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Abstract : This article aims to study the scaling behavior of the Algerian Dinar - US Dollar exchange rate using multifractal time series analysis which stems from the fractal theory first implemented by Benoît Mandelbrot in early 1960. Investigating time series properties using this technique allows us to shed light on important characteristics omitted by traditional time series analyses and highlight the usefulness of local Hölder exponents in predicting crash patterns.

Keywords : multifractal analysis, Dinar-Dollar exchange rate, Hölder exponents.

1. Introduction :

The behavior of the exchange rate has been of a crucial concern for both economists and decision makers which rely on this macroeconomic variable when building up their economic policies and the associated forecasts. In this time of economic and financial turmoil, the exchange rate is of a great importance in the worldwide economic debate, where many countries try to operate depreciative adjustments to have a favorable international competitiveness. Many studies dealt with that topic to better assess this behavior and its impact on an eventual internal/external shock.

For the case of Algeria, as for many developing countries, the exchange rate serves as the basis of the economic policy and the budget management, bearing in mind its status of single-commodity exporter (hydrocarbons), getting 98% of its revenues in US dollar and trading up to 60% with partners using the single currency (Euro). Hence, the Euro/US Dollar has a significant impact on the country's macroeconomic equilibrium.

Most of the studies targeting the nominal exchange rate in Algeria apprehended the behavioral side, stressing the use of classical time series analysis for that purpose because of the lack of macroeconomic data in low frequencies.

Hence, most academic papers were based on the Efficient Market Hypothesis (EMH)[3] which grew in popularity since early 1970. Peters[4] proposed a new approach related to market

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[3] First the works of Eugene Fama (1969, 1970) which supposed the price of an asset at the instant t contains all the past information. In its weak form, this hypothesis implies that the price of an asset, which follows a log-normal distribution,

stability rather than their efficiency, given the EMH could not predict different economic crises and stock market crashes occurring since that time[5].

We propose in this paper to use the Fractal Market Hypothesis (FMH) and the underlying fractal approach to examine the behavior of the nominal exchange rate series of the Algerian Dinar - US Dollar. This approach stems from the fractal theory first implemented by Benoît Mandelbrot in early 1960's, being widely used nowadays in different disciplines.

2. Theoretical foundations :

The fractal theory in time series analysis was linked first with the *Rescaled Range Analysis*, a method developed by British Hydrologist Harold Edwin Hurst when studying Nile river's flood. It results the Hurst statistics (Hurst exponent) which determines how the memory process behaves. For 0<H≤0.5 : the series is said to have a short-memory with anti-persistent shocks, while for 0.5<H<1 we observe long-memory phenomenon. Hence, the fractal dimension D, a statistic measuring the degree of roughness of a given series, is obtained by the relationship D=2-H.

In general, the FMH is based on market structure. Applying the self-similarity concept on a series of returns $X(t)$, we define Hurst exponent (H) by : $X(ct) = c^H X(t)$. This relationship defines a unique H, unchanged over time (unifractal case), while a more general relationship[6] defines the multifractal case $X(ct) = M(c)X(t)$ where $M(c)$ is a function of H.

Traditional methods estimating H consist of estimating, for a given interval, the ratio between the range R of the centered integrated series, and the standard error S of the original series. A particular interest is given to the difference between the minimum and the maximum of the deviations, in a certain interval, normalized by the standard error.

The Hurst exponent is reckoned as follows :

1. Calculate the mean (m) of the series X_i i=1..n
2. Generate the centered series $Y_i = X_i - m$ for i=1..n
3. Generate the series of cumulative deviations $Z_i = \sum Y_i$
4. Calculate the range $R(n) = \max(Z_1, Z_2, \dots, Z_n) - \min(Z_1, Z_2, \dots, Z_n)$

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obeys to a martingale process (random walk), whereas the returns in logarithms are independent and identically distributed.

[4] Edgar Peters : Fractal Market Analysis : Applying chaos theory to investments and economics (1994).

[5] Enrico Onali, John Goddard : Unifractality and Multifractality in the Italian Stock Market.

[6] B. Mandelbrot , A. Fisher, L. Calvet : A multifractal model of asset's returns, Cowles Foundation Discussion Paper (number 1164), 1997.

5. Calculate the standard deviation : $S(n) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(X_i - m)^2}$
6. Calculate the rescaled range series : $\frac{R(n)}{S(n)}$ and the means on each n.
7. The Hurst exponent is obtained via the power law : $E\left(\frac{R(n)}{S(n)}\right) = C.n^h$, so we regress the logarithm of $E\left(\frac{R(n)}{S(n)}\right)$ over log(n), and we obtain the slope which serves as an estimator of H.

This classical method is widely used, especially in hydrology, for series supposed to have a unifractal (monofractal) characteristic, say a unique constant exponent H over time. Its principal inconvenient lies on its sensitiveness for short time series having less than 500 observations.

Many time series, especially financial ones, are non-stationary and have a volatile behavior leading to a variable H, hence a multitude of Hurst exponents for different time scales (multifractal case). This approach is more indicated for series presenting high volatility, switching regimes and sudden shocks.

This idea recalls another classic alternative to the rescaled range analysis which is the multifractal detrended flucutation analysis (MF-DFA) that aims to detect self-similarity processes in a time series. This is done via the following steps[7] :

1. Given the initial series x_i, calculate the cumulative series $Y(j) = \sum_{i=1}^{j} x_i$
2. Divide the series $Y(j)$ into $N_s = int(\frac{N}{s})$ independent segments of the same length s . Because the length of the series N is usually not a multiple of the time scale s, we can repeat the procedure by starting from the opposite side (end of the series). We will have hence, $2N_s$ segments.
3. Calculate the local polynomial of each of the $2N_s$ segments via an ordinary least square regression of the profile $Y(j)$ on each segment $v = 0,\ldots,2N_s - 1$. We can also use high order polynomial and the method is then called *MF-DFAm* where m is the adjustment order. In our case, we set m=2 and $F_{DFAm}^2(v,s) = [Y(vs) - Y((v+1)s)]^2$ is the square root of the mean deviation of a random walk after s steps (segments).
4. We calculate the q-th order fluctuation function $F_q(s) = \left\{\frac{1}{2N_s}\sum_{v=1}^{2N_s}[F_{DFAm}^2(v,s)]^{\frac{q}{2}}\right\}^{\frac{1}{q}}$ and we focus on the relationship between the variations q and $F_q(s)$. The steps 2, 3 and 4 must be repeated over several steps s and $F_q(s)$ is only defined for $s \geq m + 2$.
5. We determine the scale bahavior of the fluctuations functions by log-regressing $F_q(s)$ over s for several values of q. $F_q(s) \sim s^{h(q)}$ where $h(q) = \frac{1+\tau(q)}{q}$ and $\tau(q)$ is called the Renyi exponent. By using a Legendre transform, we determine the degree of

[7] Jan W. Kantelhardt : Fractal and Multiractal Time Series , online document (April 2008).

singularity (Hölder exponent) $\alpha = h(q) + q * h'(q)$ et deduce the singularity spectrum $f(\alpha) = q[\alpha - h(q)] + 1$.

The Hölder exponent has a particular importance in the multifractal analysis[8]. It is defined[9] on the infinitesimal variation of a given series around an instant t. Hence, the local Hölder exponent is deduced via the relation : $|Ln\, X(t + \Delta t) - Ln\, X(t)| \sim C_t(\Delta t)^{\alpha(t)}$ where C_t is a time prefactor. The unifractal case occurs when $\alpha(t) = H$ is constant over time.

The statistic $F_q(s)$ becomes unreliable for $s \geq N/4$ and the value of $h(0)$ corresponding to the limit of h(q) for q approaching 0 cannot be obtained directly. Will use a logarithmic procedure, on average, : $F_0(s) = exp\left\{\frac{1}{4N_s}\sum_{v=1}^{2N_s} ln[F^2(v,s)]\right\} \sim s^{h(0)}$.

In the monofractal case, h(q) is independent of q because the function $F_{DFAm}^2(v,s)$ is identical for each segment v. Hence, for positive values of q, the segments v having a large variance $F^2(v,s)$ have a tendency to dominate the mean $F_q(s)$ and $h(q)$ will describe the scale behavior of segments having large fluctuations (the opposite remains true).

3. Application :

We consider the daily series of Algerian Dinar - US Dollar exchange rate during the period January 1996 to end-May 2013 (6362 observations). Figure 1 shows different periods of appreciation/depreciation of the Dinar given the prevailing economic situation. Hence, after the adoption of a new cotation system for the Dinar (base 1995)[10] based on a basket of 15 foreign currencies representing the then-trading partners, there were various breakaways from the tendency. These movements were linked to the good performance of world oil prices, given the status of Algeria, a single-commodity exporter (hydrocarbon), and the impact of the parity Euro/US Dollar.

[8] Hui-Wen Chen, Shian-Chang Huang : Multifractality Analysis for Stock Market Characteristics, Middle-Eastern Finance and Economics (2011).

[9] A function $f(x)$ belongs to the α-order class of Hölderian functions if $|f(t + h) - f(t)| < const.h^{\alpha}$ where $t, h \in \mathbb{R}\; and\; 0 \leq \alpha \leq 1$. In the case where α depends on t ($\alpha \rightarrow \alpha(t)$) then $\alpha(t)$ is said to be local Hölder exponent.

[10] In application of the IMF's structural adjustment program (1994-1998).

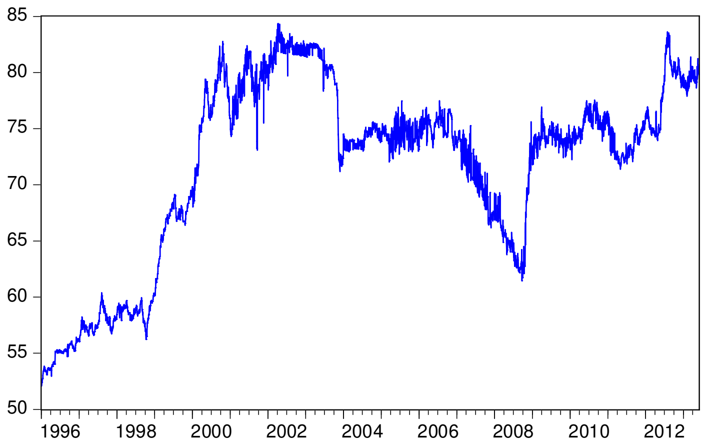

Figure 1 : Evolution of the Algerian Dinar - US Dollar exchange rate, raw data (01/01/1996 – 31/05/2013).

We will analyze the scale behavior by using the logarithm difference of the exchange rate $r_t = \log(X_t) - \log(X_{t-1})$ where X_t is the series of the daily exchange rate.

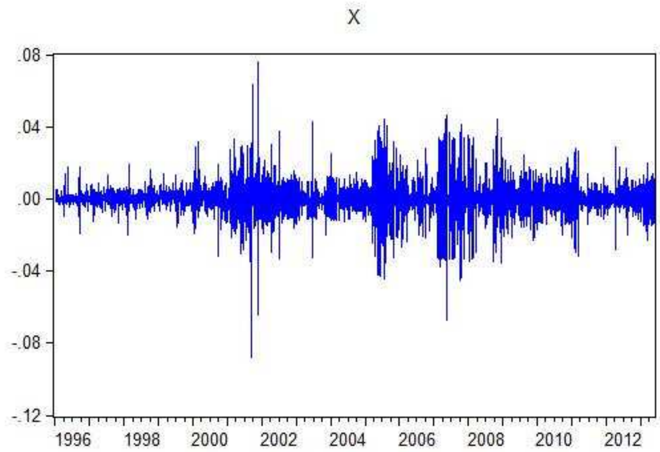

Figure 2 : Evolution of the Algerian Dinar - US Dollar exchange rate, in logarithm difference (01/01/1996 – 31/05/2013).

The raw series is highly volatile and non stationary, but the logarithm difference returns a stationary series with many clusters of volatility.

Mean	Median	Std. Dev.	Range	Skewness	Kurtosis
$6.68 * 10^{-5}$	0.000	0.0073	0.1641	-0.1587	18.9706

Table 1 : Descriptive statistics of the series in logarithm difference

The plots of the fluctuation functions $F_q(n)$[11] for a scale $16 < n < 1024$, given different values of q={-5,0,+5}, show that adjustment lines are neither superposed nor parallel, meaning many Hurst exponents could be present in the series due to possible switching regimes in the series. By retrieving the values of H for different values of q, it appears clearly the function $H(q)$ is linear, decreasing but unstable. That defends the multifractal character of the series. Moreover, the singularity spectrum shows a multifractal spectrum with a range of 0.4596 which corresponds to the difference of maximas of $H(q)$. This spectrum should be null in the case of monofractality.

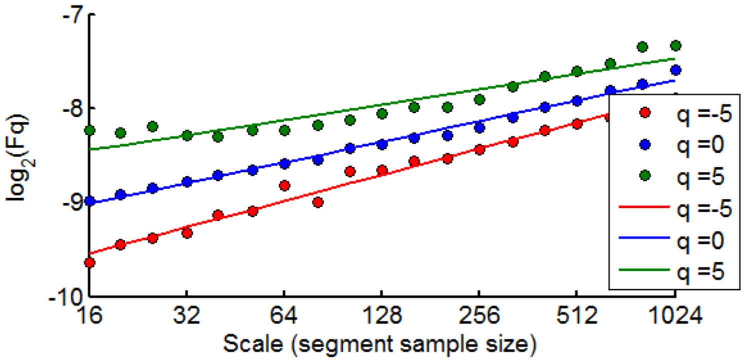

Figure 3 : Evolution F_q in function of segments s.

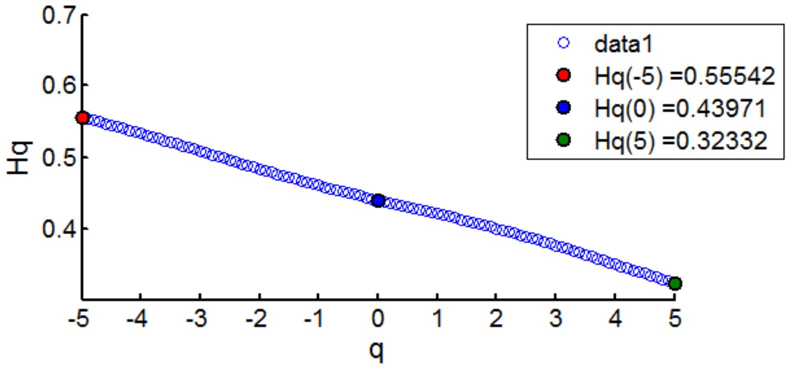

Figure 4 : Evolution of H_q in function of moments q.

[11] Matlab codes used here are provided by Espen A.F Ihlen : Introduction to multifractal detrended fluctuation analysis in Matlab, frontiers in Physiology, (June 2012).

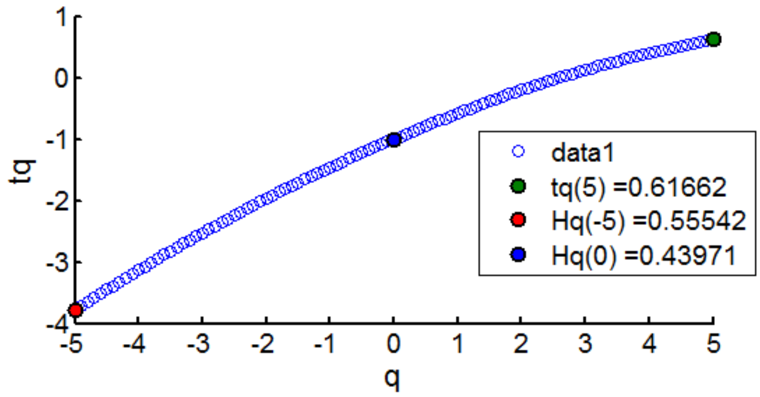

Figure 5 : Evolution of $\tau(q)$ in function of moments q.

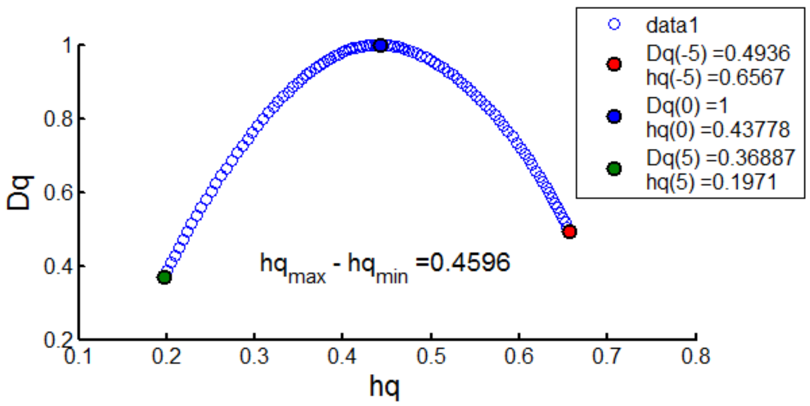

Figure 6 : Evolution of the multifractal spectrum and its range.

Recent developments of the multifractal analysis permit to determine critical moments[12] of a given series, by studying its local regularity via local Hölder exponents[13]. A Hölder exponent comprised between 0 and 1 means the signal is continued but non differentiable at a given point and a low value of that exponent indicates an irregularity in the signal. A critical event is preceded by a sudden increase of the regularity exceeding the unity, followed by small values of regularity for a long period (floor).

Figure 7 indicates clearly the series present crash patterns in different intervals[14], linked to sharp appreciations/depreciations of the Dinar, given the prevailing economic context. Thus,

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[12] called *crash patterns*.

[13] I.A.Agaev, Yu.A.Kuperin : Multifractal Analysis and Local Hoelder Exponents Approach to Detecting Stock Market Craches.

[14] The estimation of local Hölder exponents was done via the method of local oscillations provided by Matlab-toolbox *Fraclab*, developped by the research team of *Jacques-Lévy Véhel* at INRIA (France).

the figure 8 gives us a clearer view by decomposing the series in distinct intervals for the sake of clarity.

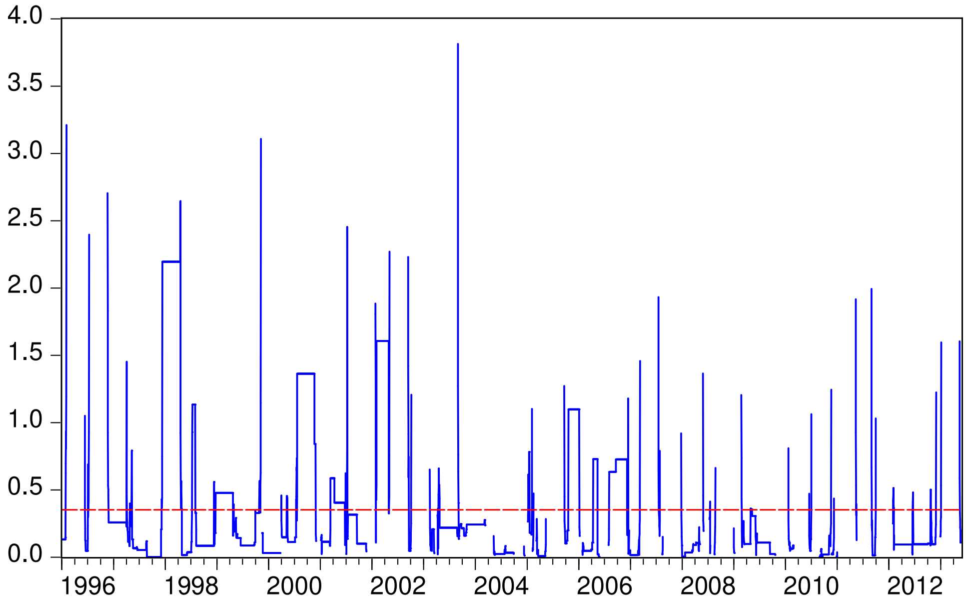
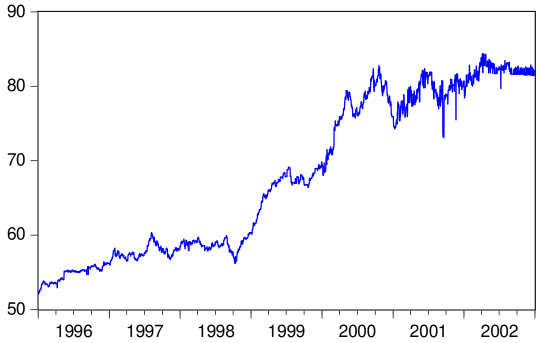
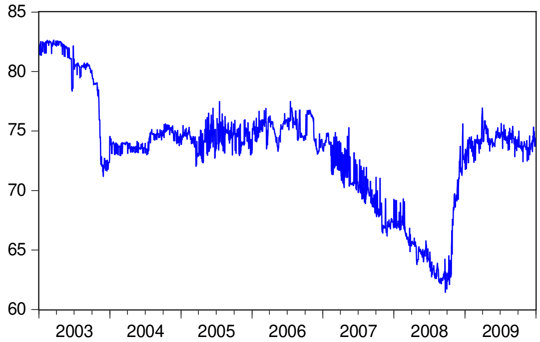


Figure 7 : Evolution of local Hölder exponents of the series in logarithmic difference (the dashed line represents the mean of Hölder exponents).

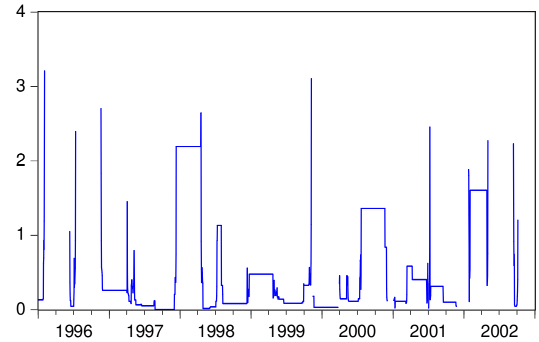
We notice local Hölder exponents' peaks preceding continued depreciations/appreciations of the Dinar, in other terms; they are harbingers of the series' trend breaks, even if it has clusters volatility. This is the case of the continued depreciation (21.26%) during the period mid-October 1998 to mid-July 1999, followed by another depreciation of 19.21% (mid-October 1999 to earl-May 2000). The crash pattern determines also the sharp volatility of the exchange rate occurred during second semester of 2001.



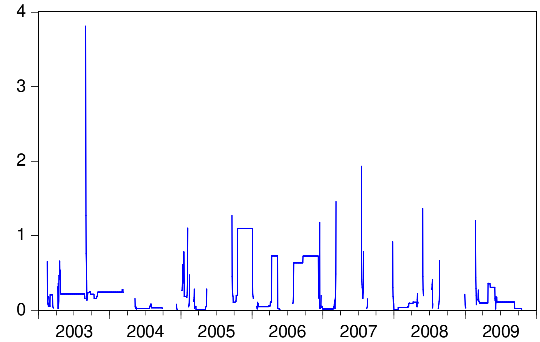
The exchange rate series Algerian Dinar - US Dollar (1996-2002)



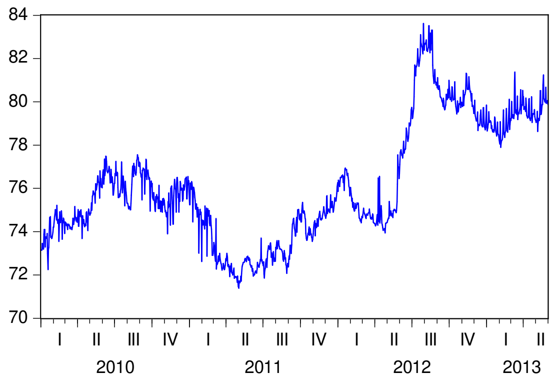
The exchange rate series Algerian Dinar - US Dollar (2003-2009)



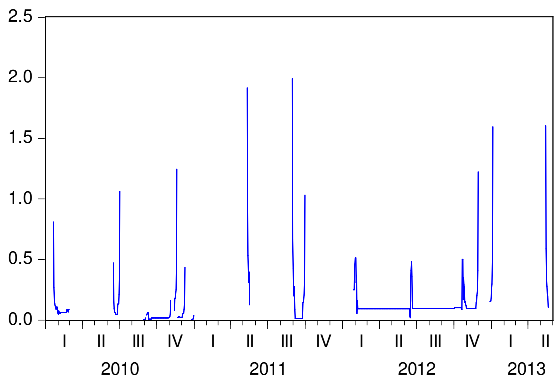
Series of local Hölder exponents (1996-2002)



Series of local Hölder exponents (2003-2009)



The exchange rate series Algerian Dinar - US Dollar (2003-2009)



Series of local Hölder exponents (2003-2009)

Figure 8 : Evolution of the Algerian Dinar – US Dollar exchange rate and the local Hölder exponents for the periods 1996-2002, 2003-2009 and 2010- May 2013.

Another event detected by local Hölder exponent is the brutal appreciation of the Dinar (11%) observed from September 2003 to November 2003, the volatile and sustained appreciation of February 2007 to September 2008 (16%) and the following depreciation of the similar amplitude (18%), which brought the exchange rate to its level reached 2 years ago. Finally, we find less pronounced variations in amplitude during the period early 2010-May 2013, but conserving a general depreciative trend.

These evolutions are correlative from one side to the good tenure of international oil prices and to the external financial position of Algeria, from another side. This translates the authorities' worrying of keeping a favorable balance of payments to face potential risks and international crises.

4. <u>Conclusion</u> :

The analysis of the daily series of the Algerian Dinar - US Dollar exchange rate demonstrated the multifractal characteristic of the series. Hence, the presence of a wide spectrum of Hurst exponents indicates that the series knew different regimes with episodes of appreciation/depreciation due to the prevailing economic situation and the monetary authority's commitment of guaranteeing a stabilization of the real effective exchange rate via the adoption of an active monetary policy. Local Hölder exponents put evidence on different tension periods presenting crash patterns, which serve as harbingers of eventual critical events that could influence the behavior of the Algerian Dinar.

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