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Majority Judgment in an Election with Borda Majority Count

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Abstract: *This paper describes aspects of the majority judgment in an election. The majority judgment is a method of election which is a new theory in social choice where voters judge candidates instead of ranking them. The paper emphasize on the works of Michel Balinski and Rida Laraki majority judgment in an election. In Arrow's impossibility theorem of social choice theory, the voters have to give a strictly preference ordering over the alternatives and hence they can not express indifference of the candidates. In the process of majority judgment the voters can express much more information than the Arrow's process does but it is not free from counter-intuitive results. The Borda majority count avoids some counter-intuitive results and an attempt has been taken here to highlight them. The paper discusses both the advantages and drawbacks of the majority judgment in an election. Sometimes tie arises in majority judgment and different processes of tie-breaking are discussed with theoretical and mathematical calculations.*

Keywords: Majority voting, drawbacks in majority voting, manipulation of voting.

Introduction

Elections are the mechanisms expressing for the wishes of individuals into a decision of the society. In the whole world the choice of one candidate from a set of candidates is performed by elections. In the elections voters can compare the candidates which one is better for them and finally make the list of preferences in their minds and apply their opinions through the elections. In the modern world there are many voting methods such as Condorcet voting, Borda voting, plurality voting, the single transferable voting, approval voting etc. The great mathematician Laplace first proposed to judge the candidates in two centuries ago (Laplace 1820) and asked the voters not to compare but to evaluate the competitors by assigning points according to the merits of the candidates. Laplace suggested the range $[0, R]$ where R is a positive integer. Usually values of R be 0, 1, 2, 5, 10 or 100. Here $R = 1$ indicates approval voting (Brams and Fishburn 1983) and $R = 100$ is the range voting introduced by Smith (Smith 2007).

In Arrow's impossibility theorem in social choice theory, the voters have to give a strictly performance ordering over the alternatives and hence they can not express indifference of the candidates. But in majority judgment voting scope is given to the voters to evaluate the candidates in some common grading system. Hence by the process of majority judgment voting the voters can express much more information than the Arrow's process does. Borda voting is one kind of majority judgment voting where voters can express their opinion by ranking the candidates.

Written evidence of voting is found since 1299 which is introduced by Ramon Llull (Hägele and Pukelsheim 2001). From that time it is seen that candidates are elected and ranked according to compare the relative merits of the candidates. A rule of voting is strategy-proof when every voter's best strategy is his true preference-order, otherwise it is manipulable.

French political philosophers Borda (1781) and Condorcet (1785) introduced modern voting system but they had not mentioned about manipulation of voting. Condorcet, Borda and even many modern politicians believe that elections are logically imperfect. In this paper majority judgment is discussed in very simple but in a detailed manner. Voting system is directly involved with economics, political science and social science. So that if one has no proper knowledge of the voting system then he can not serve the society in proper way and cannot expect the economic development of the society. In this paper we have discussed aspects of majority voting by introducing elections of French and USA. We have cited few examples and propositions with proof to make the paper easier to the readers.

Practical Experience of the Voting Result

To implement a voting system politicians face various difficulties. Hence every voting system has some drawbacks. Among the various drawbacks first we consider two voting paradoxes as follows:

Condorcet's paradox: Let us assume that there are 17 voters of three types and three alternatives x, y, and z. Let preference relations be as follows:

Type 1: $xPyPz$ by 8 voters,
Type 2: $yPzPx$ by 5 voters,
Type 3: $zPxPy$ by 4 voters.

In an election a vote between x and y, x collects 8+4 =12 votes and y collects 5 votes, so that x wins. Again a vote between y and z, y collects 8+5 =13 votes and z collects 4 votes, so that y wins. Again a vote between x and z, x collects 8 votes and z collects 4+5 =9 votes, so that z wins. We observe that there is a cycle in the voting results where x is defeated by y, y is defeated by z and also z is defeated by x which is a (Condorcet's) voting paradox.

This type of paradox is not often seen but it is observed in a Danish election (Kurrild-Klitgaard 1999). In 1976 **Judgment of Paris** where eleven voters which are well known wine experts, evaluated six Cabernet-Sauvignons of California and four of Bordeaux. By the Condorcet's majority principle, five wines including three of the four French wines all preferred to the other five wines by a majority judgment, where it is judges such a way that the output becomes $xIyPzIuPvPx$, which is a Condorcet cycle. Here for wines x, y and z we mean: xIy, x is indifferent to y and yPz, y is strictly preferred to z.

Arrow's paradox: Let there are at least three candidates x, y, and z. Suppose in an election x wins when these three candidates are competitors. If any weaker candidate between y and z, say z withdraws then y may defeats x. This type of paradox is introduced by Arrow and is called Arrow's paradox.

Arrow's paradox is seen in the US presidential election of 2000. In the first-past-the-post system the winner may change for the present or absent of irrelevant candidates which is a common situation in most of the countries of the world. In the US president election of 2000 that type of situation aroused. In that election two main competitors were George W. Bush and Albert Gore, and Ralph Nader was a candidate who has no chance of winning. But the presence of Ralph Nader changes the election outcome (table-1).

Table 1: The US presidential election of 2000.

Candidate	National vote	Electoral College	Florida vote
George W. Bush	50,456,002	271	2,912,790
Albert Gore	50,999,897	266	2,912,253

Ralph Nader	2,882,955	0	97,488

The total national vote was in favor of Gore but in Florida Nader casts 26 electoral votes which alone change the winner in the US election of 2000. Because it is estimated that if Nader remained absent in Florida election most of his 97,488 votes would have gone to Gore who had 537 votes less than Bush, obviously Gore would have won. In the majority judgment there is no chance of changing election outcome.

French Presidential Election of 2007

There were 12 candidates in 2007 French president election. In that election voters did not apply majority judgment, instead they chose one of their best preferred candidates. The total number of registered voters was 44.5 millions and 84% of these voters took part in the election of both rounds. The results of the 12 candidates after the first round are given in table-2.

Table 2: French presidential election, first round, April 22, 2007.

N.Sarkozy 31.18%	S. Royal 25.87%	F. Bayrou 18.57%	J.-M. Le Pen 10.44%
O. Besancenot 4.08%	P. de Villiers 2.23%	M.-G. Buffet 1.93%	D. Voynet 1.57%
A. Laguiller 1.33%	J. Bov´e 1.32%	F. Nihous 1.15%	G. Schivardi 0.34%

After the first round four candidates Bayrou, Sarkozy, Royal and Le Pen survived for the second round. In 2007 voters were acutely aware of the importance of who would have survived in the first round. In the second round Sarkozy defeated Royal by 18,983,138 (53.06%) to 16,790,440 (46.94%). If we follow Condorcet or Borda rule we would find the different results. The results of polls, on 28 March and 19 April, 2007 of potential second round are given in table-3. By Condorcet method Bayrou was the Condorcet winner, since he would have defeated all other candidates in pairwise contests.

Table 3: 28 March and 19 April, 2007, potential second round Polls.

	Bayrou		Sarkozy		Royal		Le Pen	
Bayrou	–	–	54%	55%	57%	58%	84%	80%
Sarkozy	46%	45%	–	–	54%	51%	84%	84%
Royal	43%	42%	46%	49%	–	–	75%	73%
Le Pen	16%	20%	16%	16%	25%	27%	–	–

From table-3 we see that there is no Condorcet cycle. Final result: Bayrou is first, Sarkozy is second, Royal is third and Le Pen is fourth. Now we use Borda rule to obtain the Borda score from table-3. On 28 March the Borda scores were as follows: Bayrou 195, Sarkozy 184, Royal 164, and Le Pen 57. On 19 April those were as follows: Bayrou 193, Sarkozy 180, Royal 164, and Le Pen 63. According to both Condorcet and Borda methods Bayrou was the winner in the French 2007 election.

In 1907 Sir Francis Galton (Galton 1907) realized that point-summing methods do not elicit honesty. He expressed that when a jury is to decide on an amount of money, for example, to allocate to a project, or in assessing damages in an insurance claim—"that conclusion is clearly not the average of all the estimates, which would give a voting power to 'cranks' in proportion to their crankiness" (Balinski and Laraki 2010b). In a voting an aggregation must be meaningful both in social-grading and social-ranking functions. To manipulate a voting successfully a voter must be able to raise or to lower a candidate's final grade by changing the grade she assigns. Voters who can both lower and raise the final grade have a much greater possibility of manipulation. Voting outcome will heed the majority's will. In an election where there are many voters and a language of relatively few grades the two middlemost order functions will have one value which is the majority grade.

Hence from the above discussion we can say that a voting system must fulfill the following six demands (Balinski and Laraki 2010b):

- avoid Condorcet's paradox,
- avoid Arrow's paradox,
- be meaningful,
- elicit honest voting,
- resist manipulation, and
- pay attention to the majority's will.

To fulfill above requirements we feel the importance of the majority judgment voting system, which is described as follows:

Majority Judgment Voting

In Arrow's impossibility theorem (Arrow 1963, Islam et al. 2009a,b, 2011) preference relation *xPy* for individual $\underline{1}$ and individual $\underline{2}$ express same preferences where *x* and *y* are two candidates. But in majority voting *xPy* may give different results. Suppose in majority Judgment voting both of the individuals' preference is *xPy*. Individual $\underline{1}$ expresses *x* is Good and *y* is Rejected. On the other hand individual $\underline{2}$ expresses *x* is Excellent and *y* is Very Good. This type of situation frequently happens in the society. We see that this gives more accurate information of the voters' opinion. Balinski and Laraki (2006, 2007, 2010a,b) first have introduced this type of judgment which is the median values of the grades given to a candidate is taken as the final grade of that alternative. They argue that an individual should choice the middlemost aggregation functions and call the resulting system majority judgment.

Let a finite set of *m* candidates is defined by $C = \{c_1, c_2, ..., c_m\}$ and a finite set of *n* voters is defined by $V = \{v_1, v_2, ..., v_n\}$. Let $G = \{g_1, g_2, ..., g_k\}$ be the set of grades where $g_1 > g_2 > ... > g_k$. Here $g_i \underset{maj}{\geq} g_j$ means that g_i is the higher grade than g_j or $g_i = g_j$. An input profile is $m \times n$ matrix (g_{ij}) of grades where each row *i* contains the grades provide by the voters to candidate c_i and each column *j* contains the grades vector v_j assign to the different candidates. A social grading function *F* is defined by $F : G^{m \times n} \to G^m$ such that (Zahid and Swart 2010);

$$\begin{pmatrix} g_{11} & g_{12} & \cdots & g_{1n} \\ g_{21} & g_{22} & \cdots & g_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ g_{m1} & g_{m2} & \cdots & g_{mn} \end{pmatrix} \rightarrow \left(f\left(g_{11}, g_{12}, \ldots, g_{1n}\right), \ldots, f\left(g_{m1}, g_{m2}, \ldots, g_{mn}\right)\right). \qquad (1)$$

That is $F(a) = \left(f^{maj}(c_1), \ldots, f^{maj}(c_m)\right)$, where $f^{maj}(c_i)$ is the majority grade which is the median value of the candidate c_i.

The k^{th} order function f^k indicates an *n*-tuple of grades and supplies an output the k^{th} highest grade (Balinski and Laraki 2007). When the number of voters is odd, the majority grade is the median. If the number of voters is even, then the lower of the two middle grades must be the majority grade. Let *a* be the number of grades is given to a candidate above its majority grade θ and *b* be the number of grades is given to a candidate below its majority grade θ. Hence we can write the majority grade θ^* as follows:

$$\theta^* = \begin{cases} \theta^+ & \text{if } a > b; \\ \theta^0 & \text{if } a = b; \\ \theta^- & \text{if } a < b. \end{cases}$$

For $g_1 > g_2 > \ldots > g_k$ a middlemost aggregate function f is defined by,
$f(g_1, g_2, \ldots, g_n) = g_{\left(\frac{n+1}{2}\right)}$ when *n* is odd, and $g_{\left(\frac{n}{2}\right)} \geq f(g_1, g_2, \ldots, g_n) \geq g_{\left(\frac{n+1}{2}\right)}$ when *n* is even.

Hence for odd *n* the order function $f^{\left(\frac{n+1}{2}\right)}$ is the middlemost aggregate function. For even *n* the upper middlemost $f^{\left(\frac{n}{2}\right)}$ is defined by $f^{\left(\frac{n}{2}\right)}(g_1, g_2, \ldots, g_n) = g_{\left(\frac{n}{2}\right)}$, and the lower middlemost aggregate function $f^{\left(\frac{n+2}{2}\right)}$ is defined by $f^{\left(\frac{n+2}{2}\right)}(g_1, g_2, \ldots, g_n) = g_{\left(\frac{n+2}{2}\right)}$.

Balinski and Laraki (2006) suggested that for even *n*, the lower of the two middle grades must be the majority grade. Hence a competitor's majority grade is defined briefly as follows:

$$f^{maj} = \begin{cases} f^{\left(\frac{n+1}{2}\right)} & \text{if } n \text{ is odd,} \\ f^{\left(\frac{n+2}{2}\right)} & \text{if } n \text{ is even.} \end{cases}$$

US Presidential Election of 2008

Within the last six weeks of the US presidential election of 2008, member of the INFORMS were invited to vote using the grades in September – early October, 2008. The results are given in table-4. Suppose we have six grades: *Reject*, *Poor*, *Acceptable*, *Good*, *Very Good* and *Excellent* to vote in grading system. We express these six grades in simple notations as: *R*, *P*, *A*, *G*, *VG* and *E*.

The majority grades and majority ranking of the results of table-4 are given in table-5. Here Bloomberg and Edwards have the same majority grade A^+. Bloomberg with a = 42.0% ranks than Edwards with a = 36.6%. From tables -4 and -5 we see that Obama is the winner in the majority judgment.

Table 4: US presidential election, INFORMS experiment, conducted September-early October, 2008.

	E	*V G*	*G*	*A*	*P*	*R*
Obama	35.9%	32.1%	12.2%	08.4%	07.6%	03.8%
Clinton	16.0%	29.0%	21.4%	16.8%	11.5%	05.3%
Powell	10.7%	22.1%	26.0%	26.7%	09.2%	22.1%
Bloomberg	03.1%	14.5%	24.4%	26.7%	09.2%	22.1%
Edwards	01.5%	13.0%	22.1%	30.5%	18.3%	14.5%
McCain	03.1%	07.6%	23.7%	21.4%	30.5%	13.7%
Romney	00.8%	07.6%	10.7%	27.5%	30.5%	22.9%
Huckabee	03.8%	03.8%	06.1%	19.8%	19.1%	47.3%

Table 5: Majority-grades and majority-ranking, U.S. presidential election, INFORMS experiment, conducted September-early October, 2008.

	a	θ	*b*
Obama	35.9%	VG^+	32.0%
Clinton	45.0%	G^+	33.6%
Powell	32.8%	G^-	41.2%
Bloomberg	42.0%	A^+	31.3%
Edwards	36.6%	A^+	32.8%
McCain	33.4%	A^-	44.2%
Romney	46.6%	P^+	22.9%
Huckabee	33.5%	P^-	47.3%

Advantages of Majority Judgment

Majority judgment election has some advantages from other voting systems. Few of them are as follows:

- It respects the election outcome of the majority. The majority grade (will be discussed later) is the unique mechanism which guarantees that when a majority of the electorate gives a grade *g* to a candidate, then that candidate's majority grade will be *g*. If everyone of a majority can give a point score of *s* to a candidate but that candidate's majority will certainly not be *s*.

- It satisfies transitive rule. Transitivity implies that for three candidates c_1, c_2 and c_3 if $c_1 \underset{maj}{>} c_2$ and $c_2 \underset{maj}{>} c_3$ then we must have $c_1 \underset{maj}{>} c_3$. The Condorcet paradox shows that the Condorcet voting is not transitive (Islam et al. 2011).
- It satisfies independent of irrelevant alternatives (IIA). IIA means that the rank order of two candidates is not influenced by the third one.
- It satisfies monotone. If every grade of a candidate is replaced by the same or a better grade, the candidate's place in the majority ranking cannot be lower. If every grade of a candidate is replaced by a strictly better grade, the candidate's majority grade must be raised. Monotonicity is not satisfied by the single transferable vote.
- It satisfies voters' utilities. In theory the motivations of voters and their satisfaction are modeled by their utilities. Given the decision mechanism and whatever information that is available, a rational voter chooses a message that maximizes her utility.
- It has freedom of expression of opinion of the candidates. Some critics have claimed that a voter should be forced to make up her mind by expressing a clear cut preference between any two candidates. But in majority judgment it is impossible.
- It does not satisfy dictatorship. In a society if the voters prefers *x* to *y*, society must prefer *x* to *y*, irrespective of the preferences else. This is called the condition of non-dictatorship. Mathematically, there is no such individual i, $\forall\, x, y \in X$ such that $xP_i y \Rightarrow xPy$ where X is a set of candidates. The dictatorship is undesirable in the society. First, it is undesirable because one's worst enemy might be dictator. Second, it is not a collective choice rule. So that dictatorship may cause the violation of human rights.
- In majority judgment every vote of all the voters must count. If two friends with opposite opinions sometimes skip voting because they think that their votes cancel each other outcome. Actually this is a wrong decision of voters in majority judgment. Their votes of course would effect if they would attend in the election.
- Every grade contributes to the determination of the majority ranking, even when a voter gives the same grade to every candidate. Again whatever may be a voter's grade (whatever may be the grades of a group of voters), there exists a situation where the voter (the group of voters) is decisive. This means that counting the voter's (the group of voters') ballot(s) gives one outcome; not counting it (them) gives another outcome.
- It reduces manipulation of voting.

The Majority Count of Borda

The Borda rule (Borda 1781) belongs to the class of point ranking rules where points are given to each candidate or alternative according to his rank in the preference of the voters. In this method if there are *n* alternatives, an elector's first choice is assigned $(m-1)$ points, his second $(m-2)$ points and so on down to his last choice, which is assigned 0 point. Once all votes have been counted and the candidate with the most points is the winner.

The majority count of Borda is a function $M(a): G^{m\times n} \to N^m$ which assigns to any profile the output $(M(c_1),..., M(c_m))$, where $M(c_i)$ is the majority count of Borda of candidate c_i. Let '*c*' be a candidate and let $G = \{ g_1, g_2, ..., g_k \}$ be the set of grades where $g_1 < g_2 < ... < g_k$. Let v_i be the number of voters, then the Borda majority count (*M*) can be defined by;

$$M(c) = v_1 \times 0 + v_2 \times 1 + ... + v_k \times (k-1)$$
$$= \sum_{i=1}^{k} v_i \times (i-1). \qquad (2)$$

We assign 0 point to the *R* grade, 1 point to the *P* grade, 2 points to the *A* grade, 3 points to the *G* grade, 4 points to the *VG* grade, and 5 points to the *E* grade. Hence this system is the Borda count.

Example 1: Suppose some journalists of a daily news paper of the USA want to survey randomly 1,000,000 people in each state to evaluate the popularity of the current president and the leader of the opposition party. The journalists can give each person a ballot paper with six grades: *Reject*, *Poor*, *Acceptable*, *Good*, *Very Good* and *Excellent*. The journalists can provide each person a ballot paper as follows:

Table 6: The Borda majority count ballot paper

	E	*VG*	*G*	*A*	*P*	*R*
The present president of USA						
The opposition leader of the USA						

The people vote with tick marks in all of the six grades or less than the six grades or even put tick in any one grade or none for two candidates and the journalists can collect all the ballot papers. Then the journalists can calculate the percentage of each grade by the Borda score mentioned above and easily can be expressed the popularity of the current president and the leader of the opposition party of the USA, and can express the results in that daily news paper.

Tie-Breaking in Majority Judgment

The general majority ranking $\underset{maj}{>}$ between two competitors c_i and c_j is determined as follows:

- If $f^{maj}(c_i) > f^{maj}(c_j)$, then $c_i \underset{maj}{>} c_j$.
- If $f^{maj}(c_i) = f^{maj}(c_j)$, then we drop one majority grade from the grades of each competitor.

If the tie is not broken then the procedure is repeated step by step, by dropping grades from lower to higher until we receive a winner between c_i and c_j. Now we set an example related to this type of tie-breaking as follows:

Example 2: Let there are two candidates *x* and *y*. They make tie in an election and the tie-breaking procedures are given as follows:

Table 7a: Tie-breaking by the majority judgment.

	a	*E*	*VG*	*G*	*A*	*P*	*R*	*b*	Total
x	37	19	18	**24**	2	17	20	39	100
y	36	17	19	**25**	20	15	4	39	100

In the example-2, both *x* and *y* have the majority grades at *G*. The type of tie is G^-. According to simple tie-breaking case the majority value of *x* is $(37, G^-, 39)$ and for *y* is $(36, G^-, 39)$. Since 37>36, obviously *x* is the majority judgment winner in the election. Now we apply the general tie-breaking rule for the example-2. First we drop the *E* grades and then we obtain the table-7b from table-7a as follows:

Table 7b: First step of tie-breaking by the majority judgment.

	a	*VG*	*G*	*A*	*P*	*R*	*b*	Total
x	18	18	**24**	2	17	20	39	81
y	19	19	**25**	20	15	4	39	83

Hence from table-7b we see that the majority value of *x* is $(18, G^-, 39)$ and for *y* is $(19, G^-, 39)$. Since 19>18, obviously *y* is the majority winner in the general tie-breaking rule.

In the case of large elections Balinski and Laraki (2007) introduce another type of tie breaking rule. Candidate's three majority values are sufficient to determine the candidate's position in the majority ranking as follows:

$$(a, \theta, b) \text{ where } \begin{cases} a = \text{the number of grades above the majority grade,} \\ \theta = \text{majority grade, and} \\ b = \text{the number of grades below the majority grade.} \end{cases}$$

The order between two majority values is defined as follows:

$$(a, \theta^*, b) \underset{maj}{>} (c, \varphi^*, d) \text{ if } \theta^* \underset{maj}{>} \varphi^*, \text{ where } \theta^* > \varphi^* \text{ if } \theta > \varphi \text{ and } \theta^+ > \theta^0 > \theta^-.$$

If $\theta^* = \varphi^*$ then $(a, \theta^+, b) \underset{maj}{>} (c, \varphi^+, d)$ if $\begin{cases} a > c \\ a = c \text{ if } b < d, \end{cases}$

and $(a, \theta^-, b) \underset{maj}{>} (c, \varphi^-, d)$ if $\begin{cases} b > t \\ b = t \text{ if } a > c, \end{cases}$

and $(a, \theta^0, b) \underset{maj}{>} (c, \varphi^0, d)$ if $a < c$ where $a = b$ and $c = d$.

Now we set an example related to this type of tie-breaking for a large election as follows:

Example 3: Let us consider an election where there are two candidates x and y, and 1259617 voters. The results are given in table-8.

Table 8: Second step of tie-breaking by the majority judgment.

	a	*E*	*VG*	*G*	*A*	*P*	*R*	*b*	total
x	361572	158976	202596	**698760**	100000	91485	7800	199285	1259617
y	361572	162848	198724	**445320**	240034	104742	8949	353725	1259617

Hence from table-8 we see that there is a tie in G^+. The majority value of *x* is $(361572, G^+, 199285)$ and for *y* is $(361572, G^+, 353725)$. Since 353725>199285, according to Balinski and Laraki (2007) the winner in the election is *y* but according to the general tie-breaking rule the winner is *x* (Balinski and Laraki 2006). From the above examples we observe that in majority tie-breaking winner in the election depends on which method is followed in the tie-breaking process.

Tie-Breaking in Borda Majority Count

Sometimes there is a tie in *M*, and then the tie can be broken by dropping the *R* grade and re-calculating for the *M*. Again if the tie arises in this case the *P* grade must be dropped and re-calculate for the *M*. This process is continue by dropping grades step by step from lower to higher until the tie is broken. The candidate, who acquires the greatest *M*, is the Borda majority winner. The following example shows the procedures of tie breaking in *M* (Zahid and Swart 2010):

Example 4: Let us consider 100 judges who give their judgments for three candidates c_1, c_2 and c_3 as in the table-9 below:

Table 9: Tie Breaking in *M*.

	E	*VG*	*G*	*A*	*P*	*R*	*M*-1	*M*-2	*M*-3
c_1	11	33	21	29	2	4	310	214	120
c_2	13	34	18	24	7	4	310	214	**125**
c_3	13	29	22	31	1	4	310	214	119

In the table-9 we see that all the three candidates with Borda score 310 tied in *M*-1. Hence we drop the *R* grades of all the candidates and then re-calculate for *M* but same condition arises in *M*-2. At this situation we drop the *P* grades of all the candidates and after re-calculating for *M* we observe that now tie breaks and candidate c_2 with highest score **125** wins in the election.

Drawbacks in Majority Judgment Voting

No voting method is stainless, so that majority judgment voting also has some counter-intuitive results. Some drawbacks of majority judgment voting are discussed with some examples as follows:

Example 5: In majority judgment sometimes the winner may loose. Consider there are 1000 voters, and two candidates *x* and *y*. Voters rank the two candidates as in the following table-10.

Table 10: Majority judgment.

	a	*E*	*VG*	*G*	*A*	*P*	*R*	*b*	total
x	500	200	300	**500**	0	0	0	0	1000
y	510	200	310	**200**	160	70	60	290	1000

Here the majority grade of *x* and *y* are G^+. The majority value of *x* is $(500, G^+, 0)$ and the majority value of *y* is $(510, G^+, 290)$. According to both tie-breaking rules *y* is the winner. Here no voter gives *x* lower grade than *G* but 290 voters do the job for *y*. Here *y* is the winner because 10 extra voters vote *VG* to *y* and not taking into account the 290 voters evaluate *y* as lower than *G*. Here *x*'s performances are *G* or better than *G* and *y*'s performances are not so but according to majority voting *y* wins. If we consider nonmajority judgment voting system then obviously x would win. Hence in majority judgment voting winner may loose.

Example 6: In majority judgment sometimes looser may be the majority judgment winner. Now we consider a 100 round competitive contest where four players x, y, z, and w are competitors. They play 100 rounds and one judge gives them grades as follows:

Table 11: Majority judgment.

	a	*E*	*VG*	*G*	*A*	*P*	*R*	*b*
x	50	20	10	20	0	0	**50**	0
y	50	0	30	10	10	0	**50**	0
z	50	0	10	10	20	10	**50**	0
w	0	0	0	0	0	**70**	30	30

At the first sight we can say that w is the looser and x is the winner because x performs five times G or better than G and w performs 0 in A or better than A. But according to Balinski and Laraki (2006, 2007) the lower majority middle grade be the majority grade. The majority grade of x, y, and z is R but the majority grade of w is P, so that w is the majority judgment winner. Hence in majority judgment voting sometimes looser may be the majority judgment winner.

Example 7: In two cities electoral cases majority judgment violates both winner consistency and rank consistency. First we consider the following example with candidates x and y, and the set of grades be {0, 1, 2, 3, 4, 5, 6}.

For city-I

Table 11a: Majority judgment for city-I.

Candidates	*Scores*					*Majority grades*
x	6	4	**3**	1	0	**3**
y	6	6	**2**	2	0	2

In city-I, $x \underset{maj}{>} y$, and x is ranked above y, so that x wins in the election.

For city-II

Table 11b: Majority judgment for city-II.

Candidates	*Scores*					*Majority grades*
x	6	6	**5**	1	0	**5**
y	6	5	**4**	4	1	4

In city-II, $x \underset{maj}{>} y$, and x is ranked above y, so that x wins in the election. Now combining the scores of two cities we get;

Table 11c: Majority judgment of combination of city-I and city-II.

Candidates	*Scores*										*Majority grades*
x	6	6	6	5	4	**3**	1	1	0	0	3
y	6	6	6	5	4	**4**	2	2	1	0	**4**

In the combination of two cities elections, $y \underset{maj}{>} x$, and y is ranked above x, so that y wins in the two cities combination elections.

Example 8: If we add same number of voters in R (*Rejected*) then election result change. Consider an example with 15 voters as follows:

Table 12a: Majority judgment.

	a	*E*	*VG*	*G*	*A*	*P*	*R*	*b*
x	4	1	3	**8**	2	1	0	3
y	3	1	2	**10**	1	1	0	2

In table-12a the majority value of *x* is $(4, G^{+}, 3)$ and the majority value of *y* is $(3, G^{+}, 2)$. Since majority value is $x \underset{maj}{>} y$, so that *x* wins in the election. Now we add two voters who vote in *R*, hence table-12a now becomes as follows:

Table 12b: Majority judgment by adding 2 voters in *R*.

	a	*E*	*VG*	*G*	*A*	*P*	*R*	*b*
x	4	1	3	**8**	2	1	2	5
y	3	1	2	**10**	1	1	2	4

Now majority value of *x* is $(4, G^{-}, 5)$ and the majority value of *y* is $(3, G^{-}, 4)$. Since majority value is $y \underset{maj}{>} x$, so that *y* wins in the election by Balinski and Laraki (2007).

Example 9: In an election two friends *A* and *B* have different opinions. *A* supports candidate *x* and *B* supports candidate *y*. The set of grades be {0, 1, 2, 3, 4, 5, 6}. Voter *A* decided to give the highest grade 6 to *x* and second highest grade 5 to *y*. Voter *B* decided to give the highest grade 6 to *y* and second highest grade 5 to *x*. Both of them think their votes will give no fruitful result, so that they decided not to cast their votes. First we consider the example due to Bishop (2010). In the election there are two candidates *x* and *y*, and their scores are as follows:

Table 13a: Majority judgment by Bishop.

Candidates	*Scores*					*Majority grades*
x	1	2	**4**	4	6	**4**
y	2	3	**3**	6	6	3

Here $x \underset{maj}{>} y$, so that *x* wins in the election. Now we apply this result for the case of voters *A* and *B*. If they would have vote then adding their votes to the table-13a of Bishop we get as follows:

Table 13b: Majority judgment if two opposite voters would vote.

Candidates	*Scores*								*Majority grades*
x	1	2	4	**4**	5	6	6	6	4
y	2	3	3	**5**	6	6	6	6	**5**

From the table-13b above we see that $y \underset{maj}{>} x$, and *y* wins in the election. In Bishop's example if both *A* and *B* votes 6 grade for *x* and 5 grade for *y*. The voting situation becomes as follows:

Table 13c: Majority judgment if two voters would vote in same grade.

Candidates	*Scores*						*Majority grades*
x	1	2	4	**4**	6	6	4
y	2	3	3	**5**	5	6	**5**

Here we observe that $y \underset{maj}{>} x$, and *y* is winner in the election. This result is the same in the following two cases:

- If both *A* and *B* be with opposite grades supporters but absent in the election.
- If both supporters *A* and *B* give the highest rank to *x*.

This is a drawback of majority judgment voting.

Theoretical Properties of the Majority Judgment and Borda Count

The majority grade of a candidate is her median grade. It is simultaneously the highest grade approved by a majority and the lowest grade approved by a majority. The winner consistency of an electorate is defined as follows: If there are two separate districts of an electorate and a candidate wins in both electorates, then he must wins in the combinations of the two districts.

Proposition 1: *Majority judgment voting is not winner consistent.*

Proof: Let there are two candidates *x* and *y*. The elections are held in two cities. Let in city-I majority judgment gives $x \underset{maj}{>} y$, so that *x* wins in the election. Again let in city-II majority judgment gives the same result i.e., *x* is also winner here. If we combine the two cities according to Balinski and Laraki (2010a) we observe that $y \underset{maj}{>} x$ always. So that *y* wins in the combination of two cities election. In example-9 we have obtained the same result. Hence majority judgment voting is not winner consistent. Q.E.D.

The majority ranking orders the candidates according to their majority grades. The rank consistency of an electorate is defined as follows: If there are two separates cities of an electorate and the ranking of two candidates *x*, *y* in two cities of a consistency are $x > y$, then in the whole electorate the ranking of the candidates will be the same, that is $x > y$ and *x* is the winner.

Proposition 2: *Majority judgment voting is not ranking consistent.*

Proof: Let there is an election in two separate districts *A* and *B*, and there are two candidates *x* and *y*. Assume in district *A* majority judgment gives $x > y$, i.e., *x* is ranked above *y* in the election, consequently *x* wins in the election. Again let in district *B* majority judgment gives the same result i.e., *x* is ranked above *y* in the election here. If we combine the two districts' outcomes according to Balinski and Laraki (2010a) we observe that $y > x$ always i.e., *y* is ranked above *x* in the election, consequently *y* wins in the election. Hence ranks of *x* higher in both separate districts but *y* is ranked above *x* in the combination of two districts election. Hence majority judgment voting is not ranking consistent. Q.E.D.

The grade consistency in majority judgment is defined as follows: If there are two separate towns of an electorate and the majority grade of a candidate in each town is θ, then the majority grade of the whole electorate will be θ always.

Proposition 3: *Grade consistency is satisfied by the majority judgment.*

Proof: Let there is an election in two separate towns *P* and *Q*, and there are two candidates *x* and *y*. According to Balinski and Laraki (2010a) let the majority grade of *x* in town *P* be θ. In another town *Q* the majority grade of *x* also be θ. Then the majority grade of *x* in the combination of *P* and *Q* must be θ. Q.E.D.

Mohajan, H.K. (2012m), Majority Judgment in an Election with Borda Majority Count, *International Journal of Management and Transformation*, 6(1): 19–31.

The Borda majority count has no so many drawbacks as the majority judgment has. It also contradicts the propositions -1 and -2. So that it is winner and rank consistent. Hence we see that even in the 21st century politicians can not present a voting system which is better than the 18th centurion politicians Condorcet and Borda provided.

Concluding Remarks

This paper analyzes the majority judgment, and its advantages and drawbacks with some examples. The theoretical properties of the majority judgment are described with some propositions. Some examples and related tables are included to show the aspects of the majority judgment in some detail. In this paper additionally we have also included Borda majority count to enrich the majority judgment. We have shown that majority judgment voting is not winner and rank consistent but the Borda majority count does not do so. Balinski and Laraki (2006) first introduced the majority judgment to avoid the problems of Arrow's theorem in social choice theory but it created some paradoxes which are unavoidable. Hence majority judgment is not a stainless voting system in social choice theory. But in many situations it gives fruitful results. Finally we want to say that voting system is a very complicated field and we have tried our best to make the majority judgment voting easier to the reader.

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