Methods of voting system and manipulation of voting

Islam, Jamal and Mohajan, Haradhan and Moolio, Pahlaj

International Journal of Management and Transformation

19 January 2010

Islam, J.N.; Mohajan, H.K. and Moolio, P. (2011), Method of Voting System and Manipulation of Voting, *International Journal of Management and Transformation:* 5(1): 10–34.

Methods of Voting System and Manipulation of Voting

J. N. Islam
Emeritus Professor, Research Centre for Mathematical and Physical Sciences
University of Chittagong, Bangladesh
Address for Correspondence: 28, Sursan Road, Chittagong, Bangladesh
Phone: +880-31-616780

H. K. Mohajan
Assistant Professor, Premier University, Chittagong, Bangladesh
Address for Correspondence: E-mail: haradhan_km@yahoo.com

P. Moolio
Professor, Paññāsāstra University of Cambodia, Phnom Penh, Cambodia
Address for Correspondence: 144-184, Norodom Blvd., Phnom Penh, Cambodia
E-mail: pahlajmoolio@gmail.com

Abstract: *In this paper an attempt has been taken to describe various types of voting system and manipulation of them. French philosophers Marquis de Condorcet (1743-1794) and Jeans-Charles Borda (1733-1799) introduced modern voting system. Duncan Black first introduced the manipulation of voting in 1958 in his book "Theory of Committee and Elections". Condorcet, Borda and even many modern politicians believe that elections are logically imperfect. In this paper this imperfection is analyzed in some detail. In this paper voting methods are discussed in very simple but in a detailed manner. Voting system is directly involved with Economics, Political Science and Social Science. So that if one has no proper knowledge of the voting system then he can not serve the society in proper way and cannot expect the economic development of the society. Some voting methods such as Arrow's theorem, median voter theorem, randomized voting, Muller-Satterthwaite theorem and Gibbard-Satterthwaite theorem are apparently non-manipulable and are included in this paper.*

Keywords: Voting system, voting paradox, manipulation of voting, Condorcet winner, dictatorship, strategy-proofness.

Introduction

A voting system is manipulable whenever some individual misrepresents his preferences in order to secure an outcome preferred to the outcome when he is honest otherwise it is strategy-proof. In voting system every voter's preference ordering i.e., the preference profile, taken collectively, form the input the output is usually a single certain winner or a set of winners. The methods of transforming preference profiles into winners i.e., mappings from the set of possible preference profiles into the set of alternatives is called voting procedures. For each preference profile the mapping produces a single winning alternative. Such a mapping is called a social decision function (SDF). The social welfare function (SWF) on the other hand, first studied by Arrow, are the rules for transforming preference profile into social preference orderings or rankings. The definition of SWF given by Arrow is as follows: Let $Y = \{a_1, a_2, ..., a_n\}$ denote a finite set of alternatives or social choice options among which the voters must select one and let $R(Y)$ denote the set of strict linear rankings on *Y*. Let $N = \{1, 2, ..., n\}$ be a finite set of individual voters. A function $f : R^N \to Y$ will be called a social choice function. A member of R^N is called a profile of rankings and its *ith* component is called individual *i*'s ranking.

A SWF is a function $f : R^N \to R$ which aggregates voters' preferences into a single preference order on *Y*. The *N*-tuple: $(R_1, ..., R_N)$ of voters' preferences is called a preference profile. Arrow declared that there exist no satisfactory SWF (Islam *et al.* 2009). A satisfactory SDF should not be a dictatorship (Feldman 1979). Gibbard (1973, 1978) and Satterthwaite (1975) independently proved this as follows: "If a satisfactory social decision function is one which is always immune to manipulation and which is non-dictatorial, there is no satisfactory social decision function". Following Myerson (1996, 2009) we have discussed some portion of this paper in some detail.

This paper is an exposition of voting system and of the manipulation of voting. French political philosophers Borda (1781) and Condorcet (1785) introduced modern voting system but they had not mentioned about manipulation of voting. Condorcet, Borda and even many modern politicians believe that elections are logically imperfect. In this paper we will explore such imperfections of the election in some detail. Duncan Black (1958) first introduced the manipulation of voting. Here we discuss in easier way of voting system and manipulation of them (Feldman 1979; Myerson (1993, 1996, 2006, 2009); Blackorby, *et al.* 1990; Blackorby, *et al*. 2002; Saporiti and Thomé 2006; McLennan 2008; Saporiti 2008; Miller 2009; Robert and Tsoukiàs 2009; Sato 2009).

The concept of Median Voter is described following Black (1948, 1958); Gans and Smart (1994); Myerson (1996); Austen-Smith and Banks (1999); Congleton (2004); Saporiti and Thomé (2006); Saporiti (2008); and Penn *et al.* (2008). In this paper we have discussed relatively simple models of voting system but the real political settings are more complex than the models seem to imply (Congleton 2004). We used simple model basically for three reasons namely: i) simple models allow knowledge to be transferred from person to person than those of more complex models, ii) simple models provide us some clear knowledge of voting whereas complex models do not always provide so, iii) from simple models we, the common people, can understand the main features of the voting system which is a theme of democracy.

Condorcet Method

A Condorcet method is any single-winner election method which always selects the Condorcet winner (i.e., an alternative that beats every other alternative in sequence of pair-wise majority contests); the candidate who would beat each of the other candidates in a run-off election if such a candidate exists. Condorcet method is named after the French political philosopher Marquis de Condorcet (1743-1794). Condorcet (1785) introduced imperfection of jury by problem in probability theory as follows: There are two alternatives *x* and *y* between which a panel of jury is to decide, who is guilty or innocent. Between the two alternatives one is guilty and the other is innocent. Since the members of jury are imperfect, so they may err. Since the jury members are efficient, so that if the numbers of jury members are more than enough the probability of correct voting will be less wrong. Suppose, *x* gets more votes than *y*, so that probability *x* of being innocent to more correct (Feldman 1979). Let probability of voting correctly be *p*, so the probability of voting incorrectly is $(1-p)$. Suppose there are three alternatives, say, *x*, *y* and *z*. Here one alternative is innocent and other two are guilty, so that the problem is complicated. Suppose the contest will be pair wise. As before if *x* gets more votes than *y*, and *y* gets more votes than *z*, and *x* gets more votes than *z*. In this case jury's decision of being innocent is x and the probability of x being innocent can be calculated as before. But the following case is not so easy which creates a paradox.

Now we discuss the Condorcet voting paradox in which there is no Condorcet winner (Condorcet 1785; Risse 2005). In this section and throughout the paper we consider each voter ranks the list of candidates in order of preference i.e., for three candidates x, y and z the preference profile of a voter may be as follows:

1. x
2. y
3. z.

Here x is one's first choice, y is second choice and z is third choice. For convenience, we will use this profile as, $xPyPz$. Let us assume that there are 17 voters of three types and three alternatives x, y, z. Let preference relations are as follows:

Type 1: $xPyPz$ by 8 voters,
Type 2: $yPzPx$ by 5 voters,
Type 3: $zPxPy$ by 4 voters.

In an election a vote between x and y, x collects 8+4 =12 votes and y collects 5 votes, so that x wins. Again a vote between y and z, y collects 8+5 =13 votes and z collects 4 votes, so that y wins. Again a vote between x and z, x collects 8 votes and z collects 4+5 =9 votes, so that z wins. We observe that there is a cycle in the voting results where x is defeated by y, y is defeated by z and also z is defeated by x which is a voter paradox.

Condorcet's ad hoc judgment is that x is the Condorcet winner, since x wins by 7 votes and defeats by 1 vote, y wins by 9 votes and defeats by 7 votes, z wins by 1 vote and defeats by 9 votes. But this is not a satisfactory and acceptable decision. Again consider the preference relations be as follows:

Type 1: $xPyPz$ by 49 voters,
Type 2: $yPzPx$ by 2 voters,
Type 3: $zPxPy$ by 48 voters.

Here y is preferred by a $50-49$ majority to x and by a $51-48$ majority to z. So, according to the Condorcet criterion, y should win, despite the fact that very few voters rank y in the first place but the plurality (will be discussed later) elects x.

Borda Count

Jean-Charles Borda (1733-1799) developed another voting method named "method of marks" (Borda 1781). Each elector ranks the alternatives according to his order of preference (ties disallowed). Once all votes have been counted and the candidate with the most points is the winner. It is currently used for the election of two ethnic minority members of the National Assembly of Slovenia, and in modified forms, to select presidential election candidates in Kiribati and to elect members of the Parliament of Nauru. It is also used throughout the world by various private organization and competitions. In this method if there are m alternatives, an elector's first choice is assigned $(m-1)$ points, his second $(m-2)$ points and so on down to his last choice, which is assigned 0 point. One property of the Borda rule is that each of the voters of each type gives $m(m-1)$ marks to the candidates. Borda votes in the above first example be as follows:

For x: 8×2+5×0+4×1 = 20 marks,
For y: 8×1+5×2+4×0 = 18 marks,
For z: 8×0+5×1+4×2 = 13 marks.

Here x gets highest marks 20, so x wins. We observed that Borda method has no voter paradox but it has some problems. Black (1958) and Satterthwaite (1975) modified the Borda method by misrepresentation of their preferences by the electors. Now we modify the example by adding two alternatives u and v. The preference relations be as follows:

Type 1: $xPyPzPuPv$ by 8 voters,
Type 2: $yPzPxPuPv$ by 5 voters,
Type 3: $zPxPyPuPv$ by 4 voters.

Now Borda counts be as follows:

For x: 8×4+5×2+4×3 = 54 marks,
For y: 8×3+5×4+4×2 = 52 marks,
For z: 8×2+5×3+4×4 = 47 marks,
For u: 8×1+5×1+4×1 = 17 marks,
For v: 8×0+5×0+4×0 = 0 mark.

So that in this case x wins again. If the type 3 voters falsely declared that their preference ordering is as,

Type 3: $zPyPuPvPx$ by 4 voters,

then the Borda counts would be,

For x: 8×4+5×2+4×0 = 42 marks,
For y: 8×3+5×4+4×3 = 56 marks,
For z: 8×2+5×3+4×4 = 47 marks,
For u: 8×1+5×1+4×2 = 21 marks,
For v: 8×0+5×0+4×1 = 4 marks.

In this case y would have won. The voters of type 3 would have been better off than when they voted honestly; the method provides a temptation for misrepresentation of preferences. The possibility of manipulation of the result of an election through the misrepresentation of preferences as described above was considered neither by Borda nor by Condorcet.

Borda Rule is Cloning Manipulable

We have seen that Borda did not use manipulation in his voting method. But we can manipulate the Borda rule by introducing a cloning candidate (Serais 2002). Suppose x would be defeated in an election following Borda count. The candidate x can manipulate the election outcome in his favor by introducing his clone y (say) in the choice set, the clone y being defined as an alternative which is ranked immediately below x in the individual preferences.

Let $N = \{1, 2, ..., n\}$ be the set of individual voters, and let $Y = \{x, y, z, ...\}$ be the finite set of alternatives. Choose a set $A \subseteq Y$ be a finite set where $n(A) = m$. Now for $A = \{x, y, z\}$ the six possible preference orderings over A will be numbered as follows:

Table 1: Possible Preference Orderings Over the Set A.

n_1	n_2	n_3	n_4	n_5	n_6
x	x	y	y	z	z
y	z	x	z	x	y
z	y	z	x	y	x

A voting situation is a vector, $s = (n_1, ..., n_6)$, where n_j $(j = 1, ..., 6)$ be the number of type j voters and $\sum_{j=1}^{6} n_j = n$. Here n is the total number of voters in an election. Let $S^n = \{s^1, ..., s^n\}$ be the set of all possible voting situations. A social choice function $f : S^n \to A$, assigns to each voting situation a non-empty subset of *A*. Let N_{xy} be the number of voters who prefer *x* to *y*, $S^x_{B,s}$ be the Borda score of *x*, and $S^{xy}_{B,s}$ be the difference of Borda score between *x* and *y* for the voting situations i.e., $S^{xy}_{B,s} = S^x_{B,s} - S^y_{B,s}$. Now we can introduce the mathematical definition of a clone as follows:

A candidate *y* is a clone of *x* for a voting situations if and only if $\forall z \in X \setminus \{x, y\}$, $\forall i \in N$ $xP_i z \Leftrightarrow yP_i z$ and $\forall i \in N$, $xP_i y$.

This type of manipulation was introduced by Dummett (1998) where he called it agenda manipulation. Dummett observed that the Borda rule may suffer from this manipulation and explained by a series of examples. Here we set an example where there are 12 voters and they have to choose preference relation among four alternatives *x*, *y*, *z* and *u*. Let the preference relations be as follows:

Type 1: *yPuPzPx* by 2 voters,
Type 2: *uPzPxPy* by 2 voters,
Type 3: *zPuPyPx* by 2 voters,
Type 4: *xPuPyPz* by 3 voters,
Type 5: *xPyPuPz* by 1 voter,
Type 6: *zPyPxPu* by 2 voters.

Borda votes in the above example be as follows:

For *x*: 2×0+2×1+2×0+3×3+1×3+2×1 = 16 marks,
For *y*: 2×3+2×0+2×1+3×1+1×2+2×2 = 17 marks,
For *z*: 2×1+2×2+2×3+3×0+1×0+2×3 = 18 marks,
For *u*: 2×2+2×3+2×2+3×2+1×1+2×0 = 21 marks.

Here *u* gets highest 21 marks, so *u* wins and *y* gets second lowest 17 marks. Dummett assumes that before the election, a fifth candidate, *v* is introduced by *y* whom every voter ranks immediately below *y*. Then the preference profile would be as follows:

Type 1: *yPvPuPzPx* by 2 voters,
Type 2: *uPzPxPyPv* by 2 voters,
Type 3: *zPuPyPvPx* by 2 voters,
Type 4: *xPuPyPvPz* by 3 voters,
Type 5: *xPyPvPuPz* by 1 voter,
Type 6: *zPyPvPxPu* by 2 voters.

Then the Borda votes would be as follows:

For *x*: 2×0+2×2+2×0+3×4+1×4+2×1 = 22 marks,
For *y*: 2×4+2×1+2×2+3×2+1×3+2×3 = 29 marks,
For *z*: 2×1+2×3+2×4+3×0+1×0+2×4 = 24 marks,
For *u*: 2×2+2×4+2×3+3×3+1×1+2×0 = 28 marks,
For *v*: 2×3+2×0+2×1+3×1+1×2+2×2 = 17 marks.

Now *y* gets highest score of 29 marks and wins in the election. Here we observed that in initial voting situation *y* scored second lowest 17 marks but after cloning a candidate *v* as fifth candidate by *y* placed him in first position. So that cloning manipulation is sufficiently powerful to win in an election by a losing candidate.

Let there are two losing candidates in an election. Now we will discuss the cloning manipulation by a single loser. Let $A = \{x, y, z\}$ then the voting situation s^1 be as follows:

Table 2: The Voting Situation of s^1.

n_1	n_2	n_3	n_4	n_5	n_6	Scores
x	x	z	z	y	y	2
y	z	x	y	z	x	1
z	y	y	x	x	z	0

From table 2 we find;
$S^x_{B,s^1} = 2(n_1 + n_2) + n_3 + n_6$, $S^y_{B,s^1} = n_1 + n_4 + 2(n_5 + n_6)$ and $S^z_{B,s^1} = n_2 + 2(n_3 + n_4) + n_5$,
$N_{xy} = n_1 + n_2 + n_3$, $N_{yx} = n_4 + n_5 + n_6$, $N_{xz} = n_1 + n_2 + n_6$, $N_{zx} = n_3 + n_4 + n_5$,
$N_{yz} = n_1 + n_5 + n_6$ and $N_{zy} = n_2 + n_3 + n_4$,
$N_{yz} + N_{zy} = n_1 + n_2 + n_3 + n_4 + n_5 + n_6 = n$,
Similarly, $N_{xy} + N_{yx} = n$ and $N_{xz} + N_{zx} = n$.
Let $S^{xy}_{B,s^1} \geq 0$ and $S^{xz}_{B,s^1} \geq 0$, so that *x* wins by Borda counts. Now suppose *y* is in the lowest position by Borda counts. The candidate *y* could introduce *u* whom every voter ranks immediately below *y*. Then the voting situation s^2 would be as follows:

Table 3: The Voting Situation of s^2

n_1	n_2	n_3	n_4	n_5	n_6	Scores
x	x	z	z	y	y	3
y	z	x	y	u	u	2
u	y	y	u	z	x	1
z	u	u	x	x	z	0

From table 3 we find:
$$\begin{aligned} S^x_{B,s^2} &= 3(n_1 + n_2) + 2n_3 + n_6 \\ &= (2(n_1 + n_2) + n_3 + n_6) + (n_1 + n_2 + n_3) \\ &= S^x_{B,s^1} + N_{xy}, \end{aligned}$$
$$\begin{aligned} S^y_{B,s^2} &= 2n_1 + n_2 + n_3 + 2n_4 + 3(n_5 + n_6) \\ &= (n_1 + n_4 + 2(n_5 + n_6)) + (n_1 + n_2 + n_3 + n_4 + n_5 + n_6) \\ &= S^y_{B,s^1} + n, \end{aligned}$$
$$\begin{aligned} S^z_{B,s^2} &= 2n_2 + 3(n_3 + n_4) + n_5 \\ &= (n_2 + 2(n_3 + n_4) + n_5) + (n_2 + n_3 + n_4) \\ &= S^z_{B,s^1} + N_{zy}, \end{aligned}$$
$$S^u_{B,s^2} = n_1 + n_4 + 2(n_5 + n_6) = S^y_{B,s^1}.$$

Since *u* is cloned by *y*, so *u* is always beaten by *y*. Now *y* beats *x* if,
$$N_{yx} > S^{xy}_{B,s^1} \text{ i.e., } S^y_{B,s^1} + n > S^x_{B,s^1} + N_{xy}$$

i.e., $2(n_1+n_4)+3(n_5+n_6)+n_2+n_3>3(n_1+n_4)+2n_3+n_6$ (1)

Now y beats z if,

$$N_{yz}\geq S^{zy}_{B,s^1} \text{ i.e., } S^{y}_{B,s^1}+N_{yz}+N_{zy}\geq S^{z}_{B,s^1}+N_{zy} \text{ i.e., } S^{y}_{B,s^1}+n\geq S^{z}_{B,s^1}+N_{zy}$$

i.e., $2(n_1+n_4)+3(n_5+n_6)+n_2+n_3>3(n_3+n_4)+2n_2+n_5$. (2)

Inequalities (1) and (2) satisfy all the properties of Borda rule, so that y wins in the election. Now we describe the cloning manipulation by both of the losing candidates. In table 2 we considered that x wins but y and z were defeated. Now both y and z could introduce cloning candidates. Let u be the clone of y and v be the clone of z and let only y would be benefited by cloning. The voting situation s^3 would be as follows:

Table 4: The Voting Situation of s^3.

n_1	n_2	n_3	n_4	n_5	n_6	Scores
x	x	z	z	y	y	4
y	z	v	v	u	u	3
u	v	x	y	z	x	2
z	y	y	u	v	z	1
v	u	u	x	x	v	0

From table 4 we find:

$$\begin{aligned}
S^{x}_{B,s^3} &= 4(n_1+n_2)+2(n_3+n_6)\\
&= (2(n_1+n_2)+n_3+n_6)+(n_1+n_2+n_3)+(n_1+n_2+n_6)\\
&= S^{x}_{B,s^1}+N_{xy}+N_{xz},\\
S^{y}_{B,s^3} &= 3n_1+n_2+n_3+2n_4+4(n_5+n_6)\\
&= (n_1+n_4+2(n_5+n_6))+(n_1+n_2+n_3+n_4+n_5+n_6)+(n_1+n_5+n_6)\\
&= S^{y}_{B,s^1}+n+N_{yz},\\
S^{z}_{B,s^3} &= n_1+3n_2+4(n_3+n_4)+2n_5+n_6\\
&= (n_2+2(n_3+n_4)+n_5)+(n_1+n_2+n_3+n_4+n_5+n_6)+(n_2+n_3+n_4)\\
&= S^{z}_{B,s^1}+n+N_{zy},\\
S^{u}_{B,s^3} &= 2n_1+n_4+3(n_5+n_6)\\
&= (n_1+n_4+2(n_5+n_6))+(n_1+n_5+n_6)\\
&= S^{y}_{B,s^1}+N_{yz},\\
S^{v}_{B,s^3} &= 2n_2+3(n_3+n_4)+n_5\\
&= (n_2+2(n_3+n_4)+n_5)+(n_2+n_3+n_4)\\
&= S^{z}_{B,s^1}+N_{zy}.
\end{aligned}$$

Since u be the clone of y, so u is always beaten by y. y beats x if $S^{yx}_{B,s^3}>0$

$$\Rightarrow S^{y}_{B,s^1}>S^{xy}_{B,s^1}+N_{xz} \text{ i.e., } 2S^{y}_{B,s^1}>S^{x}_{B,s^1}+N_{xz} \text{ i.e., } S^{y}_{B,s^1}+N_{yx}+N_{yz}>S^{x}_{B,s^1}+N_{xz}$$

i.e., $S^{y}_{B,s^1}+N_{yx}+N_{yz}+N_{xy}>S^{x}_{B,s^1}+N_{xz}+N_{xy}$

i.e., $S_{B,s^1}^{y} + n + N_{yz} > S_{B,s^1}^{x} + N_{xz} + N_{xy}$,

$$n_1 + n_4 + 2(n_5 + n_6) + n + n_1 + n_5 + n_6 > 2(n_1 + n_2) + n_3 + n_6 + (n_1 + n_2 + n_6) + (n_1 + n_2 + n_3)$$

i.e., $4(n_5 + n_6) + 3n_1 + n_2 + n_3 + 2n_4 > 4(n_1 + n_2) + 2(n_3 + n_6)$. (3)

Again v is the clone of z so that $S_{B,s^3}^{yz} \geq 0$ gives $S_{B,s^3}^{zv} \geq 0$ which implies $S_{B,s^3}^{yv} \geq 0$.

Again y beats z if $S_{B,s^3}^{yz} \geq 0 \quad \Rightarrow S_{B,s^3}^{yz} \geq N_{zy} - N_{yz} \quad \Rightarrow S_{B,s^1}^{y} + n + N_{yz} \geq S_{B,s^1}^{z} + n + N_{zy}$

i.e., $4(n_5 + n_6) + 3n_1 + n_2 + n_3 + 2n_4 > 4(n_3 + n_4) + 3n_2 + 2n_5 + n_1 + n_6$. (4)

Inequalities (3) & (4) satisfy all the properties of Borda rule, so that *y* wins in the election. In this section we have shown by calculations that Borda voting is for sincere voters and manipulation is impossible but a defeated candidate can manipulate Borda voting in his favor by introducing his clone.

Majority Voting

In section 2 we observed the voting paradox in majority voting where there is no unambiguous winner. We modify the majority voting by the introduction of an agenda (Black 1958; Feldman 1979). Let us again consider the preference relation of section 2 i.e.,

Type 1: *xPyPz* by 8 voters,
Type 2: *yPzPx* by 5 voters,
Type 3: *zPxPy* by 4 voters.

Now suppose that *x* is the *status quo*, while *y* is a motion to change the status quo and *z* is an amended version of that motion. A typical committee practice, which Black (1958) called Procedure α, is to hold a vote between *y* and *z* (the motion and the amended version), and place the winner of that vote against *x* (the status quo). If votes are sincere, Procedure α produces *y* on the first round (the amendment is defeated) and *x* on the second (the bill is defeated). In these circumstances, type 2 voters could misrepresent their preferences as,

Type 2: *zPyPx* by 5 voters.

Then, *z* could win the first round (the amendment would pass) and then *z* defeated *x* (the amended bill would be adopted). In this case type 2 voters could be benefited by manipulation of voting.

A second committee practice, which Black (1958) called Procedure β, pits each motion against the status quo. We have shown *z* defeats the status quo but *y* does not, so that *z* is adopted, provided the voters vote sincerely. We observed that under Procedure β, type 1 voters have an opportunity to gain by misrepresentation. If they vote as of their preferences were,

Type 1: *yPxPz* by 8 voters,

Then both *y* and *z* would defeat the status quo in the first round. In the second round *y* would defeat *z*. So that type 1 voters would have manipulated the choice of *y*, which they prefer, over *z*.

Single Transferable Voting System

The single transferable vote (STV) is a system of preferential voting designed to minimize wasted votes. In STV, a constituency elects two or more representatives per electorate. As a result the constituency is proportionally larger than a single member constituency from each party. Political parties tend to offer as many candidates as they most optimistically could expect to win; the major parties may nominate almost as many candidates as there are seats, while the minor parties and independents rather fewer. STV initially allocates an elector's vote for her most preferred candidate and then, after candidate have been either elected or eliminated, transfers surplus or unused votes according to the voter's stated preferences (ties disallowed). It is a many ballots electoral system and mainly used in the English-speaking countries. It can be used for either single winner or multi-winner voting system and we will discuss both methods in this section.

History of STV

The concept of transferable voting was first proposed by Thomas Wright Hill in 1821, for application in elections at his school but was not populated. Carft Andrae in 1855 proposed a transferable voting system for election in Denmark and used it in 1856 to elect the Danish Rigsdag. The English barrister Thomas Hare is generally credited with the concept of STV and proposed that electors should have the opportunity of discovering which candidate their vote had ultimately counted for, to improve their personal connection with voting. Andrew Inglish Clark was successful in persuading the Tasmanian House of Assembly to be the first parliament in the world elected by what become known as the Hare-Clark system, named after himself and Thomas Hare. The STV is a system of preferential voting designed to minimize wasted votes which provides proportional representation while ensuring that votes are explicitly expressed for individual candidates rather than party lists. In 2007 STV is used for parliamentary elections in the Republic of Ireland, North Irish Assembly and Malta. It is also used for the Australian Senate in the form of a group voting ticket, as well as certain regional and local elections in Australia, local government elections in Australia, local government elections in New Zealand. It is held up by its supports as being the best and fairest electoral system in the world, but political parties dislike it and resist to adopting it because it requires candidates to compete publicly with one another. If it is popularized in the society then there is probability of political parties to be completely abolished.

Setting the Quota

In an STV election, a candidate requires a certain minimum number of votes '*the quota*' to be elected. A number of different quotas can be used; the most common is the *Droop quota*, given by the formula (Droop 1881):

$$\left(\frac{V}{S+1}\right)+1$$

where, V = the total number of valid votes cast.

S = the number of seats to be filled.

STV is a step procedure, in each step voters cast votes for their most preferred candidate. It proceeds according to the following steps:

i) Any candidate who touched or exceeded the required quota is declared elected.
ii) If not enough candidates have been elected, the count continues.
iii) If a candidate casts more vote than the quota, then their surplus is transferred to other candidates according to the next preference on each voter's ballot.
iv) If none meets the quota, the candidate with the fewest votes is eliminated and their votes are transferred. This process continues until the last candidates survive which is the winner in the election.

Again in quota system, voting procedure is stopped when the numbers of remaining candidates instead of counting votes until all candidates have reached a quota. In STV, candidates who receive excess votes and candidates who are excluded have their votes transferred to other candidates, it is said to be minimize wasted votes. Let us introduce a simple example:

Let 40 guests (voters) are invited in party and 5 food stuffs (candidates), 3 of which will be selected. The candidates are: Beef (x), Mutton (y), Chicken (z), Fish (z) and Vegetable (v). Each of the 40 guests is given 2 ballots. In quota system, the number of votes to be elected is $\left(\frac{40}{3+1}\right)+1=11$.

In the following table 5 we have shown only first and second preferences of their food stuffs. We have excluded higher order preferences because in our election these are no needed.

Table 5: First and second Preferences of the Voters.

No. of guests	10	5	8	3	2	7
First preference	x	x	y	z	u	v
Second preference	y	v			v	

When ballots are counted the election proceeds as follows:

Step 1

Candidates	x	y	z	u	v
Collected votes	15	8	3	2	7

In step 1 x has 4 more votes than the quota, so x is declared elected. Candidate x's surplus votes transfer equally to y and v according to voters of x second choice preferences then the step 2 being as follows:

Step 2

Candidates	x	y	z	u	v
Collected votes	11	10	3	2	9

In step 2 even with the transfer of this surplus no candidate has reached the quota. Then u with the lowest votes is eliminated. The votes of u are transfer to his second preference v to reach the quota then the step 3 being as follows:

Step 3

Candidates	x	y	z	v
Collected votes	11	10	3	11

In step 3, v is elected but has no surplus to transfer. Neither of the remaining candidates meets the quota. Therefore z has lowest votes is eliminated. Candidate y is the only remaining candidate and so wins the final seat.

Final result: The winners are x, v, y i.e., Beef, Vegetable and Mutton.

The Process of Single Winner in STV and Manipulation of Voting

It is also a step procedure, in which, at each step voters cast votes for their most preferred candidate. In step 1, each voter casts vote for his most favorite candidate. Then the candidate with the fewest votes is eliminated. In step 2, each elector casts a single vote for his most favorite among the remaining candidates. As before the candidate with the fewest votes is eliminated. The process continues until one candidate remains. The last candidate is considered as the winner. Consider the preference profile as follows:

Step 1

Type 1: $xPzPyPu$ by 10 voters,
Type 2: $yPzPxPu$ by 7 voters,
Type 3: $uPzPyPx$ by 5 voters,
Type 4: $zPyPuPx$ by 3 voters,
Type 5: $uPzPxPy$ by 4 voters.

In step 1, z with 3 votes is eliminated, and then the preference profile will be as follows:

Step 2

Type 1: *xPyPu* by 10 voters,
Type 2: *yPxPu* by 7 voters,
Type 3: *uPyPx* by 5 voters,
Type 4: *yPuPx* by 3 voters,
Type 5: *uPxPy* by 4 voters.

In step 2, *u* with 9 votes is eliminated, and then the preference profile will be as follows:

Step 3

Type 1: *xPy* by 10 voters,
Type 2: *yPx* by 7 voters,
Type 3: *yPx* by 5 voters,
Type 4: *yPx* by 3 voters,
Type 5: *xPy* by 4 voters.

In step 3, *x* with 14 votes is eliminated. Finally *y* will be the remaining person and will win in the election. Type 3 voters prefer *z* to *y*. They anticipate that *y* will win then they could manipulate the preferences as follows:

Type 3: *zPxPuPy* by 5 voters.

Then as before in step 1 *u* with 4 votes would be eliminated and then the preference profile would be as follows:

Step 2a

Type 1: *xPzPy* by 10 voters,
Type 2: *yPzPx* by 7 voters,
Type 3: *zPyPx* by 5 voters,
Type 4: *zPyPx* by 3 voters,
Type 5: *zPxPy* by 4 voters.

In step 2a, *y* with 7 votes would be eliminated and then the preference profile would be as follows:

Step 3a

Type 1: *xPz* by 10 voters,
Type 2: *zPx* by 7 voters,
Type 3: *zPx* by 5 voters,
Type 4: *zPx* by 3 voters,
Type 5: *zPx* by 4 voters.

In step 3a, *x* with 10 votes would be eliminated. Finally *z* would be remaining person and would win in the election. Therefore, we have seen that STV is manipulable.

Tie-Breaking in STV

Although we mentioned above that in STV ties disallowed, sometimes ties can occur for several different reasons and the ties need to be broken (Newland and Britton 1997; O'Neill 2004). The ties can be broken simply by lottery system such as tossing coin. But this system is not the best one and usually the following four rules are used in tie-breaking.

i) Forwards Tie-Breaking (*FTB*): Choose the candidate who has the most (least) votes at the first stage where they had unequal votes.
ii) Backwards Tie-Breaking (*BTB*): Choose the candidate who has the most (least) votes at the previous stage or at the latest point in the count where they had unequal votes.
iii) Borda Tie-Breaking: Choose the candidate with the highest (least) Borda score.

iv) Cooms Tie-Breaking: Choose the candidate with the fewest (most) last place votes.

Sometimes after breaking tie by any of the above mentioned rules the candidate would still face tied. In this case it is useful to distinguish between *weak ties* and *strong ties*. A weak tie occurs when candidates have the same number of votes at a given stage. A strong tie occurs when candidates are still tied after applying a tie-breaking rule (any one rule from (i)–(iv) mentioned above). A strong tie would be broken by lottery. Here we will use ERS97 rules of tie-breaking (Newland and Britton 1997; O' Neill 2004). The difference between *FTB* and *BTB* is given in table 6 which is from Newland & Britton (1997) without any change.

Table 6: Example Tally with ERS97 Rules where 60 Voters are Electing 2 Candidates from 6.

		Surplus of x	Eliminate w	Eliminate v	Eliminate z
Stage	1	2	3	4	5
x	23	20.00	20.00	20.00	20.00
y	13	13.00	13.00	15.00	15.00
z	6	6.50	10.00	12.00	2.00
u	7	7.50	9.50	12.00	18.00
v	7	7.50	7.50	-	-
w	4	5.50	-	-	-
Non-Transferable	0	0.00	0.00	1.00	5.00

Here we have to eliminate one candidate at stage 4 and there is a tie between candidates z and u. Thus, tie-breaking needs to be used to determine which candidate is to be eliminated. We use the *FTB* following ERS97 rules. In this case we first look to the counts at stage 1. From table 6 we see that u has one more vote than z at stage 1. So that candidate z is eliminated. If z and u had been tied at stage 1, then we would have to be looked to subsequent stages. If z and u would have been tied in all stages, then we would have been a strong tie which would have been broken by lottery.

But in *BTB* we have to look at the previous stage to break ties and if necessary to the preceding stages. In table 6 we see in preceding stage 3 that z is ahead to u, so that u would be eliminated.

One problem arises with *FTB* where the elimination order is: 4, 1, 2, 3 which is not sequential and is undesirable. If we make a meaningful sequence starting from 4 then the order is: 4, 3, 2, 1 which is *BTB*. Again *FTB* does not use the most relevant information than *BTB* to break the tie. Hence *BTB* is better than *FTB* in tie-breaking.

Probability of Eliminating of Winning Candidates in Tie-Breaking STV

In tie-breaking STV, ERS97 rule sometimes eliminates winning candidates without manipulation of voting which is undesirable. Suppose there are 31 voters and 6 candidates among which one will be elected. Each of the voters is given 4 ballots (O' Neill 2004). In step 1 the preferences would be as follows:

Step 1

No. of votes	4	5	5	2	4	11
1st preference	x	y	z	u	v	w
2nd preference	y	z	y	x	x	
3rd Preference	z			y	y	
4th preference				z	z	

In step 2, u with fewest votes would be eliminated and u's votes would be transferred to his second preferred candidate x. The preference profile of step 2 would be as follows:

Step 2

Candidates	*x*	*y*	*z*	*v*	*w*
Collected votes	6	5	5	4	11

In step 3, *v* with fewest votes is eliminated and *v*'s votes would be transferred to his second preferred candidate *x*. The preference profile of step 3 would be as follows:

Step 3

Candidates	*x*	*y*	*z*	*w*
Collected votes	10	5	5	11

In step 3, a tie arises between *y* and *z* with the fewest 5 votes. Then both would be eliminated by ERS97. However, instead tie would be broken by *FTB* or *BTB* or by lottery. Suppose *z* was eliminated by lottery and *z*'s votes would be transferred to his second preferred candidate *y*. Then *y* would be tied with *x*. The preference profile of step 4 would be as follows:

Step 4

Candidates	*x*	*y*	*w*
Collected votes	10	10	11

In that situation tie would be broken by *FTB* rule. In step 1 candidate *x* has fewer votes than *y*, so that *x* would be eliminated and *y* as *x*'s second preferred candidate received all of *x*'s votes and beat *w* with 20 to 11 votes in final step. Therefore, winning candidate in ERS97 rule was eliminated which is unacceptable situation in STV election.

So that there is a flaw in STV, ERS97 *FTB* rule. This flaw could be removed in two ways: (i) by changing the rules of STV, ERS97, (ii) by using *BTB* rule which is better as we have seen before. Hence with *FTB* a winning candidate could be improperly eliminated however, with *BTB* both of these last-place candidates can not win and can thus be properly eliminated.

Plurality Voting

The plurality voting system is a single-winner voting system often used to elect executive officer or to elect members of a legislative assembly which is based on single member constituencies. This is the simplest of all voting systems for voters and vote counting officials. Generally plurality ballots can be categorized into two forms. The simplest form is a blank ballot where the name of a candidate is written in by hand. A most structured ballot will list all the candidates and allow a mark to be made for a single candidate; however a structured ballot can also include space for a write- in candidate as well. Sometimes at the end of each candidate a known symbol is enclosed and each voter votes for his favorite by sealing on the symbol of his favorite. Under this system the winner of the election acts as representative of the whole region of his constituting area. In this method there are many candidates or alternatives and there are many voters or individuals where each voter casts only one vote for one candidate. The candidate who collects highest total wins in the election; there is no requirement that the winner gain an *absolute majority* of votes. This type of voting system is prevailed in most of the countries in the world such as the USA, the UK, Canada, India, Bangladesh, Pakistan, and so on. This type of voting sometimes is called *simple plurality*, *first past the post* (FPP) or *winner-takes-all*. Plurality voting is used for local and/or national elections in about 43 of the 191 countries of the UN. The term FPP was coined as an analogy to horse racing, where the winner of the race is the first to pass a particular point as the track, after which all other runners completely lose. In some countries such as France a different plurality system is used, where there are two rounds; the *two-ballots* or *run off election* plurality system. If any candidate in the first round gains majority of the votes then there is no need of second round, otherwise the two highest-voted candidates of the first round compete in a two-candidate second round. Most of the voters cast vote for one among the favorite candidates who has a strong chance of winning.

However, some voters will want to manipulate the result by misrepresenting their votes. Consider three preference relations be as follows:

Type 1: $xPyPz$ by 15 voters,
Type 2: $yPzPx$ by 12 voters,
Type 3: $zPyPx$ by 4 voters.

In a sincere election, type 3 voters cast their votes for z, but x wins the plurality. If type 3 voters anticipate this result, they of course vote for y, as y is their second choice candidate. In this case the type 3 preference would be $yPzPx$, and then y would get 12+4 =16 votes where as x would get 15 votes and in final result y would win. Observed that here 4 voters of type 3 are manipulating the election. Manipulation by a group rather than a single individual is called *coalitional* manipulation. Unfortunately sometimes plurality voting creates tie votes. In the above example if there were 15 type 1 voters and 15 typy 2 voters, only 1 type 3 voter and one of the voters in type 1 is the leader. In a sincere election x would cast 15 votes and y would cast 15 votes z would cast 1 vote, and the leader would break the $x-y$ tie in favor of x, since, x is his most favorite candidate. Anticipating the result that x would win which is less preferred by type 3 voter, he can manipulate the election by the preference profile as $yPzPx$. Then x would cast 15 votes and y would cast 16 votes and final result that y would win. Therefore, the plurality voting is manipulable.

Exhausting Voting

This method is rarely used in modern world, which works in steps. In step 1, each voter casts a vote for his least preferred candidate. The candidate with the largest number of votes is eliminated from the list. The process continues until the last candidate survives which is the winner in the election (ties disallow). Suppose the preference relations be as follows:

Step 1

Type 1: $xPyPzPuPv$ by 5 voters,
Type 2: $yPzPxPuPv$ by 12 voters,
Type 3: $zPxPyPvPu$ by 10 voters,
Type 4: $uPvPxPyPz$ by 11 voters,
Type 5: $vPyPxPzPu$ by 8 voters,
Type 6: $zPxPuPvPy$ by 9 voters.

In step 1, u collects highest score of 10+8 =18 votes, so that u is eliminated. Now the preference relations for step 2 be as follows:

Step 2

Type 1: $xPyPzPv$ by 5 voters,
Type 2: $yPzPxPv$ by 12 voters,
Type 3: $zPxPyPv$ by 10 voters,
Type 4: $vPxPyPz$ by 11 voters,
Type 5: $vPyPxPz$ by 8 voters,
Type 6: $zPxPvPy$ by 9 voters.

In step 2, v collects highest score of 5+12+10 =27 votes, so that v is eliminated. Now the preference relations for step 3 be as follows:

Step 3

Type 1: $xPyPz$ by 5 voters,
Type 2: $yPzPx$ by 12 voters,
Type 3: $zPxPy$ by 10 voters,
Type 4: $xPyPz$ by 11 voters,

Type 5: $yPxPz$ by 8 voters,
Type 6: $zPxPy$ by 9 voters.

In step 3, z collects highest score of 5+11+8 =24 votes, so that z is eliminated. Finally the preference relation for step 4 be as follows:

Step 4

Type 1: xPy by 5 voters,
Type 2: yPx by 12 voters,
Type 3: xPy by 10 voters,
Type 4: xPy by 11 voters,
Type 5: yPx by 8 voters,
Type 6: xPy by 9 voters.

In step 4, y collects highest score of 5+10+11+9 =35 votes, so that y is eliminated. Finally x is the remaining person and wins in the election. If the type 4 voters anticipate that x will win who is not their most liking candidate and there is no chance of winning their most favorite candidate u. So they would want to manipulate the result and they would send a message that they would cast votes for z as their most preferred candidate. Then if z who had a little chance of winning would promise with them that type 4 voters would find favor of z then they would manipulate the preferences as follows:

Type 4: $zPvPuPyPx$ by 11 voters.

In that case no voters of other types would imagine such a manipulation. Similarly as before, in step 1, u would eliminate, in step 2, v would eliminate, in step 3, x would eliminate, in step 4, y would eliminate and finally z would win. So that type 4 voters are better off than when they are honest. Therefore exhausting voting is manipulable.

Approval Voting

Approval voting is a single winner voting system used for elections. In this method each voter may vote for as many of the candidates as he wishes. Let there is a set of n candidates $\{x, y, z, ...\}$. One may cast 0, 1, 2, …, or even m votes, where $m \le n$, by assigning a single vote to each candidate he approves and none to each candidate he disapproves. The candidate with the highest total wins (Brams and Fishburn 1978). The system was described in 1976 by Guy Oltewell and also by Rober J. Weber who coined the term “approval voting”. Approval voting has been adopted by the Mathematical Association of America (1986), The Institute of Management Sciences (1987), The American Statistical Association (1987), and Institute of Electrical and Electronics Engineers ((IEEE) (1987)). IEEE rescinded the approval voting in 2002 because the director of IEEE Deniel J. Senese states that “few of our members were using it and it was felt that it was no longer needed”. From 13th to 18th centuries, the Republic of Venice elected the Doge of Venice using a multi-stage process that featured random selection and voting which allowed approval of multiple candidates and required a super majority. In 19th century approval voting was used in England. The selection of the Secretary-General of the UN has involved rounds of approval polling to help discover and build a consensus before a formal vote is held in the Security Council. Approval voting usually elects Condorcet winners in practice (Brams and Fishburn 1978).

Sincere Approval Voting

An approval voting is sincere if the outcome is the same as the true preference of the voters (Brams and Fishburn 1978). Let us consider there are four candidates x, y, z and u, and a voter’s preference profile being as follows:

$$xPyPzPu.$$

We can write his possible sincere approval votes as follows:

i) vote for x, y, z and u,
ii) vote for x, y and z,
iii) vote for x and y,
iv) vote for x,
v) vote for no candidates.

If a voter be indifferent between y and z but still x is his most preferred candidate, then also (i) to (v) conditions are sincere (in this paper indifferent is not considered). Now we can also include a new combination as a sincere vote which is :

vi) vote for x and z.

Let us introduce another example as follows:

There are three electors, where elector 1 is a leader; if there are ties for first place, he breaks them and there are three alternatives x, y, z. The preference profile is as follows:

1 (leader)	2	3
x	y	z
y	z	x
z	x	y.

Each elector may cast 0, 1, 2 or 3 votes. It is foolish to cast votes equally for all or for none. Elector 1(the leader) can vote as follows:

i) vote for x, y and z,
ii) vote for x and y,
iii) vote for y,

and so on.

Here (i) and (ii) are sincere but (iii) is insincere.

Similarly voter 2's sincere strategies are as follows:

i) vote for y,
ii) vote for y and z.

Similarly one can calculate voter 3's sincere strategies.

The following discussion results that approval voting can be manipulated and can be non-manipulated.

Manipulation of the Approval Voting

Let us consider the above example. Here every voter casts 1 vote for his favorite. So that the results are: 1 for x, 1 for y, 1 for z. Person 1 (the leader) breaks the tie in favor of x, so that x wins. Voter 2 anticipates that by the leader's favorite x will win who is his less favorite, so he could vote falsely as: 1 vote for his second favorite z instead, but none for y or x, then the result would be 2 votes for z but 1 vote for x and none for y and finally z would be winning. Therefore, the approval voting is manipulable. On the other hand person 1 votes 1 for each in a sincere way but both 2 and 3 also vote sincerely in the following ways:

i) person 2 votes 1 for y, 1 for z but none for x
ii) person 3 votes 1 for z, 1 for x but none for y.

In this case the result would be 2 votes for x, 2 votes for y but 3 for z and z would be winning. For this case the voting is manipulated in a sincere way also.

Non- manipulation of the Approval Voting

Now consider that all the voters except any voter n has declared their true strategies, so that voter n can not be insincere and the result will be their best output. In this case approval voting is non-manipulated.

Arrow's Theorem

For simplicity let us consider there are two individuals in the society and three social alternatives *x*, *y*, *z*. For the preference orderings for individual 1 or 2 there are exactly 6×6=36 different constellations of individual preferences possible in the society (figure 1) where alternatives are ordered from top to bottom. For detail about Arrow's theorem see Arrow (1951, 1963); Feldman 1974; Sen 1970; Barbera 1980; Islam (1997, 2008); Bossert and Weymark (2003), Breton and Weymark 2006; Feldman and Serrano (2006, 2007, 2008); Suzumura 2007; Islam, *et al.* 2009: (Spring and Fall).

Figure 1: The Preference Orderings for Individual 1 or 2 there are Exactly 36 Different Constellations of Individual Preferences Possible in the Society.

	1	2	1	2	1	2	1	2	1	2	1	2
1st	*x*	*x*	*x*	*x*	*x*	*y*	*x*	*y*	*x*	*z*	*x*	*z*
2nd	*y*	*y*	*y*	*z*	*y*	*z*	*y*	*x*	*y*	*x*	*y*	*y*
3rd	*z*	*z*	*z*	*y*	*z*	*x*	*z*	*z*	*z*	*y*	*z*	*x*
1st	*x*	*x*	*x*	*x*	*x*	*y*	*x*	*y*	*x*	*z*	*x*	*z*
2nd	*z*	*y*	*z*	*z*	*z*	*z*	*z*	*x*	*z*	*x*	*z*	*y*
3rd	*y*	*z*	*y*	*y*	*y*	*x*	*y*	*z*	*y*	*y*	*y*	*x*
1st	*y*	*x*	*y*	*x*	*y*	*y*	*y*	*y*	*y*	*z*	*y*	*z*
2nd	*x*	*y*	*x*	*z*	*x*	*z*	*x*	*x*	*x*	*x*	*x*	*y*
3rd	*z*	*z*	*z*	*y*	*z*	*x*	*z*	*z*	*z*	*y*	*z*	*x*
1st	*y*	*x*	*y*	*x*	*y*	*y*	*y*	*y*	*y*	*z*	*y*	*z*
2nd	*z*	*y*	*z*	*z*	*z*	*z*	*z*	*x*	*z*	*x*	*z*	*y*
3rd	*x*	*z*	*x*	*y*	*x*	*x*	*x*	*z*	*x*	*y*	*x*	*x*
1st	*z*	*x*	*z*	*x*	*z*	*y*	*z*	*y*	*z*	*z*	*z*	*z*
2nd	*x*	*y*	*x*	*z*	*x*	*z*	*x*	*x*	*x*	*x*	*x*	*y*
3rd	*y*	*z*	*y*	*y*	*y*	*x*	*y*	*z*	*y*	*y*	*y*	*x*
1st	*z*	*x*	*z*	*x*	*z*	*y*	*z*	*y*	*z*	*z*	*z*	*z*
2nd	*y*	*y*	*y*	*z*	*y*	*z*	*y*	*x*	*y*	*x*	*y*	*y*
3rd	*x*	*z*	*x*	*y*	*x*	*x*	*x*	*z*	*x*	*y*	*x*	*x*

Arrow's theorem implies that there must be a dictator. Here we suppose individual 1 is a dictator so that from figure 1 individual 1's social decision function (SDF) is as follows:

Figure 2: Person 1 is a Dictator.

x	*x*	*x*	*x*	*x*	*x*
x	*x*	*x*	*x*	*x*	*x*
y	*y*	*y*	*y*	*y*	*y*
y	*y*	*y*	*y*	*y*	*y*
z	*z*	*z*	*z*	*z*	*z*
z	*z*	*z*	*z*	*z*	*z*

Here individual 2 cannot manipulate SDF of figure 2, since his preference never affects the outcome. Individual 1 cannot manipulate the result, since he always gets his first choice. Therefore Arrow's theorem is non-manipulable in any situation.

Median Voter Model

The method of majority voting prevails before the dawn of recorded history but the concept of median voter theorem came from the Black (1948). Greek Philosopher Aristotle in 330 B.C. wrote *Analysis of Political Decision Making*. Condorcet gave the idea of *pivotal voter*. But neither Aristotle nor Condorcet gave any information about the median voter and we had to wait Black's work on majority voting which was given in 1948.

Basic Concept

Consider three individuals *A*, *B*, *C* visited the U.S.A. from Bangladesh. They had to stay in a residential hotel, *A* chose a hotel which costs \$1000, *B* chose a hotel which costs \$1500 and *C* chose a luxurious hotel which costs \$3000 per night. We can say *B* as a median voter, since exactly same number of individuals prefer a more expensive hotel than *B* and as prefer a less expensive hotel than *B*, of course here one each (Congleton 2004). The weak form of the median voter theorem says the median voter casts his vote for the real situation and wins in the election. We can explain the weak form of the median voter as follows: Let us consider there are two candidates in the election. If voters cast their votes to the candidate who is closed to the median voter always wins the election. As a result the winning candidate always receives the vote of the median voter i.e., the weak form of the median voter theorem is satisfied. The strong form of the median voter theorem says that the median voter always gets his most preferred policy. We can explain strong form of median voter theorem as follows: If both candidates compete to find the favor of the median voter, the positions of both candidates converge towards the policy positions that maximize the median voter's welfare. In this case both candidates get equal number of votes. It is no matter which candidate wins in the election in this limiting case but the median voter gains what the candidates promise in election; i.e., the strong form of the median voter theorem will hold for national public choices. Although the median voter models implies that the median voter gets what he wants but in some cases gains depend on the usual Paretian sense of welfare economics. In electoral contests between two candidates if a median voter exists government policy will maximize the welfare of the median voter in equilibrium. As a result median voter plays a pioneer role in modern democracy.

Mathematical Discussion of Median Voter Model

We have two basic versions of the median voter theorem: (i) Single-peaked preference (Black 1958) and (ii) Single-crossing property (Gans and Smart 1994). Now we briefly discuss following Myerson (1996); Austen-Smith and Banks (1999); Saporiti and Thomé (2006); Saporiti (2008); Penn, *et al.* (2008) the two versions as follows:

*Single-crossing property***:** In single-crossing, preferences are assumed that the set of individuals or voters $N = \{1, 2, ..., n\}$ are finite and $\#(N) = n > 2$ is odd. The set of alternatives or social options is denoted by $Y = \{x, y, z, ...\}$, which is also finite subset of the non-negative real line R_+. Let $P(Y)^N$ be the set of alternatives which is complete, transitive and anti-symmetric binary preferences on *Y*. Let $P \in P(Y)^N$ be the preference ordering over the elements of *Y*. For any pair $x, y \in Y$, *xPy* denotes the strict preference for *x* against *y*. Here *Y* is complete, transitive and anti-symmetric; i.e., for $x, y \in Y$ completeness implies *xPy* or *yPx* such that $x \neq y$, transitivity implies if *xPy*, *yPz* then *xPz* and anti-symmetry implies *xPy* or *yPx* such that $x = y$. For $x, y \in Y$, we may write $x < y$ to mean that x is left to y in the spatial voting model. Let the voters' preferences are transitive ordered in some political spectrum say from *leftist* to *rightist*. We mean $i < j$ that voter *i* is to the left of voter j in this political spectrum. For any two voters i and *j* such that $i < j$, for any two policy alternatives *x* and *y* such that $x < y$,

$$\text{if } u_i(x) < u_i(y) \text{ then } u_j(x) < u_j(y)$$

but if $u_j(x) < u_j(y)$ then $u_i(x) < u_i(y)$.

This assumption is called the s*ingle-crossing* (SC) property. We can also define an easier way SC as follows: Let $>$ is linear order of Y and $\succ$ is a linear order of SC, and $SC \subset P(Y)^N$. Now $\forall\, x, y \in Y$ and $\forall P, P' \in SC$ the single-crossing property indicates,

$$[y > x, P' \succ P \;\&\; yPx] \Rightarrow yP'x \qquad \& \qquad [y > x, P' \succ P \;\&\; xP'y] \Rightarrow yPx .$$

SC is common and important in political economy. Austen-Smith and Banks (1999: 107) gave an example of SC as follows: "For example, in redistributive politics policy makers are concerned with reallocating resources from rich to poor people, subject to the constraint (typically) that such redistributions do not reverse the rank-order of individuals' wealth. So, while there does not exist an obvious ordering of the alternative distributions of wealth, there does exist a natural ordering of individuals and their preferences in terms of individual wealth".

Saporiti (2008) gave examples of SC as follows: "Suppose a moderately rich individual prefers a high tax rate to another relatively smaller tax rate, so that he reveals a preference for a greater redistribution of income. Then, the single-crossing property requires that a relatively poorer individual, who receives a higher benefit from redistribution, also prefers the higher tax rate. Sometimes this is interpreted in the literature by saying that there is a complementary between income and taxation, in the sense that lower incomes increase the incremental benefit of greater tax rates. For another example, consider a strong army which prefers a large territorial concession and a small probability of war to a small concession and a high probability of war. Then, under single-crossing, with a lower expected payoff from war, should also prefer the large concession".

If the number of voters is odd and their order is complete and transitive, then there is some median voter m such that

$$\#\{\, i \in N : i < m \} = \#\{\, j \in N : m < j \}.$$

For any pair of alternatives $x, y \in Y$ such that $x < y$, if the median voter m prefers x then all voters to the left of the median voter agree with him, but if the median voter prefers y then all the voters to the right of the median voter agree with him. In both cases majority grows where median voter supports. Hence, the alternative that is most preferred by the median voter must be a Condorcet winner.

*Single-peakedness***:** Single-peaked preferences have played an important role in the literature ever since they were used by Black (1948) to formulate a domain restriction that is sufficient for the exclusion of cycles according to the majority rule. A set of preference relations is single-peaked if there is linear order of the alternatives such that every preference relation has a unique most preferred alternative or ideal point, over this ordering, and the preference for any other alternatives monotonically decreases by moving away from the ideal point. Let for each voter *i*, it is assumed that there is some ideal point $\theta \in Y$ such that for every $x, y \in Y$ if $\theta_i \le x < y$ or $y < x \le \theta_i$ then, $u_i(x) > u_i(y)$. We observed that on either side of θ_i, voter *i* always prefers alternatives that are closer to θ_i. This is called the *single-peakedness* assumption. Now assume that the number of voters is odd, the median voter's ideal point is the alternative θ^* such that,

$$\frac{\#N}{2} \ge \#\left\{\, i : \theta_i < \theta^* \right\} \text{ and } \frac{\#N}{2} \ge \#\left\{\, i : \theta^* < \theta_i \right\}.$$

The voters who have ideal points at θ^* and to its left form a majority that prefers θ^* over any alternative to the right of θ^*, while the voters who have ideal points at θ^* and to its right form a majority that prefers θ^* over any alternative to the left of θ^*. So the median voter's ideal point θ^* is a Condorcet winner in *Y*.

In the light of above discussion we see that single-crossing and single-peakedness are different assumptions. Both assumptions give us a result which is "the median voter's ideal point is a Condorcet winner". On the other hand, both assumptions give different property; i.e., single-crossing property implies *the ideal point of the median voter* and the single-peakedness property implies *the median of the voters' ideal points*. Single-crossing assumption follows transitive ordering but does not follow the single-peakedness assumption.

Now we set an example to show the difference between single-crossing and single-peaked (Saporiti 2008). Consider the set of preference relations as follows:

xP_1yP_1z for individual $\underline{1}$,

xP_2zP_2y for individual $\underline{2}$,

zP_3yP_3x for individual $\underline{3}$.

We observe that this set has *SC* property on $Y=\{x, y, z\}$ with respect to $z>y>x$ and $P_3 \succ P_2 \succ P_1$ on the other hand, for every ordering of the alternatives, $\{P_1, P_2, P_3\}$ violates the single-peaked property because, every alternative is ranked less preferred in one preference relation.

Both single-crossing and single-peakedness are non-manipulable (Saporiti 2008; Penn *at el.* 2008).

Limitation of the Median Voter Model

Although median voter model plays a pioneer role in election but it does not exist always. For example we have discussed voting paradox in section-2 where we have found no median voter. The absence of median voter equilibrium may also arise in models where candidates can manipulate information and voter turnout.

Randomized Voting

This voting method sometimes is called lottery type social decision mechanism. Let Y be the set of alternatives and p_i be the probability of winning the alternative *i*. For convenient we assume that there are three alternatives $x, y, z \in Y$. Let us consider the preference profile as follows:

Type 1: *xPyPz* by 4 voters,
Type 2: *yPzPx* by 3 voters,
Type 3: *zPxPy* by 2 voters.

We see that there are 9 voters, so that $p_x=\frac{4}{9}$, $p_y=\frac{3}{9}$ and $p_z=\frac{2}{9}$. Let utility scale *U* is said to fit the preference *P* if more highly ranked alternatives give greater utility; i.e., $\forall\, x, y \in Y$, $U(x)>U(y)$ if and only if *xPy*. For any finite set *A*, $\Delta(A)$ denotes the space of probability measures on *A*. Each voter wants to maximize an expected utility function,

$$EU=\sum_{x\in Y} U(x)p(x), \text{ whenever } p\in\Delta(Y).$$

Here EU is increased most whenever the voter casts his vote for the outcome i for which $U(i)$ is largest, which is a sincere election. Hence randomized elections are non-manipulable. If there are some tied elections then these are commonly solved by tossing coins which are non-manipulable randomized elections whose domain is the set of preference profiles and whose range is the set of probability distribution over the set of alternatives. An election which Von Neumann-Morgenstern utility is consistent with the actual preference and Pareto dominated alternatives never receive any probability, then the scheme must be a random dictatorship and the decision scheme is strategy-proof.

Muller-Satterthwaite Theorem

In this section we discuss about Muller-Satterthwaite Theorem (Muller and Satterthwaite 1977) which plays an important role in Economics, Political Science and Social Science (Satterthwaite 1975; Myerson 1996; Reny 2000). We define utility function, social choice function, monotonic function and non-dictatorship as follows (Arrow 1951, 1963; Sen 1970):

Utility Function

We now define the utility function as $u(x)=u(x_1, x_2, ..., x_n)$. In preference relation we can write $u(x)>u(y) \Leftrightarrow xPy$.

Let us consider a fixed vector x_0, and consider the set of all the vectors x which are preferred to x_0. If we denote this set by $V(x_0)$, we can write $V(x_0)=\{x : xPx_0\}$.
For the utility function it can be written as, $V(x_0)=\{x : u(x)>u(x_0)\}$ where $V(x_0)$ is a convex set.

Social Choice Function and Monotonic Function

Let $N=\{1, 2, ..., n\}$ be the set of individual voters, and let $Y=\{x, y, z, ...\}$ be the complete and transitive finite set of alternatives. Let $L(Y)$ denote the set of strict transitive ordering of the alternatives in Y and $L(Y)^N$ denote the set of profiles of such preference orderings, one for each individual voter. A function $f : L(Y)^N \to Y$ will be called a social choice function. A social choice function f is monotonic if whenever $f(L_1, ..., L_N)=x$ for any alternative x and for every individual i, and every alternative y the ranking L_i' ranks x above y if L_i does, then $f(L_1', ..., L_N')=x$.

Non-Dictatorship

It is required that the SWF should not be dictatorial. That is, there should be no individual such that whenever he prefers x to y, society must prefers x to y, irrespective of the preferences else. This is called the condition of non-dictatorship. Mathematically, there is no individual i such that for every element in the domain of rule f, $\forall\, x, y \in X$ such that $xP_i y \Rightarrow xPy$. Anonymous voting systems with at least two voters satisfy the non-dictatorship property. The dictatorship is undesirable in the society. First, it is undesirable because one's worst enemy might be dictator. Second, it is not a collective choice rule. So that dictatorship may cause the violation of human rights.

Pre-requisites

Let $N=\{1, 2, ..., n\}$ be the set of individual voters, and let $Y=\{x, y, z, ...\}$ be the complete and transitive finite set of alternatives. Let $L(Y)$ denotes the set of strict transitive ordering of the alternatives in Y and $L(Y)^N$ denotes the set of profiles of such preference orderings, one for each individual voter. The utility function is $u = (u_i)_{i \in N}$, where each u_i is in $L(Y)$ and $u_i(x) > u_i(y)$ means that individual *i* prefers alternative *x* over alternative *y*. The assumption of strict preferences implies that either $u_i(x) > u_i(y)$ or $u_i(y) > u_i(x)$ must hold if $x \neq y$. For social choice function $f : L(Y)^N \to Y$ we have $f\left(L(Y)^N\right) = \left\{f(u),\ u \in L(Y)^N\right\}$. So that $\# f\left(L(Y)^N\right)$ denotes the number of elements of alternatives that would be chosen by *f* under at least one preference profile.

Muller-Satterthwaite (1977) Theorem

If $f : L(Y)^N \to Y$ is a monotonic social choice function and $\# f\left(L(Y)^N\right) > 2$, then there must exist some dictator *j* in N such that

$$f(u) = \arg \max_{x \in f\left(L(Y)^N\right)} u_j(x),\ \forall u \in L(Y)^N .$$

Discussion

Let us consider that *f* is a monotonic social choice function. Let *X* be the range of *f*; i.e., $X = f\left(L(Y)^N\right)$. Let $f(u) = x$, $x \neq y$ and $\{i : u_i(x) > u_i(y)\} \subseteq \{i : v_i(x) > v_i(y)\}$.

Now $\hat{u}$ be derived from *u* by moving *x* and *y* up to the top of every individual's preferences, keeping the order of preference between *x* and *y* unchanged. Also similarly is derived $\hat{v}$ from *v* in the same way. By monotocity we must have $x = f(\hat{u})$ and $y = f(\hat{v})$. Also monotocity implies that $f(\hat{u}) = f(\hat{v}) = x$, but $x \neq y$ so that $y \neq F(v)$ which is shown in profile 1.

Profile 1

L_1	…	L_{n-1}	L_n	L_{n+1}	…	L_N		Social choice, $f(u)$
x	…	x	x	x	…	x		
y	…	y	y	y	…	y	$\Rightarrow$	x
.		.	.	.		.		
.		.	.	.		.		

Again $f(v)$ can not be any alternative *y* that is Pareto-dominated, under the preference profile *v*, by any other alternative $x \in X$. Since if *u* be any preference profile such that $x = f(u)$, Pareto dominance gives $\{i : v_i(x) > v_i(y)\}$ is the set of all voters *N*. Now let us say that a set of voters *T* is decisive for an order pair of distinct alternatives $x, y \in X$ if and only if,

$$f(u) = x \text{ and } T = \{i : u_i(x) > u_i(y)\}.$$

That is *T* is decisive $\forall\, x, y \in X$ such that every individual in *T* prefers *x* over *y* and nobody choose *y* over *x*.

Now let *T* be a non-empty set of minimal size among all sets that are decisive for distinct pair of alternatives in *X*. Let us select an individual $j \in T$ and three alternatives $x, y, z \in X$ as before. The preference profile *u* be such that,

$$u_j(x) > u_j(y) > u_j(z),$$
$$u_i(z) > u_i(x) > u_i(y)\ \forall i \in T \setminus \{j\},$$
$$u_k(y) > u_k(z) > u_k(x)\ \forall k \in N \setminus T .$$

So that everyone prefers *x*, *y* and *z* over all other alternatives. By the above discussion decisiveness of *T* implies that $f(u) \neq y$. If $f(u)$ were *x* then $\{j\}$ would be decisive for (x, z), in this case *T* would not be minimal. If $f(u)$ were *z* then $T \setminus \{j\}$ would be decisive for (z, y), in this case *T* would not also be minimal. So there exists individual j such that $\{j\}$ is a decisive set for all pairs of alternatives. That is for any pair (x, y) of distinct alternatives in *X* there exists a preference profile *u* such that $f(u) = x$ and $\{j\} = \{i : u_i(x) > u_i(y)\}$. Also we have found $v_j(x) > v_j(y)$. Hence $f(v)$ can not be any alternative in *X* other than the one that is most preferred by individual *j*.

The theorem indicates that there is only one way to design a game that always has a unique Nash equilibrium is to give one individual all the power. Decision-making in the executive branch is often made by a single decision maker, who may be the president of that branch. But sometimes in our society a game has multiple equilibria and the decisions made by rational players may depend on culture and history of focal-point (Schelling 1960). So, sometimes we face social procedures where there are more than two possible outcomes at a time and which are not dictatorial and reflect voters collective expectations.

Gibbard-Satterthwaite Theorem

Gibbard-Satterthwaite theorem shows that, if the set of alternatives contains at least three possible outcomes and individual preferences are not restricted in any particular way, when every strategy-proof (will be discussed later) social choice rule is dictatorial (Gibbard 1973, 1978; Satterthwaite 1975; Reny 2000). That is, there is an individual whose preferences always dictate the final choice regardless of other individuals' preferences. Here we will discuss this theorem easier way as per as possible.

Pre-requisites

Let $f : L(Y)^N \to Y$ be a social choice function. $\mathbf{L}_{-i}(Y)^N$ be the set of $(n-1)$-tuples of preferences $(L_1, ...L_{i-1}, L_{i+1}, ..., L_n)$, thought of as configurations of preferences of the voters other than *i*. A social choice function $f : L(Y)^N \to Y$ is strategy-proof if $\forall\, i \in N$ and $\forall\,(L_i, \mathbf{L}_{-i}) \in L(Y)^N$, there is no $L_i' \in L(Y)$ such that $f(L_i', \mathbf{L}_{-i}) > f(L_i, \mathbf{L}_{-i})$. Since f is social choice function then $f(L_i', \mathbf{L}_{-i}) = x$ and strategy-proofness implies that $x = f(L(Y))$ is ranked $f(L_i', \mathbf{L}_{-i}) = y$ ascending to $L_i(Y)$. So that $f(L_i', \mathbf{L}_{-i}) = f(L(Y)) = x$. We have discussed in Muller-Satterthwaite theorem that $f(L(Y)) = x$ for every individual *i* every alternative *y*, the ordering $L_i'(Y)$ ranks *x* above *y* whenever $L_i(Y)$ does. If we move from $L = (L_1, ..., L_N)$ to $L' = (L_1', ..., L_N')$ by changing the ranking of each individual *i* from $L_i(Y)$ to $L_i'(Y)$ one at a time but social choice must remain unchanged so that $f(L_i'(Y)) = f(L_i(Y))$; i.e., *f* is monotonic function. Again *f* is onto, so that $f(L(Y)) = x$ for some $L(Y) \in f\left(L(Y)^N\right)$. By monotocity the social choice remains *x* whenever *x* is raised to the top of every individual's ranking. Since *x* is at the top of every individual's ranking the social choice is *x*, consequently *f* is Pareto efficiency. Hence the individual *i* is a dictator. Now we can state the Gibbard-Satterthwaite Theorem as follows.

Gibbard (1973)-Satterthwaite (1975) Theorem: If $\# f\left(L(Y)^N\right) > 2$ and $f : L(Y)^N \to Y$ is onto and strategy-proof, then *f* is dictatorial.

Concluding Remarks

This paper analyzes various types of manipulable and non-manipulable voting systems using some easier methods. We have shown that some methods have Condorcet winner, where there is no voting manipulation and the individuals sincerely declare their preferences. In this paper we have briefly introduced Arrow's theorem. Interested readers are requested to see Arrow (1951, 1963); Sen (1970); Breton and Weymark (2006); Islam, *et al*. (2009: Spring and Fall) for some details. Voting system is closely related with Political Economics and Social Science, and we have tried to show this relationship throughout the paper. We have used easier mathematical calculations and notations to discuss Muller-Satterthwaite theorem and Gibbard-Satterthwaite theorem following Myerson (1996), Reny (2000) and McLennan (2008). The paper is review of other's works but we have tried throughout the paper to discuss voting matters with simple mathematical calculations and introducing definitions where necessary. Voting system is a very complicated field but we have tried our best to make it easier.

References

Arrow, K.J. (1951), *Social Choice and Individual Values,* 1st ed., John Wiley & Sons*,* New York.

Arrow, K.J. (1963), *Social Choice and Individual Values,* 2nd ed., John Wiley & Sons, New York.

Austen-Smith, D. and Banks, J.S. (1999), *Positive Political Theory I*: *Collective Preference*, Ann Arbor, The University of Michigan Press.

Barbera, S. (1980), Pivotal Voters: A New Proof of Arrow's Theorem. *Economics Letters*, 6(1): 13-16.

Black, D. (1948), On the Rationale of Group Decision-making, *The Journal of Political Economy*, 56(1): 23-34.

Black, D. (1958), *The Theory of Committees and Elections*, Cambridge: Cambridge University Press.

Blackorby, C., Bossert, W. and Donaldson, D. (2002), Utilitarianism and the Theory of Justice, In: Arrow, K.J., Sen, A.K. and Suzumura, K. (Eds.), *Handbook of Social Choice and Welfare,* 2, Elsevier, Amsterdam.

Blackorby, C., Donaldson, D. and Weymark, J.A. (1990), A Welfarist Proof of Arrow's Theorem, *Researches économiques de Louvain*, 56: 259-286.

Borda, (1781), *Mémoire sur les élections au scrutin*, In: the Historie de l' Académie Royale des Sciences, Paris.

Bossert, W. and Weymark, J.A. (2004), Utility in Social Choice, In: Barbera, S., Hammond, P. and Seidl, C. (Eds.), *Handbook of Utility Theory*, 2: *Extensions*, Kluwer, Dordrecht.

Brams, S.J. and Fishburn, P.C. (1978), Approval Voting, *American Political Science Review,* 72 (3): 831-847.

Condorcet, M. (1785), *Essai sur l'application de l'analyse à la probabilité des décisions rendues à la pluralité des voix*, Paris.

Congleton, R.D. (2004), The median voter model, In: Rowley, C.K. and Schneider, F. (Eds.), *The Encyclopedia of Public Choice,* 2, Kluwer, Dordrecht: 382-387.

Droop, H.R. (1881), On Methods of Electing Representatives, *Journal of the Statistical Society of London*, 44 (2): 141- 202.

Dummett, M. (1998), The Borda Count and Agenda Manipulation, *Social Choice and Welfare*, 15: 289-296.

Feldman, M.A. (1974), A Very Unsubtle Version of Arrow's Impossibility Theorem, *Economic Inquiry*, 12 (4): 534-546.

Islam, J.N.; Mohajan, H.K. and Moolio, P. (2011), Method of Voting System and Manipulation of Voting, *International Journal of Management and Transformation:* 5(1): 10–34.

Feldman, M.A. (1979), Manipulating Voting Procedures, *Economic Inquiry,* 17 (3): 452-474.

Feldman, M.A. and Serrano, R. (2006), *Welfare Economics and Social Choice Theory*, 2nd ed., Springer, New York.

Feldman, M.A. and Serrano, R. (2007), *Arrow's Impossibility Theorem: Preference Diversity in a Single-Profile World,* Working Paper No. 2007-12: Department of Economics, Brown University.

Feldman, M.A. and Serrano, R. (2008), *Arrow's Impossibility Theorem: Two Simple Single-Profile Versions*, Working Paper, Department of Economics, Brown University.

Gans, J.S. and Smart, M. (1996), Majority Voting with Single-Crossing Preferences, *Journal of Public Economics,* Elsevier, 59 (2): 219-237.

Gibbard, A. (1973), Manipulation of Voting Schemes: A General Result, *Econometrica*, 41: 587-601.

Gibbard, A. (1978), Straightforwardness of Game Forms with Lotteries as Outcomes, *Econometrica*, 46(3): 595-614.

Islam, J.N. (1997), Aspects of Mathematical Economics and Social Choice Theory, In: Islam, J.N. (Ed.), *Proceedings of the Second Chittagong Conference on Mathematical Economics and its Relevance for Development,* University of Chittagong, Bangladesh.

Islam, J.N. (2008), An *Introduction to Mathematical Economics and Social Choice Theory,* Book to Appear.

Islam, J.N., Mohajan, H.K. and Moolio, P. (2009), Preference of Social Choice in Mathematical Economics, *Indus Journal of Management & Social Sciences*, 3(1): 17-38.

Islam, J.N., Mohajan, H.K. and Moolio, P. (2009), Political Economy and Social Welfare with Voting Procedure, *KASBIT Business Journal,* 2(1): 42-66 (spring and fall).

Le Breton, M. and Weymark, J.A. (2002), Arrovian Social Choice Theory on Economics Domains, In: Arrow, K.J., Sen, A.K. and Suzumura, K. (Eds.), *Handbook of Social Choice and Welfare,* 2, Elsevier, Amsterdam.

McLennan, A. (2008), Manipulation in Elections with Uncertain Preference, *Unpublished Manuscript.*

Miller, M.K. (2009), Social Choice Theory without Pareto: The Pivotal Voter Approach, *Mathematical Social Sciences,*58(2009): 251-255.

Myerson, R.B. (1985), An Introduction to Game Theory, Discussion Paper No. 623, *Center for Mathematical Studies in Economics and Management Science*, Northwestern University.

Myerson, R.B. (1993), Effectiveness of Electoral Systems for Reducing Government Corruption: A Game-Theoretic Analysis, *Games and Economic Behavior,* 5(1): 118-132.

Myerson, R.B. (1996), *Fundamentals of Social Choice Theory*, Discussion Paper No. 1162, *Centre for Mathematical Studies in Economics and Management Science*, Northwestern University.

Myerson, R.B. (2006), Federalism and Incentives for Success of Democracy, *Quarterly Journal of Political Science*, 1(1): 3-23.

Myerson, R.B. (2009), Constitutional Structures for a Strong Democracy: Considerations on the Government of Pakistan, *Presentation Paper at Lahore University of Management Sciences.*

Islam, J.N.; Mohajan, H.K. and Moolio, P. (2011), Method of Voting System and Manipulation of Voting, *International Journal of Management and Transformation:* 5(1): 10–34.

Muller, E. and Satterthwaite, M.A. (1977), The Equivalence of Strong Positive Association and Strategy-proofness, *Journal of Economic Theory*, 14(2): 412-418.

Newland, R.A. and Britton, F.S. (1997), *How to Conduct an Election by the Single Transferable Vote*: ERS, 3rd edition.

O'Neill, J.C. (2004), Tie-Breaking with the Single Transferable Vote, *Voting Matters*, 18: 14-17.

Penn, E.M., Gailmard, S. and Patty J.W. (2008), Strategic Manipulation in Representative Institution: *Conference Paper/Unpublished Manuscript*.

Reny, P.J. (2000), *Arrow's Theorem and Gibbard-Satterthwaite Theorem: A Unified Approach*, *Mimeo*, Department of Economics, University of Chicago.

Risse, M. (2005), Why the count de Borda cannot beat the Marquis de Condorcet, *Social Choice and Welfare*, 25(October): 95-113.

Robert, F. and Tsoukiàs, A. (2009), Voting Theory and Preference Modeling, *Mathematical Social Sciences*, 57(3): 289-291.

Saporiti, A. and Thomé, F. (2006), Single-Crossing, Strategic Voting and the Median Choice Rule, *Social Choice and Welfare*, 26(2): 363-383.

Saporiti, A. (2008), Strategy-Proof and Single-Crossing, *Unpublished Manuscript*, University of Manchester.

Sato, S. (2009), Strategy-Proof Social Choice with Exogenous Indifference Classes, *Mathematical Social Sciences*, 57(1): 48-57.

Satterthwaite, M.A. (1975), Strategy-Proofness and Arrow's Conditions: Existence and Correspondence Theorems for Voting Procedures and Social Welfare Functions, *Journal of Economic Theory*, 10: 198-217.

Serais, J. (2002), Cloning Manipulation of the Borda Rule, *Meeting SCW*, GEMMA-CRÈME, University of Caen, France.

Sen, A.K. (1970), *Collective Choice and Social Welfare,* Holden-Day: Oliver and Boyd.

Suzumura, K. (2007), Choice, Opportunities and Procedures, *Collected Papers of Kotaro Suzumura*, Institute of Economic Research, Hitotsubashi University, Kunitatchi, Tokyo.