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Is Health Trade of Mode 3 in Public Health Care Gainful?

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Abstract: This paper attempts to analyze the aspect of public health care in the context of increasing globalization. This paper examines the importance of health trade through mode 3 of General Agreement on trade in services (GATS) in the presence of a public funding health care. For that purpose the paper develops a static three sector general equilibrium trade model of a small open economy where health sector is defined as a public subsidized health care. Evidence shows that trade liberalization in the form of an increase in foreign health capital inflow leads to an expansion of public health care. This paper also shows that health trade through commercial presence may deteriorate the wage inequality. Apart from that in this study we have shown that the level of welfare of the small open economy increases due to trade in health services of mode 3.

Key words: Public Health sector, International health capital mobility, Social welfare and General Equilibrium.

JEL Classification: I18, F11, H51, I31, D58

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1. Introduction

Literature to social sector is gaining more importance among social scientists due to its inherent features that will enhance the economy to move towards their desirable path of economic development. Sectors like health, education, etc, are the most crucial parts of a social sector of any developing economy. It has been rightly pointed out in various articles and news papers that for the last decade health care becomes one of the most favorable destinations for foreign investors\(^2\). Not only that the overall health care that includes private as well as public health care have generated huge amount of employment opportunity\(^3\). These facts and figures insist us to analyze the aspect of health care with deep attention.

It is to be noted that although private health care dominates the overall domain of health care but relevance of public health care should give proper attention especially in the context of developing economies. Intuition behind the above statement is that purchasing power of individual belongs to a developing economy are very low and hence they can not afford private health services as these are costlier relative to public health services\(^4\). So there exists enough demand for public health services, but what about supply of public health care? To match up the

\(\text{\footnotesize{2}}\) Funds such as ICICI Ventures, IFC, Ashmore and Apax Partners invested about US$ 450 million in the first six months of 2008-2009 compared to US$ 125 million during the same period of the previous year. Feedback Ventures expects private equity funds to invest at least US$ 1 billion during 2009-2013. 12 percent of the US$ 77 million venture capital investments in July-September 2009 were in the healthcare sector. GE plans to invest over US$ 3 billion on R&D, US$ 2 billion to drive healthcare information technology and health in rural and underserved areas, US$ 1 billion in partnerships, content and services, over the next six years.

\(\text{\footnotesize{3}}\) Indian Healthcare market is estimated to touch US$ 77 billion by 2013(Pricewaterhouse Coopers, 2007). Healthcare industry accounted for 5.1% of India’s GDP in 2006. The compound annual growth rate of Indian healthcare sector was 16% during the 1990s. (Pricewaterhouse Coopers, 2007) and is expected to grow at a compound annual growth rate (CAGR) of 15% over the next 15 years (Ernst & Young: Fostering quality healthcare for all, 2008). It is also expected to generate employment to 9 million people in 2012. (Ernst & Young: Fostering quality healthcare for all, 2008).

\(\text{\footnotesize{4}}\) It is to be noted that Government of India has been worried to see the trend of foreign players taking over domestic players in the health care sector (pharmaceutical firms, etc). India today allows 100 per cent FDI in the health sector, but the policy is being reviewed in the wake of fears over the takeover of these domestic companies by MNCs leading to the fact that essential medicines becoming costlier and thereby impacting public health programmes, including the universal immunisation programme. Keeping in view the need to exercise a certain degree of supervision over takeovers, the Ministry has recommended that prior approval of the Foreign Investment Promotion Board (FIPB) be made mandatory.
demand of public health care, it has to develop modern and efficient health infrastructure. Now, modern health infrastructure needs huge mounts of capital and funds. There exists enough evidence that state of most of the developing economies are less willing to finance modern health facilities\(^5\). It also implies that the problem of disequilibrium will persist in public funding health care. To way out from this problem state may allow large amount of foreign direct investment (FDI) in public health care\(^6\). It implies that if health trade of mode 3\(^7\) is possible in public health care in a developing economy then the above mentioned disequilibrium phenomena will be vanished.

Thus it is become important to analyze the issues related to public health care and international trade in health services. Apart from that point of view we can also say that at the theoretical level there exists almost no work related to FDI and public health care in a general equilibrium trade model and hence we are trying to fill up this lacuna in this paper. To examine the above mentioned facts that health trade of mode 3 may enhance the supply side of public health care and the welfare impact of such type trade in health services, we have developed a three sector general equilibrium trade model. The present model is an extension of Beladi-Marjit(1992) as in this model as a third sector a public funding health sector has been introduced\(^8\). In this paper we

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\(^5\) National Health Accounts (NHA) has Shown that in India public health expenditure as a share of GDP increased from 0.96 per cent in 2004-05 to just 1.01 per cent in 2008-09 as compared to 5 per cent for developed economies. The public health sector is characterized by economically inefficient along with poor physical infrastructure. The mismatch between demand and supply of healthcare services and infrastructure has triggered the emergence of private participation in the Indian health sector through FDI.

\(^6\) State have taken some steps in a post globalization period and they are reduction of import duty for medical equipments and devices to 5% with countervailing duty (CVD) 0f 4%, over 50% of which are being imported. Assistive devices, rehabilitation aids, etc. have been completely exempted from CVD (Union Budget 2010-2011), relaxed rules for the NRI medical practitioners to invest and provide medical services in India (Baru, 1998) and depreciation rates for essential equipments and consumables increasing from 25% to 40%, giving tax saving incentive to the healthcare institutions, introduction of Medical Visa and Medical Attendant Visa for a period of one year with four multiple entries in 2005 and allowing 100% FDI in Healthcare sector in 2000, making long term loans and capital cheaper for Healthcare institutions due to 2002-2003 and 2003-2004 union budget and introduction of 100% Income Tax exemption for a period of five years, for new hospitals with more than 100 beds and located outside eight agglomerations (Finance Act, 2008, GOI), etc.

\(^7\) It has been rightly pointed out in General Agreement on Trade in Services (GATS) that trade in health services occur through four modes. These are namely, cross border delivery of trade, consumption of health services abroad, commercial presence (mode 3) and movement of health personnel.

\(^8\) In the three-sector models on foreign capital and welfare the third sector may either be an export processing zone as in the work of Beladi-Marjit(1992) or it may be the urban informal sector as in the works of Grinols(1991), Gupta(1997) etc or it may be intermediate goods producing sector as in the works of Marjit and Beladi(1997) and Marjit, Broll and Mitra (1997).
have examined the impact of health trade through mode 3 of General Agreement on trade in services (GATS) on the size of a public funding health care, skilled- unskilled wage gap and social welfare of the small open economy.

The paper is organized in the following manner. Section 2 considers the model. Section 3 considers the drive towards health capital mobility. It has one subsection. Subsection 3.1 considers the Impact of Foreign Health Capital Inflow on Social Welfare. Finally, the concluding remarks are made in section 4.

2. The Model

Consider a small open economy with three sectors. Among these three sectors, one produces exportable composite good (X_A) with unskilled labour (L) and capital (K). The second one produces an import competing good (X_M) with skilled labour (S), unionized unskilled labour, hired at a fixed wage \( \bar{W} \) and capital. The third sector is a public health good (X_H) producing sector. X_H is produced with skilled labour and health capital (N). It is to be noted that the cost of production of public health sector is totally financed by government revenue per unit of health output (G). Here we have also assumed that the budget of government is a balance budget type. The budget is balance in the sense that tax revenue per unit of health output (G) that come from two different sources, the first one come from return of foreign capital (rK_F) with tax rate t_F and the second one come from return of foreign health capital(RN_F) with tax rate t_N. Markets are competitive, technology is neoclassical and resources are fully employed. Note that there is no open unemployment as workers cannot survive without jobs and hence both the unskilled and skilled labour markets always clear.

The following is the list of symbols used in the model.

- \( P^*_A \) = world price of commodity A; \( P_A \) = domestic price of commodity A. we assume \( P_A = P^*_A = 1 \); \( P^*_M = P_M \) = world price of good M; \( N_D \) = domestic health capital stock of the economy; \( N_F \) = foreign health capital stock of the economy; \( K_F \) = foreign capital stock; \( K_D \) = domestic capital stock; \( a_{ji} \) = quantity of the jth factor for producing one unit of output in the ith sector, \( j=L,K,N \) and \( i=A,M,H \); \( \theta_{ji} \) = distributive share of the jth input in the ith sector; \( \lambda_{ji} \) = proportion of the jth factor used in the production of the ith sector; \( W \) = competitive unskilled wage rate; \( W_S \) =
competitive skilled wage rate; \( r \) = rate of return to capital; \( R \) = rate of return to health capital; \( Y \) = national income at domestic price; \( \sigma_i \) = elasticity of factor substitution in sector \( i \), \( i = A, M, H \).

The competitive price equations are:

\[
\begin{align*}
\text{a}_{LA}W + \text{a}_{KR} &= 1 \quad (1) \\
\text{a}_{SM}W + \text{a}_{LM}W + \text{a}_{KM} &= P_M \quad (2) \\
\text{a}_{SH}W + \text{a}_{NH}R &= G \quad (3)
\end{align*}
\]

Full-employment conditions are:

\[
\begin{align*}
\text{a}_{NH}X &= N_D + N_F = N \quad (4) \\
\text{a}_{KA}X + \text{a}_{KM} &= K_D + K_F \quad (5) \\
\text{a}_{LA}X + \text{a}_{LM} &= L \quad (6) \\
\text{a}_{SM}X + \text{a}_{SH} &= S \quad (7)
\end{align*}
\]

Balanced budget equation can be written as

\[
G = t_FK_F + t_NRN_F \quad (8)
\]

Given \( P_M \), \( N \), \( K \), \( L \), \( S \), \( t_F \), \( t_N \), we can determine \( W \), \( r \), \( W_S \), \( R \), \( X_A \), \( X_M \), \( X_H \) and \( G \) from eight equations. Determination of the general equilibrium proceeds as follows.

From equation (1) and (2) we can express \( W \) and \( r \) in terms of \( W_S \). Using equation (8) in equation (3) we can derive \( R \) as a function of \( W_S \). Thus it can be derived that \( W \), \( r \) and \( W_S \) are function of \( R \) only. Since all factor prices are expressed in terms of \( R \), \( a_{ij} \)s are also be expressed in terms of \( R \).

Let us start with a rise in \( R \). From equation (4) we can say that a rise in \( R \) implies a fall in \( a_{NH} \). Given \( N \) a fall in \( a_{NH} \) implies \( X_H \) has to be increased for maintaining health capital market equilibrium. Thus the locus of \( R \) and \( X_H \) for which health capital market will be in equilibrium is positively sloped and named by NN schedule. Mathematically the slope of NN curve can written as,

\[
\frac{dR}{dX_H} \bigg|_{NN} = (R / A_6X_H) > 0 \quad (4.3)
\]

Given \( N_F \) and \( t_N \), from equation (4) we can write
Similarly, we can find another locus of \( R \) and \( X_H \) which maintains equilibrium in the market for skilled workers. It is called by SS schedule. The slope of the SS schedule can be obtain from equation (7)

Given \( N_F \) and \( t_N \),

\[
\frac{dR}{dX_H} \bigg|_{SS} = \left( \frac{X_H}{A_{22} R} \right) < 0
\]

(7.3)

As \( A_{22} < 0 \), we can conclude that The SS schedule is negatively sloped.

![Figure 1](image_url)

The intersection of NN and SS schedule gives us the equilibrium values of \( R \) and \( X_H \). Once \( R \) is known \( W, W_S, r, X_A, X_M \) and \( G \) are also known.

### 3. Drive towards Health Capital Mobility

In this section we want to examine the impact of trade liberalization through foreign health capital inflow on output levels of different sectors in the presence of welfare state. Apart from that here we will show the effect of foreign health capital inflow on wage inequality.

Before going towards the comparative statics, we have to consider following assumptions.
**Assumption 1:** Share of government expenditure on health capital is greater than share of government revenue from health capital, i.e., $\theta_{NH} > t_N N_F R / G$.

**Assumption 2:** Change in the stock of foreign health capital due to trade liberalization will be greater compare to the change in the demand for health capital, i.e., $X_H \, d\alpha_{NH} < dN_F$.

Differentiation of equation (8) gives us

$$\hat{G} = A_4 \hat{R} + A_5 \hat{N}_F$$  \hspace{1cm} (8.2)

Where, $A_4 = (\theta_1 / A_2 + \theta_2) > 0$, $A_5 = (\theta_2 - \theta_1 A_3/A_2) > 0$.

Given $N_F$, we can write

$$\frac{dR}{dG} \bigg|_{\text{GG}} = \left( \frac{R}{A_4 G} \right) > 0$$  \hspace{1cm} (8.3)

Thus we can say that there exists a positive relationship between $R$ and $G$. Intuition behind this result is that an increase in $R$ implies a rise in tax revenue from the earnings of foreign health capital and hence $G$ will also go up. Thus the locus of different combinations of $R$ and $G$ will be positively sloped and balance-budget condition will also be maintained along this locus.

Using equations (1), (2) and (3), we can write
\[ \hat{W}_s - \hat{W} = (\theta_{KA} / \theta_{LA} - \theta_{KM} / \theta_{SM}) \left[ \frac{1}{A_2} \hat{R} - \frac{A_3}{A_2} \hat{N}_F \right] \tag{9} \]

Here, we have assumed that, \((\theta_{KA} / \theta_{LA} \theta_{KM} / \theta_{SM})\)

From equations (8.2) and (9) for given \(N_F\) we can say that \(\frac{\hat{W}_s - \hat{W}}{G} \rightarrow 0\), as \(\frac{\hat{G}}{R} \rightarrow 0\), i.e.,

\[ \frac{d(W_s / W)}{dG} \bigg|_{W_1-G} > 0. \]

Thus the locus of \((W_s/W)G\) is positively sloped and it is referred to \(W_1-G\) schedule.

Before going towards the main parts of comparative static we have to derive the relationship between \(R\) and \(N_F\) through the following lemma.

**Lemma 1:** An increase in \(N_F\) leads to a fall in \(R\) iff \(SSRFHNNRFH\), \(0^\hat{0}\), \(0^\hat{0}\), \((\hat{X}_s / \hat{X}_F)\), \(\hat{G} = \hat{N}_F\), \((\hat{X}_s / \hat{X}_F)\), \(\hat{G} = \hat{N}_F\).

**Proof of lemma 1:** Using equations (7.2) and (4.2) and for given \(t_N\) one can obtain [For details see Appendix]

\[ A_8 \hat{N}_F - A_6 \hat{R} - A_7 \hat{N}_N = A_{22} \hat{R} + A_{23} \hat{N}_F + A_{23} \hat{N}_N \]

\[ (A_{22} + A_6) \hat{R} = (A_8 - A_{23}) \hat{N}_F - (A_7 + A_{23}) \hat{N}_N \]

For given \(t_N\), i.e. \(\hat{N}_N = 0\)

\[ \hat{R} = (A_8 - A_{23} / A_{22} + A_6) \hat{N}_F \]

Thus we can say that \(\hat{R} / \hat{N}_F < 0\), if \(A_8 > A_{23}\), that is, the shift parameter of SS curve due to increase in \(N_F\) is greater than the shift parameter of NN curve due to increase in \(N_F\).

Let us start with a inflow of foreign health capital to our small open economy. With larger inflow of foreign health capital into our stylized economy, the level of output of the sector where it is specific will increase. Thus for given \(R\) and hence for given \(a_{NH}\) from health capital market equilibrium we can argue that an inflow of \(N_F\) will lead to a rise in \(X_H\), that is, NN locus will shift rightward. Similarly we can examine the shift of SS schedule due to foreign health capital inflow.
To do so at first we have to examine the effect of an upward movement of $N_F$ on the output levels of sectors A and M. This can be explained with help of following figure.

The shift of KK and LL schedules can be analyzed as follows.

Differentiation of equations (5) and (6)

\[
\lambda_{KA} \dot{X}_A + \lambda_{KM} \dot{X}_M = A_{12} \hat{R} + A_{13} \dot{N}_F \tag{5.1}
\]

\[
\lambda_{LA} \dot{X}_A + \lambda_{LM} \dot{X}_M = A_{14} \hat{R} + A_{15} \dot{N}_F \tag{6.1}
\]

For given $X_M$, using equations (5.1) and (6.1) one can easily show that the KK and LL schedules will shift leftward and rightward respectively. Thus from figure-3 we can conclude that the output of agricultural sector will rise and of manufacturing sector will reduce. The economic interpretation behind this argument is that an increase in $N_F$ implies a fall in $R$. Reduction in $R$ will lead to a rise in $W$, $W$ and a fall in $r$. Again a fall in $r$ and an increase in $W$ imply fall in $a_{LA}$ and $a_{LM}$. Thus labour availability to sectors A and M will increase and hence sector A will go up and sector M will go down due to Rybczynski-type effect.

Using this facts and for given $R$, from equation (7) we can argue that $X_H$ will go up. Hence SS locus will shift rightward. Thus from figure-2 we can show that $X_H$ will go up and $R$ will go
down due to foreign health capital inflow. This result may be interpreted economically as well. Let us start with an increase in $N_F$. A rise in foreign health capital stock implies the return from it will decline (see lemma 1 also), that is, $R$ will go down. Again an inflow of $N_F$ to health sector implies the infrastructure of the corresponding health sector will develop and hence this sector can also expand due to infrastructural development force. On the other hand a reduction in $R$ will lead to a rise in $W_S$, $W$ and a fall in $r$. and thus sector $M$ will go down due to Rybczynski-type effect. Contraction of sector $M$ implies skilled labour availability to health sector will increase and hence $X_H$ will go up mainly due to factor price force.

**Proposition 1:** A movement towards trade liberalization through foreign health capital inflow leads to: i) a rise in the government expenditure per unit of health output, ii) expansion of both export and public health sectors and contraction of traditional import competing sector, under the sufficient conditions that $[t_N N_F R/(G \theta_{NH} - t_N N_F R)] > 0$ and $\lambda_{LM} < \lambda_{KM} (\lambda_{LA} / \lambda_{KA})$.

From the general equilibrium structure we can also infer about the movement of wage inequality due to foreign health capital inflow.

Reconsideration of equation (9)

$$\hat{W}_S - \hat{W} = (\theta_{KA} / \theta_{LA} - \theta_{KM} / \theta_{SM}) \left[ \frac{1}{A_2} \hat{R} - \frac{A_3}{A_2} \hat{N}_F \right]$$

Using lemma 1, from the above expression we can show that wage inequality will rise due to inflow of foreign health capital inflow (see figure-2), under the sufficient condition that $(\theta_{KA} / \theta_{LA} < \theta_{KM} / \theta_{SM})$.

**Proposition 2:** If $\theta_{LM} < \theta_{KM} (\theta_{LA} / \theta_{KA})$ and $[\theta_{SM} - \theta_{KM} (\theta_{LA} / \theta_{KA})] < 0$, $(W_S / W)$ will rise due to trade liberalization through foreign health capital inflow.

3.1 **Impact of Foreign Health Capital Inflow on Social Welfare**

We now consider the impact of foreign health capital inflow on the social welfare of our stylized small open economy. To solve the purpose we have to consider following equations.

The national income of the economy at domestic prices is given by
\[ Y = X_A + P_M X_M + G X_H - r K_F - R N_F \]  
\[ \text{Or } Y = (\bar{W} - W) a_L M X_M + W_L + W_S S + R N_D + r K_D + t_{\text{fr}} K_F X_H + t_N R N_F X_H \]  

(10.1)  

(10.2)

The demand side of the model is represented by a social utility function. Here we have assumed that the utility function \( U \) depends upon consumption demand of all the goods that have considered in our model and it also depends on the supply of health care. Hence the social utility function can be written as,

\[ U = W (D_A, D_M, D_H) + V (X_H) \]  

(11)

With \( W_A > 0, W_M > 0, W_H > 0, W_{AA} < 0, W_{MM} < 0, W_{HH} < 0, V_H > 0 \)

The balance of trade equilibrium requires that

\[ D_A + P_M D_M + G D_H = X_A + P_M X_M + G X_H - r K_F - R N_F \]  

(12)

To analyze the welfare implication of an inflow of foreign health capital totally differentiating equations (11) and (12), we obtain [for details see appendix A.2]

\[ \Omega(\hat{\Omega}/\hat{N}_F) = \Gamma(\hat{X}_M / \hat{N}_F) + \Pi((\hat{W}_S - \hat{W})/\hat{N}_F) + \Xi_1(\hat{X}_H / \hat{N}_F) \]  

(13)

From the above expression we can infer that social welfare depends on three different terms that have shown in the RHS of equation (13). As a result of foreign health capital inflow we find \( X_M \) decrease and it will cause a negative impact on social welfare. We call it Output Effect of Manufacturing Sector (OEMS). From the second term of RHS of equation (13) we can show that trade liberalization may aggravate wage income gap. Since wages of both skilled and unskilled labour have increased due to such type of trade liberalization, it implies wage gap causes a positive impact on welfare. We call it Wage Gap Effect (WGE). Finally we can show that welfare may also increase due to expansion of public subsidized health service sector. It is very easy to explain that welfare will improve as infrastructure and size of public funding health care increases due to foreign health capital inflow. We refer it Health Efficiency Augmented Positive Effect (HEAPE)\(^{10}\). As \( \Gamma, \Pi, \) and \( \Xi_1 \) are positive, from equation (13) we can find that the social

\[^{9}\text{The production function for sector A is } F^A(L_A, K_A), \text{ for sector M is } F^M(S_M, L_M, K_M) \text{ and } F^H(S_H, N_H) \text{ is the production function for sector H. We need these production functions for derivation of equation (36) as shown in Appendix A.2.}\]

\[^{10}\text{The introduction of private health care in a general equilibrium structure implies there exists a positive labour productivity effect on the production side of the economy. However, such type of productivity effect become negligible in the context of public health care, since public health care affect positively more on consumer side. So, to capture the positive effect of public health care on consumer we have introduced } V(X_H) \text{ on the R.H.S of (11).}\]
welfare will improve due to health trade of mode 3 if the composite effects of WGE and HEAPE dominates over OEMS. Thus the above analysis can be summarized in the form of following proposition.

**Proposition 3:** Health trade of mode 3 in a public funding health care leads to an improvement in welfare if 
\[
\{(\bar{W} - W)A_{LM}X_M + (rK_F / A_2 + RN_F)A_2 / (\theta_{KM} - \theta_{KA} - \theta_{LA}) + (V_h / W_h)X_H\} > 0.
\]

4. Concluding Remarks

This paper deals with a very crucial aspect of a developing economy, that is, the aspects of health sector. Moreover here we are incorporating the phenomenon of government intervention in health sector and we are very keen to analyze the impact of trade liberalization through foreign health capital inflow not only on the output level of government improvising health care sector but also on the government revenue account. Apart from that we have also interested to find out the movement of wage inequality among skilled and unskilled workers.

Here we have shown that inflow of foreign health capital leads to expansion of both public health care and agricultural sector and contraction of manufacturing sector. Moreover we have derived that government revenue account will also enhance due to implementation of such type of trade policy. It is to be noted that a rise in government revenue may affect the public health care positively. It is very often to argue that private health sector dominates over public health sector in most of the developing economy. The reason behind it has been clarified by several economists as well as by politicians is trade liberalization. On the contrary of that argument in this paper we have shown that trade liberalization through foreign health capital inflow lead to an expansion of public health care in our stylized economy. Apart from that the present study offer clear reasons to show that even in the presence of welfare state, wage inequality may rise after trade liberalization, which is also going against the traditional myth. Finally from this paper we have shown that the welfare of the small open economy in the presence of a public funding health care will improve due to trade in health services of mode 3 and thus ensures the gains from such type of health trade.
References


Appendix A

Appendix A.1

Differentiation of equation (1) gives us,

$$\dot{W} = -\left(\frac{\theta_{K_a}}{\theta_{L_a}}\right) \dot{r}$$

(14)

Using $\text{da}_{LM} = dW = 0$ and envelop condition, that is, $\text{da}_{SM} + \text{da}_{KM} = 0$ in equation (2),

$$\dot{W}_S = -\left(\frac{\theta_{KM}}{\theta_{SM}}\right) \dot{r}$$

(15)

Differentiating equation (3) we obtain,

$$\theta_{SH} \dot{W}_S + \theta_{KM} \dot{R} = \dot{G}$$

(16)

Similarly, differentiation of equation (8)

$$\dot{G} = \theta_1 \dot{r} + \theta_2 \dot{R} + \frac{\theta_2}{\theta_{NH} - \theta_2} \dot{N}_F + \frac{\theta_2}{\theta_{NH} - \theta_2} \dot{N}_T$$

(17)

Where, $\theta_1 = (t_N K_F / G) > 0$, $\theta_2 = (t_N N_F R / G) > 0$.

Using equation (17) in equation (16),

$$\dot{R} = \left\{A_1/(\theta_{NH} - \theta_2)\right\} \dot{r} + \left\{\theta_2/(\theta_{NH} - \theta_2)\right\} \dot{N}_F + \left\{\theta_2/(\theta_{NH} - \theta_2)\right\} \dot{N}_T$$

$$\dot{R} = A_2 \dot{r} + A_3 \dot{N}_F + A_3 \dot{N}_T$$

$$\dot{r} = \left\{(1/A_2)\right\} \dot{R} - (A_3/A_2) \dot{N}_F - (A_3/A_2) \dot{N}_T$$

(18)

Where, $A_1 = (\theta_1 + \theta_{KM}/\theta_{SH}) > 0$, $A_2 = \left\{A_1/(\theta_{NH} - \theta_2)\right\} > 0$, $A_3 = \left\{\theta_2/(\theta_{NH} - \theta_2)\right\} > 0$. Here we have assumed that $\theta_{NH} > t_N N_F R / G$.

Inserting the value of $\dot{r}$ in equation (17)

$$\dot{G} = A_4 \dot{R} + A_5 \dot{N}_F + A_5 \dot{N}_T$$

(19)

Where, $A_4 = (\theta_1 / A_2 + \theta_2) > 0$, $A_5 = (\theta_2 - \theta_1 A_3 / A_2) > 0$.

Differentiation of equation (4),
\[ \lambda_{NH} \hat{X}_H + \lambda_{NH} \hat{a}_{NH} = \mu \hat{N}_F \]  

(20)

From envelop theorem we get

\[ R \, da_{NH} + W \, da_{SH} = 0 \]

\[ \hat{a}_{NH} = - \left( \frac{\theta_{NH}}{\theta_{SH}} \right) \hat{a}_{SH}. \]

Using the elasticity of factor substitution of sector H and inserting the value of \( \hat{W}_S \) we can get,

\[ \sigma_H = (\hat{a}_{NH} / \hat{W}_S - \hat{R}) \]

\[ \hat{a}_{NH} = [-\{(\sigma_H \, \theta_{KM} \, A_1 / \theta_{NH} \, \theta_{SM} \, A_2) + 1\}] \hat{R} + (\theta_{KM} \, A_3 / \theta_{SM} \, A_2) \hat{N}_F + (\theta_{KM} \, A_3 / \theta_{SM} \, A_2) \hat{i}_N \]

\[ \hat{a}_{NH} = A_6 \hat{R} + A_7 \hat{N}_F + A_7 \hat{i}_N \]  

(21)

Where, \( A_6 = [-\{(\sigma_H \, \theta_{KM} \, A_1 / \theta_{NH} \, \theta_{SM} \, A_2) + 1\}] < 0, \]

\( A_7 = (\theta_{KM} \, A_3 / \theta_{SM} \, A_2) > 0. \)

Using equations (20) and (21)

\[ \hat{X}_H = \{(\mu / \lambda_{NH}) - A_7\} \hat{N}_F - A_6 \hat{R} - A_7 \hat{i}_N \]

\[ \hat{X}_H = A_8 \hat{N}_F - A_6 \hat{R} - A_7 \hat{i}_N \]  

(22)

Where, \( A_8 = \{(\mu / \lambda_{NH}) - A_7\} > 0. \) Here, \( (\frac{\mu}{\lambda_{NH}} - A_7) > 0 \) as per the following assumption.

Differentiating equations (5) and (6) and after some simplification

\[ \lambda_{KA} \hat{X}_A + \lambda_{KA} \hat{a}_{KA} + \lambda_{KM} \hat{X}_M + \lambda_{KM} \hat{a}_{KM} = 0 \]  

(23)

Using the envelop condition \( W \, da_{LA} + r \, da_{KA} = 0 \) and elasticity of factor substitution of sector A, that is, \( \sigma_A = (\hat{a}_{KA} / \hat{W}_L - \hat{r}) \)

\[ \hat{a}_{KA} = (-\sigma_A / A_2) \hat{R} + (\sigma_A A_3 / A_2) \hat{N}_F + (\sigma_A A_3 / A_2) \hat{i}_N \]  

(24)

Similarly, Using the envelop condition \( W_S \, da_{SM} + r \, da_{KM} = 0 \) and elasticity of factor substitution of sector A, that is, \( \sigma_M = (\hat{a}_{KM} / \hat{W}_S - \hat{r}) \)

\[ \hat{a}_{KM} = (-\sigma_M / \theta_{KM} \, A_2 A_0) \hat{R} + (\sigma_M A_3 / \theta_{KM} \, A_2 A_0) \hat{N}_F + (\sigma_M A_3 / \theta_{KM} \, A_2 A_0) \hat{i}_N \]
\[ \hat{a}_{KM} = A_{10}\hat{R} + A_{11}\hat{N}_F + A_{11}\hat{t}_N \]  

(25) 

Where, \( A_9 = (\theta_{SM} + \theta_{KM})/\theta_{KM} \geq 0 \), \( A_{10} = (-\sigma_M/\theta_{KM} A_2 A_9) < 0 \) and \( A_{11} = (\sigma_M A_3/\theta_{KM} A_2 A_9) > 0 \). 

Using (24) and (25) in (23) 

\[ \lambda_{KA} \hat{X}_A + \lambda_{KM} \hat{X}_M = ((\lambda_{KA} \sigma_A/\theta_{LA} A_2) - \lambda_{KM} A_{10}) \hat{R} - ((\lambda_{KA} \sigma_A A_3/\theta_{LA} A_2) - \lambda_{KM} A_{11}) \hat{N}_F - ((\lambda_{KA} \sigma_A A_3/\theta_{LA} A_2) - \lambda_{KM} A_{11}) \hat{t}_N \] 

(26) 

Similarly, we can show that 

\[ \lambda_{LA} \hat{X}_A + \lambda_{LM} \hat{X}_M = -((\lambda_{LA} \theta_{KA} \sigma_A/\theta_{LA} A_2)) \hat{R} + (\lambda_{LA} \theta_{KA} \sigma_A A_2/\theta_{LA} A_2) \hat{N}_F + (\lambda_{LA} \theta_{KA} \sigma_A A_2/\theta_{LA} A_2) \hat{t}_N \] 

(27) 

Where, \( A_{14} = -((\lambda_{LA} \theta_{KA} \sigma_A/\theta_{LA} A_2)) \) < 0, \( A_{15} = (\lambda_{LA} \theta_{KA} \sigma_A A_2/\theta_{LA} A_2) > 0 \). 

Here, \( |\lambda| = (\lambda_{KA} \lambda_{LM} - \lambda_{KM} \lambda_{LA}) < 0 \). 

From the above equation one can obtain 

\[ \hat{X}_A = A_{16}\hat{R} + A_{17}\hat{N}_F \] 

(28) 

Where, \( A_{16} = \frac{1}{|\lambda|}(\lambda_{LM} A_{12} - \lambda_{KM} A_{14}) < 0 \) and \( A_{17} = \frac{1}{|\lambda|}(\lambda_{LM} A_{13} - \lambda_{KM} A_{15}) > 0 \). 

And, \( \hat{X}_M = A_{18}\hat{R} + A_{19}\hat{N}_F \) 

(29) 

Where, \( A_{18} = \frac{1}{|\lambda|}(\lambda_{KA} A_{14} - \lambda_{LA} A_{12}) > 0 \), \( A_{19} = \frac{1}{|\lambda|}(\lambda_{KA} A_{15} - \lambda_{LA} A_{13}) < 0 \). 

Differentiation of equation (7) 

\[ \lambda_{SM} \hat{X}_M + \lambda_{SH} \hat{X}_H = -\lambda_{SM} \hat{a}_{SM} - \lambda_{SH} \hat{a}_{SH} \] 

(30)
Now, \( \hat{a}_{SM} = (\sigma_M / A_2) \hat{R} - (\sigma_M A_3 / A_2) \hat{N}_F - (\sigma_M A_3 / A_2) \hat{t}_N \)

and \( \hat{a}_{SH} = (-\theta_{NH} A_6 / \theta_{SH}) \hat{R} - (\theta_{NH} A_7 / \theta_{SH}) \hat{N}_F - (\theta_{NH} A_7 / \theta_{SH}) \hat{t}_N \)

Using both \( \hat{a}_{SM} \) and \( \hat{a}_{SH} \) in equation (30) one can obtain

\[
\lambda_{SM} \hat{X}_M + \lambda_{SH} \hat{X}_H = \{(\lambda_{SH} \theta_{NH} A_6 / \theta_{SH}) - (\lambda_{SM} \sigma_M / A_2)\} \hat{R} + \{(\lambda_{SM} \sigma_M A_3 / A_2) + (\lambda_{SH} \theta_{NH} A_7 / \theta_{SH})\} \hat{N}_F + \{(\lambda_{SM} \sigma_M A_3 / A_2) + (\lambda_{SH} \theta_{NH} A_7 / \theta_{SH})\} \hat{t}_N
\]

\[
\lambda_{SM} \hat{X}_M + \lambda_{SH} \hat{X}_H = A_{20} \hat{R} + A_{21} \hat{N}_F + A_{21} \hat{t}_N \quad (31)
\]

Where, \( A_{20} = \{(\lambda_{SH} \theta_{NH} A_6 / \theta_{SH}) - (\lambda_{SM} \sigma_M / A_2)\} \langle 0, A_{21} = \{(\lambda_{SM} \sigma_M A_3 / A_2) + (\lambda_{SH} \theta_{NH} A_7 / \theta_{SH})\} \rangle 0 \)

Using equation (29) in equation (31)

\[
\hat{X}_H = \left\{ \frac{1}{\lambda_{SH}} \right\} (A_{20} - \lambda_{SM} A_{18}) \hat{R} + \left\{ \frac{1}{\lambda_{SH}} \right\} (A_{21} - \lambda_{SM} A_{19}) \hat{N}_F + \left\{ \frac{1}{\lambda_{SH}} \right\} (A_{21} - \lambda_{SM} A_{19}) \hat{t}_N
\]

\[
\hat{X}_H = A_{22} \hat{R} + A_{23} \hat{N}_F + A_{23} \hat{t}_N \quad (32)
\]

Where, \( A_{22} = \{1 / \lambda_{SH} \} (A_{20} - \lambda_{SM} A_{18}) \} < 0, A_{23} = \{1 / \lambda_{SH} \} (A_{21} - \lambda_{SM} A_{19}) \} > 0 \)

**Appendix A.2. The impact on welfare**

Total differentiation of equations (11) and (12) gives us

\[
(dU/U_A) = dD_A + P_M dD_M + G dD_H + \Xi dX_H \quad (33)
\]

\[
dD_A + P_M dD_M + G dD_H = dX_A + P_M dX_M + G dX_H - rdK_F - K_Fdr - RdN_F - N_Fdr \quad (34)
\]

Differentiating equation (10.1) one can obtain

\[
dY = [dX_A + P_M dX_M + G dX_H - rdK_F - K_Fdr - RdN_F - N_Fdr] + X_H dG \quad (35)
\]

By differentiating production functions (see footnote 8) and using (4), (5), (6) and (7)

\[
[dX_A + P_M dX_M + G dX_H - rdK_F - K_Fdr - RdN_F - N_Fdr] + X_H dG
\]

\[
= (F_L^A dL_A + F_K^A dK_A) + P_M (F_S^M dS_M + F_L^M dL_M + F_K^M dK_M) + G (F_S^H dS_H + F_N^H dN_H) - rdK_F - K_Fdr - RdN_F - N_Fdr] + X_H dG
\]

\[
= (WdL_A + \overline{W} dL_M) + r (dK_A + dK_M - dK_F) - K_F dr + W_S (dS - dS_M - dS_H) + (RdN - RdN_F - N_Fdr) + X_H dG
\]
\[(\overline{W} - W) a_{LM} dX_M - K_F dr - N_F dR + t_N RX_H dN_F + t_N X_H N_F dR + t_K R X_H dK_F + t_K X_H K_F dr\]

(assuming \(dK_A + dK_M = dK_F, \ dN_F = dN \text{ and } dL = 0\))

\[= (\overline{W} - W) a_{LM} dX_M - (t_K X_H - 1) K_F dr + (t_N X_H - 1) N_F dR + t_N RX_H dN_F\]  \hspace{1cm} (36)

Using (36) in equation (35)

\[dY = (\overline{W} - W) a_{LM} dX_M - (t_K X_H - 1) K_F dr + (t_N X_H - 1) N_F dR + t_N RX_H dN_F\]  \hspace{1cm} (37)

Using (37),(36) and (33)

\[\frac{1}{d(U_A)} dU = (\overline{W} - W) a_{LM} dX_M - K_F dr - N_F dR + \Xi dX_H\]  \hspace{1cm} (38)

Using (18) and (9) in (38)

\[\Omega(\hat{\Omega}) = \Gamma(\hat{X}_H) + \Pi(\hat{W}_H - \hat{W}) + \Xi_1(\hat{X}_H)\]  \hspace{1cm} (39)

Where, \(\Omega = dU / W_A\),

\[\Gamma = (\overline{W} - W) a_{LM} X_M, \Pi = (t_K f / A_2 + R N_F) A_2 / (t_K \theta_A / \theta_{LM} - \theta_{KM} / \theta_{SM}), \Xi = V_H / W_A,\]

\[\Xi_1 = (V_H / W_A) X_H\]