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Single transferable vote in local and national elections

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Abstract: *The single transferable vote (STV) is a system of preferential voting designed to minimize wasted votes. In STV, a constituency elects two or more representatives per electorate. As a result the constituency is proportionally larger than a single member constituency from each party. Political parties tend to offer as many candidates as they most optimistically could expect to win; the major parties may nominate almost as many candidates as there are seats, while the minor parties and independents rather fewer. STV initially allocates an elector's vote for her most preferred candidate and then, after candidate have been either elected or eliminated, transfers surplus or unused votes according to the voter's stated preferences (ties disallowed). The paper deals with different STV counting methods so that one can compare among them and analyze advantages and drawbacks of them. Since in STV ties are disallowed, so that tie-breaking in STV is important and are discussed in some details. In STV manipulation of voting is sometimes possible and this paper has taken an attempt to give a clear concept of STV manipulation.*

Keywords: Single transferable vote, Tie-breaking in STV, ERS97, Hare and Droop quota.

INTRODUCTION

Most of the politicians believe that elections are logically imperfect. Perhaps STV is one of the best and fairest voting systems which has least such imperfection. It is a many ballots electoral system and mainly used in the English-speaking countries. But political parties dislike it and resist to adopting it because it requires candidates to compete publicly with one another. If it is popularized in the society then there is a probability of political parties to be completely abolished. It can be used for either single winner or multi-winner voting system and in this paper steps are taken to discuss both methods in details.

Plurality voting is criticized for the property, **winner-takes-all,** but it has a good side in vote counting. In plurality voting if vote counting is conducted in many ways, the result remains same in every case. But in STV different counting systems give different results. So in STV the results vary depending upon how the counting is conducted. The counting rule commonly used is ERS97 rule, was given by Newland and Britton (1997). Droop quota is one of the best method in STV counting. We have elucidated this following Droop (1881), Woodall (1994), Tideman (1995) and Lundell and Hill (2007). Meek's (1969, 1970) feedback method is a good procedure to reduce wasted votes. Warren's counting (Warren 1994) in STV is another kind of easier counting process and in STV election it gives more accurate result.

In Meek system premature exclusion remains but it is a serious drawback in STV. Hill's sequential counting system takes an attempt to repair these types of drawbacks. It consists of a number of main phases and sub-phases and is discussed later.

In STV ties disallowed but sometimes ties arise. Ties must be broken by various methods. We have highlighted different processes of tie-breaking following Hill (1994), Newland and Britton (1997), Kitchener (2000) and O'Neill (2004). Woodall (1982) proposed computer counting whenever human counting is comparatively complicated. In STV the question of elimination of candidate came when no candidate has surplus above the quota, in order to allow the count to continue. The eliminate candidate is he who obtains least votes. In STV sometimes winning candidates may be eliminated, which is a serious drawback in this method.

STV uses in parliamentary or local government elections in many countries of Europe, Australia, New Zealand and America. Here we only elucidate electoral STV result of Northern Ireland in the Appendix.

HISTORY OF STV

The concept of STV was first proposed by Thomas Wright Hill in 1821, who was a school master, for application in elections at his school but was not populated. Carl Andrae proposed a transferable voting system for elections in Denmark. Andrae's system was used in 1856 to elect the Danish Rigsdag and by 1866 it was also adopted for indirect elections to the second chamber, the Landsting, until 1915.

Though STV was rejected in British Parliament but Tasmanian used it in 1907 and then widely spread throughout the Europe. In 2007 STV is used for parliamentary election in the Republic of Ireland (see Appendix), Northern Irish Assembly and Malta. It is also used for the Australian Senate in the form of a group of tickets, as well as certain regional and local elections in Australia, local government election in Australia, local government election in the Republic of Ireland, local government elections in Scotland and some local government elections such as Dunedin and the capital city of Wellington in New Zealand.

In the USA, it is used for city elections in Cambridge, Massachusetts, various student government elections, and also has been used for certain city elections in Minneapolis, Minnesota in 2009. It is set for adoption in British Columbia in a 2005 referendum, and it is put to the votes a second time in May 2009. It has been used for local elections in Scotland since May 2007. The English barrister Thomas Hare is generally credited with the concept of STV and proposed that electors should have the opportunity of discovering which candidate their vote had ultimately counted for, to improve their personal connection with voting. Tasmanian Attorney General, Andrew Inglish Clark was successful in persuading the Tasmanian House of Assembly to be the first parliament in the world elected by what become known as the Hare-Clark system, named after himself and Thomas Hare.

DIFFERENT STV COUNTING METHODS

STV system differs in a number of ways, such as the exact size of the quota used for determining winners and how the votes are transferred. So that some of the politicians consider STV as a set of voting systems. At first STV is counted by Hare quota (total votes are divided by seats) which is a largest quota such that some candidates (equal to number of seats) can be elected. As a result it is not populated in the societies. Now this is considered as an inferior method. On the other hand the Droop quota (the total votes are divided by seats plus one, and is added to 1) is the most commonly used quota. This method ensures majority rule while maintaining the condition that no more candidates can reach a quota than there are seats to be filled.

The above two methods are used when number of candidates and voters are comparatively small. For larger case such as Senate elections Gregory rule (also known as Newland-Britton or Senatorial rules) is used which eliminates randomness by allowing for the transfer of fraction of votes (Newland and Britton 1997). However the above rules do not treat all the votes equally, but Meek's method (Meek 1969, 1970) and Warren's method (Warren 1994) do more accurately. For the vote counting the latter (comparatively complicated) methods sometimes conducted by computer but for the former (comparatively simpler) methods only hand counting is sufficient.

The most recent refinements of STV involve attempting to remove the problems of sequential exclusions, which mean that sometimes STV eliminates, at an earlier stage in the count, a candidate who might have gone on to be elected later had they been allowed to remain in the contest. CPO-STV, Schulze STV, and Sequential STV have been invented to overcome this problem by incorporating elements of Condorcet methods (Islam et al. 2011) into STV. Here we have discussed only sequential STV (section-3.5).

A new method RTV-STV deals with the problem differently and more simply than these systems by simply making sure no such candidate could possibly be eliminated. Unfortunately none of these new methods has been used in a government election. We hope in future these methods will be used when the technologies of the voting system and public consciousness will be developed. Now we will discuss some methods of STV counting as follows:

Droop Quota

In an STV election, a candidate requires a certain minimum number of votes '*the quota*' to be elected. A number of different quotas can be used; the most common is the *Droop quota*. Henry Droop (Droop 1881, Islam et al. 2011) himself defined his quota as

$$\frac{mV}{n+1}+i,$$

where, V = the total number of voters have m votes each,

n = the number of seats to be filled,

i = the number necessary to reach the smallest integer greater than $\frac{mV}{n+1}$.

If $m = 1$, $i = 1$ and $n = S$ then we can write the Droop quota as:

$$\left(\frac{V}{S+1}\right)+1$$

where, V = the total number of valid votes cast,

S = the number of seats to be filled.

The Droop quota is the smallest quota such that no more candidates can be elected than there are seats to be filled. The Droop quota is often rounded to an integer. The calculation of Droop quota in STV is as follows: STV is a step procedure, in each step voters cast votes for their most preferred candidate. It proceeds according to the following steps:

- Any candidate who touched or exceeded the required quota is declared elected.
- If not enough candidates have been elected, the count continues.
- If a candidate casts more vote than the quota, then their surplus is transferred to other candidates according to the next preference on each voter's ballot.
- If none meets the quota, the candidate with the fewest votes is eliminated and their votes are transferred. This process continues until the last candidate survives which is the last winner in the election.

Again in quota system, voting procedure is stopped when the numbers of remaining candidates instead of counting votes until all candidates have reached a quota. In STV, candidates who receive excess votes and candidates who are excluded have their votes transferred to other candidates. That is why it is said to be minimizing wasted votes. Let us introduce a simple example as follows:

Let 40 guests (voters) are invited in a party and from 5 soft drinks (candidates), 3 of which will be selected. The candidates are: Tea (x), Coffee (y), Pepsi (z), Coke (z) and Mineral water (v). Each of the 40 guests is given 2 ballots. In quota system, the number of votes to be elected are, $\left(\frac{40}{3+1}\right)+1=11$.

In the following table-1 we have shown only first and second preferences of their soft drinks. We have excluded higher order preferences because in our election these are no needed.

Table 1: The first and second preferences of the soft drinks.

No. of guests	10	5	8	3	2	7
First preference	x	x	y	z	u	v
Second preference	y	v			v	

When ballots are counted the election proceeds as follows:

Step-3.1

Candidates	x	y	z	u	v
Collected votes	15	8	3	2	7

In step-3.1 x has 4 more votes than the quota, so x is declared elected. Candidate x's surplus votes transfer equally to y and v according to voters of x second choice preferences then the step-3.2 being as follows:

Step-3.2

Candidates	x	y	z	u	v
Collected votes	11	10	3	2	9

In step-3.2 even with the transfer of this surplus no candidate has reached the quota. Then u with the lowest votes is eliminated. The votes of u are transfer to his second preference v to reach the quota then the step-3.3 being as follows:

Step-3.3

Candidates	x	y	z	v
Collected votes	11	10	3	11

In step-3.3, v is elected but has no surplus to transfer. Neither of the remaining candidates meets the quota. Therefore z has lowest votes is eliminated. Candidate y is the only remaining candidate and so wins the final seat.

Final result: The winners are x, v, y i.e., Tea, Mineral water and Coffee.

Problems with Droop Quota

The Droop quota has following problems (Lundell and Hill 2007).

Too many winners: We have mentioned above that Droop quota is rounded to an integer, so that;

- We have to adjust the quota upward by rounding as much as to the next higher integer such a way that winning candidates will be less than or equal to S.
- Sometimes we face tie and tie could be break (section-4). But in this case there is a probability of winning more than the S candidates. So that counting may be stop until for all the S seats are filled or the winning candidates will be less than S seats.

Let there are 2 seats and 100 voters then $\frac{100}{2+1}$ can not be exactly represented in an integer. ERS97, uses two decimal digits of precision. So that $\frac{100}{2+1}$ is represented by 33.34 and $\frac{100}{3+1}$ is represented by 25.00. Integer-based methods use the Droop quota as $\left(\frac{V}{S+1}\right)+1$, so that for the above two cases the quota will be 34 and 26 respectively. In both of the cases there are probabilities of winners will be greater than S and we have to adjust by ignoring truncation and rounding errors, and also break the ties when too many candidates reach the quota.

Failure of Droop Proportionality Criterion (*DPC*)

Woodall (1994) defined the *DPC* as follows: "If V votes are cast in an election to fill S seats, then the quantity $\left(\frac{V}{S+1}\right)$ is called the Droop quota. For some numbers k and m satisfying $0<k\le m$, more than k Droop quotas of votes put the same m candidates as the top m candidates in their preference listing, then at least k of those m candidates should be elected".

Robert Newland states that Droop quota does not satisfy the *DPC*. For example (Woodall 1994),

Party A:	1001	1001	1001	998	(Total 4001 votes),
Party B:	1000	1000	1000	999	(Total 3099 votes).

If the quota is 1000, $\left(\frac{V}{S+1}\right)$, Party A takes 4 seats and Party B takes 3. If the quota is 1001, $\left(\frac{V}{S+1}+1\right)$, Party A takes 3 seats and Party B takes 4, which is violation of *DPC*.

Premature Election

Tideman (1995) indicates that the exact quota $\left(\frac{V}{S+1}\right)$ creates an additional difficulty. Suppose from 15 candidates 2 to be selected. The preference relation be as follows:

5 x
5 y

$$3 \;\; u \;\; z$$
$$2 \;\; z \;\; u.$$

The quota is 5, so that x and y are elected which does not violate Woodall's *DPC* but we observe that there is a x-y-u tie, so that it is a premature election. This difficulty can be overcome by eliminating z at first stage or the candidates exceed the exact quota.

Meek System: An Step to Reduce the Problems

Minimum wasted votes

The aim of STV is to minimize the wasted votes. If there are S seats to be filled in an election the quota q is the smallest number such that S candidates have q votes each, it is not possible for an (S+1)-*th* candidate to have as many as q votes. Let the total votes are V, so that $V - Sq < q$ but $V - S(q-1) \geq (q-1)$. Considering equality we find $q = \dfrac{V}{S+1} + 1$ which is Droop quota mentioned above. If W is the total wasted votes then $W < \dfrac{V}{S+1}$. But in trivial cases when $S \geq V$ there are no wasted votes. Also in STV it is not possible to guarantee that all votes will be taken equally into account. The following feedback method (Meek 1969, 1970) is a process which attempts to reduce such problems partially.

The Feedback Process

STV rules are very complicated. The feedback process established on the basis of the principles as follows:

i) If a candidate is eliminated, all ballots are treated as if that candidate had never stood.
ii) If a candidate has achieved the quota, he attains a fixed proportion of every vote received, and transfers the remainder to the next non-eliminated candidate, the retained total equaling the quota.

Now, suppose that a voter's preference relation being as, *xPyPzPu…*. If x is eliminated then by principle (i), the preference relation of that voter will be as, *yPzPu…*, which indicates that x had not stood at all and y is the first preference of that voter. Suppose fortunately in an earlier count y had been reached the quota, so by principle (ii), the same proportion of this vote must retain by y as for the others, passing the rest to z. Hence total retained by y is now greater than the quota. So that, the proportion of y's votes to be retained, must be recalculated. If p_y is the proportion y transfers, and p_z that which z transfers, supporters of both y and z have their votes transferred to their preferences at value $p_y \times p_z$. Those putting y first have $(1-p_y)$ retained by him and $p_y \times (1-p_z)$ retained by z; those putting z first have $(1-p_z)$ retained by him and $p_z \times (1-p_y)$ retained by y.

Computer Counting in STV

Although STV is the fairest among the electoral systems there are some anomalies which are of course annoying and unavoidable (Woodall 1982). In STV when a candidate is already elected by getting the necessary votes for the quota, no votes are transferred to him. Suppose a voter z is sure to be eliminated early, then a voter can increase the effect of his genuine second choice by putting z first. Let two voters both want x as first choice and y

as second, and x is declared elected on the first count, then the voter who lists his choices as xPy will have $\frac{1}{3}$ (say) of his votes transferred to y, whereas the other voter who lists his preference as $zPxPy$… will have all of his votes transferred to y, since he knows that z is eliminated and x is already won.

Since one aim of an electoral system should be to discourage tactical voting. This is a serious drawback in STV. When the candidates are very few then no doubt hand counting is better but when candidates and voters are too large, for example, in a parliamentary elections the hand counting in STV is very complicated. Some cases it is impossible to count the votes by hand accurately within a limited time.

In such situation we can use computer to calculate very long calculations precisely, concisely and safely. Sometimes fraction of a vote (say millionth of a vote) is transferred, so that in this case hand counting will be tedious. But computer counting makes it easy. The rules and process of vote counting must be precisely installed in computer by software, otherwise counting must give error.

Warren Counting in STV

Warren (1994) expressed STV vote counting as follows: After fulfill the quota extra votes are retained as usual rule of STV. Let an elected candidate's retain amount is a_1 (say), the remaining amount $(1-a_1)$ is transferred to the voters' expressed second preferences. If an expressed second preference has an amount retained of a_2 (say), and if (a_1+a_2) is less than unity, then the voter still has an amount remaining of $(1-a_1-a_2)$, which is then transferred to the expressed third preference, and so on. At the end of counting of the first stage of the count, some candidates have just the quota, whereas the remainders have varying amounts of votes less than the quota. At the end of the first stage the candidate whose vote is least is eliminated. However if his name appears on a ballot paper, it is '*passed over*'. This helps the other unelected candidates to exceed the quota. Similarly preceding as like first stage, at the end of the second stage some more candidates will have just the quota, and the remainder will have varying amounts of vote less than the quota. If at any stage a ballot paper contains no sufficient preferences to transfer votes, then balance of vote is ascribed 'non-transferable', and the quota is recalculated excluding the non-transferable vote. The count continues until the total number of seats is filled.

Hill's Sequential STV

In Meek system premature exclusion remains but it is a serious drawback in STV. Exclusion of the lowest candidate is needed to continue the counting but it is not a real solution. Hill (1994) proposed the sequential STV as follows: Let the election be to fill k seats from n candidates and let $m = n - k$. Sequential STV then consists of a number of main-phases and sub-phases, each being an STV election for k seats but with varying selections of candidates.

Main-phase-1: First we do not divide all n candidates into elected and excluded but divide them into probables and others respectively. Set all n candidates to 'unmarked'.

Sub-phase 1.1: The k probables plus any other one candidate. Set the winners to Sub-phase 1.2 . The same k probables plus any other one candidate not yet tested. Set any unmarked winners to 'marked'. Similarly we can write,

Sub-phase 1.*m* : The same k probables plus the least candidate not yet tested. Set any unmarked winners to 'marked'. Sometimes a tie may arise and it will be settled using random selection, then all $k+1$ of the candidates involved are set to 'marked'.

Main-phase-2: In this phase all the marked candidates, dividing into probables and others. If the resulting set of probables is the same as a previous set, those candidates are elected and the process finishes. Otherwise reset all n candidates to 'unmarked' and continue.

Sub-phase 2.1 to 2. m are same as before but must be used new probables. Main-phase-3 is same as main phase 2 and so on.

Now we give an example with 5 candidates for 2 seats (Hill 1994). Suppose the preference relation be as follows:

$x\ v\ y\ z\ u$ by 1005 votes,
$y\ v\ z\ u\ x$ by 1004 votes,
$z\ v\ u\ y\ x$ by 1003 votes,
$u\ v\ y\ z\ x$ by 1002 votes,
$v\ x\ y\ z\ u$ by 10 votes,
$v\ y\ z\ u\ x$ by 10 votes,
$v\ z\ u\ y\ x$ by 10 votes,
$v\ u\ y\ z\ x$ by 10 votes.

By plurality voting candidate x would win. Again any one candidate of x, y, z or u were to withdraw v would be the first elected. Under simple STV v with fewest votes 40 would be excluded. But under sequential STV we observe as follows:

Phase	Candidates	Winners	Probables	Marked
1	$x\ y\ z\ u\ v$	$y\ z$	$y\ z$	___
1.1	$y\ z\ x$	$y\ z$	___	$y\ z$
1.2	$y\ z\ u$	$y\ z$	___	___
1.3	$y\ z\ v$	$y\ v$	___	v
2	$y\ z\ v$	$y\ v$	$y\ v$	___
2.1	$y\ v\ x$	$y\ v$	___	$y\ v$
2.2	$y\ v\ z$	$y\ v$	___	___
2.3	$y\ v\ u$	$y\ v$	$y\ v$	___

3	y v	y v	y v	___ .

Final result: y and v are consequently elected.

Tie-breaking in STV

Although we mentioned above that in STV ties disallowed, sometimes ties may occur for several different reasons and the ties need to be broken (Newland and Britton 1997, O' Neill 2004). The ties can be broken simply by lottery system such as tossing a coin. But this system is not the best one and usually the following four rules are used in tie-breaking.

i) Forwards Tie-Breaking (*FTB*): Choose the candidate who has the most (least) votes at the first stage where they had unequal votes.
ii) Backwards Tie-Breaking (*BTB*): Choose the candidate who has the most (least) votes at the previous stage or at the latest point in the count where they had unequal votes.
iii) Borda Tie-Breaking: Choose the candidate with the highest (least) Borda score (see section-4.3).
iv) Cooms Tie-Breaking: Choose the candidate with the fewest (most) last place votes.

Sometimes after breaking tie by any of the above mentioned rules the candidate would still face tied. In this case it is useful to distinguish between *weak ties* and *strong ties*. A weak tie occurs when candidates have the same number of votes at a given stage. A strong tie occurs when candidates are still tied after applying a tie-breaking rule (any one rule from (i) to (iv) is mentioned above). A strong tie would be broken by lottery. Here we will use ERS97 rules of tie-breaking (Newland and Britton 1997, O' Neill 2004). The difference between *FTB* and *BTB* is given in table-2 which is from Newland and Britton (1997) without any change.

Table 2: Example tally with ERS97 rules where 60 voters are electing 2 candidates from 6.

		Surplus of x	Eliminate w	Eliminate v	Eliminate z
Stages	1	2	3	4	5
x	23	20.00	20.00	20.00	20.00
y	13	13.00	13.00	15.00	15.00
z	6	6.50	10.00	12.00	2.00
u	7	7.50	9.50	12.00	18.00
v	7	7.50	7.50	-	-
w	4	5.50	-	-	-
Non-Transferable	0	0.00	0.00	1.00	5.00

Here we have to eliminate one candidate at stage-4 and there is a tie between candidates z and u. Thus, tie-breaking needs to be used to determine which candidate is to be eliminated. We use the *FTB* following ERS97 rules. In this case we first look to the counts at stage-1. From table-2 we see that u has one more vote than z at stage-1. So that candidate z is eliminated. If z and u had been tied at stage-1, then we would have to be looked to subsequent stages. If z and u would have been tied in all stages, then we would have been a strong tie which would have been broken by lottery.

But in *BTB* we have to look at the previous stage to break ties and if necessary to the preceding stages. In table-2 we see in preceding stage-3 that z is ahead to u, so that u would be eliminated.

One problem arises with *FTB* where the elimination order is: 4, 1, 2, 3 which is not sequential and is undesirable. If we make a meaningful sequence starting from 4 then the order is: 4, 3, 2, 1 which is *BTB*. Again *FTB* does not use the most relevant information than *BTB* to break the tie. Hence *BTB* is better than *FTB* in tie-breaking.

Unintended Tie-Breaking

Let us consider an STV where total votes are 12, and 2 to be elected from 3 candidates as follows:

$$x \quad y \qquad 8$$
$$z \qquad\quad 4.$$

The exact quota is 4. If we round the quota up to 4.01 then x is elected in first round, the surplus of 3.99 is to be transferred to y, as a result z beats y. But by exact quota both y and z tied. Obviously this tie-breaking is unintended.

Probability of Eliminating of Winning Candidates in Tie-Breaking STV

In tie-breaking STV, ERS97 rule sometimes eliminates winning candidates without manipulation of voting which is undesirable. Suppose there are 31 voters and 6 candidates among which one will be elected. Each of the voters is given 4 ballots (O' Neill 2004). In step-4.1 the preferences would be as follows:

Step-4.1

No. of votes	4	5	5	2	4	11
1st preference	x	y	z	u	v	w
2nd preference	y	z	y	x	x	
3rd Preference	z			y	y	
4th preference				z	z	

In step-4.2, u with fewest votes would be eliminated and u's votes would be transferred to his second preferred candidate x. The preference profile of step-4.2 would be as follows:

Step-4.2

Candidates	x	y	z	v	w
Collected votes	6	5	5	4	11

In step-4.3, v with fewest votes is eliminated and v's votes would be transferred to his second preferred candidate x. The preference profile of step-4.3 would be as follows:

Step-4.3

Candidates	x	y	z	w
Collected votes	10	5	5	11

In step-4.3, a tie arises between y and z with the fewest 5 votes. Then both would be eliminated by ERS97. However, instead tie would be broken by *FTB* or *BTB* or by lottery. Suppose z was eliminated by lottery and z's votes would be transferred to his second preferred candidate y. Then y would be tied with x. The preference profile of step-4.4 would be as follows:

Step-4.4

Candidates	x	y	w
Collected votes	10	10	11

In that situation tie would be broken by *FTB* rule. In step-4.1 candidate x has fewer votes than y, so that x would be eliminated and y as x's second preferred candidate received all of x's votes and beat w with 20 to 11 votes in final step. Therefore, winning candidate in ERS97 rule was eliminated which is unacceptable situation in STV election.

So that there is a flaw in STV, ERS97 *FTB* rule. This flaw could be removed in two ways: (i) by changing the rules of STV, ERS97, (ii) by using *BTB* rule which is better as we have seen before. Hence with *FTB* a winning candidate could be improperly eliminated however, with *BTB* both of these last-place candidates can not win and can thus be properly eliminated.

Tie-Breaking in STV by Borda Count

We have seen that the main feature of STV that later preferences should not affect the fate of earlier ones (Kitchener 2000) but sometimes ties arise and breaking ties give unreasonable results. Suppose in a vote of one seat all the partners want to elect a senior partner. They are using Borda system (Borda 1781, Islam et al. 2011). Borda count is as follows: If there are m alternatives, an elector's first choice is assigned $(m-1)$ points, his second $(m-2)$ points and so on down to his last choice, which is assigned 0 point. We consider four candidates for one place. The preference relation being as follows (Hill 1994):

$$\begin{array}{rl} x\ y\ u & 1 \\ y\ x\ z & 1 \\ x\ u\ z\ y & 1 \\ y\ u\ x & 1. \end{array}$$

The quota is two. By Condorcet method (Condorcet 1785) we see that there is a tie between x and y. But by Borda count we see;

$$\begin{array}{lll} x \text{ casts} & 2\times1+1\times1+3\times1+0\times1 & = 6 \text{ votes,} \\ y \text{ casts} & 1\times1+2\times1+0\times1+2\times1 & = 5 \text{ votes,} \\ \text{z casts} & 0\times1+1\times1 & = 1 \text{ vote,} \\ u \text{ casts} & 0\times1+2\times1+1\times1 & = 3 \text{ votes.} \end{array}$$

So that x wins by Borda count. If candidate y could anticipate that x would win then y would not place x in second preference so that y would win or may be a tie. Then tie-breaking by Borda scores is not a sensible way of conducting elections, which cause more trouble than it is worth (Hill 2000). Sometimes both candidates z and u can manipulate the result, and then tie- breaking by Borda count will be difficult.

THE PROCESS OF SINGLE WINNER IN STV AND MANIPULATION OF VOTING

It is a step procedure, in which, at each step voters cast votes for their most preferred candidates (Feldman 1979, Islam et al. 2011). In step-5.1, each voter casts vote for his most favorite. Then the candidate with the fewest votes is eliminated. In step-5.2, each elector casts a single vote for his favorite among the remaining candidates. As before the candidate with the fewest votes is eliminated. The process continues until one candidate remains. The last candidate consider as the winner. Consider the preference profile be as follows:

Step-5.1

Type 1: $xPzPyPu$ by 10 voters,
Type 2: $yPzPxPu$ by 7 voters,
Type 3: $uPzPyPx$ by 5 voters,
Type 4: $zPyPuPx$ by 3 voters,
Type 5: $uPzPxPy$ by 4 voters.

In step-5.1, z with 3 votes is eliminated, and then the preference profile will be as follows:

Step-5.2

Type 1: $xPyPu$ by 10 voters,
Type 2: $yPxPu$ by 7 voters,
Type 3: $uPyPx$ by 5 voters,
Type 4: $yPuPx$ by 3 voters,
Type 5: $uPxPy$ by 4 voters.

In step-5.2, u with 9 votes is eliminated, and then the preference profile will be as follows:

Step-5.3

Type 1: xPy by 10 voters,
Type 2: yPx by 7 voters,
Type 3: yPx by 5 voters,
Type 4: yPx by 3 voters,
Type 5: xPy by 4 voters.

In step-5.3, x with 14 votes is eliminated. Finally y will be the remaining person and will win in the election. Type-3 voters prefer z to y. They anticipate that y will win then they could manipulate the preferences as follows:

Type 3: $zPxPuPy$ by 5 voters.

Then as before in step-5.1 u with 4 votes would be eliminated and then the preference profile would be as follows:

Step-5.2a

Type 1: $xPzPy$ by 10 voters,
Type 2: $yPzPx$ by 7 voters,

Type 3: $zPyPx$ by 5 voters,
Type 4: $zPyPx$ by 3 voters,
Type 5: $zPxPy$ by 4 voters.

In step-5.2a, y with 7 votes would be eliminated and then the preference profile would be as follows:

Step-5.3a

Type 1: xPz by 10 voters,
Type 2: zPx by 7 voters,
Type 3: zPx by 5 voters,
Type 4: zPx by 3 voters,
Type 5: zPx by 4 voters.

In step-5.3a, x with 10 votes would be eliminated. Finally z would be remaining person and would win in the election. Therefore, we have seen that STV is manipulable.

CONCLUDING REMARKS

This paper analyzes aspects of STV using various methods. We have shown that STV is a good method in voting system. The method can be used for single winner or multi-winner voting systems. Since every voter gives her own preferences so that STV reflects every individual's idea to elect candidates. It minimizes the wasted votes than any other voting method. The main drawback of STV is in vote counting system. Different systems give different winners. So that winner in STV depends on which method is followed in the election. Again the STV is not manipulation free, which is a drawback of the system. Another drawback of STV is tie-breaking and we have shown different methods of tie-breaking. This paper has discussed various advantages and drawbacks of STV in every vote counting method. We have tried throughout the paper to discuss STV with simple mathematical calculations, introducing definitions, and displaying tables and step procedures where necessary. Voting system is a very complicated field but we have tried our best to make it easier.

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APPENDIX

PROPORTIONAL REPRESENTATION AND STV IN NORTHERN IRELAND

We know in STV the proportional representation of seats depend only on the eligible electorate. But in the Assembly election in Northern Ireland, in March 2007 had been made by Sainte-Lagu¨e rule did not follow this rule. Instead it follows that proportional representation of seats depend on the valid votes (The Northern Ireland Assembly Education Service Website http://education.niassembly.gov.uk).

The Assembly consists of 108 MLAs (Members of the Legislative Assembly) representing 18 constituencies. Elections are held every four years. Voting at an Assembly election is by secret ballot using a system of Proportional Representation (PR), known as STV. STV is also used in Northern Ireland Local Government and European Parliament elections and in elections in the Republic of Ireland.

STV is a type of PR system. In an election using STV, constituencies elect a set number of candidates. In Northern Ireland this is 6. A party standing in an election can put forward as many candidates as it likes per constituency.

Voters have as many preferences as there are candidates. They mark the candidates 1, 2, 3, etc in order of preference, with 1 for their first choice candidate, 2 for their second, and so on. Voters do not have to rank-order all candidates - they can choose as many or as few as they like.

With STV, seats are awarded in proportion to votes cast, with later preferences expressed taken into account. STV has advantages over the 'first past the post' system used in Westminster elections:

- it increases voter choice (voters can vote for more than one candidate and can choose between candidates as well as between parties); and
- ensures that more voters have an effect on the outcome (over 80% of all valid votes are used to determine the 6 successful candidates) and therefore a result that is more representative of the views of the electorate.

Each voting paper is checked to see if it has been correctly filled in. Those that are not (spoilt papers) are removed from the count to give the number of **valid votes,** which will be used to calculate the **quota**.

A **quota** is calculated for each constituency. This is the number of votes needed by a candidate to get elected. The quota is calculated by using the Droop quota (section-3.1). In Northern Ireland all our constituencies are 6-member, i.e. the number of seats (S) is 6, this means the quota is $\frac{1}{7}th$ of the votes cast plus 1 vote.

Example 1A: In the 2007 Assembly Election, 41,822 valid votes were cast in the constituency of Lagan Valley. The quota of votes required therefore for a member to be elected was, $\frac{41{,}822}{6+1}+1=5{,}975$.

In the elections to the Northern Ireland Assembly the whole number is always used in calculating the quota. Should there be a fraction, the numbers after the decimal point are ignored, e.g., in calculation above, 5,974.5714 becomes 5,974.

Voting papers are sorted into bundles according to first preferences and counted. Any candidate reaching or exceeding the quota is elected. If they are elected with more 1st preference votes than the quota, their extra votes are called a surplus. Surplus votes from candidates who exceed the quota are transferred to the remaining candidates who were chosen as number 2 (second preference) on the elected candidates' ballot papers (which show a second preference). All votes are transferred at a fractional value. The surplus is calculated as follows:

$$\text{Surplus} = \text{Number of valid votes received} - \text{Quota}.$$

Example 2A: The quota in constituency X is 6,300 votes and candidate A received 7,000 votes.

$$\text{Surplus} = 7{,}000 - 6{,}300 = 700.$$

Candidate A was selected at the first count, having exceeded the quota. It would not be a fair system to transfer just candidate A's 700 surplus papers to other candidates. If only the 'extra' papers were transferred there would be no way of ensuring that the 2nd preferences on these 700 papers were representative of all the 7,000 ballot papers that candidate A had received: 6,300 people would not have their second preferences considered. For fairness, all the candidate's ballot papers with a 2nd choice are redistributed. These are called transferable ballot papers as the voter has indicated a 2nd preference.

The transferable ballot papers are reallocated to the next choice candidates at a transfer value (a fractional percentage of one vote). This reduces the value of each vote transferred, so that the total redistributed vote is not worth more than the value of the candidate's surplus. So when we talk about transferring the surplus we really mean transferring the value of the surplus (across all the transferable papers) rather than transferring the actual surplus papers.

If we take the example of candidate *A* again, if all their papers have a 2nd preference then there are 7,000 transferable papers to be reallocated. This will be at a total transfer value of their surplus – 700. So 7,000 papers transferred to equal a total value of 700 means that each ballot paper has an individual transfer value of 0.10 i. e., 700/7,000 = 1/10 = 0.10. This transfer value is calculated as follows:

$$\text{Transfer value} = \frac{\text{Surplus}}{\text{Total number of transferable ballot papers for candidate}}.$$

Example 3A: Candidate *B* receives 1,000 votes. The quota in their constituency is 850; this means they have a surplus of 150, (1,000 - 850). The transfer value is calculated by dividing the surplus (150) by the total number of transferable ballot papers. If all 1,000 ballot papers Candidate *B* received were transferable that would be 150/1,000 = $\frac{3}{20}$*th* or 0.15 of a vote (2 decimal places). So in this example the 1,000 ballot papers would be re-distributed to the next available preferences at the value of $\frac{3}{20}$*th* of a vote.

Using the figures from examples-2A and -3A, and assuming all papers are transferable, the transfer value at the first stage of the count be as follows:

Candidate's Vote	Quota	Surplus	Transfer Value
7,000	6,300	700	0.10
1,000	850	150	0.15.

If no candidate reaches the quota when the 1st preferences votes have been counted, the candidate with the lowest number of 1st preferences is eliminated. Their next available preferences are redistributed to the candidates left. The transfer value of each transferable paper is still 1 vote, as the 1st preference was not used.

The second count adds the number of 1st preference votes for the candidates not selected in the first count with the value of the second preferences transferred to them. Again, if a candidate reaches the quota at this stage they are elected and any surplus over the quota is redistributed at transfer (fractional) value according to the next available preference. This process is repeated until all 6 seats have been filled. If no one reaches the quota in a particular stage of the count, the candidate with the lowest vote is eliminated and their votes redistributed to the next preference candidate. There will be as many counts as are needed to fill all 6 seats. The first 6 candidates to reach or come closest to reaching the quota will be successful.

Table-1A based upon the eligible electorate and table-2A based upon the valid votes. Table-2A shows what this would have done, compared with table-1A; East Antrim, Lagan Valley and North Down would each have lost a seat as a result of poor turnout, while Fermanagh and South Tyrone, Mid Ulster and Newry and Armogh would each have gained one for good turnout.

Table 1A: Northern Ireland constituencies at the March 2007 Assembly election and the seats that each would have had if based on eligible electorate, under the Sainte-Lagu¨e rule.

Constituency	*Electorate due*	*Seats due*
North Antrim	72814	7
South Down	71704	7
Newry and Armagh	70823	7
Upper Bann	70716	7
Lagan Valley	70101	7
Strangford	66648	6
Fermanagh and South Tyrone	65826	6
South Antrim	65654	6
Foyle	64889	6
Mid Ulster	61223	6
West Tyrone	58367	6
North Down	57525	6
East Antrim	56666	6
East Londonderry	56104	5
Belfast West	50792	5
Belfast North	49372	5
Belfast South	48923	5

Table 2A: Northern Ireland constituencies at the March 2007 Assembly election and the seats that each would have had if based on valid votes, under the Sainte-Lagu¨e rule.

Constituency	*Valid Votes*	*Seats due*
Newry and Armagh	49619	8
Fermanagh and South Tyrone	46442	7
South Down	46110	7
North Antrim	44331	7
Mid Ulster	44277	7
Upper Bann	42882	7
Lagan Valley	41822	6
West Tyrone	41454	6
Foyle	41036	6
South Antrim	38175	6
Strangford	36019	6
East Londonderry	33922	5
Belfast West	33790	5
North Down	30707	5
Belfast South	30344	5
East Antrim	30039	5
Belfast North	29715	5
Belfast East	29629	5