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Tenryu, Yohei

Institute of Economic Research, Kyoto University.

October 2013

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Yohei Tenryu†

October 9, 2013

Abstract

Voracious behavior is one of the excess uses of the commons. It is known that the voracity effect can be observed in the economy with common and private capital. We explore another cause of voracious behavior and investigate the effects of voracious behavior on the economy. For this purpose, we introduce a new direction of capital flow. A government mandates that all groups invest their private capital in the common sector to mitigate the effects of excess use of the commons. We show theoretically that there is no standard voracity effect in the sense that Tornell and Lane (1999) define and that a group's equilibrium consumption strategy is the Markov control-state complementarity. We observe numerically that an increase in the contribution of the private sector into the common sector has a negative effect on growth. This implies that the policy for preservation of the commons leads to the harmful effect on the economy.

Keywords: differential game, Markov perfect equilibrium, voracity effect.

JEL Classification: C73, O10, O40

*We are grateful to Akihisa Shibata, Takashi Komatsubara, Takuma Kunieda, Real Arai, Akitoshi Muramoto, Tetsuya Hoshino, and participants in the 87th WEAI annual conference and GCOE seminars for their helpful comments and suggestions. All errors are our own. Our research is financially supported by JSPS Grant-in-Aid for Specially Promoted Research (No. 23000001) and the Keio-Kyoto joint G-COE program, "Raising Market Quality-Integrated Design of Market Infrastructure."

†Institute of Economic Research, Kyoto University, Yoshida Honmachi, Sakyo-Ku, Kyoto, Japan. 606-8501; e-mail: mail@yoheitenryu.jp

1 Introduction

Interest in studying the relationship between the growth rate of an economy and economic institutions has been increasing. Insecure property rights is one of the most interesting fields of study noted among economists. Developing countries generally have a weak property rights system, and it is thought that the system becomes a set of shackles that cripples economic progress. Some developing countries share a common capital that everyone can access, which is not secured property. The common-pool problem is widely used to analyze such economies.

Regarding the common-pool problem, the excess use of common resources is an interesting phenomenon. In an economy with common capital, each agent freely extracts the resource without taking the protection of it into account. As a result, the growth rate of the economy is lower than that in an economy with secure property rights. This is called the tragedy of the commons. Excess use of the commons is also a cause of the voracity effect. This was first studied by Tornell and Velasco (1992), Lane and Tornell (1996), and Tornell and Lane (1999), and they define it thusly: a positive technology shock in the common sector leads to an increase in appropriation and thus slows economic growth. However, we suggest that this is not the only cause of the voracity effect and explore another cause of voracious behavior.

Tornell and Velasco (1992) and Tornell and Lane (1999) are also concerned with the role of private capital. They introduce private capital into the economy with multiple interest groups and the common sector. Each group appropriates the resource from the common sector and can use it not only for its consumption but also for investment to accumulate private capital stock. The private capital is secured property not accessible by the other groups, but it is less productive. In other words, groups in the economy have respective private sectors and accumulate their own capital. Tornell and Velasco (1992) and Tornell and Lane (1999) show that under some circumstances, the introduction of secure but less productive capital stock increases the growth rate of the common sector. They also find that the voracity effect occurs.

In their model, there is only one direction of capital flow from the common sector to the private sector. They do not consider the other direction from the private sector to the common sector. In practice, consideration of the interaction between both sectors is important. It is plausible that a portion of private assets is used in the common sector. Schneider

(1998) show empirically that a fraction of the earnings in the informal sector are immediately spent in the private sector.[1] The private sector is interpreted as informal or shadow sector in a country. In what follows, we represent the sector in which assets are secured as the private sector and the sector in which assets are not secured as the common sector. Schneider (1998) shows that the private sector has a positive effect on economic growth. Loayza (1996) uses an endogenous growth model to show that an increase in the size of the private sector negatively affects growth. They also find this result to be observable empirically by using data from Latin America. The role of the private sector in economic growth is, therefore, ambiguous. We, therefore, introduce the capital flow from the private sector to the common sector and study how interest groups' voracious behaviors change.

Furthermore, in the existing literature (e.g., Tornell and Velasco (1992) and Tornell and Lane (1999)), the ratio of private capital stock to common capital stock diverges to infinite in the long run. This is not a realistic situation. We investigate how the introduction of another direction of capital flow changes this result.

The aims of the present paper are to explore another cause of voracious behavior and to investigate the effects of voracious behavior on the economy. We extend the Tornell and Velasco (1992) model by introducing a capital flow from the private sector to the common sector; a fraction of each interest group's private capital stock is invested in the common sector. In this situation, the obtained results are as follows. First, we show theoretically that the balanced growth rates are independent of the technology level in the common sector. This implies that there is no standard voracity effect in the sense that Tornell and Lane (1999) define. Second, we also find that the opponents' private capital has a positive effect on a group's equilibrium consumption strategy called Markov control-state complementarity. Third, we observe numerically that an increase in the contribution rate leads to an increase in appropriation, and hence the balanced growth becomes slow. The paper predicts that the contribution of the private sector to the common sector has a negative effect on economic growth. Finally, the ratio of private capital stock to common capital stock on the balanced growth path is likely to be a U-shaped function of the contribution rate.

Other lines of literature on the dynamic common-pool problem are as follows. Mino (2006) and Itaya and Mino (2007) introduce labor into the economy without the private sec-

[1] It is necessary to be careful about the term, the private sector. Since he deals with the data not only on developing countries but also on developed countries, he regards the national sector as the private sector. In our paper, however, we focus on developing countries without secure property rights, and we represent the national sector as the common sector and the informal sector as the private sector.

tor and consider variable labor-leisure choices by changing the linear production function to an increasing-returns production function. They find that the effects of a rise in productivity and the number of interest groups would be significantly different from the results obtained in the basic framework. Strulik (2011) reconsiders the voracity effect by introducing basic needs matter in consumption. It is shown that interest groups are, ceteris paribus, more likely to generate the voracity effect due to more appropriation when an economy is in decline and sufficiently close to stagnation. Tornell (1997) and Lindner and Strulik (2008) use trigger strategy equilibria in economic growth models with common access to capital to analyze the features of endogenous property rights. Long and Sorger (2006) extend the Tornell and Velasco (1992) model by adding the following three features. First, extracting the common property asset involves a private appropriation cost. Second, each group derives utility from wealth as well as from consumption. Finally, each group can be heterogeneous. They show that an increase in appropriation cost and an increase in the degree of heterogeneity of these costs under different appropriation cost across interest groups lower the growth rate of the common capital stock.

There are four remaining sections of the present paper. The model, a solution concept, and each group's maximization problem are described in section 2. Section 3 characterizes the balanced growth equilibrium. In section 4, the balanced growth comparative statics will be numerically analyzed. Section 5 contains some conclusions.

2 The Model

Our framework builds on the models of Tornell and Velasco (1992) and Tornell and Lane (1999). We consider a continuous time model. There is a developing economy organized by multiple interest groups. The number of multiple interest groups is $n \geq 2$. We suppose that each group is homogeneous in the sense that each group has the same preference, and the subjective rate of discount and the technology level of the private sector are common among all groups. Within each group, there is a set of people who cooperate with other people belonging to the same group. They do not cooperate with those who do not belong to the same group, and they cannot move and belong to other groups. The reason may be that each group has different beliefs or belongs to different ethnic, religious, or occupational categories, so it has no incentive to cooperate with other groups. We can, therefore, interpret a group as the representative agent.

Since each group has the same preference, it has the same utility function. The utility function is assumed to be CRRA. The discounted sum of the utility is, therefore, represented as follows.

$$\int_0^\infty \frac{c_i(t)^{1-\theta}}{1-\theta} e^{-\rho t} dt, \quad \theta > 0, \; \theta \neq 1, \; i = 1, 2, \cdots, n \tag{1}$$

where $c_i(t)$ is group i's consumption at instantaneous time t, θ is the inverse of the intertemporal elasticity of substitution in consumption, and ρ is the subjective rate of time preference. Each group i maximizes (1) subject to some restrictions explained below.

2.1 Secure and Insecure Property Rights

Using the concepts of secure and insecure property rights, we introduce the common sector and the private sector. The common capital stock is generally regarded as the insecure property right asset; e.g., a big, clean fisheries; underground oil; or forests. In Tornell and Velasco (1992), Tornell and Lane (1996), and Long and Sorger (2006), private capital is interpreted as small, private, and stagnant lakes or bank accounts in foreign developed countries that cannot be deprived by other groups. The common capital stock is assumed to allow each group to have a larger marginal profit than the private-access capital does. In the case of the fisheries, common fisheries are large and highly nutritious. The marginal productivity of fish in common fisheries is, therefore, larger than that in small, private, and stagnant fisheries. In the case of bank accounts, the interest rate in foreign developed countries is lower than that of the developing (home) country.

Each group decides how much common capital is appropriated, consumed and invested in order to accumulate its own private capital. Taking the opponents' behavior into account, each group can appropriate any share it desires from the common capital stock. The resource appropriated by a group is used for consumption of the group or investment in private capital.

We consider, however, the interaction between the common sector and the private sector. For this purpose, we assume that for each group, a portion of its private capital stock must be used for production of the output in the common sector. Since a government or the society in the economy knows that excess use of the resource occurs, it requires all groups to invest in the common sector in order to avoid that phenomenon. An alternative interpretation can be considered. First, the government might permit each group to accumulate its own private capital stock in exchange for dedicating a part, which can be regarded as a kind of bribe. Such a government is called a Predatory state or Kleptocracy state. [2] Second, in the fishery case, some fish are moved to the bountiful fisheries — the common sector— because the private fisheries are stagnant. This implies that there exists a positive spillover into the common sector.

In the common sector, an output is produced from the aggregate capital, which is composed of the common capital stock and the sum of a part of group i's private capital stock. Following Tornell and Velasco (1992) and Tornell and Lane (1999), we assume that production technology is linear. In addition, we assume that the production function is additively separable for analytical simplicity. The common capital stock is insecure property: each group can extract it to for consumption and investment. The common-access capital stock, therefore, evolves according to the following differential equation,

$$\dot{K}(t) = A\left[K(t) + \sum_{i=1}^{n} u_i h_i(t)\right] - \sum_{i=1}^{n} d_i(t), \tag{2}$$

where $K(t) \in \mathbb{R}_+$ is the common capital stock, $A \in \mathbb{R}_{++}$ is the productivity of the common sector, $u_i \in (0,1)$ is the rate of the private sector contribution to the common sector, and $d_i(t) \in \mathbb{R}_+$ is the amount appropriated by interest group i. The aggregate capital stock is represented as $K(t) + \sum_{i=1}^{n} u_i h_i(t)$.

As for the private sector, the resource extracted by each interest group can be either consumed or invested in its private and secure capital, but a fraction of the private capital is used for investment in the common sector. The private capital stock of group i, therefore, evolves according to the following differential equation:

$$\dot{h}_i(t) = B(1-u_i)h_i(t) + d_i(t) - c_i(t),\ i = 1, 2, \cdots, n, \tag{3}$$

where $h_i(t) \in \mathbb{R}_+$ is group i's private capital stock, $B \in \mathbb{R}_{++}$ is the technology level of the private sector, and $c_i(t) \in \mathbb{R}_+$ is group i's consumption. It is plausible that the technology level of the private sector is common because of the assumption of symmetric groups.

Note that we assume that the government sets the rate, u_i, before each group i solves its problem. This means that u_i is assumed to be an exogenous and constant parameter. In addition, since we focus on homogeneous interest groups, the contribution rate,u, is assumed to be common to all interest groups. In the present model, we make the following

[2] Bayart, Stephan, and Hibou (1999) researched such a government.

assumption.

Assumption 1. *The marginal product of the common sector is larger than that of the private sector;* $A > B$. *The contribution rate is common to all groups;* $u_i = u$ *for all* i. *The parameters* B *and* ρ *satisfy* $B > \rho$.

The first condition is followed from Tornell and Velasco (1992) and Tornell and Lane (1999). The second is for simplicity, and thus we set that each u_i is common among all groups, that is $u_i = u$ for all i. The last guarantees that the balanced growth rate is positive, and the transversality condition is satisfied.[3]

2.2 The Solution Concept: Markov Perfect Equilibrium

We focus on a symmetric Markov perfect equilibrium (henceforth, MPE) of the noncooperative insecure property rights game. In the present model, each group i has two stationary Markov strategies; consumption strategy ψ_i and appropriation strategy ϕ_i. These strategies are functions $\psi_i : \mathbb{R}_+^{n+1} \to \mathbb{R}_+$ and $\phi_i : \mathbb{R}_+^{n+1} \to \mathbb{R}_+$, respectively. This means that group i chooses its consumption and appropriation according to the feedback rules $c_i(t) = \psi_i(K(t), h(t))$ and $d_i(t) = \phi_i(K(t), h(t))$. Let us define h as an n-dimensional vector; that is $h = (h_1, h_2, \cdots, h_n)$. Strategies ψ_i and ϕ_i are called symmetric if for all i and $j(\neq i)$ the relations $\psi_i = \psi_j$ and $\phi_i = \phi_j$ hold. Therefore the definition of MPE is as follows.

Definition 1. *The Markov strategies* $c_i^*(t) = \psi_i^*(K(t), h(t))$ *and* $d_i^*(t) = \phi_i^*(K(t), h(t))$ *constitute MPE if and only if each group i's problem maximizing* (1) *subject to* $(2) - (3)$*, any given initial stock* K_0 *and* h_{i0}*, and* $c_j^*(t) = \psi_j^*(K(t), h(t))$ *and the opponents' strategies* $d_j^*(t) = \phi_j^*(K(t), h(t))$ *for all* $j(\neq i)$ *have an optimal solution.*

From the above discussion, one can understand the information structure defined in the present paper. The government and each interest group can observe not only the common-access capital stock but also all the private capital stocks due to the introduction of the contribution ratio, $u \neq 0$. Therefore, both strategies in our model depend on the common-access capital stock and all private-access capital stocks. On the other hand, the existing literature (e.g., Tornell and Velasco (1992) and Tornell and Lane (1999)) implicitly assumes that all the groups cannot observe or are not interested in the opponents' private capital

[3]This relates to Assumption 2, which is amplified below.

stock. This is because they have assumed that the strategies of each group do not depend on the opponents' private capital. Namely, in their model the following situations are considered. There is no contribution of the private sector to the common sector, i.e. $u = 0$, and thus the appropriation strategy depends only on the common capital. The consumption strategy of group i depends only on the common capital stock and its own private capital stock. Although the former is justified by the fact that there is no direct influence of the private capital on the common sector in the present model, the latter is based on a stronger assumption. Therefore, we can consider another situation: the consumption strategy depends on the opponents' private capital.[4]

2.3 The Hamilton-Jacobi-Bellman Equation: Group i's Problem

Each group chooses the optimal levels of consumption and appropriation in each instant time t to maximize (1) subject to (2), (3), the opponents' strategies, and the initial levels of capital. Our model is, thus, a differential game among n interest groups where the control variables are c and d, and the state variables are the common capital stock K and the private capital stock h. Since we consider only a symmetric group case, we focus on one group, group i, in the discussion below.

An MPE is generally derived through the dynamic programming technique and must satisfy the Hamilton-Jacobi-Bellman (HJB) equation. The HJB equation of group i is as follows: for all $t \geq 0$ and $i = 1, 2, \cdots, n$,

$$\begin{aligned}\rho V_i(K, h) = \max_{c_i, d_i} \Bigg\{ & \frac{c_i^{1-\theta}}{1-\theta} + \frac{\partial V_i}{\partial K} \cdot \left(A \left[K + u \sum_{i=1}^{n} h_i \right] - d_i - \sum_{j \neq i} \phi_j \right) \\ & + \frac{\partial V_i}{\partial h_i} \cdot (B(1-u)h_i + \phi_i - \psi_i) + \sum_{j \neq i} \frac{\partial V_i}{\partial h_j} \cdot \left(B(1-u)h_j + \phi_j - \psi_j \right) \Bigg\}. \end{aligned} \tag{4}$$

Furthermore, the value function V_i must satisfy the following boundary condition:

$$\lim_{t \to \infty} V_i(K, h) \exp(-\rho t) = 0. \tag{5}$$

[4]Tenryu (2013) considers this problem.

Differentiating the HJB equation with respect to c_i and d_i yields optimal conditions,

$$c_i^{-\theta} = \frac{\partial V_i}{\partial h_i}, \tag{6}$$

$$\frac{\partial V_i}{\partial h_i} = \frac{\partial V_i}{\partial K}, \tag{7}$$

for all i. Equations (6) and (7) constitute a set of MPE solutions. Note that due to the assumption of the utility function, the above maximization problem satisfies the second-order conditions as well.

3 Balanced Growth Equilibrium

The Markov strategies simultaneously satisfy (6) and (7). Substituting these conditions into the HJB equation and using the envelope theorem, we obtain the following equations.

$$\begin{aligned}\rho\frac{\partial V_i}{\partial K} = &\frac{\partial^2 V_i}{\partial K^2}\cdot\left(A\left[K+u\sum_{i=1}^{n}h_i\right]-\phi_i^*-\sum_{j\neq i}\phi_j^*\right)+\frac{\partial V_i}{\partial K}\cdot\left(A-\sum_{j\neq i}\frac{\partial\phi_j^*}{\partial K}\right)\\ &+\frac{\partial^2 V_i}{\partial K\partial h_i}\cdot(B(1-u)h_i+\phi_i^*-\psi_i^*)+\sum_{j\neq i}\frac{\partial V_i}{\partial h_j}\cdot\left(\frac{\partial\phi_j^*}{\partial K}-\frac{\partial\psi_j^*}{\partial K}\right)\\ &+\sum_{j\neq i}\frac{\partial^2 V_i}{\partial K\partial h_j}\cdot\left(B(1-u)h_j+\phi_j^*-\psi_j^*\right),\end{aligned} \tag{8}$$

$$\begin{aligned}\rho\frac{\partial V_i}{\partial h_i} =&\frac{\partial^2 V_i}{\partial h_i\partial K}\cdot\left(A\left[K+u\sum_{i=1}^{n}h_i\right]-\phi_i^*-\sum_{j\neq i}\phi_j^*\right)+\frac{\partial V_i}{\partial K}\cdot\left(Au-\sum_{j\neq i}\frac{\partial\phi_j^*}{\partial h_i}\right)\\ &+\frac{\partial^2 V_i}{\partial h_i^2}\cdot(B(1-u)h_i+\phi_i^*-\psi_i^*)+\frac{\partial V_i}{\partial h_i}\cdot B(1-u)\\ &+\sum_{j\neq i}\frac{\partial^2 V_i}{\partial h_i\partial h_j}\cdot\left(B(1-u)h_j+\phi_j^*-\psi_j^*\right)+\sum_{j\neq i}\frac{\partial V_i}{\partial h_j}\cdot\left(\frac{\partial\phi_j^*}{\partial h_i}-\frac{\partial\psi_j^*}{\partial h_i}\right),\end{aligned} \tag{9}$$

and

$$
\begin{aligned}
\rho\frac{\partial V_i}{\partial h_j} =& \frac{\partial^2 V_i}{\partial h_j \partial K}\cdot\left(A\left[K+u\sum_{i=1}^{n}h_i\right]-\phi_i^*-\phi_j^*-\sum_{k\neq i,j}\phi_k^*\right)+\frac{\partial V_i}{\partial K}\cdot\left(Au-\frac{\partial\phi_j^*}{\partial h_j}-\sum_{k\neq i,j}\frac{\partial\phi_k^*}{\partial h_j}\right)\\
&+\frac{\partial^2 V_i}{\partial h_j\partial h_i}\cdot\left(B(1-u)h_i+\phi_i^*-\psi_i^*\right)+\frac{\partial^2 V_i}{\partial h_j^2}\cdot\left(B(1-u)h_j+\phi_j^*-\psi_j^*\right)\\
&+\frac{\partial V_i}{\partial h_j}\cdot\left(B(1-u)+\frac{\partial\phi_j^*}{\partial h_j}-\frac{\partial\psi_j^*}{\partial h_j}\right)+\sum_{k\neq i,j}\frac{\partial V_i}{\partial h_k}\cdot\left(\frac{\partial\phi_k^*}{\partial h_j}-\frac{\partial\psi_k^*}{\partial h_j}\right)\\
&+\sum_{k\neq i,j}\frac{\partial^2 V_i}{\partial h_j\partial h_k}\cdot\left(B(1-u)h_k+\phi_k^*-\psi_k^*\right). \qquad (10)
\end{aligned}
$$

The functions with an asterisk represent the optimal strategies in the model. In the following analysis, we focus on the symmetric MPE and show that the growth rates of c_i, d_i, and h_i, for all i and K grow at a positive constant. Before proceeding to the balanced growth analysis, we refer to a restriction of strategy space for consumption and appropriation.

3.1 Linear Markov Strategy

We restrict the consumption strategy $\psi(K,h)$ and the appropriation strategy $\phi(K,h)$ to be linear strategies, i.e., $\psi_i(K,h)=a'+aK+eh_i+bZ_i$ and $\phi_i(K,h)=\gamma\left[K+uh_i+uZ_i\right]$, where a', a, b, γ, and e are unknown constants. For notational simplicity, we define the aggregate private capital of the opponents' group, $\sum_{j\neq i}h_j$, as Z_i. The consumption strategy is a standard linear strategy. Since we focus on the symmetric MPE, it is assumed to be the equal coefficient b among all the opponents' private capital h_j for all $j(\neq i)$. As for the appropriation strategy, we assume that it depends on the aggregate capital in the common sector, following the existing literature. It is noteworthy that in our model, each group can observe and is interested in the opponents' private capital stock, and thus the aggregate capital is composed of not only the common capital but also the sum of a ratio of the respective private capital stock.

Next, we conjecture the value function as follows.

$$
V_i(K,h)=\frac{\xi(K+\alpha h_i+\beta Z_i)^{1-\theta}}{1-\theta}, \qquad (11)
$$

where ξ, α, and β are unknown constants. Note that although ξ and α are usually positive, β

can be either positive, negative, or zero, depending on the model. In what follows, we solve for unknown parameters by using the strategies and the value function and then discuss the sign of β in detail.

Substituting the strategies and (11) into equations $(8)-(10)$, we can rewrite them as

$$\{\rho - A + (1-\beta)(n-1)\gamma + a\beta(n-1)\}\frac{\partial V_i}{\partial K} = \frac{\partial^2 V_i}{\partial K^2}\cdot F(K,h), \tag{12}$$

$$\{\rho - u[A-(1-\beta)(n-1)\gamma] - B(1-u) + a\beta^2(n-1)\}\frac{\partial V_i}{\partial K} = \frac{\partial^2 V_i}{\partial K^2}\cdot F(K,h), \tag{13}$$

and

$$\{\beta\rho - u[A-(1-\beta)(n-1)\gamma] - \beta B(1-u) + a\beta[1+\beta(n-2)]\}\frac{\partial V_i}{\partial K} = \beta\frac{\partial^2 V_i}{\partial K^2}\cdot F(K,h), \tag{14}$$

where the function F represents

$$\begin{aligned} F(K,h) = {} & \{A-(1-\beta)(n-1)\gamma - a[1+\beta(n-1)]\}K \\ & + \{u[A-(1-\beta)(n-1)\gamma] + B(1-u) - a[1+\beta^2(n-1)]\}h_i \\ & + \{u[A-(1-\beta)(n-1)\gamma] + \beta B(1-u) - a\beta[2+\beta(n-2)]\}Z_i. \end{aligned}$$

Furthermore, we summarize the three equations as follows:

$$(1-\beta)(n-1)(1-u)\gamma = (A-B)(1-u) - a\beta(n-1)(1-\beta), \tag{15}$$

$$(1-\beta)(n-1)(\beta-u)\gamma = A(\beta-u) - \beta[B(1-u) - a(1-\beta)]. \tag{16}$$

Let us consider the solution candidate of the model. The unknown parameters, a, β, and γ, must satisfy above both of the above equations simultaneously. First, if $\beta = 1$, the above conditions require that the contribution rate u must be unity because of the assumption $A > B$. This contradicts the assumption $u \in (0,1)$, and thus this is not equilibrium. We are required to state a qualification for this point. Although we can relax the assumption and set $u = 1$, in this situation, each interest group is forced to serve all the private capital stock to the common sector except for its consumption. This implies that there is no property right for the private capital; i.e., the common capital stock has no discrimination from the private capital stock. As a result, the economy is reduced to a one-sector economy. This is not the interesting case, and therefore we remove it from our analytical consideration.

Second, we consider the possibility that β is zero. Tornell and Velasco (1992) and Tornell and Lane (1999) consider this situation.[5] They assume implicitly that each group i cannot observe the opponents' capital stock or is not interested even if it can observe. Substituting $\beta = 0$ into (15) and (16), we get two equations, $(n-1)\gamma = A - B$ and $(n-1)\gamma = A$. For the two equations to be satisfied simultaneously, B must be zero, which contradicts the positivity of B. Therefore, $\beta = 0$ is not an equilibrium.

Finally, we consider the case $\beta \neq 0, 1$. The result is obtained in the following lemma.

Lemma 1. *The candidates of optimal parameters are obtained as follows.*

$$\begin{aligned}
a &= \frac{uB(1-u)}{\beta[(n-1)\beta + 1 - un]}, \\
\beta &= \frac{y \pm \sqrt{y^2 + 4x\theta uB(1-u)}}{2x}, \\
\gamma &= \frac{A[(n-1)\beta + 1 - un] - B(1-u)[(n-1)\beta + 1]}{(1-\beta)(n-1)[(n-1)\beta + 1 - un]}, \\
a' &= 0, \quad \xi = a^{-\theta}, \quad \alpha = 1, \quad \textit{and} \quad b = a\beta,
\end{aligned}$$

where

$$x \equiv (n-1)[\rho + (1-u)(\theta - 1)B] \ \textit{and} \ y \equiv ux - (\rho + (\theta - 1)B)(1-u).$$

Proof. See Appendix A. □

The Markov strategy ψ_i is represented as $\psi_i = a(K + h_i + \beta Z_i)$ and shows that a group's optimal rate of consumption c_i depends on the common and its own private capital stock and the opponents' private capital. Although the coefficient a is positive[6] there are two candidates for β for the MPE; i.e., one is positive and the other is negative.[7] The sign of β is of importance for groups' consumption strategies because different signs have different effects on them. We define the concepts of substitutability and complementarity by following Long (2010).

[5] If the opponents' private capital stock is observable, there are two equilibrium solutions. One is the same as that of Tornell and Velasco (1992), which implies that each group chooses its strategies without taking the opponents' private capital into account. In this case, each group competes for the only common capital stock, K. Another equilibrium is the case that β is positive. For a detailed discussion, see Tenryu (2013).

[6] See Appendix C.

[7] See Appendix A.

Definition 2 (Long (2010, Chapter 5)). *The Markov strategy $c_i = \psi^i(K(t), h(t))$ is said to display Markov control-state complementarity (respectively, Markov control-state substitutability) if and only if $\partial\psi_i^*/\partial h_j > 0$ (respectively, < 0).*

In the next subsection, we show that one of these candidates can be ruled out by considering the dynamic system of the model.

3.2 Dynamic System and Stability

With the linear strategy rules, the state dynamics of the game are represented as follows.

$$\dot{K} = (A - n\gamma)K + (Au - n\gamma u)h_i + (Au - n\gamma u)Z_i, \tag{17}$$

$$\dot{h}_i = (\gamma - a)K + (B(1-u) + \gamma u - a)h_i + (\gamma u - a\beta)Z_i, \tag{18}$$

and

$$\dot{h}_j = (\gamma - a)K + (\gamma u - a\beta)h_i + (B(1-u) + \gamma u - a)h_j + (\gamma u - a\beta)\sum_{k \neq i,j} h_k. \tag{19}$$

In the symmetric equilibrium, the amount of group i's private capital stock is equal to that of all the other groups j ($\neq i$) so that $h_i = h_j$ and thus the dynamic system of h_j is identical to that of h_i. This implies that the $n-1$ state equations of private capital are redundant. Therefore, we can represent the dynamic system as the following two equations composed by the common capital, K, and group i's private capital, h_i:

$$\begin{pmatrix} \dot{K} \\ \dot{h}_i \end{pmatrix} = \begin{pmatrix} A - n\gamma & n(A - n\gamma)u \\ \gamma - a & B(1-u) + n\gamma u - a - (n-1)a\beta \end{pmatrix} \begin{pmatrix} K \\ h_i \end{pmatrix}. \tag{20}$$

To confirm the stability of our model, we compute the eigenvalues of the 2-by-2 matrix in (20) by using the condition that the determinant of the characteristic matrix equals 0. The quadratic equation is

$$\begin{aligned} \lambda^2 - (A{-}n(1-u)\gamma + B(1-u) - a[(n-1)\beta + 1])\lambda \\ + (A - n\gamma)\{B(1-u) - a[(n-1)\beta + 1 - un]\} = 0, \end{aligned} \tag{21}$$

where we define λ as the eigenvalues.

From the discussion in Appendix C, the term $A - n\gamma$ must be positive. The sign of the constant term depends on $B(1-u) - a[(n-1)\beta - 1 - un]$ being either positive or negative. Using Lemma 1, it is rewritten as $B(1-u)(1-u/\beta)$. If β is positive, we can prove that u/β is less than 1. This implies that the constant term is positive. On the other hand, if β is negative, it is clear that the term is positive.

Next, let us check the sign of the coefficient of λ. In the case that β is positive, to verify the sign, we can rewrite the coefficient as follows:

$$[A - n\gamma] + nu(\gamma - a) + B(1-u)\left(1 - \frac{u}{\beta}\right).$$

As discussed above, we know that the first and the third terms are positive. The second term is also positive due to the proof of Lemma 3. Therefore, the coefficient of λ is negative, and the constant term is positive. On the other hand, in the case that β is negative, it is clear that the $A - n(1-u)\gamma + B(1-u) - a[(n-1)\beta + 1]$ is positive.

From this relationship, we can verify that if β is positive, the characteristic equation has two positive real roots and thus the dynamical system, (20), is unstable and that if β is negative, the characteristic equation has two imaginary roots and the dynamic system is an unstable focus. The relationship can also be discussed by illustrating the phase diagram, which is given in Figure 1.[8]

[Figure 1 is here: The phase diagram]

Figure 1(a) illustrates that the positive root β leads to balanced growth given the initial states K_0 and h_0. In other words, there is no transition path, and the economy immediately achieves balanced growth. All the variables grow at the same positive constant (See Proposition 1). In the negative root, however, the dynamic system does not ensure the positivity of the state variables over time. The case is not an equilibrium.

Therefore, we obtain the following lemma.

Lemma 2. *The optimal parameter, β^*, is*

$$\beta^* = \frac{y + \sqrt{y^2 + 4x\theta uB(1-u)}}{2x} \tag{22}$$

where

$$x \equiv (n-1)[\rho + (1-u)(\theta-1)B] \ \text{ and } \ y \equiv ux - (\rho + (\theta-1)B)(1-u).$$

[8] $A - n\gamma$ and $B(1-u) + n\gamma u - a - (n-1)a\beta$ are always positive. $\gamma - a$ is positive (negative) if β is positive (negative). See Appendix C.

Proof. The proof follows the discussion above. □

3.3 Characteristics of the Balanced Growth Path

In this subsection, we characterize the balanced growth path. Before proceeding to the discussion, we impose the following assumption.

Assumption 2. *The following conditions are assumed to be satisfied,*

$$\max\left\{u, \frac{-z+\sqrt{z^2-4s(n-1)Bu(1-u)}}{2s}\right\} < \beta^* < \frac{nB(1-u)-A(1-un)}{A(n-1)},$$

where

$$s \equiv (n-1)[A-B(1-u)] \;\; \textit{and} \;\; z \equiv [A-B(1-u)](1-un) - Bu(1-u).$$

For the left inequality, the contribution rate u is smaller than β^* under the third condition of Assumption 1, $B > \rho$. It also makes the balanced growth rates of all the variables positive. The second term in curly brackets is smaller than β^*, which is one of the conditions ensuring the positivity of the ratio of private capital stock to common capital stock. Furthermore, the appropriation rate γ is positive if it is satisfied. The right inequality, on the other hand, is the other condition associated with the balanced growth ratio between private capital and common capital.

Under Assumptions 1 and 2, we obtain the following lemma. The lemma states that the economy achieves balanced growth immediately, and thus the MPE growth rate of group i's consumption is constant over time.

Lemma 3. *The growth rate of consumption is given by*

$$g = \frac{\dot{c}_i}{c_i} = \frac{B(1-u)(\beta^*-u)[\beta^*(n-1)+1]}{\beta^*[\beta^*(n-1)+1-un]}. \tag{23}$$

Proof. See Appendix B. □

In what follows, we show that the growth rates of all the other variables correspond to that of consumption and characterize the balanced growth path. To characterize the balanced growth, we define the balanced growth ratio of private capital stock to common capital stock as $\chi \equiv h_i/K$. We obtain the following proposition.

Proposition 1. *Under Assumptions 1 and 2, the strategy profile $\{(\phi_i, \psi_i)\}_{i=1}^n$ defined by $\phi_i(K,h)$ and $\psi_i(K,h)$ forms a symmetric MPE. In the equilibrium, the optimal strategies are*

$$\psi_i^* = a^*K + a^*h_i + b^*Z_i \;\; and \;\; \phi_i^* = \gamma^* \left[K + uh_i + uZ_i\right].$$

On the balanced growth path, the growth rates are

$$g = \frac{\dot{c}_i}{c_i} = \frac{\dot{d}_i}{d_i} = \frac{\dot{K}}{K} = \frac{\dot{h}_i}{h_i} = \frac{B(1-u)(\beta^* - u)[\beta^*(n-1)+1]}{\beta^*[\beta^*(n-1)+1-un]}, \tag{24}$$

and the ratio of private capital stock to common capital stock is

$$\chi^* = \frac{g - (A - n\gamma^*)}{nu(A - n\gamma^*)}. \tag{25}$$

Proof. See Appendix C. □

Note that the marginal productivity in the common sector and that in the private sector are constant due to the assumption of a linear technology, so that balanced growth is achieved without transitional dynamics. In the economy, the growth rate of common capital is equivalent to the growth rate of private capital. All the variables grow at the same positive and constant rate regardless of the initial level of common-private capital ratio (see Figure 1). In the existing literature, however, since the evolution of the common capital is not dependent on the private capital stock, the case does not exist that both growth rates are equivalent. The growth rate of the private sector becomes higher than that of the common sector, and thus χ diverges to infinity in the long run. On the other hand, in the model, χ^* has a finite positive value unless $A - n\gamma^*$ is close to zero. This enables us to discuss the relative size of both the common capital and the private capital.

At the end of this section, we derive another proposition. From Lemma 2, β^* is positive. Differentiating the consumption strategy, $\psi_i^* = a^*K + a^*h_i + a^*\beta^*Z_i$, with respect to h_j yields

$$\frac{\partial \psi_i^*}{\partial h_j} = a^*\beta^*,$$

where $Z_i \equiv \sum_{j \neq i} h_j$. Since a^* is positive, the partial differential coefficient is positive. Therefore, we obtain the following proposition.

Proposition 2. *The consumption strategy is Markov control-state complementarity.*

Two features are worth noting. First, Tenryu (2013) considers the case that $u = 0$ and derives that the consumption strategy ψ_i is Markov control-state substitutability. He considers only one direction of capital flow, from the common sector to the private sector, like Tornell and Velasco (1992), Tornell and Lane (1999), and Long and Sorger (2006). In this situation, once groups extract the resource, it cannot be returned to the common sector, and the more of the resource a group extracts, the less of it the other groups can obtain. Furthermore, the marginal product of the common sector is assumed to be larger than that of the private sector. These lead to Markov control-state substitutability.

Second, there is a crucial difference between the present paper and Tenryu (2013). We consider the interaction between the common sector and the private sector by introducing u; i.e., a fraction of the private capital is used to produce output in the common sector. A group is not only forced to contribute its own capital but also the other groups are forced to. For the group, the situation is equivalent to the positive externality in the common sector. As a consequence, the proposition derives the result that the strategy ψ_i is Markov control-state complementarity.

4 Balanced Growth Comparative Statics

In this section, we consider the effect of the contribution ratio, u, on the parameters, a^*, β^*, γ^*, g, and χ^*; we will explore how these parameters change as the ratio increases. As discussed above, all the parameters depend on the equilibrium value of β^*. However, deriving the derivative of β^* with respect to u is so complicated that the changes are analyzed numerically. We first need to assert values to the structural parameters of the model. In the numerical analysis below, we use the following values as the baseline: $\theta = 2$, $\rho = 0.04$, $A = 1.0$, $N = 5$, and $B = 0.3$. The elasticity of intertemporal substitution, the discount rate, and the technology level of the common sector are followed by the values in Mulligan and Sala-i-Martin (1992). The number of interest groups is equal to that in Lindner and Strulik (2004) and Strulik (2011). We set the technology level of the private sector to 0.3 in order to characterize the balanced growth comparative statics well. Our aim is to analyze the effect of the contribution ratio on the equilibrium parameters. At the same time, we vary the values of exogenous parameters, θ, ρ, B, and N, to check the sensitivity of the results with regard to different parameter choices.

4.1 A Numerical Example

4.1.1 Results of Parameter β

To begin, we experiment with changes in the equilibrium value of β^* as the contribution ratio increases. The transition is illustrated in Figure 2. The upper left of Figure 2 illustrates the transitions under different levels of technology in the private sector. The upper right shows the transitions under different values of the inverse of the intertemporal elasticity of substitution. In the bottom left, the values for β^* at different discount rates are illustrated. The bottom right reports those for β^* are illustrated under different numbers of interest groups. All the figures show that the relationship between β^* and the contribution ratio is monotonic within the region where Assumption 2 is satisfied. Outside the region, β^* is not monotonic but rather is inverted U-shaped, which is maximized at around $u = 1$. Furthermore, for a fixed contribution rate β^* takes a lower value as B, ρ, and n increase or θ decreases.

[Figure 2 is here: The effect of u on β^*]

4.1.2 Results of Other Parameters

Using the transition of β^*, we can understand the effect of the contribution ratio on endogenous parameters, which are the coefficients of common capital and each group's own private capital in the consumption strategy, a^*, the appropriation rate, γ^*, the balanced growth rate, g, and the ratio of private capital stock to common capital stock, χ^*. At the same time, we investigate how the behavior of β^* changes by varying each exogenous parameter, B, θ, ρ, and n. Figures 3-6 depict these numerical results.

First, we find the same behavior of parameters concerning the effect of u that (i) a^* is an increasing function,[9] (ii) γ^* is an increasing function, and (iii) g is a decreasing function and

[9]This behavior is obtained in the case $\theta > 1$. If $\theta < 1$, a^* is a decreasing function with respect to u. This difference, however, has no effect on other results.

that (iv) χ has U-shaped, i.e., χ^* is decreasing with respect to u when u is relatively low, and χ^* is increasing when u is relatively high. The last result is interpreted as follows. When u is relatively low, the marginal increase of appropriation is dominated by that of u. On the other hand, when u is relatively high, the marginal increase of appropriation dominates that of u. Therefore, there exists a point where both effects are set off.

Second, we observe the effects of exogenous parameters, B, θ, ρ, and n, on β^*. Figure 3 provides the relationship between the technology level in the private sector and β^*. We observe that for a given u, the coefficient of common capital and each group's private capital in the consumption strategy and the growth rate increase when B increases, whereas the appropriation rate and the ratio of private capital stock to common capital stock decrease. The former is a standard phenomenon because a more efficient technology generally leads to an increase in the growth rate and then to an increase in its consumption. The latter is, on the other hand, interesting. The direct effect of an increase in B leads to a production increase in the private sector. This is enjoyed by respective groups and motivates them to reduce their incentives to appropriate the common capital. As a result, more common capital is accumulated and the ratio of private capital stock of common capital stock decreases due to $A > B$.

[Figure 3 is here: Change in B]

Figure 4 shows the relationship between the inverse of the intertemporal elasticity of substitution and β^*. We observe that for a given contribution rate when θ decreases parameters γ^*, g, and χ^* increase, whereas a^* decreases. These results are equal to those obtained in standard neoclassical growth; when the intertemporal elasticity of substitution is higher,[10] the economy grows at a higher rate. Furthermore, χ^* increases due to the increasing appropriation rate.

[Figure 4 is here: Change in θ]

Figure 5 illustrates the relationship between the discount rate and β^*. The results are analogous to those in Figure 4. For a given u, when ρ decreases, parameters γ^*, g, and χ^* increase, while a^* decreases. These results are also the same as those obtained in standard neoclassical growth; as the discount rate becomes low, the economy grows at a higher rate.

[Figure 5 is here: Change in ρ]

[10]It is represented by a lower θ.

Figure 6 presents the relationship between the number of interest groups and β^*. It is observed that for a given u, all parameters decrease when n increases. The interpretation is as follows. The potential conflict becomes high for more fractionalized societies, which leads to decreases in the appropriation rate and in the the ratio of private capital stock to common capital stock. Therefore, each group has less incentive to invest in common and private capital, so that the balanced growth rates become low.

[Figure 6 is here: Change in n]

Note that in each case when function χ^* shifts down, Assumption 2 is not satisfied within the range of relatively low u. This implies that the positivity of χ^* is not satisfied. Therefore, we get the following observation.

Result 1. *The ratio of private capital to common capital, χ^*, is a U-shaped function of the contribution rate, u, except for the cases of relatively high B, θ, ρ, and n.*

4.1.3 The Voracity Effect

In this subsection, we consider the voracity effect. The voracity effect is one of the most interesting results in the literature. The voracity effect is the phenomenon that countries with multiple interest groups respond to a positive technology shock in the common sector by increasing the appropriation rate, and thus the growth rates become slow. In the existing literature (e.g., Tornell and Velasco (1992), Tornell and Lane (1999), and Long and Sorger (2006)), under some circumstances, the voracity effect is observed.

From (24), the balanced growth rate is not dependent on the marginal productivity of the common sector because β^* is also independent of A. We can verify that in our model, there is no effect of a positive technology shock in the common sector on the growth rate as Tornell and Lane (1999) define. However, we can confirm that the contribution rate plays the same role as technology in the common sector. The rate is determined by the government in this economy and is an exogenous variable for each group. When u increases, a group is forced to invest its private capital in the common sector. At the same time, however, the remaining $n-1$ groups also are forced to invest their private capital, this is regarded as a positive externality for the group. The externality dominates the impact of an increase in u on the group and, hence, makes it extract the resource more. This leads to the reduction of balanced growth rates. This is another channel of the voracity effect.

Result 2. *An increase in the contribution ratio,u, leads to an increase in the appropriation rate and a decrease in the balanced growth rate.*

The voracity effect, therefore, can be interpreted as that the positive external effect on the common sector leads to an increase in appropriation by interest groups and slows the growth rate.

5 Conclusion

We analyzed a developing economy with multiple interest groups. There are the common sector without secure property rights and the private sectors with secure property rights. A government requires each group to invest a fraction of its own private capital in the common sector in order to protect the commons. In this situation, we explore another cause of voracious behavior and investigate the effects of voracious behavior on the economy. First, we show theoretically that the balanced growth rates are independent of the technology level in the common sector. This implies that there is no standard voracity effect in the sense that Tornell and Lane (1999) define. We also find that the opponents' private capital has a positive effect on a group's equilibrium consumption strategy, called Markov control-state complementarity. In addition, we observe numerically that an increase in the contribution rate leads to an increase in appropriation, and hence the balanced growth becomes slow. The paper predicts that the contribution of the private sector to the common sector has a negative effect on economic growth and that the policy for preservation of the commons leads to the harmful effect on the economy. Finally, the ratio of private capital stock to common capital stock on the balanced growth path is likely to be a U-shaped function of the contribution rate.

Our model has some limitations and directions of possible extensions. First, we assumed that the contribution rate is exogenously chosen by a government for analytical simplicity. It is possible that the government or another agent chooses the contribution rate endogenously. Second, since we assumed homogeneous interest groups, we could not analyze what happens when there are heterogeneous interest groups. Introducing some kinds of asymmetry into the model would be an important issue. Third, we assumed simplified production, i.e., linear technology. We can consider other types of production and utility functions. For example, it is interesting to use the production with externality, as Mino

(2006) and Itaya and Mino (2007) used, and to add appropriation costs and wealth effects to the utility function, as Long and Sorger (2006) did. Finally, we have treated only the linear Markov strategies. Characterizing equilibrium under other Markov strategies, including non-linear Markov strategies, would be important.

Appendix A. Proof of Lemma 1

First, from (6) and (11) we obtain

$$\xi(K + h_i + \beta Z_i)^{-\theta} = (a' + aK + ah_i + bZ_i)^{-\theta},$$

which leads to

$$a' = 0, \quad \xi = (a)^{-\theta}, \ \ and \ \ b = a\beta.$$

Next, using (15) and (16) yields the following equation:

$$a\beta^2(n-1) - ua\beta(n-1) + a\beta(1-u) - uB(1-u) = 0. \tag{26}$$

We solve this for a,

$$a = \frac{uB(1-u)}{\beta[(n-1)\beta + 1 - un]}.$$

Substituting it into (11), we obtain the appropriation rate γ:

$$\gamma = \frac{A[(n-1)\beta + 1 - un] - B(1-u)[(n-1)\beta + 1]}{(1-\beta)(n-1)[(n-1)\beta + 1 - un]}.$$

Finally, we substitute these two parameters, a and γ, into (12), and after some manipulation, we obtain the following equation:

$$\begin{aligned}&\left[\rho - B + \frac{(n-1)uB(\beta - u)}{(n-1)\beta + 1 - un}\right]\frac{\partial V_i}{\partial K}\\ &\qquad = \frac{\partial^2 V_i}{\partial K^2}\left[\frac{B(1-u)(\beta-u)[\beta(n-1)+1]}{\beta[(n-1)\beta + 1 - un]}\right](K + h_i + \beta Z_i).\end{aligned} \tag{27}$$

Since $\partial V_i/\partial K = \xi(K + h_i + \beta Z_i)^{-\theta}$ and $\partial^2 V_i/\partial K^2 = -\theta\xi(K + h_i + \beta Z_i)^{-1-\theta}$, it is rewritten as follows:

$$x\beta^2 - y\beta - \theta uB(1-u) = 0, \tag{28}$$

where $x \equiv (n-1)[\rho + (1-u)(\theta-1)B]$ and $y \equiv ux - (\rho - (\theta+1)B)(1-u)$. Solving the quadratic equation for β,

$$\beta = \frac{y \pm \sqrt{y^2 + 4x\theta uB(1-u)}}{2x}.$$

From (28), when $\beta = 0$, we confirm that the constant term is always negative. This implies that if the equation (28) has two different real roots, one is negative,

$$\beta_1 = \frac{y - \sqrt{y^2 + 4x\theta u B(1-u)}}{2x} < 0,$$

and the other is positive,

$$\beta_2 = \frac{y + \sqrt{y^2 + 4x\theta u B(1-u)}}{2x} > 0.$$

Appendix B. Proof of Lemma 3

Let us derive the growth rate of consumption. The consumption of group i is represented by $c_i = \psi_i^* = a^*(K + h_i + \beta^* Z_i)$. Differentiating this with respect to t and dividing it by c_i yields

$$\frac{\dot{c}_i}{c_i} = \frac{\dot{K} + \dot{h}_i + \beta^* \dot{Z}_i}{K + h_i + \beta^* Z_i}.$$

Substituting equations $(17) - (19)$ into the numerator, we obtain

$$\begin{aligned} \dot{K} + \dot{h}_i + \beta^* \dot{Z}_i = & [A - \gamma^*(n-1) - a^* + \beta^*(n-1)(\gamma^* - a^*)]K \\ & + [Au - \gamma^* u(n-1) + B(1-u) - a^*\beta^*(n-1)(\gamma^* u - a^*\beta^*)]h_i \\ & + [Au - \gamma^* u(n-1) - a^*\beta^* + \beta^*(B(1-u) + \gamma^* u - a^*) \\ & \qquad + \beta^*(n-2)(\gamma^* u - a^*\beta^*)]Z_i. \end{aligned} \tag{29}$$

We arrange these three coefficients by using the parameters a^* and γ^*. First, the coefficient of the common capital K is

$$A - \gamma^*(n-1) - a^* + \beta^*(n-1)(\gamma^* - a^*) = \frac{B(1-u)(\beta^* - u)[\beta^*(n-1) + 1]}{\beta^*[\beta^*(n-1) + 1 - un]}.$$

Second, the coefficient of the group i's capital stock h_i is

$$\begin{aligned} & Au - \gamma^* u(n-1) + B(1-u) - a^*\beta^*(n-1)(\gamma^* u - a^*\beta^*) \\ & \qquad = \frac{B(1-u)(\beta^* - u)[\beta^*(n-1) + 1]}{\beta^*[\beta^*(n-1) + 1 - un]}. \end{aligned}$$

Finally, the coefficient of the others' capital stock Z_i is

$$\begin{aligned} & Au - \gamma^* u(n-1) - a^*\beta^* + \beta^*(B(1-u) + \gamma^* u - a^*) + \beta^*(n-2)(\gamma^* u - a^*\beta^*) \\ & \qquad = \beta^* \left[\frac{B(1-u)(\beta^* - u)[\beta^*(n-1)+1]}{\beta^*[\beta^*(n-1)+1-un]} \right]. \end{aligned}$$

Using the coefficients above, the numerator is rewritten as follows.

$$\dot{K} + \dot{h}_i + \beta^* \dot{Z}_i = \frac{B(1-u)(\beta^* - u)[\beta^*(n-1)+1]}{\beta^*[\beta^*(n-1)+1-un]}(K + h_i + \beta^* Z_i).$$

Therefore, we obtain the growth rate of consumption,

$$\frac{\dot{c}_i}{c_i} = \frac{B(1-u)(\beta^* - u)[\beta^*(n-1)+1]}{\beta^*[\beta^*(n-1)+1-un]}.$$

For the growth rate to be positive, it is necessary that $(n-1)\beta^* + 1 - un$ must be positive. We know $\beta^* > u$, which guarantees that the condition holds because

$$\beta^*(n-1) + 1 - un > u(n-1) + 1 - un = 1 - u \tag{30}$$

Therefore, the growth rate is positive.

Appendix C. Proof of Proposition 1

First, we derive the symmetric MPE strategies. Substituting β^* into parameters obtained in Lemma 1 yields MPE parameters. Therefore, the optimal strategies are obtained as follows:

$$\psi_i^* = a^* K + a^* h_i + b^* Z_i \;\; and \;\; \phi_i^* = \gamma^* \left[K + u h_i + u Z_i \right].$$

From lemma 2 and equation (30), we confirm that a^* is positive.

Second, we derive the balanced growth ratio χ. From (20), the growth rates of common capital and private capital are represented by

$$\frac{\dot{K}}{K} = (A - n\gamma^*) + nu(A - n\gamma^*)\frac{h_i}{K}, \tag{31}$$

$$\frac{\dot{h}_i}{h_i} = (\gamma^* - a^*)\frac{K}{h_i} + B(1-u) + n\gamma^* u - a^*[(n-1)\beta^* + 1]. \tag{32}$$

Let us define the ratio h_i/K as χ. Using lemmas 1 and 2, we can derive the growth rate of χ,

$$\begin{aligned}\frac{\dot{\chi}}{\chi} &= \frac{\dot{h}_i}{h_i} - \frac{\dot{K}}{K} \\ &= (\gamma^* - a^*)\frac{1}{\chi} + B(1-u) + n\gamma^* u - a^*[(n-1)\beta^* + 1] - nu(A - n\gamma^*)\chi - (A - n\gamma^*).\end{aligned}$$

On the balanced growth path, the growth rate of χ is zero, i.e., $\dot{\chi} = 0$ and the value of χ must be positive. This leads to

$$nu(A - n\gamma^*)\chi^2 + \{A - n\gamma^* - B(1-u) - n\gamma^* u + a^*[(n-1)\beta^* + 1]\}\chi - (\gamma^* - a^*) = 0. \tag{33}$$

Let us solve it for χ. Before doing so, we first consider the signs of the constant term and the coefficient of χ. For notational convenience, we derive the following. First, substituting (20) into $A - n\gamma^*$ and doing some manipulation, we obtain

$$A - n\gamma^* = \frac{[\beta^*(n-1) + 1]\{nB(1-u) - A[\beta^*(n-1) + 1 - un]\}}{(1-\beta^*)(n-1)[\beta^*(n-1) + 1 - un]}. \tag{34}$$

The term $nB(1-u) - A[\beta^*(n-1) + 1 - un]$ in the numerator is positive from Assumption 2. Therefore, $A - n\gamma^*$ is positive. Second, we subtract $A - n\gamma^*$ from the growth rate:

$$g - (A - n\gamma^*) = \frac{\begin{aligned}[(n-1)\beta^* + 1]\{A\beta^*[(n-1)\beta^* + 1 - un] \\ -B(1-u)[nu + (\beta^* - u)(1 + (n-1)\beta^*)]\}\end{aligned}}{(n-1)(1-\beta^*)\beta^*[(n-1)\beta^* + 1 - un]}. \tag{35}$$

Under Assumption 2, this is positive. Using the expression, we can represent the constant term[11] as follows,

$$\begin{aligned}\gamma^* - a^* &= \frac{A\beta^*[(n-1)\beta^* + 1 - un] - B(1-u)[nu + (\beta^* - u)(1 + (n-1)\beta^*)]}{(n-1)(1-\beta^*)\beta^*[(n-1)\beta^* + 1 - un]}, \\ &= \frac{g - (A - n\gamma^*)}{(n-1)\beta^* + 1} > 0.\end{aligned} \tag{36}$$

[11] This is negative if $\beta < 0$. We use it to describe Figure 1(b).

The coefficient of χ can be rewritten as follows.

$$A - n\gamma^* - B(1-u) - n\gamma^* u + a^*[(n-1)\beta^* + 1] = A - n\gamma^* - g + \frac{nu(A - n\gamma^*)}{(n-1)\beta^* + 1}.$$

Therefore, equation (33) can be simplified:

$$\left\{ nu(A - n\gamma^*)\chi + A - n\gamma^* - g \right\} \left\{ \chi + \frac{1}{(n-1)\beta^* + 1} \right\} = 0. \tag{37}$$

The solution of the equation is

$$\chi^* = \frac{g - (A - n\gamma^*)}{nu(A - n\gamma^*)} > 0,$$

because the common capital stock and the private capital stock must be positive and the other candidate, $\chi = -1/[(n-1)\beta^* + 1]$, is negative.

Third, we derive the common and private capital growth rates. Substituting (25) into (31), we obtain

$$\frac{\dot{K}}{K} = \frac{B(1-u)(\beta^* - u)[\beta^*(n-1) + 1]}{\beta^*[\beta^*(n-1) + 1 - un]}.$$

Also, substituting (25) into (32), we obtain

$$\begin{aligned} \frac{\dot{h}_i}{h_i} &= \frac{1}{(n-1)\beta^* + 1} \left[\{g - (A - n\gamma^*)\} \frac{K}{h_i} - nu(A - n\gamma^*) \right] + g \\ &= \frac{B(1-u)(\beta^* - u)[\beta^*(n-1) + 1]}{\beta^*[\beta^*(n-1) + 1 - un]}. \end{aligned}$$

Fourth, we derive the growth rate of appropriation. The appropriation of group i is represented by $d_i = \gamma^*[K + uh_i + uZ_i]$. As in the discussion above, since we focus on the symmetric MPE, $h_i = h_j$ for all $j \neq i$. Therefore, differentiating this with respect to t yields

$$\frac{\dot{d}_i}{d_i} = \frac{\dot{K} + un\dot{h}_i}{K + unh_i}.$$

On the balanced growth path, the growth rate of the common capital is equivalent to that of the private capital, $\dot{K}/K = \dot{h}_i/h_i$, and thus

$$\frac{\dot{d}_i}{d_i} = \frac{\frac{\dot{K}}{K}\left(1 + nu\frac{h_i}{K}\right)}{1 + nu\frac{h_i}{K}} = \frac{\dot{K}}{K}. \tag{38}$$

Finally, we check the boundary condition. Note that since the value function $V_i(K,h)$ has the properties $V_i(0,0) = 0$ and strict concavity, holding the boundary condition (5) guarantees that the transversality conditions are satisfied. Using equation (11) and lemmas 1-3, the value function is calculated by

$$V_i(K,h) = \frac{c_i(0)^{1-\theta}}{a^*(1-\theta)} \exp\left[(1-\theta)\left\{\frac{B(1-u)(\beta^*-u)\{(n-1)\beta^*+1\}}{\beta^*[(n-1)\beta^*+1-un]}\right\}t\right].$$

$c_i(0)^{1-\theta}/a^*(1-\theta)$ is constant. Thus, for the boundary condition to be satisfied,

$$\lim_{t\to\infty} \exp\left[(1-\theta)\left\{\frac{B(1-u)(\beta^*-u)\{(n-1)\beta^*+1\}}{\beta^*[(n-1)\beta^*+1-un]}\right\} - \rho\right]t.$$

must converge to zero. If $\theta > 1$, it is easy to verify that this is satisfied. If $0 < \theta < 1$, the power function is rewritten as follows:

$$\lim_{t\to\infty} \exp\left[\left\{\frac{Bu(1-u)(1-\theta)(\beta^*-1)}{\beta^*[(n-1)\beta^*+1-un]}\right\} - \{\rho + (\theta-1)(1-u)B\}\right]t.$$

The first term is negative because of Assumption 2, and $\rho + (\theta-1)(1-u)B$ is positive because of Assumption 1. This indicates that the power function converges to zero, and therefore the boundary condition is satisfied.

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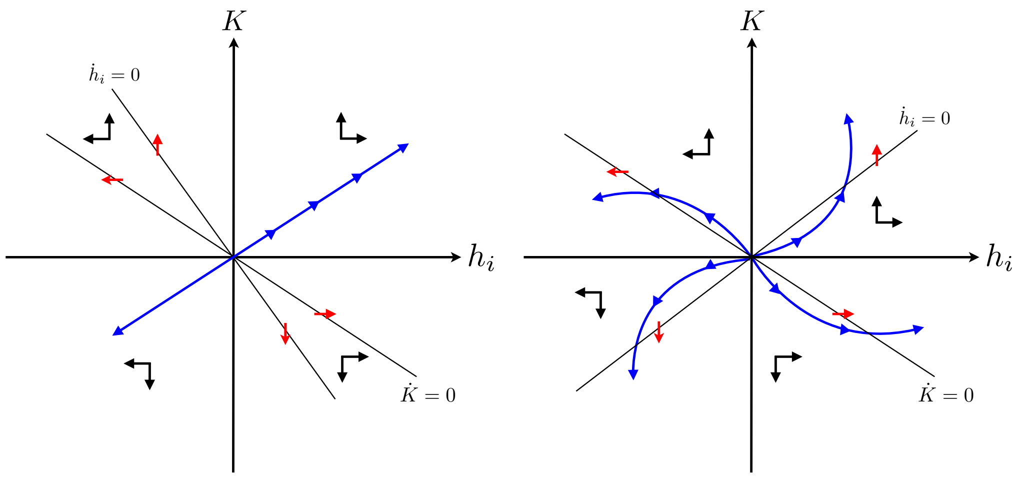

(a): $\beta > 0$ case (b): $\beta < 0$ case

Figure 1: The phase diagrams

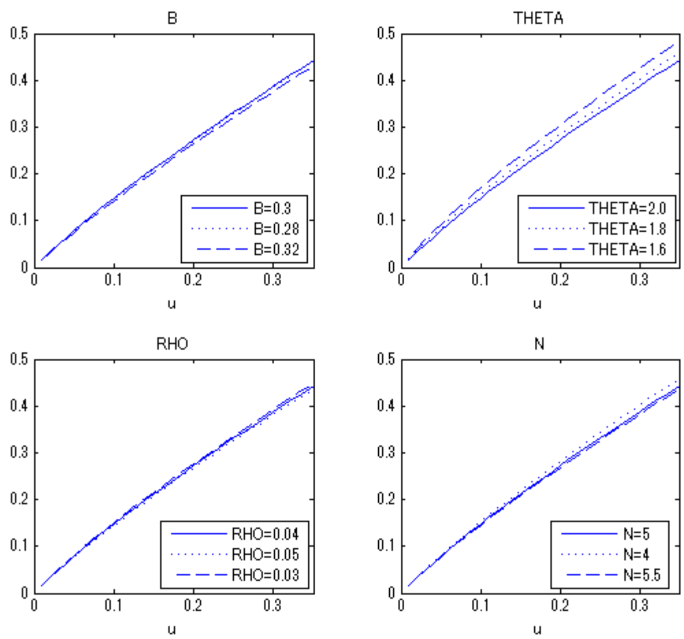

Figure 2: The effect of u on β^*

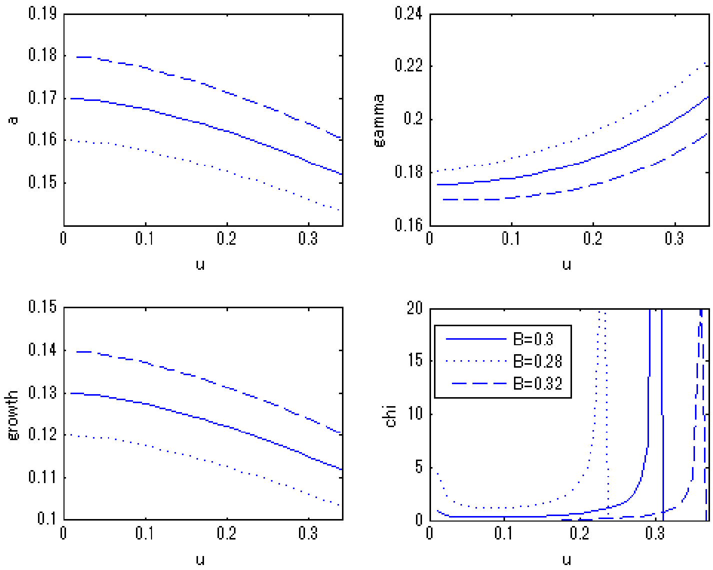

Figure 3: Change in B

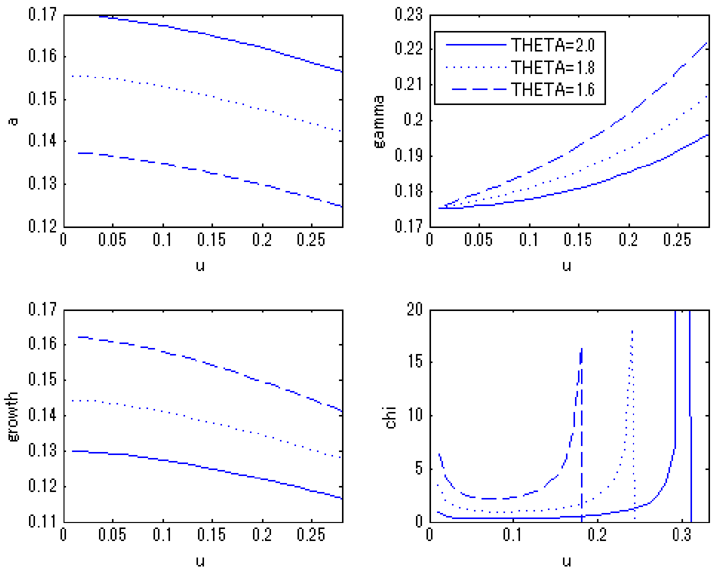

Figure 4: Change in θ

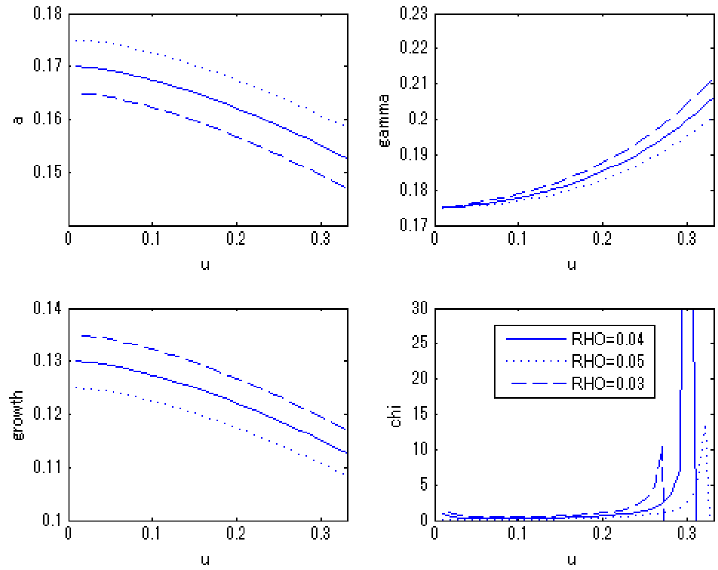

Figure 5: Change in ρ

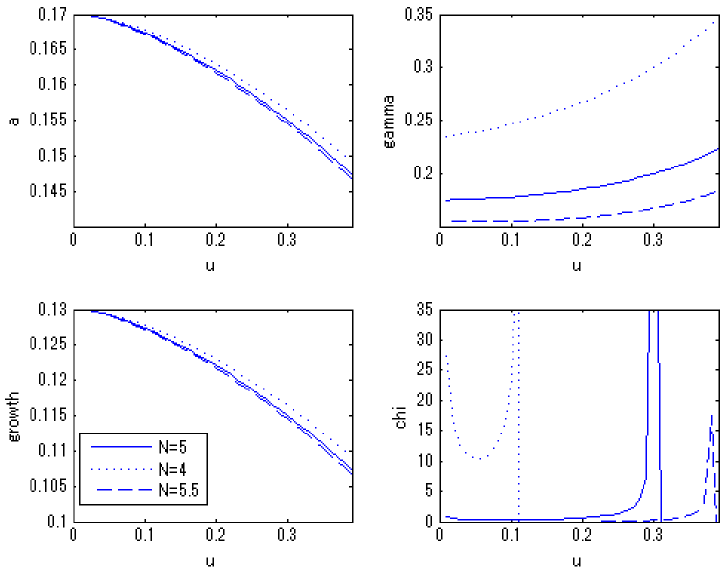

Figure 6: Change in n