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Green Net National Product for the Sustainability and Social Welfare

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Abstract: *This paper discusses the theory of green national accounting and, emphasizes on social welfare and sustainable accounting. Weitzman provides a foundation for net national product as the stationary equivalent of a wealth maximizing path when there is a constant interest rate and no exogenous technological progress. An attempt has been taken here to make the relationship with different incomes and green net national product, under no exogenous technological progress and a constant utility discount rate. The paper shows that green net national product measures the gross social profit rather than net social profit.*

Keywords: Green NNP, National accounting, Social welfare and Sustainability.

Introduction

Net national product (NNP) is an important item for a country. In the aftermath of the World Commission on Environment and Development (WCED 1987), it became important to investigate whether the concept of NNP can serve as an indicator of sustainability. Martin Weitzman published his seminal paper (Weitzman 1976) on the significance for dynamic welfare of comprehensive national accounting aggregates, where he had included important theoretical contributions on welfare and sustainable accounting. In this paper we also emphasis on social welfare comparisons based on national accounting aggregates, in tradition of Weitzman (1976). Weitzman (1976), Solow (1986), Hartwick (1990), and Mäler (1991) lay the foundation for a concept of NNP which is adjusted for the depletion of natural and environmental resources. NNP represents the maximized value of flow of goods and services that are produced by the productive assets of the society. If NNP of a society increases usually it is thought that the society is in better position but according to green NNP it is an apparent concept of the society. In Dasgupta-Heal-Solow model of capital accumulation and resource depletion, eventually the welfare of the society is optimally decreases along the discounted utilitarian path (Dasgupta and Heal 1974, 1979, and Solow 1974). Green national accounting includes depletion and degradation of natural capital as negative components to the vector of investment goods and adds flows of environmental amenities to the vector of consumption goods. The paper is prepared following Weitzman (1970, 1976), Dasgupta and Heal (1974), Solow (1974), Hartwick (1977, 1990), Asheim (1994, 1996, 2000, 2010, 2011), Dasgupta (2008), and Asheim and Wei (2009).

We have introduced some propositions with proof to clarify the concept of green NNP and social welfare.

Gross National Product and National Income

In the modern world the state of the economy of a country is determined by its gross domestic product (GDP). In the closed economy, gross national product (GNP) is measured by all final goods that are produced there. When a commodity is produced and sold the price paid for the purchase is sent to the pocket of the seller. Hence by adding everyone’s incomes such as interests, profit, wages, salaries and government income we find the GNP. The sum of all the incomes is called gross national income. Hence in closed economy we can say GNP is the same as gross national income. Now we should note two points:

- GNP does not measure wealth. GNP is a flow but wealth is a stock.
- Although it has become a commonplace to regard GNP as a welfare index, it is an aggregate measure of the output of final goods and services, nothing more.

The interpreting national income as a measure of well-being has some problems which are as follows (Dasgupta 2008):

i) According to GNP, a dollar in the hands of the poor is measured the same weight as a dollar in the hands of the rich.

But in our common knowledge we can say that the income of a dollar going to a poor household is measured a greater weight than a dollar going to a rich household. So that GNP does not reflect concerns about income inequality.

ii) Following Sen (1987), Dreze and Sen (1990), Anand and Ravallion (1993) and UNDP (1994) we have criticized those who regard GNP to be a welfare index on grounds that it is instead a measure of a country's opulence, and they remark that opulence is not the same as well-being.

The criticism is fault in two ways (Dasgupta 2008). First, opulence is a stock concept, and GNP is not a return on any index of opulence. Second, and more importantly, it is not a mistake to seek to measure well-being in terms of an index of opulence. The point is not that opulence misleads, but rather that we should search for the right measure of opulence.

iii) An educated population produces greater output (Schultz 1961, Becker 1983) as does a healthy population (Dasgupta and Ray 1986, 1987; Fogel 1994, 2004; Dasgupta 1997). So it would seem that GNP responds to improvements in education and health. It has been countered though that, as they reflect mere instrumental virtues of human capital, GNP does not adequately reflect the well-being people enjoy from becoming educated (Sen 1987, 1999), or from being in good health (Bauer 1971). Health and education are simultaneously aspects of human well-being and factors that produce human well-being.

The word *gross* means that GNP ignores the depreciation of capital assets. Among natural resources, that depreciation can range from a full 100% of the services drawn from oil and natural gas, to the depreciation experienced by ecosystems from mismanagement. Due to over use of natural resources seriously criticize the GNP which causes environmental and natural resource in problems. Sometimes environmental problems are identified in terms of pollution but not the depletion of natural resources. Actually environmental pollutants are the reverse side of natural resources. For environmental resources, the assumption of free disposal of investment flows means that the positive valued resources can be freely destroyed. Hence, negative valued waste products can be freely generated but not freely disposed of (Asheim 1994). Finally we can say that there is no reason to distinguish resource economics from environmental economics, nor resource management problems from pollution management problems, nor resource accounting from environmental accounting. Roughly speaking, *resources* are *goods*, while *pollutants* (the degrader of resources) are *bads*. Pollution is the reverse of conservation (Dasgupta 2008).

Net National Product

Following Hicks (1946, Ch-14), Asheim (1994) stated that
"*NNP should measure what can be consumed in the present period without reducing future consumption possibilities and, in line with this, to argue that the NNP should equal the maximum per capita consumption level that can be sustained* ".

Net national product (NNP) of a country expresses the maximized value of the flow of goods and services that are produced by the productive assets of that country. If NNP increases, then the capacity of the country to produce has increased, and we consider then the economy is better off. Although such an interpretation is often made in public debate, the assertion has been subject to controversy in the economic literature. Weitzman (1976) showed that greater NNP indicates higher welfare if;

- dynamic welfare equals the sum of utilities discounted at a constant rate, and
- current utility equals the market value of goods and services consumed.

Weitzman's result is truly remarkable but we need to relax the assumption of discounted utilitarianism. In the real world societies are conscious about whether welfare is improving, both in terms of what proponents of economic growth may refer to as *progress* and in terms of what environmentalists call *sustainability*.

Asheim and Buchhold (2004) extended Weitzman's (1976) result by developing a framework for national accounting that is sufficiently general to include, in addition to discounted utilitarianism, cases like (i) maximin, (ii) undiscounted utilitarianism and (iii) discounted utilitarianism with a sustainability constraint. In these cases, non-decreasing current welfare does entail that current utility can be sustained indefinitely, as opposed to the case of unconstrained discounted utilitarianism (Asheim 1994 and Pezzey 1994).

Human economic activity frequently depletes the natural capital. Hence it is important to see whether our accumulation of man-made capital is sufficient to make up for the decreased availability of natural capital. The stocks of natural capital will tend to be accumulated.

The Notations and Mathematical Definitions

Let $C(t)$ be the consumption at time t and we consider $C(t)\geq 0$ be the indicator of well-being at time t. The vector of capital stocks at time t be $\mathbf{K}(t)$ and the vector of investment at time t be $\mathbf{I}(t)=\dot{\mathbf{K}}(t)\geq 0$ and $\mathbf{K}(t)\geq 0$ indicates not only different kinds of man-made capital, but also stock of natural capital, environmental assets, human capital and other durable productive assets. Let $F(t)$ be a time-dependent closed and convex feasible triple set (Dixit et al. 1980). Let $(C,\mathbf{K},\mathbf{I})$ is feasible if and only if $(C,\mathbf{K},\mathbf{I})\in F(t)$ where F is smooth and convex set. The consumption $C(t)\geq 0$ generates utility $u(t)=u(C(t))$, where u is a time-invariant strictly increasing , concave and differentiable function. Let $p(t)$ denotes the present value price of consumption at time t, and let $\mathbf{q}(t)$ denote the vector of present value prices of the capital stocks at time t. We assume that;

$$p(t)=p(0)e^{-rt} \tag{1}$$

is an exponentially decreasing function, where r is constant interest rate. Differentiating (1) with respect to t we get;

$$\dot{p}(t) = -r\ p(0)e^{-rt} = -r\ p(t) \qquad \Rightarrow r = -\frac{\dot{p}(t)}{p(t)}. \qquad (2)$$

The instantaneous interest rate is;

$$r_0(t) = -\frac{\dot{p}(t)}{p(t)}, \qquad (3)$$

and the infinitely long-term interest rate is;

$$r_\infty(t) = \frac{p(t)}{\int_t^\infty p(\tau)\,d\tau}. \qquad (4)$$

The term;

$$p(t)C + \mathbf{q}(t)\mathbf{I} + \dot{\mathbf{q}}(t)\mathbf{K}, \qquad (5)$$

indicates instantaneous profit. We write;

$$Q(t) = \frac{\mathbf{q}(t)}{p(t)} \qquad (6)$$

for the capital prices in terms of current consumption. Differentiating (6) with respect to t we get;

$$\dot{Q}(t) = \frac{d}{dt}\left(\frac{\mathbf{q}(t)}{p(t)}\right) = \frac{\dot{\mathbf{q}}(t)}{p(t)} - \frac{\mathbf{q}(t)}{(p(t))^2}\dot{p}(t) = \frac{\dot{\mathbf{q}}(t)}{p(t)} - \frac{\dot{p}(t)}{p(t)}\frac{\mathbf{q}(t)}{p(t)} = \frac{\dot{\mathbf{q}}(t)}{p(t)} + r_0(t)Q(t),$$

$$\frac{\dot{\mathbf{q}}(t)}{p(t)} = r_0(t)Q(t) - \dot{Q}(t). \qquad (7)$$

From (5) we obtain; $C + \dfrac{\mathbf{q}(t)}{p(t)}\mathbf{I} + \dfrac{\dot{\mathbf{q}}(t)}{p(t)}\mathbf{K} = (C + Q(t)\mathbf{I}) - (r_0(t)Q(t) - \dot{Q}(t))\mathbf{K}$,

where $(C + Q(t)\mathbf{I})$ is the current value of production and $(r_0(t)Q(t) - \dot{Q}(t))\mathbf{K}$ is the current cost of holding capital. Hence (5) indicates instantaneous profit. We now define the competitive path as follows (Asheim 2000): The path $(C^*(t), \mathbf{K}^*(t), \mathbf{I}^*(t))_t^\infty$ is competitive at present value prices $(p(t), \mathbf{q}(t))_t^\infty$ and utility discount factor $(\lambda(t))_t^\infty$ if at each t:

C-1: instantaneous utility is maximized i.e., $C^*(t)$ maximizes $\lambda(t)u(C) - p(t)C$.

C-2: instantaneous profit is maximized i.e., $(C^*(t), \mathbf{K}^*(t), \mathbf{I}^*(t))$ maximizes $p(t)C + \mathbf{q}(t)\mathbf{I} + \dot{\mathbf{q}}(t)\mathbf{K}$ subject to $(C, \mathbf{K}, \mathbf{I}) \in F(t)$.

Here C-2 combines with the assumption of free disposal of investment flows which implies that the vector $\mathbf{q}(t)$ is non-negative. Now we define the regulation of competitive path as follows (Asheim 2000): The competitive path $(C^*(t), \mathbf{K}^*(t), \mathbf{I}^*(t))_t^\infty$ is regular at present value prices $(p(t), \mathbf{q}(t))_t^\infty$ and utility discount factor $(\lambda(t))_t^\infty$ if at each t:

R-1: $\int_0^\infty \lambda(t)u(C^*(t))dt$ exists and is finite,

R-2: $q(t)K^*(t) \to 0$ as $t \to \infty$.

Here R-2, implies that the value of the capital stocks along regular path equals the present value of the rents that arise from the future productivity and depletion of the stocks:

$$\mathbf{q}(t)\mathbf{K}^*(t) = \int_t^\infty \left[\left(-\dot{\mathbf{q}}(\tau)\mathbf{K}^*(\tau) + \mathbf{q}(\tau)\left(-\mathbf{I}^*(\tau)\right)\right)\right] d\tau$$

where the imputed rents to the assets are equal to $(-\dot{\mathbf{q}}(t))$ which measures the marginal productivity of the capital stocks (Asheim 1994). A regular path is efficient and maximizes $\int_0^\infty \lambda(t) u(C^*(t)) dt$ over all feasible paths $(C, \mathbf{K}, \mathbf{I})_{t=0}^\infty$ with given initial stocks $\mathbf{K}$ for each *t*, $\lambda(t) > 0$, $\mathbf{q}(t)\mathbf{K}(t) \geq 0$. By C-1 and C-2 we get;

$$\lambda(t)\left[u\left(C - u\left(C^*(t)\right)\right)\right] \leq \frac{d}{dt}\left(\mathbf{q}(t)\mathbf{K}^*(t) - \mathbf{q}(t)\mathbf{K}(t)\right),$$

i.e., $\int_0^T \lambda(t)\left[u\left(C - u\left(C^*(t)\right)\right)\right] dt \leq \mathbf{q}(T)\mathbf{K}^*(T)$.

Hence, a regular path provides price information for the net output vector and capital stocks, and can be realized as a competitive equilibrium if the intergenerational altruism of each generation *t* is represented by (Asheim 1994);

$$\int_0^\infty \lambda(\tau) u(C(\tau)) d\tau.$$

Two welfare Theorems

Now we can state two theorems of welfare as follows (Asheim 2000):

First Welfare Theorem: *If* $\left(C^*(t), \mathbf{K}^*(t), \mathbf{I}^*(t)\right)_t^\infty$ *is regular at present value prices* $(p(t), \mathbf{q}(t))_t^\infty$ *and utility discount factors* $\lambda(t)_{t=0}^\infty$, *then* $\left(C^*(t), \mathbf{K}^*(t), \mathbf{I}^*(t)\right)_t^\infty$ *maximizes* $\int \lambda(t) u(C^*(t)) dt$ *subject to* $(C, \mathbf{K}, \mathbf{I}) \in F(t)$ *for all t and* $\mathbf{K}(0) = \mathbf{K}_0$ (given).

From this theorem we can say that the utility discount factors are positive, which means that any competitive path satisfying the regularity conditions R-1 and R-2 is efficient. So that, with these qualifications, any inter-temporal competitive equilibrium is Pareto efficient.

Second Welfare Theorem: *If* $\left(C^*(t), \mathbf{K}^*(t), \mathbf{I}^*(t)\right)_t^\infty$ *maximizes* $\int_{t=0}^\infty \lambda(t) u(C^*(t)) dt$ *subject to* $(C, \mathbf{K}, \mathbf{I}) \in F(t)$ for all *t* and $\mathbf{K}(0) = \mathbf{K}_0$, (given) *then there exists present value prices* $(p(t), \mathbf{q}(t))_t^\infty$ *such that* $\left(C^*(t), \mathbf{K}^*(t), \mathbf{I}^*(t)\right)_t^\infty$ *is competitive at* $(p(t), \mathbf{q}(t))_t^\infty$ *and* $(\lambda(t))_t^\infty$.

From this theorem we can say that any utility path that is supported by utility discount factors is supported by present value prices of consumption and capital stocks. So that, any Pareto-efficient path can be seen to be the outcome an inter-temporal competitive equilibrium where the intergenerational distribution is given by the consumption path $\left(C^*(t)\right)_t^\infty$.

Some Related Definitions

Now we will cite some definitions which are related to this paper as follows:

Welfare Equivalence Income

Let a generation t has inherited the capital stocks **K**. The generation wants to maximize a social welfare functional, $\int_t^\infty \lambda(t)u(C(t))\,dt$, over all feasible paths. Weitzman (1970) considers the level of utility $U(t)$ at maximizing path $\left(C^*(\tau),\mathbf{K}^*(\tau),\mathbf{I}^*(\tau)\right)_{\tau=t}^{\infty}$. Let the welfare at time t be $\int_t^\infty \frac{\lambda(\tau)}{\lambda(t)u(C^*(\tau))}\,d\tau$. This welfare is the same as the welfare of the utility $U(t)$ is held constant will yield the same welfare as the welfare maximizing path $\left(C^*(\tau),\mathbf{K}^*(\tau),\mathbf{I}^*(\tau)\right)_{\tau=t}^{\infty}$, so that;

$$U(t)=\frac{\int_t^\infty \lambda(\tau)u(C^*(\tau))\,d\tau}{\int_t^\infty \lambda(\tau)\,d\tau}. \tag{8}$$

The consumption index of welfare $W(t)$ is defined by (Asheim 2000);

$$W(t)=\frac{1}{u}U(t)=\frac{1}{u}\frac{\int_t^\infty \lambda(\tau)u(C^*(\tau))\,d\tau}{\int_t^\infty \lambda(\tau)\,d\tau}. \tag{9}$$

The notation W refers to Weitzman (1970), who first suggested stationary welfare equivalence. We assume that $W(t)$ is continuous and differentiable everywhere.

Sustainable Income

A sustainable income $s(t)$ at time t is the maximum consumption that can be sustained from time t on, given the capital stocks **K** that generation t has inherited:

$$s(t)=\sup\left(\inf_{\tau\geq t}\left(C(\tau)\right)\right).$$

If the present management of natural and environmental resources compatible with sustainable development then $s(t)$ could be measured.

Sustainable income is the best process in welfare economics, since the future generations will not suffer for national stocks. A sustainability requirement to the effect that $C(t)$ should not exceed $s(t)$ is satisfied if $\dot{C}(\tau)\geq 0$ or $\dot{u}(\tau)\geq 0$ for all $\tau\geq t$. If the welfare at time t be, $\int_t^\infty \frac{\lambda(\tau)}{\lambda(t)}u(C^*(\tau))\,d\tau$, then one can argue that a requirement of sustainability should instead be imposed on the path of welfare equivalent income. Consider a case where $\lambda(\tau)=\lambda(0)e^{-\delta\tau}$, then the welfare sustainability is given by $\dot{W}(\tau)\geq 0$ for all $\tau\geq t$.

Green Net National Product

Green NNP is the sum of consumption and the value of net investments:

$$g(t) = C^*(t) + Q(t)\mathbf{I}(t) \tag{10}$$

where the vector of capital goods, **K**, comprises all kinds of man-made capital and all kinds of natural capital. We see that green NNP includes current consumption and the value of net investments, but are not included capital gains $\dot{Q}(t)\mathbf{K}(t)$.

Weitzman (1976) stated that if the own interest rate of consumption good is constant, the present value of future consumption equals the present value of consuming $C^*(t) + Q(t)\mathbf{I}(t)$ for all $\tau \geq t$. Weitzman claimed that it is feasible to sustain a consumption equal to $C^*(t) + Q(t)\mathbf{I}(t)$ which is correct in fact. But in a closed economy with a constant population and no exogenous technological progress, NNP defined as $C(t) + Q(t)\mathbf{I}(t)$ is not in general an exact indicator of sustainability, except in the uninteresting case with only one capital good (Asheim 1994).

Hartwick (1977) found that, in a closed economy with a constant population and a stationary technology steering along an efficient path with $\mathbf{q}(t)\mathbf{I}^*(t) = 0$ for all *t*, the utility level is constant and equal to the maximal sustainable level. Hartwick's rule states that a competitive equilibrium leads to a completely eagliterian utility path if and only if, at all times, the values of depleted natural capital measured in competitive prices equals the reinvestment in man-made capital. But Hartwick's rule did not claim that a competitive economy which, for the moment, at market value reinvests depleted natural capital in man-made capital, manages its stocks of natural and man-made capital in a sustainable manner. For it is conceivable that such reinvestment is achieved. If future generations are poorer than we are, they will be unable to bid highly through the inter-temporal competitive equilibrium for the depletable natural capital we manage, leading to low prices of such capital today. Hence, Hartwick's rule characterizes a sustainable development; it is not a prescriptive rule for a sustainable development (Asheim 2000). $g(t) = C^*(t) + Q(t)\mathbf{I}(t)$ does not in general indicate the maximum sustainable consumption level, it does equal the maximum sustainable level along an efficient consumption path which happens to be eagliterian (Asheim 1996).

Net Social Profit

Let the path $\left(C^*(\tau), \mathbf{K}^*(\tau), \mathbf{I}^*(\tau)\right)_{\tau=0}^{\infty}$ be a regular path at present values prices $\left(p(\tau), \mathbf{q}(\tau)\right)_{\tau=0}^{\infty}$ and utility discount factors $\left(\lambda(\tau)\right)_{\tau=0}^{\infty}$, given the capital stocks $\mathbf{K}_0$ that generation 0 has inherited. Let $\left(C^*(\tau; \mathbf{K}, t)\right)_{\tau=t}^{\infty}$ be a consumption path maximizing $\int_t^{\infty} \lambda(\tau) u(C(\tau))\, d\tau$ subject to feasibility if generation *t* inherits **K**. For the generation 0 we can write it as $\left(C^*(\tau; \mathbf{K}_0, 0)\right)_{\tau=0}^{\infty} = \left(C^*(\tau)\right)_{\tau=0}^{\infty}$. A social cost-benefit is an index which has the property that the acceptance of a small policy change increases the index if and only if the policy change leads to a welfare improvement (Dasgupta et al. 1972 and Dasgupta et al. 1995, 1997). Let a policy change at time *s* refer to the substitution of an alternative feasible set, $\overline{F}(\tau)$, for $F(\tau)$ at time τ. A policy change for the time interval [0, *t*] is welfare improving if and only if there exists a path $\left(\overline{C}(\tau), \overline{\mathbf{K}}(\tau), \overline{\mathbf{I}}(\tau)\right)_{\tau=0}^{t}$ satisfying $\left(\overline{C}(\tau), \overline{\mathbf{K}}(\tau), \overline{\mathbf{I}}(\tau)\right) \in \overline{F}(\tau)$ and $\overline{\mathbf{K}}(0) = \mathbf{K}_0$ such that (Asheim 2000);

$$\int_0^t \lambda(\tau)\left(u\left(\overline{C}(\tau)\right)-u\left(C^*(\tau)\right)\right)d\tau+\int_t^\infty \lambda(\tau)\left(u\left(C^*(\tau;\mathbf{K}(t),t)\right)-u\left(C^*(\tau)\right)\right)d\tau>0.$$

Hence by a small policy change the net social profit $\pi(\tau)$ can be written as (Asheim 2000);

$$\pi(\tau)=C^*(\tau)+\frac{\mathbf{q}(\tau)}{p(\tau)}\mathbf{I}^*(\tau)+\frac{\dot{\mathbf{q}}(\tau)}{p(t)}\mathbf{K}^*(\tau).$$

Wealth Equivalence Income

If a regular path $\left(C^*(\tau),\mathbf{K}^*(\tau),\mathbf{I}^*(\tau)\right)_{\tau=t}^{\infty}$ is followed then $\int_t^\infty p(\tau)C(\tau)\,d\tau$ is maximized over all feasible paths given the capital stocks $\mathbf{K}$ that generation t has inherited. Let wealth at time t is $\int_t^\infty \frac{p(\tau)}{p(t)}C^*(\tau)\,d\tau$. Then the wealth equivalent income $h(t)$ at time t is the consumption that if held constant will yield the same wealth as the wealth maximizing path $\left(C^*(\tau),\mathbf{K}^*(\tau),\mathbf{I}^*(\tau)\right)_{\tau=t}^{\infty}$ i.e.,

$$\int_t^\infty p(\tau)h(\tau)\,d\tau=\int_t^\infty p(\tau)C^*(\tau)\,d\tau,$$

$$h(t)=\frac{\int_t^\infty p(\tau)C^*(\tau)\,d\tau}{\int_t^\infty p(\tau)\;d\tau}. \tag{11}$$

Relation between Green NNP and Wealth Equivalent Income

By the relations (3) and (10) we can write;

$$g(t)=\int_t^\infty r_0(\tau)\frac{p(\tau)}{p(t)}C^*(\tau)\,d\tau. \tag{12}$$

Again by the relations (4) and (11) we can write;

$$h(t)=r_\infty(t)\int_t^\infty \frac{p(\tau)}{p(t)}C^*(\tau)\,d\tau. \tag{13}$$

Suppose there is no exogenous technological progress. For constant interest rate i.e., $r_0(\tau)=r_\infty(t)=r$ for all τ, we get;

$$g(t)=h(t)=r\int_t^\infty \frac{p(\tau)}{p(t)}C^*(\tau)\,d\tau \tag{14}$$

which is Weitzman's (1976) fundamental result on green national accounting. Again for constant consumption i.e., for $C^*=C$ and for all τ, we get;

$$g(t)=h(t)=C^*. \tag{15}$$

Since it follows from the definitions of $r_0(\tau)$ and $r_\infty(t)$ that,

$$\int_t^\infty r_0(\tau)\frac{p(\tau)}{p(t)}C^*(\tau)\,d\tau = r_\infty(t)\int_t^\infty \frac{p(\tau)}{p(t)}C^*(\tau)\,d\tau = 1.$$

If no exogenous technological progress holds, and equations (14) and (15) do not hold; then wealth equivalent income exceeds green NNP whenever consumption tends to increase (decrease) and interest rates tend to decrease (increase). Again green NNP exceeds wealth equivalent income whenever both consumption and interest rates tend to decrease. This can occur in the Dasgupta-Heal-Solow model (Dasgupta and Heal 1974, Solow 1974 and Asheim 1994). We have,

$$\frac{d}{dt}\left(\int_t^\infty \frac{p(\tau)}{p(t)}C^*(\tau)\,d\tau\right) = -C^*(\tau)+r_0(t)\int_t^\infty \frac{p(\tau)}{p(t)}C^*(\tau)\,d\tau\,,$$

$$\int_t^\infty \frac{p(\tau)}{p(t)}C^*(\tau)\,d\tau = \frac{1}{r_0(t)}\left[C^*(\tau)+\frac{d}{dt}\left(\int_t^\infty \frac{p(\tau)}{p(t)}C^*(\tau)\,d\tau\right)\right]. \qquad (16)$$

Using (16), we can write (13) as follows:

$$h(t) = \frac{r_\infty(t)}{r_0(t)}\left[C^*(\tau)+\frac{d}{dt}\left(\int_t^\infty \frac{p(\tau)}{p(t)}C^*(\tau)\,d\tau\right)\right]. \qquad (17)$$

Constant returns to scale (CRS) explained by Lindahl (1933), which states that all factors of production, including labor, are dealt with as capital that is evaluated by the present value of future earnings. So that CRS implies that wealth is equal to the value of current capital stocks;

$$\int_t^\infty \frac{p(\tau)}{p(t)}C^*(\tau)\,d\tau = q(t)\frac{\mathbf{K}^*(t)}{p(t)} = Q(t)\mathbf{K}^*(t). \qquad (18)$$

Using (18), the relation (17) becomes (Asheim 2000);

$$\begin{aligned} h(t) &= \frac{r_\infty(t)}{r_0(t)}\left[C^*(\tau)+\frac{d}{dt}\left(Q(t)\mathbf{K}^*(t)\right)\right] \\ &= \frac{r_\infty(t)}{r_0(t)}\left[C^*(\tau)+Q(t)\dot{\mathbf{K}}^*(t)+\dot{Q}(t)\mathbf{K}^*(t)\right] \\ &= \frac{r_\infty(t)}{r_0(t)}\left[C^*(\tau)+Q(t)\mathbf{I}^*(t)+\dot{Q}(t)\mathbf{K}^*(t)\right] \\ &= \frac{r_\infty(t)}{r_0(t)}\left[g(t)+\dot{Q}(t)\mathbf{K}^*(t)\right]. \qquad (19) \end{aligned}$$

Relation (19) implies that to find wealth equivalent income $h(t)$ we must add capital gains $\dot{Q}(t)\mathbf{K}^*(t)$ to the green NNP, $g(t)$ and the sum $g(t)+\dot{Q}(t)\mathbf{K}^*(t)$ must be adjusted for interest rate effects if there is not a constant interest rate, in which case $r_\infty(t) / r_0(t)$ need not equal 1.

Proposition 1: *If there is no exogenous technological progress, then wealth equivalent income is related to green NNP according to resource allocation, consumption of resources and interest.*

Proof: If exogenous technological exists then we may face open economy condition and a regular path $\left(C^*(\tau), \mathbf{K}^*(\tau), \mathbf{I}^*(\tau)\right)_{\tau=t}^{\infty}$ is not followed that $\int_t^\infty p(\tau) C(\tau) d\tau$ is maximized over all feasible paths given the capital stocks **K** that generation *t* has inherited. So that both conditions R-1 and R-2 will not satisfy. As a result equation (15) does not satisfy i.e., $h(t) \neq g(t)$ for constant consumption, since some part of the instantaneous return on a country's capital stock must be use to augment the countries national wealth (Asheim 1996). Again $h(t) \not> g(t)$ whenever both consumption and interest rate tend to decrease which violates Dasgupta- Heal-Solow model. Moreover CRS rule will not also satisfy, because wealth will not be equal to the value of current capital stocks and obviously (19) will not satisfy. Therefore in every case $h(t) \not> g(t)$ for all $\tau \geq t$. Q. E. D.

Relation between Green NNP and Net Social Profit

The net social profit $\pi(\tau)$ can be written as (Asheim 2000);

$$\pi(\tau) = C^*(\tau) + \frac{\mathbf{q}(\tau)}{p(\tau)} \mathbf{I}^*(\tau) + \frac{\dot{\mathbf{q}}(\tau)}{p(t)} \mathbf{K}^*(\tau) \tag{20}$$

where $\pi(\tau)$ is discounted by $p(\tau)$. Again we have,

$$\begin{aligned} \dot{Q}(\tau) &= \frac{d}{d\tau}\left(\frac{\mathbf{q}(\tau)}{p(\tau)}\right) \\ &= \frac{\dot{\mathbf{q}}(\tau)}{p(\tau)} - \frac{\dot{\mathbf{q}}(\tau)}{p(\tau)} \frac{\mathbf{q}(\tau)}{p(\tau)} \\ &= \frac{\dot{\mathbf{q}}(\tau)}{p(\tau)} + r_0(\tau) Q(\tau), \end{aligned}$$

$$\frac{\dot{\mathbf{q}}(\tau)}{p(\tau)} = \dot{Q}(\tau) - r_0(\tau) Q(\tau). \tag{21}$$

Using (21), the equation (20) becomes;

$$\pi(\tau) = C^*(\tau) + Q(\tau)\mathbf{I}^*(\tau) - \left\{r_0(\tau) Q(\tau) - \dot{Q}(\tau)\right\}\mathbf{K}^*(\tau)$$

$$\pi(t) = g(t) - \left\{r_0(t) Q(t) - \dot{Q}(t)\right\}\mathbf{K}^*(t). \tag{22}$$

So that net social profit is obtained by subtracting the cost of holding capital $\left\{r_0(t) Q(t) - \dot{Q}(t)\right\}\mathbf{K}^*(t)$ from the green NNP. The actual difference between $\pi(\tau)$ and $g(t)$ is that $g(t)$ measures gross social profit while $\pi(\tau)$ measures net social profit. Vellinga and Withagen (1996) showed that $g(t) = C^*(t) + Q(t)\mathbf{I}^*(t) = \left\{p(t)C^*(t) + q(t)\dot{\mathbf{K}}^*(t)\right\} / p(t)$ is a cost-benefit index for a small policy change lasting only an instance. For a small policy change we get;

$$\begin{aligned} &\lambda(t)\left[u\overline{C}(t) - uC^*(t)\right] + \frac{d}{dt}\left(\int_t^\infty \lambda(\tau)\left[uC^*(\tau; \overline{\mathbf{K}}(t), t) - uC^*(\tau)\right] d\tau\right) \\ &= p(t)\left[u\overline{C}(t) - uC^*(t)\right] + \frac{d}{dt}\left(q(t)\left[u\,\overline{\mathbf{K}}(t) - u\,\mathbf{K}^*(t)\right]\right) \\ &= p(t)\left(u\overline{C}(t) - uC^*(t)\right) + q(t)\left(u\,\overline{\mathbf{I}}(t) - u\,\mathbf{I}^*(t)\right). \end{aligned} \tag{23}$$

Since $\mathbf{K}(t)=\mathbf{K}^*(t)=\mathbf{K}$, then (23) gives;

$$\lambda(t)\left[u\,\bar{C}(t)-u\,C^*(t)\right]+\frac{d}{dt}\left(\int_t^\infty \lambda(\tau)\left[u\,C^*(\tau;\bar{\mathbf{K}}(t),t)-u\,C^*(\tau)\right]d\tau\right)=p(t)\left(u\,\bar{C}(t)-u\,C^*(t)\right).$$

Hence, the change in the value of consumption measures the current change in utility, while the change in the value of investments measures the time derivative of the discounted inter-temporal sum of future changes in utility. This implies that green NNP can be used to verify that no policy change should be implemented at any point in time, if it can be shown that any small policy change would contribute non-positively to green NNP. The above results on the measurement of net social profit are general; i.e., they do not depend on there being a constant utility discount rate or there being no exogenous technological progress (Asheim 2000).

Proposition 2: *Net Social profit is always smaller than green NNP.*

Proof: Suppose $\pi(\tau)\geq g(\tau)$ then equation (22) violates, since $g(\tau)$ is gross social profit and $\pi(\tau)$ is net social profit for all τ. For a small policy change $g(\tau)$ is not a cost-benefit index, as $\pi(\tau)$ is general, so that always $\pi(\tau)\geq g(\tau)$ is impossible. We have mentioned that opulence is not the same as well-being, so that always we get $\pi(\tau)< g(\tau)$. Q. E. D.

Relation between Green NNP and Welfare Equivalent Income

If there is a constant utility discount rate under no exogenous technological progress, then it follows from a generalization of Weitzman's (1976) result that (Weitzman 1970; Kemp and Long 1982 and Asheim 2000):

$$\int_t^\infty \lambda(\tau)\left(u\left(C^*(t)\right)+\frac{\mathbf{q}(t)}{\lambda(t)}\mathbf{I}^*(t)\right)d\tau=\int_t^\infty \lambda(\tau)u\left(C^*(t)\right)d\tau. \qquad (24)$$

Which indicates that green NNP in terms of utility $u\left(C^*(t)\right)+\frac{\mathbf{q}(t)}{\lambda(t)}\mathbf{I}^*(t)$, is equal to the utility derived from welfare equivalent income, $u(W(t))$, i.e.,

$$u\left(C^*(t)\right)+\frac{\mathbf{q}(t)}{\lambda(t)}\mathbf{I}^*(t)=\frac{\int_t^\infty \lambda(\tau)u\left(C^*(\tau)\right)d\tau}{\int_t^\infty \lambda(\tau)d\tau}=u(W(t)).$$

As a foundation for using green NNP, $g(t)$, Hartwick (1990) observed that $u'\left(C^*(t)\right)\times g(t)$ is a linear approximation of green NNP in terms of utility and thus of $u(W(t))$;

$$u'\left(C^*(t)\right)\times g(t)=u'\left(C^*(t)\right)\times C^*(t)+u'\left(C^*(t)\right)\times\frac{\mathbf{q}(t)}{p(t)}\mathbf{I}^*(t)=u'\left(C^*(t)\right)\times C^*(t)+\frac{\mathbf{q}(t)}{\lambda(t)}\mathbf{I}^*(t), \qquad (25)$$

since $\lambda(t)u'\left(C^*(t)\right)=p(t)$. If u is linear, then we can write;

$$u'\left(C^*(t)\right)\times\left(W(t)-C^*(t)\right)=u\left(W(t)\right)-u\left(C^*(t)\right). \qquad (26)$$

At this situation we can write $g(t)=W(t)$. If the value of net investment $Q(t)\mathbf{I}(t)=0$, then we can write also $g(t)=C^*(t)=W(t)$. Again if there is no exogenous technological progress and the utility discount factor $\lambda(\tau)$ is an exponentially decreasing function, then $W(t)\geq g(t)$ (Asheim 2000).

Proposition 3: *Green NNP never exceeds welfare equivalent income in closed economy.*
Proof: For closed economy there is no exogenous technological progress and utility discount rate is constant. Hence for closed economy, maximum welfare equivalent income of a country never affected by the other countries. As a result $g(t)=W(t)$ if *u* is linear or if $Q(t)\mathbf{I}(t)=0$ then $g(t)=C^*(t)=W(t)$ i.e., both *g*(*t*) and *W*(*t*) are equal and are in maximum level. Again if utility discount factor $\lambda(\tau)$ is an exponentially decreasing function, then we get $W(t)\geq g(t)$. Therefore *g*(*t*) never exceeds *W*(*t*) in closed economy. Q.E.D.

Relation between Green NNP and Sustainable Income

In a closed economy with a non-linear technology, and at the level of a large open economy that influences international prices, wealth equivalent income overestimates sustainable income, since turning the actual consumption path into a constant consumption path leads to a loss of present value (Asheim 2000). In the Ramsey model, one capital good technology is described by $C+\mathbf{I}\leq f(K)$, with f being an increasing and strictly concave function, hence green NNP measures sustainable income. Let $g(t)=C^*(t)=\mathbf{I}(t)$, while $f(\mathbf{K}^*(t))$ is the sustainable income given that generation *t* has inherited the capital stock $\mathbf{K}^*(t)=\mathbf{K}$. Since efficiency indicates that $C^*(t)+\mathbf{I}^*(t)=f(\mathbf{K}^*(t))$, which implies that $g(t)=C^*(t)+\mathbf{I}^*(t)=f(\mathbf{K}^*(t))=s(t)$.

If there is no exogenous technological progress then for constant consumption $C^*(\tau)=C^*$ for all τ, $Q(t)\mathbf{I}^*(t)=0$. So that $g(t)=C^*=s(t)$ for a constant consumption path in a stationary technology. Asheim (1994) showed by Dasgupta-Heal-Solow model, that $g(t)>C^*(t)>s(t)$.

Proposition 4: *If no exogenous technological progress then for constant consumption or for constant interest rate green NNP is greater than or equal to sustainable income.*
Proof: For no exogenous technological progress and constant consumption we have $g(t)=s(t)$. Again for constant interest rate and no exogenous technological progress, $g(t)=W(t)$, but we have $W(t)>s(t)$ which implies that $g(t)>s(t)$. Hence for constant consumption or for constant interest rate and no technological progress we get $g(t)\geq s(t)$. Q. E. D.

Concluding Remarks

In this paper we have shown the relations of green NNP with some other incomes. National accounting seeks to measure income based on current prices. As like Weitzman (1976) we have assumed that there is no technological progress. There are two reasons to assume it in such a way. First, it requires that accumulated knowledge is represented by augmented capital stocks. Second, it excludes open economies whose technology due to changing terms of trade. We have used mechanisms of Dasgupta-Heal-Solow model of capital accumulation and resource depletion. If green NNP approaches zero then unconstrained development is no longer sustainable. We have shown all the calculations in some detail and included some propositions with proof. Asheim (2000) showed that $W(t)>h(t)>s(t)$ as a general result and $h(t)>g(t)$ under no exogenous technological progress and constant utility discount rate.

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