as follows: The first part proposes a review of methodologies to be implemented in the second part, which presents empirical studies. The methodological part explores the Malmquist productivity index, including the implication of taking into account undesirable outputs in defining technological production in a DEA context and, finally, major advances in enhancing the Malmquist index in order to make it able to compute total factor productivity in a comprehensive way while minimizing infeasibility cases that could arise. This result is obtained by implementing a Malmquist-Luenberger index. In addition, a method to avoid technical regression of the production frontier - computing the so called Sequential Malmquist-Luenberger index - is discussed.

Methodologies are implemented in the empirical part. Detailed results and tests allow drawing some conclusions about competitiveness over the period 1995-2010 of 15 European countries and the US. Most of the hypothesis of the empirical study has been statistically tested. Notably, several tests have been deployed in order to analyze convergence dynamics of the efficiency score of the countries over the time period. In addition to the country comparison, general trends for all countries and specific results achieved by Luxembourg are highlighted.
Methodology

I. Measuring TFP using Malmquist productivity index.

Productivity is the ratio of production (output) to factors used to obtain it (inputs). Total factor productivity (TFP) measures the production obtained with respect to all factors of production; i.e. labor, capital and intermediate inputs.

Measuring the productivity of a DMU relative to a production frontier requires a distance measurement. Moreover, in a competitive environment, it is crucial to measure performance by comparing productivity as well as its evolution. In other words, the matter is not only the relative positioning to the efficient frontier at time t but also changes in pattern of relative positioning between units and shifts of the frontier itself over time. Using the quantity index of Sten Malmquist (1953) based on the radial type distance functions, and following [4] and [19] define the Malmquist productivity index output oriented for time t+1 by multiplying 2 ratios: the first refers to the distance of the DMU between t and t+1 compared to the technology of period t, while the second measures the distance to the frontier at time t+1. The Malmquist index is given by the geometric mean of the two ratios:

\[
M_{t+1}^T = \left[ \frac{d_0^t(x^{t+1}, y^{t+1})}{d_0^t(x^t, y^t)} \times \frac{d_0^{t+1}(x^{t+1}, y^{t+1})}{d_0^{t+1}(x^t, y^t)} \right]^{1/2}
\]

[20] have shown how to decompose this index in order to highlight the different sources of productivity growth - namely the technical efficiency change (EFFCH) measuring changes in the distance to the frontier of units and the technological change (TECH) corresponding to a shift in the efficiency frontier itself:

\[
M_{t+1}^T = \frac{d_0^{t+1}(x^{t+1}, y^{t+1})}{d_0^t(x^t, y^t)} \left[ \frac{d_0^t(x^{t+1}, y^{t+1})}{d_0^{t+1}(x^{t+1}, y^{t+1})} \times \frac{d_0^{t+1}(x^t, y^t)}{d_0^t(x^t, y^t)} \right]^{1/2}
\]

\[
M_{t+1}^T = EFFCH_{t+1}^T \times TECH_{t+1}^T
\]

EFFCH and TECH are computed using Data Envelopment Analysis techniques (EA) with linear programming.

Direct calculation of the Malmquist index and the construction of a nonparametric frontier have many advantages but also significant limitations.

Advantages:

- The Malmquist index remains consistent with economic theory even with no hypothesis regarding profit-maximization behavior producers. Indeed, there is no need to approximate marginal productivity though prices, unlike a traditional Divisia index [41];
- Data on prices and costs of the fixed factors are not needed;
- No econometric estimation is required, only an approximation of the production envelope, which is much simpler to implement.
Drawbacks

Thus, the assumptions used in these approaches are less restrictive than alternative measures of TFP. However, the use of the Malmquist index encounters two major limitations.

On one hand, when taking into account environmental impacts, the implementation of a measure of radial distance cannot easily handle negative productions or undesirable outputs.

On the other hand, setting a distinct frontier from observations (DMU) annually available may lead to a shifting back of the frontier seen as a "technological regress". The concept has to be justified from an economic point of view. One can raise an objection, as [40] did. These authors emphasize the weakness of a theoretical base for technological decline in the standard DEA model.

The Malmquist productivity index has a pioneering role in using non-parametric approaches for measuring productivity growth. The main advantage of these approaches is that they do not require data on prices of inputs and outputs (desirable and undesirable) or a functional form to describe the entire production. However, based on a radial distance measure, the Malmquist index results from a distance ratio calculated by an optimization program that considers only the way to increase maximum positive amounts of outputs produced (i.e. along a radial axis), while maintaining constant amounts of inputs. However, considering undesirable outputs, one would like the program to allow good outputs to increase while reducing bad output quantities produced. In this respect, [36], based on [28], suggest using a directional distance function. These contributions were widely quoted in subsequent work which applied the suggested methods: [7], [22], [42], [34], and [31], [43] and [27] for instance.

Several approaches have been explored to overcome limitations of the Malmquist productivity index. Among them, a Malmquist index enhanced into a Malmquist-Luenberger (ML) index has to be carefully considered. Indeed, DMU are able to increase the production of desirable outputs while reducing the outputs of events. In addition, the Index Sequential Malmquist-Luenberger (SML) introducing a sequential method when setting the production frontier prevents any regression technique. These two approaches are now detailed in the following paragraphs.

II. Production technology with pollutants

Pollutants are negative environmental externalities of production. It can be seen as an undesirable output tied to those of the goods and services commonly taken into account in the measurement of productivity.

The main issue is to find a way to consider negative outputs that should be minimized instead of being maximized as usual when considering outputs. One can proceed by modifying variables measuring pollutants as negative outputs applying a monotonically decreasing function [37].

In this respect, undesirable transformed outputs can be introduced into the model alongside desirable outputs and maximized as they are. Thus, the original values of the undesirable outputs are indeed minimized.

[44] introduced the axiom of weak disposition of undesirable outputs for their model based on radial distance measurements. Nevertheless, the authors argue that in some circumstances it may be
difficult to compare some DMUs only throughout their environmental performance index (EPIs) because of the weak discriminating power of radial DEA efficiency measures. Since non-radial DEA models usually have a higher discriminating power in evaluating the efficiencies of DMUs in practice it may be more practical to incorporate different environmental DEA technologies with the non-radial DEA efficiency scores [44]. A directional distance functions approach has been developed by [36] to further the work [28]. These metrics allow asymmetric treatment of desirable and undesirable outputs. Formally, for the DMUs producing a vector of desirable outputs $y$ and a vector of undesirable output $b$ from a vector of inputs $x$, the directional distance functions are defined as follows:

$$
\tilde{D}_o(x, y, b, g) = \max \{ \delta: (x, y, b) + (\delta g_x, \delta g_y, \delta g_b) \in P(x) \} \tag{4}
$$

Where $g = (g_x, g_y, g_b)$ is a vector defining direction. Several directions can be considered. For instance [7] use the following:

$$
g = (g_y, g_b) = (y, -b), \tag{5}
$$

Those directional distance functions measure the maximum increase of desirable outputs simultaneously to a proportional reduction of the production of undesirable outputs, given a fixed amount of inputs. Formally, such directional distance is defined as:

$$
\tilde{D}_o(x, y, b; y, -b) = \max \{ \delta: (x, y, b) + (\delta y, -\delta b) \in P(x) \} \tag{6}
$$

The calculated distance $\delta$ whose value of 0 to 1. Finally, directional distance functions appear to be general framework including the radial distance function as a special case.

### III. Measuring TFP using Malmquist-Luenberger productivity Index

[7] develop the Malmquist-Luenberger index (ML) based on directional distance functions, which is defined as:

$$
ML_{t+1} = \left[ \left( \frac{1+\tilde{D}_o^t(t)}{1+\tilde{D}_o^t(t+1)} \right) \left( \frac{1+\tilde{D}_o^{t+1}(t)}{1+\tilde{D}_o^{t+1}(t+1)} \right) \right]^{1/2} \tag{7}
$$

The first term in brackets measures the «shift» of the DMU occurred from time $t$ to $t+1$ compared to the technology of period $t$.

The Second term measures the movement of the DMU from the technology of period $t+1$. As noted above, the index results of the calculation of a geometric mean and its interpretation is the same as for the Malmquist index. In addition, it can also be broken down to show the main sources of productivity change.

The graph below illustrates the calculation of the ML TFP index from the point of view of any DMU $A$. It describes the shift from $t$ to $t+1$, for $A$, a DMU producing a desirable output $y$ and an undesirable output to $b$ from an amount of input $x$ fixed over time.

**Figure 1: ML calculation**
From t to t+1, DMU A reaches a higher level of y with less b, nevertheless it is closer to the best practices available in t. It means that the DMU lost in efficiency.

**Drawback**

Despite its widespread use, the Malmquist-Luenberger has some weaknesses:

- As well as the usual Malmquist index, the ML index results of the geometric mean of two terms. The first measures the move relative to the technological frontier of the period. The second term measures the shift toward the t+1 frontier. Therefore, “ML index faces potential LP infeasibility problems in measuring cross-period directional distance functions. Furthermore, a geometric mean of two contemporaneous ML indexes is not circular.” [33]
- ML index tolerates technological decline, in effect, a theoretical point of view, even if it seems less likely, a deterioration of technical progress cannot be excluded. Improvement ML index consists in choosing the sequential approach which eliminates the possibility of measuring the technological decline.

These shortcomings of the conventional ML index that may provide biased measures of productivity growth, led to the introduction of an alternative methodology that can use both desirable and undesirable outputs and inputs to the measure of the environmental performance in order to overcome the drawbacks of the conventional ML index.

Because reducing Greenhouse gas (GHG) regulations differ from country to country, it is needed to take into account impact of country-specific regulations on productivity growth. [36] developed the directional distance function based on the work of [28] allowing an asymmetric treatment of good "outputs" and bad "outputs".

**The underlying assumptions**

First, let’s introduce the assumptions on technology production and definitions of directional distance functions providing the components for computing productivity indicators.
Production technology transforms:

Inputs: \( x = (x_1, \ldots, x_n) \in \mathbb{R}_+^n \) into desirable outputs \( y = (y_1, \ldots, y_m) \in \mathbb{R}_+^m \) and undesirable outputs \( b = (b_1, \ldots, b_j) \in \mathbb{R}_+^j \)

For each time period \( t \), the production possibility set \( T \) summarizes the set of all feasible input and output vectors and is defined as follows:

\[
T = \{(x, y, b) \in \mathbb{R}_+^{n+m+j} : x \text{ can produce } (y, b)\}
\] (8)

Alternatively, technology can be characterized by its output set

\[
P(x) = \{(y, b) : x \text{ can produce } (y, b), x \in \mathbb{R}_+^n \}.
\] (9)

Standards axioms

**P1: Inaction**

\[(0,0) \in P(x), \forall x \in \mathbb{R}_+^n \text{ et } (y, b) \notin P(0) \text{ if } (y, b) \geq 0 \text{ et } (y, b) \neq 0\]

**P2: \( P(x) \) is compact**

\( x \in \mathbb{R}_+^n, P(x) \text{ is bounded } \) and \( P(x) \) is a closed correspondence, \( \forall x \in \mathbb{R}_+^n \Rightarrow P(x) \text{ is compact} \)

This mathematical property ensures achieving optimum during any optimization programs.

**P3 Strong Input Disposability:**

\[x' \geq x \Rightarrow P(x') \supset P(x)\]

If some inputs are increased, outputs do not decrease

Axioms related to the presence of bad outputs.

**P4 Weak Output Disposability:**

\[(y, b) \in P(x) \Rightarrow (\theta y, \theta b) \in P(x) \forall \theta \in [0,1]\]

For a given level of inputs, it is always possible to proportionally reduce the level of outputs (each of the outputs can be reduced by factor \( \theta : 0 \leq \theta \leq 1 \)). This assumption states that a reduction of the bad outputs is not costless and negatively influences the production level of the good outputs.

**P5 Strong desirable output Disposability:**

\[\forall (y, b) \in P(x) \text{ if } (0,0) \leq (y', b) \leq (y, b) \text{ then } (y', b) \in P(x)\]

This assumption implies that a reduction of the good outputs is feasible without a simultaneous reduction of the bad outputs. With axiom P4, P5 emphasizes the asymmetry between the good and the bad outputs insofar as good outputs are costless disposable but bad outputs are not (Färe et al., 2001).
**P6: null-jointness**

\[(y, b) \in P(x) \text{ if } b = 0 \text{ then } (y, b) = 0\]

This means that if no bad outputs are produced, then there can be no production of good outputs. Alternatively, if one wishes to produce some good outputs then there will be undesirable byproducts of production.

A production technology that satisfies these assumptions can be represented by a directional output distance function. Introduced by Chambers et al. (1996, 1998), it can be formally defined on \(P(x)\) as:

\[
D_o(x, y, b; g_y, g_b) = \max\{\beta: (y, b) + (\beta g_y, \beta g_b) \in p(x)\}
\]  

(10)

where \(g = (g_y, g_b) = (y, -b)\), and \(\beta\), respectively, represent the direction and proportion in which the output vector \((y, b)\) is scaled to reach the boundary or frontier of the output set \(P(x)\). The directional output distance function value \(\beta\) is bounded below by zero. A value of zero identifies the observed output vector as located on frontier and, hence, as being technically efficient. Values greater than zero belong to output vectors within the frontier, indicating technical inefficiency.

Figure 2: Directional distance

The production frontier shown in the figure above can be used to illustrate the meaning of directional measurement functions. The directional distance function aims to expand good output and to contract bad output simultaneously. Moreover, the function treats good output and bad...
output asymmetrically. It means that, increasing $y$ can be obtained with simultaneous decreasing in $b$ at the same factor $\theta$ applied to 2 different amounts $y$ and $b$.

- Line segment $DD''$ depicts the case when the good and bad output are treated symmetrically (i.e., good and bad output production expand).
- The line segment $DD'$ depicts the directional distance function in which the good and bad outputs are asymmetrically treated (i.e., good output production is expanded while bad output production contracts). Strong disposability (P5) implies that a DMU can reduce bad output without incurring any abatement costs.
- This result comes from the specific axiomatic underlying directional functions measurement.

$P5 \Rightarrow P4 \text{ but } P4 \nRightarrow P5$ Because axiom of strong desirable output disposability (P5) hold then weak output disposability (P4) holds too, good output defined throughout constraints resulting of P4 axiom may be reduced. Indeed, for a given level of inputs, it is always possible to proportionally reduce the level of output by the same factor either if it is a good or a bad output.

### IV. Preventing Technological regress using Sequential Malmquist-Luenberger (SML)

As mentioned above, [40] firstly expressed their doubt regarding technological decline concept that may arise from the data in the standard DEA approach. [21] argued "one of the disadvantages of the standard DEA model is that this method allows the DMUs to ignore all previous technologies (no memory process)." Of course, such a situation is theoretically possible- it may be the case of countries economy seriously affected by the war. Nevertheless, it cannot be reflect the more general situation. In order to avoid technological decline as result of yearly data envelopment process, alternative approach has been suggested [40]. Thus, the frontier of period $t$ "envelopes" all the data observed so far- [39] named sequential this method. Against conventional contemporaneous ML productivity index [7], building output set $P^t(x^t)$ in time period $t$ by using only observations of each period, SML index incorporates past information including all observations from period 1 up to period $t$. Sequential output set in period $t$ is defined as:

$$\bar{P}^t(x^t) = P^1(x^1) \cup P^2(x^2) \cup \ldots \cup P^t(x^t),$$

(11)

where $1 \leq t \leq T$

In order to calculate the SML productivity index, four directional distance functions have to be specified:

- two functions based on observations and sequential output set generated into the same period, $D^t_0(x^t, y^t, b^t; g^t_0, g^t_b)$ and $D^{t+1}_0(x^{t+1}, y^{t+1}, b^{t+1}; g^{t+1}_y, g^{t+1}_b)$
- and two functions using observations and the sequential output set coming from different periods, $D^t_0(x^{t+1}, y^{t+1}, b^{t+1}; g^{t+1}_y, g^{t+1}_b)$ and $D^{t+1}_0(x^t, y^t, b^t; g^t_0, g^t_b)$.

Let $D^t_0(t), D^{t+1}_0(t + 1), D^t_0(t + 1)$ and $D^{t+1}_0(t)$, respectively, be those four functions. SML index of productivity changes from period $t$ to $t+1$ is the geometric mean of Malmquist-Luenberger productivity index over two periods:
The first term in brackets measures the shift of the DMU analyzed from $t$ to $t+1$ compared to the technology of period $t$. The second term measures the movement of the DMU compared to the technology of period $t+1$. Interpretation of the results is the same as the Malmquist index and it can be also decomposed in order to express the main sources of productivity change:

$$
SML_{t}^{t+1} = SMLEC_{t}^{t+1} \times SMLTC_{t}^{t+1}
$$

(13)

Where $SMLEC_{t}^{t+1}$ factors increase technical efficiency, comparing the distances to the frontier of best practices and measuring the growth of technical efficiency from time period $t$ to $t+1$, while $SMLTC_{t}^{t+1}$ measures the technological progress as a geometric mean, $SML_{t}^{t+1} = 1$ means that there is no change in the inputs and outputs ratios from periods $t$ to $t+1$.

$SML_{t}^{t+1} > ( < ) 1$ results from an increase (decrease) in productivity. One can noticed that the above condition implies a stable relationship between the two types of outputs.

In addition, $SMLTC_{t}^{t+1} > 1$ indicates a catching up or convergence process to the frontier to period $t+1$. It can be interpreted as a technical efficiency improvement.

Similarly, $SMLTC_{t}^{t+1} < 1$ indicates that the DMU moves away from the $t+1$ frontier with respect to $t$. That is to say, it is divergent and less efficient.

Technological change in the SML index is always greater than unity since $\vec{D}_{0}^{t+1}(x^s, y^s, b^s) \geq \vec{D}_{0}^{t}(x^s, y^s, b^s)$. If technical change allows greater production of desirable outputs and less production of undesirable outputs than $SMLTC_{t}^{t+1} > 1$, otherwise $SMLTC_{t}^{t+1} = 1$ (given a sequential index that not allows technical regress i.e. $SMLTC_{t}^{t+1} < 1$ is not possible).

The sequential directional output distance functions can be worked out using linear programming techniques.

$$
D_{0}^{t}(x_{k0}^{t}, y_{k0}^{t}, b_{k0}^{t}, y_{k0}^{t}, -b_{k0}^{t}) = \max \beta
$$

s.t.\[
\begin{align*}
\sum_{t=1}^{T} \sum_{k=1}^{K} \lambda_{k}^{T} y_{k,m}^{t} & \geq (1 + \beta) y_{k0,m}^{t}, & m = 1, ..., M, \\
\sum_{t=1}^{T} \sum_{k=1}^{K} \lambda_{k}^{T} b_{k,j}^{t} & = (1 - \beta) b_{k0,j}^{t}, & j = 1, ..., J, \\
\sum_{t=1}^{T} \sum_{k=1}^{K} \lambda_{k}^{T} x_{k,n}^{t} & = x_{k0,n}^{t}, & n = 1, ..., N, \\
\lambda_{k}^{T} & \geq 0, & k = 1, ..., K,
\end{align*}
\]

(14)

where $\lambda_{k}^{T}$ is an intensity variable to shrink or expand the individual observed activities of DMU $k_0$ for the purpose of constructing convex combination of the observed inputs and outputs.
The first and third inequality constraints made to the DMU $k_0$ does not produce more good outputs and does not use fewer inputs than its efficient benchmark on the frontier. The first inequality constraint (good) outputs and the second strict equality constraint (bad outputs) impose weak disposability of the good and the bad outputs.

Finally, the zero bound binds constraints on the intensity variables indicate that the production technology exhibits constant returns to scale [7]. The solution to the program $D^g_i(t)$ above; a maximum value for $\beta$, indicates how much the good and the bad outputs can be proportionally expanded or contracted relative to the efficient benchmark on the frontier, given inputs.

The linear programs for the other three directional distance functions, $D^g_i(t + 1)$, $D^{i+1}_e(t)$, $D^{i+1}_e(t + 1)$, are obtained respectively by substituting $t$ with $t+1$ respectively on the right hand side, on the left hand side, and on both sides of the constraints into the three constraints of the linear program.

### Empirical Study

This empirical study has been developed implementing methods presented above. The dataset uses mainly official information provided by public bodies and published. Before going further, the hypothesis used have been tested in order to select the one which seems to best fit our dataset. Thus, a return to scale has been tested, as well as convergence or divergence movement of the DMUs against the efficiency frontier. Finally, the most important choice is the method used to define the frontier. Applying yearly data to the ML or sequential approach with SML may lead to drastically different conclusions, notably for the smallest countries. That is the reason why results for both methods are systematically presented and compared.

When contents of the database and return to scale hypothesis are presented, we will analyze results summarized in geometrical means by country then by year. Finally we will highlight rankings and trend for one country: Luxembourg.

### Data base

The data come from various tables provided by EUROSTAT, EUKLEMS, the UNFCCC and the National Accounts Division of STATEC.

For the purposes of calculation, we use four sets of variables for the period 1995 to 2010, and a total of 15 countries of the European Union and the United States (U.S.). Gross Domestic Product is used as a proxy variable of the desirable output, and CO$_2$ is a proxy of the undesirable output. Labor force, capital stock, and commercial energy consumption are describe the inputs of production technology-

1. Production (GDP): The series are converted in purchasing power parity (PPP) using rates provided on EUROSTAT website\(^1\), to ensure the comparability of aggregates between countries.

\(^1\) [http://epp.eurostat.ec.europa.eu/portal/page/portal/purchasing_power_parities/data/database](http://epp.eurostat.ec.europa.eu/portal/page/portal/purchasing_power_parities/data/database)
2. The (Employment): The use retained reference to a concept of domestic employment which includes both resident and non-residents. It is measured by the number of jobs furnished by Eurostat.

3. K (capital stock): Estimates of capital stock are constructed from the data of capital stock of the database EUKLEMS and investment series obtained from Eurostat, except for Luxembourg for which data are from STATEC.


The data are used to calculate the Malmquist-Luenberger index (ML) conducive to the consideration of undesirable outputs (GHG) and the Sequential Malmquist-Luenberger index (SML) which minimize cases of infeasibility and prohibit technological regression.
II. Testing returns to scale

Computing the Malmquist Total Factor Productivity index does not require specifying a technology satisfying any type of returns to scale.

Method used to select the return to scale hypothesis that best fit our data is explained in the Box 1. After testing the equalities of the means of scores estimated by different models under constant return to scale (CRS), variable return to scale (VRS), non-increasing return to scale (NIRS) and non-decreasing return to scale (NDRS). The table below summarizes the results obtained when testing return to scale:

<table>
<thead>
<tr>
<th>All observations</th>
<th>H0</th>
<th>t-statistic</th>
<th>p-value one-tail</th>
<th>p-value two-tail</th>
<th>Reject HD</th>
<th>interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>ML</td>
<td>VRS=CRS</td>
<td>10.71</td>
<td>0,000</td>
<td>0,000</td>
<td>Yes</td>
<td>significantly differ</td>
</tr>
<tr>
<td></td>
<td>NIRS=NDRS</td>
<td>7.92</td>
<td>0,000</td>
<td>0,000</td>
<td>Yes</td>
<td>significantly differ</td>
</tr>
<tr>
<td>SML</td>
<td>VRS=CRS</td>
<td>11.52</td>
<td>0,000</td>
<td>0,000</td>
<td>Yes</td>
<td>significantly differ</td>
</tr>
<tr>
<td></td>
<td>NIRS=NDRS</td>
<td>10.33</td>
<td>0,000</td>
<td>0,000</td>
<td>Yes</td>
<td>significantly differ</td>
</tr>
</tbody>
</table>

The table shows that all assumptions of equality (Ho) are rejected since the p-value is less than 0.05. Indeed, test statistics are larger than 1.645, the upper 5% value from standard normal distribution. Thus, in all cases, rejecting the null hypothesis is accepted VRS on one hand, and the NIRS on the other hand. Since NIRS is nothing other than (DRS, CRS), the hypothesis VRS can be accepted.

To summarize the main findings of the test, production frontier coming out of our data is characterized by non-increasing return to scale. It means that return to scale may be constant or decreasing depending on the part of the frontier. Therefore, returns to scale vary and the following results have been obtained under VRS hypothesis. Results are summarized on the following graph where both ML indexes and SML indexes are calculated against a frontier with VRS. Respective TFP trend of geometric means for all countries are presented using 1995 as reference year.
Calculating the Malmquist TFP does not generally require a specific type of technology. In principle, one may calculate Malmquist productivity indexes relative to any type of technology (i.e., satisfying any type of re-turns to scale). All the distances can be computed whether the technology exhibits variable returns to scale or constant returns to scale.

However, [15] use a simple one-input one-output example to illustrate that the Malmquist TFP index may not correctly measure TFP changes under variable returns to scale technology.

Most of the studies adopt the constant returns to scale frontier as a benchmarking technology. There are several studies that find constant returns to scale in developing countries and increasing returns to scale in developed countries-[16], [17], [35], [6], [1] and [14] find for Haryana state of India for the years 1996-97 to 1998-99 that wheat cultivation in the state experienced constant returns to scale, as the sum of input elasticities (in the Cobb-Douglas production function) was 1.01.

As has been said above, there are some models with different assumptions in DEA; CCR, IRS, DRS, and BCC. [8]. A CCR model is proposed by [5] and assumes the frontier to be constant returns to scale.

This paper is based on tests using relationships among the means of scores estimated by different models, for trying to clarify the issue of returns to scale.

It consists to test the equality of \( \bar{\theta}^{\text{NIRS}} = \bar{\theta}^{\text{NDRS}} \) and \( \bar{\theta}^{\text{VRS}} = \bar{\theta}^{\text{CRS}} \) using the test of equality of means or sign test. The reason why we adopt sign test is that means can be easily affected by outliers. When the alternative hypothesis that \( \theta^{\text{NIRS}} > \theta^{\text{NDRS}} \) is accepted, the true frontier function can be regard as increasing returns to scale. When the hypothesis \( \theta^{\text{NIRS}} = \theta^{\text{NDRS}} \) is not rejected, the true frontier function can be regard as constant returns to scale.

The test statistics used to compare two means is the student test given by:

\[
\frac{\bar{Y}_1 - \bar{Y}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}
\]

Where \( S \) stands for standard deviation and \( n \) is the number of DMUs.

Most statistical software provides the ability to write procedures to automate calculations, like R through the function t.test (...).

The test score is calculated annually for all DMUs at first and then for the entire panel where each observation is considered as a DMU.

The following testing procedures using are suggested.

**Test of equality of the mean of NIRS and that of NDRS scores.**

The tests are based on two assumptions.

i. \( H_0: \theta^{\text{NIRS}} = \theta^{\text{NDRS}} \) \( \text{Vs.} \ H_1: \theta^{\text{NIRS}} > \theta^{\text{NDRS}} \)

ii. \( H_0: \theta^{\text{NIRS}} = \theta^{\text{NDRS}} \) \( \text{Vs.} \ H_1: \theta^{\text{NDRS}} > \theta^{\text{NIRS}} \)

**Test of equality of the mean of CRS and that of VRS scores**

The tests are based on two assumptions.

i. \( H_0: \theta^{\text{CRS}} > \theta^{\text{VRS}} \) \( \text{Vs.} \ H_1: \theta^{\text{CRS}} > \theta^{\text{VRS}} \)

ii. \( H_0: \theta^{\text{CRS}} > \theta^{\text{VRS}} \) \( \text{Vs.} \ H_1: \theta^{\text{CRS}} > \theta^{\text{VRS}} \)

In general and particularly in social research, we use the 5% p-value as a boundary at which to assume we have evidence to reject the null hypothesis, i.e. if \( p \leq 5\% \), the difference in our samples is unlikely enough, given the null hypothesis, that we infer there is a difference in the population; if \( p > 5\% \) we do not think there is sufficient evidence of a difference in the population and stick with the null hypothesis. This boundary is, of course, arbitrary and it makes sense to interpret data with intelligence and reflection rather than making decisions with a simplistic rule. Indeed, in some applications, \( p \leq 10\% \) is deemed acceptable, in others 0.1% is used. Also, sample size affects the p-value, with larger sample sizes tending to give smaller p-values, so it makes sense to interpret them along with the difference (or effect size) measured in your sample data.
Results clearly show that, despite the variations, outcomes are similar and trends go up and down the same year for both indices, the sequential index growth is much faster. Over a 15 year period, the latter accumulated more than 8 percentage points over the non-sequential Malquist-Luenberger index.

The cumulative productivities of our sample using the two productivity indices are depicted in Figure 3. In this chart, the productivity growth of the first year is adjusted to unity so that the developments of the two measures are easily compared. Even though temporal developments of productivity growth measured by the two methodologies are similar to each other, as discussed earlier, their cumulative versions are apparently different in two ways. First, the productivity measures diverge over time. The cumulative productivity growth for the study period measured by the SML index is 6.7%, and the one measured by the ML index is 1.3%. Second, the cumulative productivity of the SML index is larger than unity for whole study period, whereas that of the ML index is less than unity in 1996.

By taking into account the environmental performance, the positively cumulated productivity growth of the SML index appears to reflect changes better than that of the ML index.
III. Countries analysis

Results are first presented by country. Average growth of productivity, technical efficiency change and technological progress are calculated for the countries using a Luenberger-Malmquist Index and a Malmquist-Luenberger sequential index.

Analysing this results, we will show:

1. The selection of the method for building the frontier and hence, the model we believe is prevalent, is an important matter because results are slightly different.
2. Productivity measurements scored by their geometric mean are also different country by country. The second step of the analysis aims to highlight findings related to countries heterogeneity.
3. Finally, the concept developed by [22] in defining innovator countries has been used to identify countries drivers in building the frontier for benchmark purpose.

Ranking countries

Figures represent the geometric mean changes for the whole period in total factors productivity for each country.

Both ML and SML index have been computed. Results for each are presented in two different tables. In each table, changes in TFP are presented together with their component and ranking on behalf of the TFP mean. The measurements obtained are presented in the following tables. It may be recalled that when the value of the geometric mean of the index is greater (less) than 1, this means a positive growth rate (negative), that is to say an improvement (deterioration) of the technical efficiency (EC), technical progress (TC) or total factor productivity (TFP). In addition, EC and TC are the components of TFP. When expressed as rate of growth TFP is the sum of the two.
When comparing ML and SML indices, both taking into account environmental performance throughout, energy consumption into the input list and CO2 as negative output, it appears that:

1. Since technical progress is always positive or zero, the average growth for this component obtained with the sequential index (SML) is always higher than or equal to the non-sequential measurement (ML).

2. All countries observe a decline in their average technical efficiency (except IE). Luxembourg, Ireland, Sweden, United States and Germany have the highest growing technical progress when technological declines are no longer allowed in the model.

3. The ranking of 4 less performing countries are the same i.e. Greece, Portugal, Italy, and UK. Nevertheless, by using sequential frontier, Greece is the only one that keeps a negative geometric mean for its TFP from 1995 to 2010 while countries having experienced a decline in the evolution of TFP measured by the geometric mean of annual changes in the ML index are the four above mentioned.

4. Moreover, the most dramatic move in the ranking is experienced by Ireland, Luxembourg and the U.S. Instead on being ranked in 10th to 12th ranking with ML, all of them come up in the 5 first places with SML index.

Additionally, technical efficiency gains are often lower than the non-sequential measurement, except for Ireland where technical efficiency gain is almost identical in the two measurements.
Finally, an implementing sequential index gives TFP measurements that are, on average, always higher than ML index. This findings result in a huge modification of the ranking that has to be statistically tested. When testing hypothesis of equality of TFP index and its components calculated using the non-sequential (ML) or sequential method (SML), the equality of ranking hypothesis is indeed rejected.

<table>
<thead>
<tr>
<th>Box 2 – Test of equality of ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>The ranking of countries according to the average annual growth rate of their TFP is given for the growth rates obtained from the calculation of the Malmquist-Luenberger (ML-EP) and the sequential index that integrates environmental performance (SML-EP).</td>
</tr>
<tr>
<td>The comparison of the ranks of the top 5 changes (only DE, SE remain in this group, losing two positions each. At the bottom of the ranking changes for the last five with the same countries (GR, IT, PT, UK SE remain in this group; IE is replaced by ES)</td>
</tr>
<tr>
<td>The ranking changes when comparing ML and SML indices.</td>
</tr>
<tr>
<td>This is confirmed by the Wilcoxon test where Ho hypothesis of equality is rejected.</td>
</tr>
<tr>
<td><strong>Ho:</strong> sml = ml</td>
</tr>
<tr>
<td>z = 3.103</td>
</tr>
<tr>
<td>Prob &gt;</td>
</tr>
</tbody>
</table>

**Countries heterogeneity**

One important question in the literature regarding productivity measurement and international comparison aims to specify degree and sources of heterogeneity of countries performance. From a statistical point of view testing heterogeneity copes with concept of beta and sigma convergence. We are going to test heterogeneity of DMUs (i.e. countries) in our data set by using the concept of beta and sigma convergence. Results for beta convergence are significant presence of transversal convergence – that is to say that less performing countries seems to catch up to the best performing countries following a process of various length. In order to check and confirm the result of beta convergence test that is necessary but not sufficient, a sigma test has been implemented. It shows that the process is not continuous over the time period and convergence process has been occasionally stopped. It is probably because the overall process covers different convergence path for each country. It is suggesting that panel convergence test should enhance understanding of the on-going dynamics. Finally, panel convergence test allows to conclude that each country converge though its steady state over time. Step of this tests and analysis are now explained.

**Beta and sigma –convergence**

The results in the table show a movement of absolute convergence for all countries in the sample. The β coefficients are negative and significantly different from zero. Hypothesis testing H0: β=0 against H1: β>0 leads us to reject the null hypothesis. We can conclude that
the absolute convergence of the least effective to the production frontier formed by the more powerful.

Box 3 - Beta-convergence

Beta-convergence refers to a process in which less performing DMUs grow faster than more performing ones and therefore catch up on them. It consists of regression of the score efficiency of the series on the initial level of the series. One can say that, there is convergence if the coefficient of the initial level Beta is negative. Mathematics form is given by:

\[
\frac{1}{T} \ln \left( \frac{\theta_{k,T}}{\bar{y}_{k,T_0}} \right) = \alpha_k + \beta \ln(\theta_{k,T_0}) + \varepsilon_k e_i - i.i.d(0,\sigma^2)
\]

Where:

- \(0 < \beta < 1\)
- \(\theta_k\) is the score efficiency and \(T\) is the total period of analysis.

The speed of convergence to the benchmark is calculated from the equation: \(\Phi = - \ln(1+\beta T) / T\)

The half-life is computed by the formula:

\[
\tau = \frac{-\ln(2)}{-\ln(1+\beta)} = \frac{-\ln(2)}{-\Phi}
\]

\(\tau\) years is the time it would take to halve the gap between the efficiency scores and the total benchmark. The greater \(\beta\), the faster is the convergence process.

\(\beta\) is negative both for ML and SML. The estimated coefficients are significant when considering both ML and SML model.

Results are given in the following table:

<table>
<thead>
<tr>
<th>Result of the Beta-convergence test</th>
<th>(\beta)</th>
<th>Std.Err</th>
<th>t</th>
<th>R²</th>
<th>(\Phi) (%)</th>
<th>(\tau) (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ML Efficiency</td>
<td>-0.0325</td>
<td>0.0034</td>
<td>-9.6320</td>
<td>0.86</td>
<td>4.58</td>
<td>21</td>
</tr>
<tr>
<td>SML Efficiency</td>
<td>-0.0309</td>
<td>0.0125</td>
<td>-2.4663</td>
<td>0.30</td>
<td>4.26</td>
<td>22</td>
</tr>
</tbody>
</table>

Rhythms of convergence to the steady state are evaluated annually at a rate of 4.58% for ML and 4.26% for SML. The calculated half-life of \(\tau=21\) for ML and \(\tau=22\) years, this means that we should under 21 years and 22 years under ML SML to halve the gap between technical efficiency scores and benchmark.

The drawbacks of this test have been extensively discussed in the literature. The harshest criticisms were addressed by Quah in a series of articles (1993a, b, 1994, 1996), they relate to the interpretation of the results and seriously call into question the use of beta-convergence test.
Box 4 - Sigma-convergence

Sigma-convergence [26] refers to a reduction of disparities among the DMUs in time. Beta-convergence is necessary but not sufficient for Sigma-convergence. The most frequently used summary measures of Sigma convergence are the standard deviation or the coefficient of variation. [9] criticized the method of sectional regressions showing that this type of regression suffered so-called errors Galton. According to him, the best way to assess the convergence hypothesis is to exploit the temporal information included in the cross-sectional variance. [32] argues that the hypothesis of convergence is checked only if the variance of the observations is decreasing over time. Indeed, in this case, there is a reduction of disparities in levels of income per capita of all countries in the sample considered. [26] introduced the concept of sigma-convergence to explain this idea. While Beta-convergence focuses on detecting possible catching-up processes, Sigma-convergence simply refers to a reduction of disparities among DMU in time. The two concepts are of course closely related. Formally, Beta-convergence is necessary but not sufficient for Sigma-convergence. Intuitively, this is either because DMUs can converge towards one another but random shocks push them apart or because, in the case of conditional Beta-convergence, economies can converge towards different steady-states. The most frequently used summary measures of Sigma-convergence are the standard deviation or the coefficient of variation. However, other indices exist and present interesting properties (Gini coefficient, Atkinson index, Theil index and Mean Logarithmic Deviation).

In this paper, we use the coefficient of variation which indicates a high or low degree of variability only in relation to the mean value.

![Coefficient of variation](image)

**Coefficient of variation decreases 1996 and 2000 and increase in 2000 and 2010.**

Panel convergence

Results of sigma convergence presented on the figure show a curve partly increasing and decreasing. Thus, the sigma-convergence is not observed continuously. The phenomenon of σ-convergence is observed as discontinuous declines alternating with increases throughout the study period.

This leads us to look at another type of test more relevant: the test of convergence panel.

In addition new procedure of the convergence hypothesis using panel data has been developed. These procedures bring together cross-sectional and time series analysis. Two main approaches have been proposed. A first approach extends the methodologies designed for cross-sectional data, to the
analysis of panel data [29], [25] and [3]. The second approach uses unit root testing procedures for panel data.

The test summary is fairly detailed and reports the panel test result as well as the individual. ADF tests statistics. In this case the test rejects the null quite strongly. Thus, each country converges to its steady state over time.

<table>
<thead>
<tr>
<th>Box5 - Panel-convergence</th>
</tr>
</thead>
<tbody>
<tr>
<td>[18] method proposed to test convergence in panel has been applied. It characterizes</td>
</tr>
<tr>
<td>Let $y_{it} = y_{it} - \tilde{y}_t$, be the data generating process proposed by [18]</td>
</tr>
<tr>
<td>General process to test the convergence hypothesis is defined by the:</td>
</tr>
<tr>
<td>$y_{it} = \alpha_i + \lambda y_{it-1} + \epsilon_{it}$</td>
</tr>
<tr>
<td>[18] use the following functional for of the previous general process defined by:</td>
</tr>
<tr>
<td>$\Delta(y_{it} - \tilde{y}<em>t) = \alpha_i + \rho_i(y</em>{it-1} - \tilde{y}<em>{t-1}) + \sum</em>{j=1}^{p} \gamma_{ij}\Delta(y_{ijt-1} - \tilde{y}<em>{ijt-1}) + \epsilon</em>{it}$</td>
</tr>
<tr>
<td>Where all parameters $\rho_i$ are negative if the N economies converge and zero if they diverge, and where the roots of the polynomial $\sum_{j=1}^{p} \gamma_{ij} L^j$ are outside the unit circle. Parameters $\alpha_i$ denote individual effects without time dimension. Residual $\epsilon_{it}$ are assumed to be asymptotically uncorrelated in the individual dimension.</td>
</tr>
<tr>
<td>Considering the test advocated in [24], we obtain the following result:</td>
</tr>
<tr>
<td>Table: IPS test results</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>lags</th>
<th>obs</th>
<th>rho</th>
<th>trho</th>
<th>mean</th>
<th>var</th>
</tr>
</thead>
<tbody>
<tr>
<td>AT</td>
<td>3</td>
<td>-1.99972318</td>
<td>-3.77261096</td>
<td>-1.3300</td>
<td>1.5278</td>
</tr>
<tr>
<td>BE</td>
<td>0</td>
<td>-0.23428927</td>
<td>-0.98072146</td>
<td>-1.5140</td>
<td>0.9230</td>
</tr>
<tr>
<td>DE</td>
<td>0</td>
<td>-0.28553005</td>
<td>-2.73620281</td>
<td>-1.5140</td>
<td>0.9230</td>
</tr>
<tr>
<td>DK</td>
<td>2</td>
<td>-0.85293744</td>
<td>-3.43378414</td>
<td>-1.3598</td>
<td>1.2152</td>
</tr>
<tr>
<td>ES</td>
<td>3</td>
<td>-0.43768754</td>
<td>-3.20239047</td>
<td>-1.3300</td>
<td>1.5278</td>
</tr>
<tr>
<td>FI</td>
<td>1</td>
<td>-0.40368939</td>
<td>-2.78988849</td>
<td>-1.5000</td>
<td>1.0598</td>
</tr>
<tr>
<td>FR</td>
<td>1</td>
<td>-0.92875344</td>
<td>-3.26001822</td>
<td>-1.5000</td>
<td>1.0598</td>
</tr>
<tr>
<td>GR</td>
<td>1</td>
<td>-0.56410831</td>
<td>-2.46416801</td>
<td>-1.5000</td>
<td>1.0598</td>
</tr>
<tr>
<td>IE</td>
<td>0</td>
<td>-1.13426206</td>
<td>-4.43311611</td>
<td>-1.5140</td>
<td>0.9230</td>
</tr>
<tr>
<td>IT</td>
<td>1</td>
<td>-0.32409848</td>
<td>-2.92617071</td>
<td>-1.5000</td>
<td>1.0598</td>
</tr>
<tr>
<td>LU</td>
<td>0</td>
<td>0.02757409</td>
<td>0.10538631</td>
<td>-1.5140</td>
<td>0.9230</td>
</tr>
<tr>
<td>NL</td>
<td>2</td>
<td>-0.92425185</td>
<td>-3.55430553</td>
<td>-1.3598</td>
<td>1.2152</td>
</tr>
<tr>
<td>PT</td>
<td>0</td>
<td>-0.26107241</td>
<td>-1.74786636</td>
<td>-1.5140</td>
<td>0.9230</td>
</tr>
<tr>
<td>SE</td>
<td>1</td>
<td>-0.65185408</td>
<td>-1.60811202</td>
<td>-1.5000</td>
<td>1.0598</td>
</tr>
<tr>
<td>UK</td>
<td>0</td>
<td>-0.01130295</td>
<td>-0.05936313</td>
<td>-1.5140</td>
<td>0.9230</td>
</tr>
<tr>
<td>US</td>
<td>0</td>
<td>-0.59943819</td>
<td>-2.54789625</td>
<td>-1.5140</td>
<td>0.9230</td>
</tr>
</tbody>
</table>

Exogenous variables: Individual Intercepts ;Automatic selection of lags using AIC: 0 - 3 lags (max: 4);statistic : -3.837;p-value : 0

In order to determine which countries in which periods are ‘innovators’ the following set of conditions defined by [22]) is used:

**Innovators**
\[ SMLTC_{t}^{t+1} > 1 \]  
\[ D_{0}^{t}(t + 1) < 0 \]  
\[ D_{0}^{t+1}(t + 1) = 0 \]

The first condition indicates an outward shift of the best-practice frontier from period t to period t+1. That is in our model a shift towards more good outputs and fewer CO2 emissions (bad output). The second condition states that the country’s production in period t + 1 is located above the best-practice frontier of period t. This means that the country itself made technological progress. Finally, the third condition implies that the country’s production in period t+1 is located on the best-practice frontier in t+1.

The figure below lists the frontier shifting innovative countries for each consecutive two-year period in our three models.

<table>
<thead>
<tr>
<th>Years</th>
<th>Innovator countries</th>
</tr>
</thead>
<tbody>
<tr>
<td>1996</td>
<td>PT</td>
</tr>
<tr>
<td>1997</td>
<td>FR, UK, US</td>
</tr>
<tr>
<td>1998</td>
<td>FR, UK</td>
</tr>
<tr>
<td>1999</td>
<td>FR, US</td>
</tr>
<tr>
<td>2000</td>
<td>FR, IE, US</td>
</tr>
<tr>
<td>2001</td>
<td>FR, UK, US</td>
</tr>
<tr>
<td>2002</td>
<td>FR, UK, US</td>
</tr>
<tr>
<td>2003</td>
<td>US</td>
</tr>
<tr>
<td>2004</td>
<td>SE, UK, US</td>
</tr>
<tr>
<td>2005</td>
<td>UK, US</td>
</tr>
<tr>
<td>2006</td>
<td></td>
</tr>
<tr>
<td>2007</td>
<td>US</td>
</tr>
<tr>
<td>2008</td>
<td>SE</td>
</tr>
<tr>
<td>2009</td>
<td></td>
</tr>
<tr>
<td>2010</td>
<td>IE</td>
</tr>
</tbody>
</table>
Figure 4: Innovators countries shifting the frontier

Sources: Statec Eurostat, EUKLEMS, and UNFCC – Calculations by the authors using R
IV. Analysis of Trends

Changes in SML and ML productivity indices show similar trends during the period under review. Thus, the geometric mean of growth rates of TFP for all countries experiences a sharp decline for both indices between 2000 and 2001 and between 2004 and 2005 then a dramatic drop between 2007 and 2008 which continued until 2009. The rise is also spectacular for both indices between 2009 and 2010. However, the evolution of the indices decomposition is very different, while the ML index tends to allocate a portion of these declines regression technology in 2001 and in 2008 and 2009, the index sequential imputes more widely to loss of efficiency.

Box 6 - Test of equality of indices

We observe that the SML methodology, while excluding technological regress, has a similar profile to ML, in terms of total factor productivity, with a very strong correlation (0.9436). Observable deteriorations relate to the same years (2001, 2003, 2008 and 2009); except in 2002 for ML Equality between the different parameters could not be rejected, as shown in the table below.

<table>
<thead>
<tr>
<th>Null hypothesis</th>
<th>p-value</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>ML_EC = SML_EC</td>
<td>0.3066</td>
<td>H₀ is not rejected (p&gt;0.05)</td>
</tr>
<tr>
<td>ML_TC = SML_TC</td>
<td>0.0076</td>
<td>H₀ is rejected (p&lt;0.05)</td>
</tr>
<tr>
<td>ML = SML</td>
<td>0.0031</td>
<td>H₀ is rejected (&lt;0.05)</td>
</tr>
</tbody>
</table>

Observation: the p-value may vary if we change the sample, and generally, we cannot say that our sample is comprehensive for such a test.
### Geometric mean of TFP growth rates

<table>
<thead>
<tr>
<th>Years</th>
<th>Eff. Ch.</th>
<th>Tech. Ch.</th>
<th>TFP-SML</th>
<th>Years</th>
<th>Eff. Ch.</th>
<th>Tech. Ch.</th>
<th>TFP-ML</th>
</tr>
</thead>
<tbody>
<tr>
<td>1996</td>
<td>-0.43%</td>
<td>0.75%</td>
<td>0.32%</td>
<td>1996</td>
<td>0.03%</td>
<td>-0.57%</td>
<td>-0.54%</td>
</tr>
<tr>
<td>1997</td>
<td>0.42%</td>
<td>1.46%</td>
<td>1.89%</td>
<td>1997</td>
<td>0.55%</td>
<td>0.90%</td>
<td>1.46%</td>
</tr>
<tr>
<td>1998</td>
<td>-0.46%</td>
<td>0.99%</td>
<td>0.53%</td>
<td>1998</td>
<td>0.34%</td>
<td>-0.22%</td>
<td>0.13%</td>
</tr>
<tr>
<td>1999</td>
<td>0.39%</td>
<td>1.97%</td>
<td>2.37%</td>
<td>1999</td>
<td>0.53%</td>
<td>1.12%</td>
<td>1.66%</td>
</tr>
<tr>
<td>2000</td>
<td>0.79%</td>
<td>2.29%</td>
<td>3.10%</td>
<td>2000</td>
<td>0.45%</td>
<td>1.61%</td>
<td>2.07%</td>
</tr>
<tr>
<td>2001</td>
<td>-1.54%</td>
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<td>2001</td>
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<td>1.59%</td>
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</table>

Sources: Statec Eurostat, EUKLEMS, and UNFCC – Calculations by the authors using R

Figure 5: Decomposition of TFP according to the ML index-annual growth of the geometric mean-All countries

Sources: Statec Eurostat, EUKLEMS, and UNFCC – Calculations by the authors using R
Figure 6: Decomposition of TFP according to the SML index-annual growth of the geometric mean-All countries

Sources: Statec Eurostat, EUKLEMS, and UNFCC – Calculations by the authors using R
V. Results for Luxembourg

For Luxembourg, the measurement of TFP and its components by ML (Malmquist-Luenberger) and SML (Sequential Malmquist-Luenberger) index is presented in the tables below.

The ML index for Luxembourg detects 8 cases of deterioration (1997, 2001, 2003, 2005, 2006, 2007, 2008, and 2009) which is an almost equal number of cases than observed when considering the geometric mean of growth rates of countries, indicating that the means presented so far do cover a wide heterogeneity in the evolution of the situation in each country. In contrast, the total productivity SML index for Luxembourg is closer to the average with 5 cases of deterioration (2001, 2003, 2005, 2008 and 2009) as for all countries over the period 1995 to 2010.

Finally, there were 5 cases of damage to the SML index against 8 cases of damage to the ML index in the case of Luxembourg.

The sequential approach is particularly interesting in the case of Luxembourg. Indeed, when the technological regressions are authorized, Luxembourg is still on the frontier, and variations in results are entirely attributed to the movements of the efficient frontier and interpreted as technological declines. If we accept that a level of production reached in the past is part of the whole production achievable in subsequent periods, then a deterioration observed in Luxembourg can also be attributable to a decrease in technical efficiency. The charts below illustrate the point clearly.

In the sequential approach, Luxembourg experiences a continuous period of TFP growth entirely due to technical progress from 1996 to 1998. Then, the evolution of TFP deteriorates in 2001 in the absence of technical progress and with a sharp decrease in technical efficiency, the same phenomenon prevails in 2003, 2005 and again in 2008 and 2009. These results seem more consistent with the hypothesis of a loss of technical efficiency due to the delay in factor adjustments following a decrease in production.
### Table 6: Geometric mean of growth rates of the sequential Malmquist-Luenberger index

<table>
<thead>
<tr>
<th>Year</th>
<th>SMLEC</th>
<th>SMLTC</th>
<th>TFP - SML</th>
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<tr>
<td>2006</td>
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<td>1.6%</td>
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<tr>
<td>2010</td>
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### Table 7: Malmquist-Luenberger index for Luxembourg

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<th>Year</th>
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<tr>
<td>1997</td>
<td>0.0%</td>
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<tr>
<td>1998</td>
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<td>3.6%</td>
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<tr>
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<td>0.9%</td>
<td>0.9%</td>
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<td>2000</td>
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<td>2001</td>
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<td>2002</td>
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<tr>
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<td>0.8%</td>
</tr>
</tbody>
</table>

Sources: Statec Eurostat, EUKLEMS, and UNFCC – Calculations by the authors using R

**Figure 7: Environmental performance of Luxembourg - TFP and its components with the ML index**

Sources: Statec Eurostat, EUKLEMS, and UNFCC – Calculations by the authors using R
Figure 8: Environmental performance of Luxembourg - TFP and its components with the sequential ML index

Sources: Statec Eurostat, EUKLEMS, and UNFCC – Calculations by the authors using R
Conclusion

In this paper, different distance functions have been used building on the seminal radial distance function [38]. Indeed, taking into consideration negative externalities implies defining the order and the way to allow the modification in quantities of factors. Directional distance function implemented, has been proposed by [36]. It allows focusing on one or more factors of production with respect to others and works simultaneously in input and output. These methodological contributions have been used to measure the evolution of total factor productivity in 15 European countries and in the United States between 1995 and 2010. In addition, returns to scale have been tested. As a result, using variable returns to scale rather than constant returns to scale seems to fit better the data used in the empirical study. Indeed, [15] show that a Malmquist index can cause a bias in the measurement of productivity growth if the variable returns to scale characterize in fact the DMU technology analyzed.

Results of the study show that the consideration of environmental performance measured through the emission of greenhouse gases improves total factor productivity. Moreover, eliminating possibility of technological decline through the implementation of the sequential Malmquist-Luenberger index results in a TFP growth always higher than Malmquist-Luenberger index.

The analysis of temporal trends in SML and ML productivity indices for all countries reveals similar patterns of evolution during the period under review (1995-2010). However, this overall TFP trend covers very different developments of its components as it has been shown. While the ML index tends to attribute a greater part of TFP source of growth to technological changes. The sequential index tends to attribute decreasing of TFP to losses of efficiency gains.

When technological regressions are not allowed, Luxembourg reaches a higher level of performance measured throughout its ranking by geometrical means computed across years. Variations in results are partially attributed to efficiency loss.

An important limitation of the approach is its sensitivity to specific DMUs involved in the computation. In this respect it would be more accurate to develop a worldwide frontier including most countries to avoid too much impact on ranking when adding or suppressing one of them. In order to arrive at conclusions more accurately future contributions should use capacity utilization instead of stock of capital and the number of employees. Conducting the analysis would be deploy these measures in the context of an international comparison of performance across sectors of activity. Nevertheless, there will be a trade-off between integrating much more DMUs and availability of richer data for all of them.
References

[21] H. Forstner and A. Isaksson, Productivity, technology, and efficiency: an analysis of the world technology frontier when memory is infinite, SIN Working