Schwarzschild Geometry from Exact Solution of Einstein Equation

Mohajan, Haradhan

Journal of Environmental Treatment Techniques

11 August 2013

Journal of ENVIRONMENTAL Treatment Techniques

Enviro. Treat. Tech.
ISSN: 2309-1185
Journal web link: http://www.jett.dormaj.com

Schwarzschild Geometry from Exact Solution of Einstein Equation

Haradhan Kumar Mohajan

Premier University, Chittagong, Bangladesh.

Received: 11/08/2013 Accepted: 16/10/2013 Published: 18/10/2013

Abstract

An exact solution of Einstein equation is easier than actual solution. The Schwarzschild metric is established on the basis of Einstein's exact solution and it is also a static and stationary solution. The Schwarzschild solution expresses the geometry of a spherically symmetric massive body's (star) exterior solution. It predicts small observable departures from the Newtonian gravity. It also represents theory of black holes when sufficiently massive stars unable to support themselves against the pull of self gravity and must undergo a complete gravitational collapse when they have exhausted their internal nuclear fuel. Various sides of Schwarzschild geometry, such as, Kruskal–Szekeres extension, space-time singularities and black hole formation, are discussed with simple but detail calculations. The black hole is a region from which no causal signals can reach to the external observers and it contains a space-time singularity hidden within the event horizon.

Key words: Einstein equation, Schwarzschild solution, Black hole, Space-time singularity.

1 Introduction

Karl Schwarzschild (1873–1916) discovered Schwarzschild metric in December 1915. In 1916 he had died by a disease (perhaps by Pneumonia). Schwarzschild metric is established assuming a star isolated from all the gravitating bodies. In this paper we consider the procedure of Schwarzschild geometry and singularities therein. By the exact solution of Einstein equation prediction of general theory of relativity, such as, the perihelion shift of planets, when a light ray passes near a heavy mass then the ray is deflected by the gravitational effect of that body a shift towards the red end of spectrum, verified experimentally. The Schwarzschild solution of Einstein equation is also important for the interpretation of black hole.

Einstein field equation is highly non-linear and it is combination of ten equations connecting the total twenty quantities and to solve Einstein equation need to place constraints on the simultaneous choice of these twenty quantities. One generally needs to impose several symmetry assumptions on the space-time in order to work out the metric components as a solution to the Einstein equation. In the actual universe the energy-momentum tensor will be made up of contributions from a large number of different matter fields, so that we need a precise measurement of it to solve Einstein equation. In fact one has little idea of the behavior of matter under extreme conditions of density and pressure (i.e., near big bang). Hence exact solutions of Einstein equation are better than actual solutions [1].

Haradhan Kumar Mohajan, Premier University, Chittagong, Bangladesh. E- mail: haradhan_km@yahoo.com

Schwarzschild established his metric by considering asymptotically flat solutions to Einstein's equation. Hence in our study we will consider all the known solutions of null geodesics of the space-time. This provides a complete illustration of the light cone from an arbitrary apex in the space-time [3].

2 Einstein field Equation

Einstein's field equation can be written as [5];

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^4} T_{\mu\nu} . \quad (1)$$

where G is the gravitational constant and c is the velocity of light. A metric is defined as;

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} \quad (2)$$

where $g_{\mu\nu}$ is an indefinite metric in the sense that the magnitude of non-zero vector could be either positive, negative or zero. Ricci tensor is defined as;

$$R_{\mu\nu} = g^{\lambda\sigma} R_{\lambda\mu\sigma\nu} . \quad (3)$$

Further contraction of (3) gives Ricci scalar;

$$\hat{R} = g^{\lambda\sigma} R_{\lambda\sigma} . \quad (4)$$

The energy momentum tensor $T^{\mu\nu}$ is defined as;

$$T^{\mu\nu} = \rho_0 \; u^{\mu} u^{\nu} \quad (5)$$

where ρ_0 is the proper density of matter, and if there is no pressure, and $u^\mu = X^\mu = \frac{dx^\mu}{dt}$ is a tangent vector. A perfect fluid is characterized by pressure $p = p(x^\mu)$, then;

$$T^{\mu\nu} = (\rho + p)\, u^\mu u^\nu + p g^{\mu\nu} \tag{6}$$

where ρ is the scalar density of matter. The principle of local conservation of energy and momentum states that;

$T^{\mu\nu}_{;\nu} = 0$.

Riemann curvature tensor

$$R^\alpha_{\mu\nu\sigma} = \Gamma^\alpha_{\mu\sigma;\nu} - \Gamma^\alpha_{\mu\nu;\sigma} + \Gamma^\alpha_{\beta\nu}\Gamma^\beta_{\mu\sigma} - \Gamma^\alpha_{\beta\sigma}\Gamma^\beta_{\mu\nu} \tag{7}$$

is a tensor of rank four.

Einstein introduced a cosmological constant $\Lambda(\approx 0)$ for static universe solutions as;

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda\ g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}. \tag{8}$$

It is clear that divergence of both sides of (1) and (8) is zero. For empty space $T_{\mu\nu} = 0$ then $R_{\mu\nu} = \Lambda\ g_{\mu\nu}$, then;

$$R_{\mu\nu} = 0 \text{ for } \Lambda = 0 \tag{9}$$

which is Einstein's law of gravitation for empty space.

3 Minkowski Space-time

The Minkowski space-time is the manifold $\mathcal{M} = \Re^4$ with Lorentzian metric is given by;

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 \tag{10}$$

where $-\infty < t, x, y, z < \infty$. Here coordinate *t* is timelike and other coordinates *x*, *y*, *z* are spacelike. This is a flat space-time manifold with $R^\mu_{\nu\lambda\sigma} = 0$. In spherical polar coordinates (10) becomes;

$$ds^2 = -dt^2 + dr^2 + r^2\left(d\theta^2 + \sin^2\theta\ d\phi^2\right) \tag{11}$$

with $0 < r < \infty,\ 0 < \theta < \pi,\ 0 < \phi < 2\pi$. Here coordinate *t* is timelike and other coordinates r, θ, ϕ are spacelike.

Under Lorentz transformation the Minkowski metric preserves both time and space orientations. The geodesics of Minkowski space-time are straight lines of the Euclidean geometry. If a future directed non-spacelike curve γ has a future end point *p*, then $I^-(\gamma) = I^-(p)$ ([2], [5]). Again, if γ is future inextensible without any future ideal point, the set $I^-(\gamma)$ determine a 'point at infinity' of $\mathcal{M}$. Two such curves γ_1 and γ_2 determine the same point or a point at infinity if $I^-(\gamma_1) = I^-(\gamma_2)$. This defines future ideal points, and the past ideal points are defined dually. In Minkowski space-time, $I^-(\gamma) = \mathcal{M}$ .Hence all such timelike curves determines single future ideal point i^+ (figure 2), called the future timelike infinity. The past timelike infinity is defined dually. Any timelike geodesic originated at i^- is finished at i^+ (figure 1). Let the

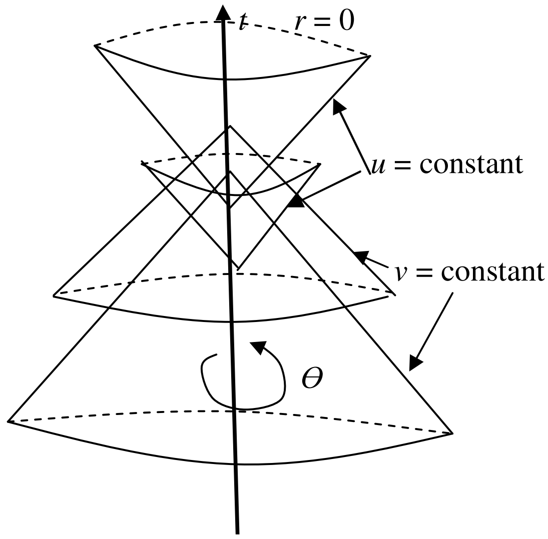

Figure 1: Future and past light cones are given respectively by the null surfaces *u* = constant and *v* = constant.

Collection of future ideal points be denoted by $\mathcal{J}^+$, and $\mathcal{J}^+$ is a three-dimensional manifold with topology $S^2 \times \Re$. For Minkowski space-time three-dimensional null hypersurface $\mathcal{J}^+$ and $\mathcal{J}^-$ are called future and past null infinities respectively [3].

If in equation (11) the advanced and retarded null coordinates are given by (figure 1);

$$v = t + r, \qquad u = t - r \tag{12}$$

then (11) becomes;

$$ds^2 = -du dv + \frac{1}{4}(u - v)^2\left(d\theta^2 + \sin^2\theta\ d\phi^2\right) \tag{13}$$

with $-\infty < u < \infty$ and $-\infty < v < \infty$.

Absence of du^2 and dv^2 in (13) indicates that {u = constant} and {v = constant} are null.

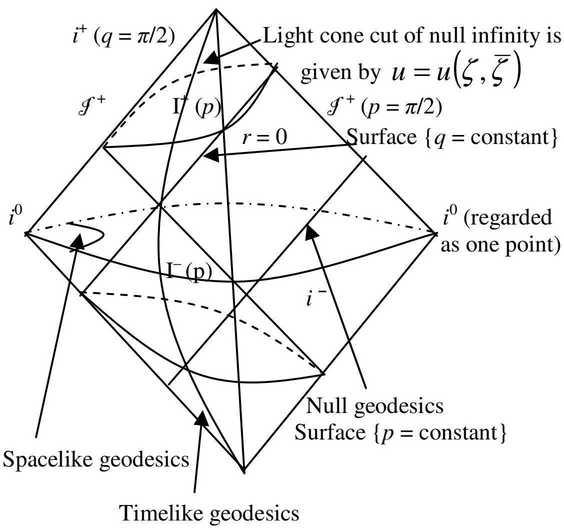

Figure 2: Conformal infinity in the Minkowski space-time. The future and past null infinities are $\mathscr{I}^{\pm}$.

4 Schwarzschild Geometry

The Schwarzschild metric represents the empty exterior solution where the Ricci tensor vanishes and which is matched to the interior solution inside the body. In (t, r, θ, ϕ) coordinates, the Schwarzschild metric is given by;

$$ds^2 = -\left(1 - \frac{2m}{r}\right) dt^2 + \left(1 - \frac{2m}{r}\right)^{-1} dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right) \qquad (14)$$

where $r = 2m$ is the Schwarzschild radius, for the earth r = 1 cm and for the sun r = 3 km. Here coordinate t is timelike and other coordinates r, θ, ϕ are spacelike. The range of coordinate t is $-\infty < t < \infty$. From (14) we see that for t = constant and r = constant we find the two-sphere;

$$ds^2 = r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right) \qquad (15)$$

and the area of such two-sphere would be $4\pi\, r^2$. Here we have;

$$m = \frac{GM}{c^2}, \qquad (16)$$

where M is the point mass at the origin which gives rise to the Newtonian gravitational potential ϕ. From (16) we observe that the Schwarzschild solution is interpreted as describing the gravitational field of a point particle with mass m situated at the centre, in relativistic units G =c = 1 [2], The coordinate r is restricted by the condition $r > 2m$, as the metric (14) has an apparent singularity at $r = 2m$. If r_0 is the boundary of a star then $r > r_0$ gives the outside metric as in (14). If there is no surface, (14) represents a highly collapsed object viz. a black hole of mass m [5]. The metric (14) has singularities at $r = 0$ and $r = 2m$, so it represents patches $0 < r < 2m$ or $2m < r < \infty$. If we consider the patches $0 < r < 2m$ then it is seen that as r tends to zero, the curvature scalar, is divergent and it follows that the point $r = 0$ is a real space-time singularity.

$$R^{\mu\nu\sigma\lambda} R_{\mu\nu\sigma\lambda} = \frac{48m^2}{r^6}$$

At $r = 2m$ the curvature scalars are well behaved at this point, so it is a singularity due to inappropriate choice of coordinates. The maximal extension of the manifold (14) with $2m < r < \infty$ was obtained by [4] and [6]. We now discuss about this, which uses suitably defined advanced and retarded null coordinates. For null geodesics $ds^2 = 0$, $d\theta^2 = 0$, $d\phi^2 = 0$, then (14) takes the form,

$$\left(1 - \frac{2m}{r}\right) dt^2 = \left(1 - \frac{2m}{r}\right)^{-1} dr^2$$

$$t = \pm\int \frac{r}{r - 2m} dr$$

$$= \pm\left[r + 2m \log\left(\frac{r}{2m} - 1\right)\right] + \text{constant}$$

$$t = r^* + \text{constant}. \qquad (17)$$

The null coordinates u and v are defined by;

$$u = t - r^*, \qquad v = t + r^*$$

$$r^* = \frac{v - u}{2} \qquad (18)$$

Now;

$$\left(1 - \frac{2m}{r}\right) dt^2 + \left(1 - \frac{2m}{r}\right)^{-1} dr^2$$

$$= -\frac{2m}{r}\left(\frac{r}{2m} - 1\right)\left(dt^2 - dr^{*2}\right)$$

$$= -\frac{2m}{r} e^{-r/2m} e^{(v-u)/2m} \, du dv$$

Hence (14) becomes;

$$ds^2 = -\frac{2m}{r} e^{-r/2m} e^{(v-u)/4m} \, du dv + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2 \right). \qquad (19)$$

As $r \to 2m$ corresponds to $u \to \infty$ or $v \to -\infty$, we define new coordinates U and V by;

$$U = -e^{-u/4m}, \; V = e^{v/4m}$$

$$dUdV = \frac{1}{16m^2} e^{(v-u)/4m} \, du dv.$$

Hence (19) becomes;

$$ds^2 = -\frac{32m^3}{r} e^{-r/2m} \, dUdV + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2 \right). \qquad (20)$$

Hence there is no singularity at U = 0 or V = 0 which corresponds to the value at $r = 2m$.

Let us take a final transformation by;

$T = \frac{U+V}{2}$ and $X = \frac{V-U}{2}$, then (20) becomes;

$$ds^2 = \frac{32m^3}{r} e^{-r/2m} \left(-dT^2 + dX^2 \right) + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2 \right) \qquad (21)$$

which is Kruskal–Szekeres form of Schwarzschild metric. Then transformation (t, r) to (T, X) becomes;

$$X^2 - T^2 = -UV = e^{(v-u)/2m} = e^{r/2m} \left(\frac{r}{2m} - 1 \right) \qquad (22)$$

$$\frac{T}{X} = \tanh \frac{t}{4m} \quad \Rightarrow t = 4m \tanh^{-1} \frac{T}{X}. \qquad (23)$$

From (22), $r > 0$ gives $X^2 - T^2 > -1$. The physical singularity at r = 0 gives $X = \pm \left(T^2 - 1 \right)^{1/2}$, and we observe that there is no singularity now at $r = 2m$.

The original Schwarzschild solution for $r > 2m$ corresponds to the region I which is interpreted as the exterior gravitational field of a collapsing body. Regions I and I' are asymptotically flat and of identical properties. We use a conformal diagram of figure 3 as figure 4.

There is no causal communication between regions I and I'; any observer or photon from region I either goes away to infinity or crosses the null line X = T and enters region II. When an observer falls into the closed trapped surfaces region II, cannot escape from it and within a finite proper time the observer must fall into the singularity at $X = \left(T^2 - 1 \right)^{1/2}$.

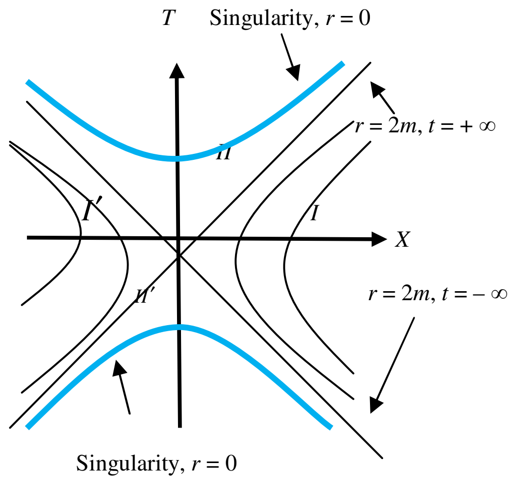

Figure 3: The Kruskal–Szekeres extension of the Schwarzschild metric.

Any light signal emitted from region II must fall into the singularity and never cross to region I. Hence region II is termed *black hole*.

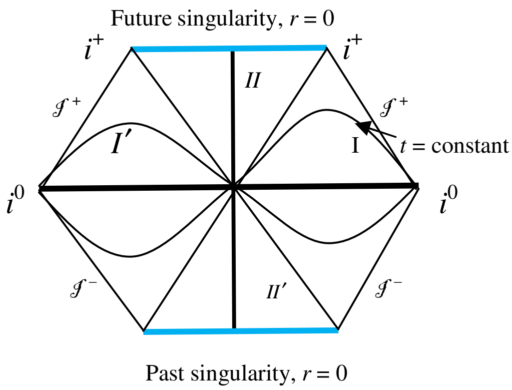

Figure 4: A conformal representation of the Schwarzschild geometry.

The region I' is another asymptotically flat universe on the other side of the Schwarzschild 'throat'. The S^2, r = constant surfaces are almost Euclidean for large values of r, but for small r, their areas decreases to a minimum corresponding to that of the value $r = 2m$, and then it increases again as the S^2.

Expand in the other region of asymptotically flat three space. However, if we consider a complete gravitational collapse of a spherically symmetric homogeneous dust cloud the regions I' and II' are no longer relevant as they are replaced by the interior metric which is not vacuum Schwarzschild, $T_{\mu\nu}$ being non-zero there (figure 5).

The region II' has time reversed properties of region *II* and also called a *white hole*. A particle emitted by the singularity at $X = -(T^2 - 1)^{1/2}$ must leave this region within a finite proper time. Each point of figure 3 is S^2 in space-time.

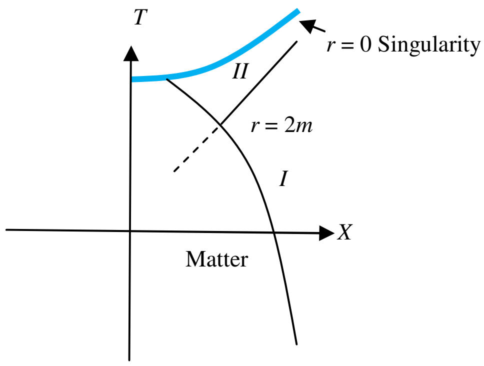

Figure 5: Complete gravitational collapse of a homogeneous dust cloud represented in the Kruskal picture.

If a source at a point *p* in the region $r > 2m$ emits a flash of light from *p*, then there will form two S^2, one by the outgoing wave front and the other by ingoing wave front, the area of outgoing sphere will be greater than ingoing one. But in region $r < 2m$ both areas of the ingoing and outgoing sphere will be less than that at *p*. Then *p* is a *closed trapped* surface. In the extended Schwarzschild manifold, the surface $r = 2m$ is a null hypersurface and at each point there is S^2 surface of area $16\pi m^2$. The uncovered portions of region *I* and *II* represent the vacuum Schwarzschild geometry exterior to the collapsing matter. The portion of region *II* indicates that a Schwarzschild black hole is always produced in the complete gravitational collapse which fully covers the resulting space-time singularity of infinite curvature and density, which plays an important role in cosmic censorship and the black hole formation. The straight lines $X = \pm T$ are odd labeling in the Schwarzschild solution, since on this line $t = \pm\infty$.

Let us consider the Schwarzschild solution in spacelike surface *r* = constant and the equatorial plane $\theta = \frac{\pi}{2}$ then (14) becomes;

$$ds^2 = \left(1 - \frac{3m}{r}\right)^{-1} dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right).$$

which is embedded in the ordinary Euclidean geometry. Here, $0 < r < r_b$ is filled by matter of spherical star with a boundary at $r = r_b$. We have retarded time;

$$u = t - r - 2m\log(r - 2m)$$

$$du^2 = dt^2 - \frac{r^2}{(r-2m)^2}dr^2 - \frac{2r}{(r-2m)}dudr,$$

$$-\left(1 - \frac{2m}{r}\right)du^2 - dudr = -\left(1 - \frac{2m}{r}\right)dt^2 + \left(1 - \frac{2m}{r}\right)^{-1}dr^2.$$

Then (14) takes the form;

$$ds^2 = -\left(1 - \frac{2m}{r}\right)du^2 - 2\,dudr + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right). \tag{24}$$

Let us put $\ell = \frac{1}{\sqrt{2}\,r}$ and $\sqrt{2}\,u' = u$. Let us introduce complex stereographic coordinates ζ and its complex conjugate $\bar{\zeta}$ such that $u = u(\zeta, \bar{\zeta})$ on the sphere are defined by;

$$\zeta = e^{i\phi}\cot\frac{1}{2}\theta, \quad \bar{\zeta} = e^{-i\phi}\cot\frac{1}{2}\theta$$

$$d\zeta = -e^{i\phi}\left(\frac{1}{2}\cos ec^2\frac{1}{2}\theta\, d\theta - i\cot\frac{1}{2}\theta\, d\phi\right)$$

$$d\bar{\zeta} = -e^{-i\phi}\left(\frac{1}{2}\cos ec^2\frac{1}{2}\theta\, d\theta + i\cot\frac{1}{2}\theta\, d\phi\right)$$

$$\left(\frac{1+\zeta\bar{\zeta}}{2}\right)^2 = P_0^2 = \frac{1}{4}\cos ec^4\frac{1}{2}\theta$$

$$\frac{d\zeta\, d\bar{\zeta}}{P_0^2} = d\theta^2 + \sin^2\theta\, d\phi^2.$$

Hence the conformal transformation of (24) becomes;

$$d\bar{s}^2 = \Omega^2 ds^2 = -\left(1 - 2m\sqrt{2}\ell\right)4\ell^2 du'^2$$

$$+ 2\sqrt{2}.\sqrt{2}du'd\ell + \frac{d\zeta d\bar{\zeta}}{P_0^2}$$

$$= -4\left(\ell^2 - 2\sqrt{2}m\ell^3\right)du'^2 + 4du'd\ell + \frac{d\zeta d\bar{\zeta}}{P_0^2} \tag{25}$$

where $\Omega = r^{-1} = \sqrt{2}\ell$.

The new coordinate ℓ is now finite at infinity and $\mathscr{I}^+$ is described by the hypersurface $\ell = 0$, which corresponds to $r = \infty$. The Lagrangian for the geodesics is written as;

$$L = -\frac{1}{2}\frac{d\bar{s}^2}{ds^2} = 2\left(\ell^2 - 2\sqrt{2}m\ell^3\right)\dot{u}^2 - 2\dot{u}\dot{\ell} - \frac{d\dot{\zeta}d\dot{\bar{\zeta}}}{2P_0^2} \quad (26)$$

where dot denotes derivative with respect to affine parameter *s*.

The known equations for null geodesics are [2];

$$2\left(\ell^2 - 2\sqrt{2}m\ell^3\right)\dot{u} - \dot{\ell} = 1,$$

$$\ddot{u} + 2\left(\ell - 3\sqrt{2}m\ell^2\right)\dot{u} = 0,$$

$$\ddot{\zeta}\left(1 + \zeta\bar{\zeta}\right) - 2\bar{\zeta}\dot{\zeta}^2 = 0, \quad (27)$$

$$\ddot{\bar{\zeta}}\left(1 + \zeta\bar{\zeta}\right) - 2\dot{\bar{\zeta}}^2\zeta = 0,$$

$$4\left(\ell^2 - 2\sqrt{2}m\ell^3\right)\dot{u}^2 - 4\dot{u}\dot{\ell} - \frac{\dot{\zeta}\dot{\bar{\zeta}}}{P_0^2} = 0.$$

where the last equation corresponds to $ds^2 = 0$. For simplicity let us take equatorial plane $\theta = \frac{\pi}{2}$. From a fixed apex this yields an S^1 worth of null geodesics. For $\theta = \frac{\pi}{2}$, $\zeta = e^{i\phi}$, then third equation of (27) gives

$$\ddot{\phi} = 0,\ \dot{\phi} = b.$$

From the first equation of (27) we get;

$$\dot{u} = \frac{1 + \dot{\ell}}{2\left(\ell^2 - 2\sqrt{2}m\ell^3\right)}. \quad (28)$$

From the last equation of (27) we get;

$$\dot{\ell}^2 = 2\sqrt{2}mb^2\ell^3 - b^2\ell^2 + 1 = A \text{ (say)},$$

$$\dot{\ell} = \pm\sqrt{A}, \quad ds = \pm\frac{d\ell}{\sqrt{A}}.$$

Again,

$$d\phi = bds = \pm b\frac{d\ell}{\sqrt{A}}. \quad (29)$$

The null rays coming from an arbitrary apex are divided into two sets (the two sheets of *A*) that are given initially by $\dot{\ell} < 0$ and $\dot{\ell} > 0$.

For the first sheet ($\dot{\ell} < 0$) the geodesics continue with a decreasing ℓ (increasing *r*) until intersection with $\mathscr{I}^+$. For the rays which begin with $\dot{\ell} > 0$, that is, those rays with initially increasing ℓ (decreasing *r*), some reach a minimum ℓ (when *A* = 0) and then begin to move outwards, and eventually intersect $\mathscr{I}^+$. For others depending on *b* they continue to move towards increasing ℓ and eventually fall within the horizon and do not reach $\mathscr{I}^+$.

For the second sheet ($\dot{\ell} > 0$), the range is again from b_m to *b* = 0, but now there is a critical value b_c such that for all $b < b_c$ the rays continue past the horizon, so that range for *b* is $b_c < b < b_m$. From (28) we get for the first sheet ($\dot{\ell} < 0$);

$$\dot{u} = \frac{1 - \sqrt{A}}{2\left(\ell^2 - 2\sqrt{2}m\ell^3\right)} = \frac{b^2\left(1 - \sqrt{A}\right)}{2(1 - A)} = \frac{b^2}{2\left(1 + \sqrt{A}\right)}$$

$$du = \frac{b^2 d\ell}{2\sqrt{A}\left(1 + \sqrt{A}\right)}.$$

Integrating we get,

$$u = u_0 - \frac{1}{2}\int_{\ell_0}^{\ell}\frac{b^2 d\ell'}{\sqrt{A}} + \frac{1}{2}\int_{\ell_0}^{\ell}\frac{b^2 d\ell'}{1 + \sqrt{A}}. \quad (30)$$

Since *A* is a cubic, both the integrals of (30) are elliptic integrals due to $m \neq 0$. If *m* = 0 then (29) reduce to flat space-time case. If $\ell = 0$ $(r = \infty)$ then (30) becomes by dropping primes;

$$u = u_0 + \frac{1}{2}\int_0^{\ell_0}\frac{b^2 d\ell}{\sqrt{A}} - \frac{1}{2}\int_0^{\ell_0}\frac{b^2 d\ell}{1 + \sqrt{A}}. \quad (31)$$

From (31) we get;

$$\phi = \int_0^{\ell_0} b\frac{d\ell}{\sqrt{A}}. \quad (32)$$

We now consider the second sheet ($\dot{\ell} > 0$). The integration procedure is slightly complicated than $\dot{\ell} < 0$.

We integrate (29) with $+\sqrt{A}$ (for fixed *b*) to the 'bounce' point ℓ_b after which we return to the first sheet ($-\sqrt{A}$) and integrate to $\ell = 0$ to ℓ_0;

$$u = u_0 + \frac{1}{2}\int_{\ell_0}^{\ell_b} \frac{2b^2 d\ell}{\sqrt{A}} + \frac{1}{2}\int_{0}^{\ell_0} \frac{b^2 d\ell'}{\sqrt{A}\left(1+\sqrt{A}\right)} \qquad (33)$$

$$\phi = 2\int_0^{\ell_b} b\frac{d\ell}{\sqrt{A}} + \int_0^{\ell_0} b\frac{d\ell}{\sqrt{A}}. \qquad (34)$$

Equation (33) and (34) are very complicated and are difficult to integrate, since *A* is cubic.

Conclusions

In this paper we have tried to give a simple explanation of the Schwarzschild geometry. We have avoided complex calculations thinking about the common readers. Those who want to know more about the Schwarzschild geometry we refer to see them [1] and [2].

References

1- Hawking S.W., Ellis, G.F.R., *The Large Scale Structure of Space-time*, Cambridge University Press, Cambridge. 1973.
2- Joshi P.S., *Global Aspects in Gravitation and Cosmology*, Clarendon Press, Oxford. 1993.
3- Joshi P.S., Kozameh C.N., Newman, E.T., Light Cone Cuts of Null Infinity in Schwarzschild Geometry, *Journal of Mathematical Physics*, 1983. 24(10): 2490–2497.
4- Kruskal, M.D., Maximal Extension of Schwarzschild Metric, *Physical Review*, 1960.119: 1743.
5- Mohajan H.K., *Singularity Theorems in General Relativity*, M. Phil. Dissertation, Lambert Academic Publishing, Germany. 2013.
6- Szekeres, P., On the Singularities of a Riemannian Manifold, *Publ. Math. Debrecen*. 1960. 7: 285.