Indirect Taxation, Public Pricing and Price Cap Regulation: a Synthesis

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INDIRECT TAXATION, PUBLIC PRICING AND PRICE CAP REGULATION: A SYNTHESIS[1]

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1 Introduction

Some welfare properties of price cap regulation have been recently analyzed by a number of papers that, especially during the first decade of the current millennium, moved from the seminal contribution of Vogelsang and Finsinger (1979), to analyze the price cap's ability to guarantee welfare maximization, welfare improvements and/or poverty reduction (Iozzi, Poritz and Valentini, 2002; Valentini, 2006; Makdissi and Wodon, 2007). These papers contributed to the extant literature by extending some familiar results on price cap regulation in frameworks where efficiency and equity issues can be dealt with simultaneously.

As we will see, there is a strong parallelism between the price cap results that will be surveyed in this paper and those originating from the well-established theories on optimal indirect taxation and tax reforms, as well as public pricing. As a matter of fact, it is well known that many standard results on optimal taxation and tax reforms have a straightforward counterpart in the monopoly pricing context and the Ramsey-Boiteux pricing rule represents the most obvious and well known example of this connection. This sort of parallelism started with the contributions of Ramsey (1927) and Boiteux (1956) and it has gone on with Diamond and

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Mirrlees (1971a, 1971b) and Feldstein (1972) - who proposed the optimal structure of, respectively, indirect taxation and public pricing when distributional concerns are accounted for in the social welfare function -, Ahamad and Stern (1984) and Ross (1984) – who proposed, independently but almost contemporaneously, an identical method to infer social welfare weights from, respectively, indirect taxation and regulated prices - and so on for the subsequent contributions in these research fields. What is less acknowledged, maybe even by many regulatory economists, is that this parallelism exists also with respect to a number of properties that characterize some types of price cap regulation. This paper reviews the economic literature that explored such properties, showing that the links between optimal taxation, optimal pricing and price cap mechanisms go beyond the well-known adjustment process of price capped prices towards Ramsey prices firstly proposed by Vogelsang and Finsinger (1979) and further analyzed by Brennan (1989).

This paper deals with the normative properties of price cap regulation but it has no pretension to deliver an exhaustive survey of the articles in this area where most of the literature is concerned with price cap's efficiency properties from a productive point of view. Most of the papers that, especially during the 80's and the 90's, studied price caps also from a social welfare perspective (Bradley and Price, 1988; Neu, 1993; Cowan, 1995, among the others) refer, explicitly or implicitly, to Vogelsang and Finsinger (1979) and to the Ramsey-Boiteux pricing rule as the benchmark for their welfare evaluations. In all these papers, however, the normative analysis is neither based on ethical judgments on the utilitarian welfare function that underlines Ramsey prices, nor on the distributional consequences of its implementation, which are, in contrast, the issues characterizing the papers reviewed in this survey.

Other surveys on price cap regulation have been published in the last years. For instance, both Sappington (2002), who reviews the theoretical and practical characteristics of the various incentive regulatory plans that have been mostly used in telecommunications markets, and Armstrong and Sappington (2005), who provide a review of the most influential theoretical work on the design of regulatory policy, devote several pages on the design of price cap regulation. Vogelsang (2002) and Sappington and Weisman, (2010), instead, report very detailed and critical reviews of price cap regulation in the experience of its applications in, respectively, public utility and, more specifically, telecommunications industries. However, to the best of our knowledge, no other paper has ever attempted to give a unified vision of the literature reviewed in the present survey.

The paper is organized as follows. In the next section we review very briefly some important contributions in the field of optimal public pricing and indirect taxation in order to highlight the strong correspondence between the results of these two strands of literature. As we will see in the following sections, this correspondence can be extended also to the theory of price cap regulation. In section 3, indeed, we start from the Feldstein generalization of the Ramsey-Boiteux pricing rule in order to show how an ad hoc generalization of the traditional Laspeyres-type price cap can guarantee second best prices that can incorporate distributional concerns on consumers. Also, we will see that, given this more general formulation of price cap, it can be possible to rescue the regulator preferences over different groups of consumers from the implemented price cap formula. The final part of section 3 shows what are the sufficient conditions guaranteeing that a marginal price cap reform is welfare improving and the necessary and sufficient conditions guaranteeing that it is poverty reducing. We will stress that these assessments are not contingent on any given social welfare function. Finally, section 4 concludes and points out the possible future researches in this area.

2 A short tour of indirect taxation and public pricing

Since the pioneering articles of Ramsey (1927) and Boiteux (1956), some hundreds of paper have contributed, more or less independently, to add insight to the theories of indirect taxation and public pricing. As both the original Ramsey problem and its application to monopolistic markets deal with how prices should depart from marginal costs in order to maximize social welfare subject to a constraint (tax revenue in Ramsey, profit in Boiteux), it is not surprising that any result obtained in the taxation context has its equal in public pricing and viceversa. So, Feldstein (1972) extended the Ramsey-Boiteux pricing rule and proposed the optimal structure of public pricing for the case when the social welfare function accounts for distributional concerns; in the same spirit, Diamond (1975) developed the analysis of Diamond and Mirrlees (1971) to derive a many-person Ramsey tax rule which enables to take into account the trade-off between efficiency and equity objectives.

2.1 Optimal indirect taxation vs. optimal pricing

To give an analytic synthesis of these strands of literature we may consider the following individualistic social welfare function

$$W(\mathbf{p},\mathbf{y}) = W[v^1(\mathbf{p}, y_1),...v^H(\mathbf{p}, y_H)] \qquad (1)$$

where $\mathbf{p}$ is the price vector faced by any of the H households, $\mathbf{y}$ is the vector of the households' incomes, $v^h(\mathbf{p}, y_h)$ is the indirect utility function of household h (h=1,…H), with y_h which is the income of household h. Now, if we assume that $\mathbf{p} = \mathbf{s} + \mathbf{t}$, where $\mathbf{s}$ is the vector of producers' prices and $\mathbf{t}$ the vector of specific taxes, we can formulate the optimal taxation problem as

$$\begin{aligned} &\max_{\mathbf{t}} \quad W(\mathbf{p},\mathbf{y}) \\ &\text{s.t. } T(\mathbf{t}) \geq \overline{T} \end{aligned} \qquad (2)$$

where $T(t) = \mathbf{t}\cdot\mathbf{x} = \sum_{i=1}^{I} t_i x_i$ is the tax revenue constraint, I is the number of goods and $x_i = \sum_{h=1}^{H} x_i^h(\mathbf{p})$ is the aggregate consumption of good i. Let $\mathbf{t}^* = (t_1^*,..,t_M^*)$ be a tax rates' vector that solves this problem; $\mathbf{t}^*$ is implicitly given by the I + 1 conditions

$$\begin{cases} T(\mathbf{t}^*) = \overline{T} \\ \left.\dfrac{\partial W}{\partial t_i}\right|_{\mathbf{t}^*} + \lambda \left.\dfrac{\partial T}{\partial t_i}\right|_{\mathbf{t}^*} = 0 \qquad \text{for } i = 1, .., I \end{cases} \tag{3}$$

where λ is the Lagrange multiplier. We further assume that this vector exists and is unique for any level of tax revenue $\overline{T}$ in (2).

The optimal taxation problem is essentially equivalent to the following maximization problem

$$\begin{aligned} &\max_{\mathbf{p}} \; W(\mathbf{p}, \mathbf{y}) \\ &\text{s. t. } \Pi(\mathbf{p}) \geq \overline{\Pi} \end{aligned} \tag{4}$$

where the main difference is in the nature of the constraint that in (4) represents a minimum level of profits, $\overline{\Pi}$, that must be guaranteed to a multi-product monopolist that produces I goods in order to maximize profits given by $\Pi(\mathbf{p})$ = $\sum_i p_i \; q_i(\mathbf{p}) - c(\mathbf{q}(\mathbf{p}))$. We let $\mathbf{q}(\mathbf{p})$ be the I-dimensional vector whose elements are the market demand functions $q_i(\mathbf{p})$ (i = 1, .., I) which are assumed to be continuous and downward sloping, and $c(\mathbf{q})$ denoting production costs which are assumed to be continuously differentiable in q_i, for any i = 1, .., I. Now the price vector $\mathbf{p}^* = (p_1^*, .., p_M^*)$ that solves problem (4) is implicitly given by the I + 1 conditions

$$\begin{cases} \Pi(\mathbf{p}^*) = \overline{\Pi} \\ \left.\dfrac{\partial W}{\partial p_i}\right|_{\mathbf{p}^*} + \mu \left.\dfrac{\partial \Pi}{\partial p_i}\right|_{\mathbf{p}^*} = 0 \qquad \text{for } i = 1, .., I \end{cases} \tag{5}$$

where μ is the Lagrange multiplier.

It is straightforward to show that conditions defined in (3) are exactly equivalent to those defined in (5) as long as we limit problem (2) to the case of constant return to scale. Indeed, under constant return to scale **s** is constant and we can interpret the problem of selecting a tax structure as equivalent to choosing a structure of consumer prices (Sandmo, 1976).

2.2 The Ramsey- Boiteux condition

To provide a convenient interpretation of (3) and (5), we consider the further assumptions that *i*) for any given pair of goods i,j=1,…I, $i \neq j$, there is no demand cross elasticity and *ii*) $W(\mathbf{p},\mathbf{y})$ is defined as the simple sum the quasi-linear indirect utility functions of the H individuals purchasing the I goods, that is

$$W(\mathbf{p},\mathbf{y}) = \sum_{h=1}^{H} v_h(\boldsymbol{p}, y_h) = \sum_{h=1}^{H} u_h(\boldsymbol{p}) + y_h \tag{6}$$

Quasi-linear indirect utility function implies that the Roy's identity takes the form $\partial v_h/\partial p_i = -q_{i\,h}$, for any h= 1, …H , and any i= 1, …I (see for instance Varian, 1992 or Mas Colell et al., 1995) so that

$$-q_i(\mathbf{p}^*) = \left.\frac{\partial W(\mathbf{p},\mathbf{y})}{\partial p_i}\right|_{\mathbf{p}^*} \tag{7}$$

Under these assumptions the first order conditions defined by both (3) and (5) imply the well-known Ramsey-Boiteux condition

$$\frac{\dfrac{p_i^* - c_i}{p_i^*}}{\dfrac{p_j^* - c_j}{p_j^*}} = \frac{\eta_j}{\eta_i} \quad \forall i, j = 1,...,I, i \neq j \tag{8}$$

where η_i and η_j are the demand elasticity of good i w.r.t. p_i and the demand elasticity of good j w.r.t. p_j, respectively, and $p_i^* - c_i$ (i=1,…I) can be seen as the optimal departure from marginal costs either in terms of taxation (i.e. $t_i^* = p_i^* - c_i$ when we limit problem (2) to the case of constant return to scale) or in terms of monopoly pricing.

Condition (8) provides an operational rule telling us that when the demand elasticity of one good is higher than the demand elasticity of another good, the distance from the marginal cost should be less for the former than for the latter.

2.3 Distributional issues

Several authors (see Atkinson and Stiglitz, 1972 and Feldstein, 1972 among the others), however, noticed that the Ramsey-Boiteux condition may imply conflict between allocative efficiency and distributional objectives. Typically, commodities with low price elasticity are necessities while those with high elasticity are luxuries. Then condition (8) might imply that necessities should be taxed at higher rates than luxuries which may result undesirable from the distributional point of view since, typically, necessities represent a large share of expenditure for lower income consumers. This undesirable result, however, depends on the characterization of the social welfare function given in (6): the choice of a simple sum of quasi-linear indirect utility functions implies that the social welfare weight attached to any household is always the same or, equivalently, that the consumer side of the economy can be treated as if there were just one representative household.

Therefore, in order to extend the analysis to a many person economy and to combine both distribution and allocation, we follow Diamond and Mirrlees (1971) and go back to the more general individualistic social welfare function defined in (1). Differentiating (1) w.r.t. p_i we obtain

$$\frac{\partial W(\mathbf{p},\mathbf{y})}{\partial p_i} = \sum_{h=1}^{H} \frac{\partial W}{\partial v_h} \frac{\partial v_h}{\partial p_i} \tag{9}$$

where, by the Roy's identity, $\frac{\partial v_h}{\partial p_i} = -q_{i,h}\alpha_h$, for any h= 1, …H , and any i= 1, …I

and $\alpha_h = \frac{\partial v_h}{\partial y_h}$ is the marginal utility of income of consumer h. Therefore, we can rewrite (9) as

$$\frac{\partial W(\mathbf{p},\mathbf{y})}{\partial p_i} = -\sum_{h=1}^{H} \beta_h q_{i,h} \tag{9'}$$

where $\beta_h = \frac{\partial W}{\partial v_h} \alpha_h$ is the marginal social utility of income, or social welfare weight, of consumer h. We can use (9') to rewrite conditions (3) and (5), respectively, as follow:

$$\begin{cases} T\left(\mathbf{t}^*\right) = \overline{T} \\ -\sum_{h=1}^{H} \beta_h q_{i,h}\left(\mathbf{t}^*\right) + \lambda \frac{\partial T}{\partial t_i^*} = 0 \qquad \text{for } i = 1,..,I \end{cases} \tag{10}$$

and

$$\begin{cases} \Pi\left(\mathbf{p}^*\right) = \overline{\Pi} \\ -\sum_{h=1}^{H} \beta_h q_{i,h}\left(\mathbf{p}^*\right) + \mu \frac{\partial \Pi}{\partial p_i^*} = 0 \qquad \text{for } i = 1,..,I \end{cases} \tag{11}$$

2.4 The Feldstein optimal structure of public prices

The many household optimal taxation problem derived by Diamond and Mirrlees (1971) is formally equivalent to the less general framework used by Feldstein (1972) who tackles with an optimal public pricing problem in a multi-product context where the social planner aims at maximizing a welfare function expressed as weighted sum of the households' consumer surpluses. Formally, we can define (1) as $W(\mathbf{p};\mathbf{y}) = \sum_h S_h(\mathbf{p}) \cdot u'(y_h)$ and restate (11) as

$$\begin{cases} \Pi\left(\hat{\mathbf{p}}\right) = \overline{\Pi} \\ \sum_h \frac{\partial S_h}{\partial \hat{p}_i} u'(y_h) + \mu \frac{\partial \Pi}{\partial \hat{p}_i} = 0 \qquad \text{for } i = 1,..,I \end{cases} \tag{11'}$$

where $S_h(\mathbf{p})$ is the consumer surplus of household h, with $\frac{\partial S_h(\mathbf{p})}{\partial p_i} = -q_{i,h}(p_i)$, $\mathbf{y}$ is the H-dimensional vector of households' income, y_h is household's h income, and $u'(y_h)$ is the marginal social utility due to a small increase in the income of household h. Under standard assumptions on the shape of $u(y_h)$ ($u'>0$ and $u''<0$), the use of u' as welfare weights implies that society values more a marginal increase in utility by a low-income household than an equal increase for a high-income household. To make easier the comparison between the Ramsey-Boiteux conditions and those derived under Feldstein's (and Diamond and Mirrlees') model it is useful to define

$$R_i = \frac{\sum_{h=1}^{H} q_{i,h}(\mathbf{p}) \cdot u'(y_h)}{\sum_{i=1}^{H} q_{i,h}(\mathbf{p})} \qquad \text{for any } i = 1, .., I \qquad (12)$$

as the distributional characteristic of the good i. R_i is a weighted average of the marginal social utilities, where each household's marginal social utility is weighted by the quantity of good i consumed by that household. The conventional welfare assumption that $u''(y)<0$ implies that the value of R_i will be greater for goods that take a larger share of the budget of households with lower income (necessities) than for goods that take a larger share of the budget of high income households (luxuries).

In particular, when η_i denotes the own-price elasticity of good i and $\frac{\partial q_i}{\partial p_j} = 0$ for any $i, j = 1, .., I$ and $i \neq j$, we can get

$$\frac{\frac{\hat{p}_i - c_i}{\hat{p}_i}}{\frac{\hat{p}_j - c_j}{\hat{p}_j}} = \frac{\eta_j}{\eta_i} \frac{(R_i - \mu)}{(R_j - \mu)} \qquad \text{for any } i, j = 1, ..., I, i \neq j \qquad (13)$$

which corresponds to Feldstein's equation (9) (see Feldstein, 1972, p 34) and can be easily compared to the Ramsey-Boiteux condition defined above by (8). Also condition (13) provides an operational rule telling us how we must depart from the Ramsey-Boiteux condition when $R_i \neq R_j$, that is when goods differ for their distributional characteristics.

2.5 Marginal commodity tax and price reforms

Feldstein (1976) and most of the following literature shifted the emphasis from optimal commodity tax design to marginal commodity tax reforms. Marginal commodity tax reforms have been investigated by Ahamad and Stern (1984) as a viable approach to evaluate empirically a tax system. This approach consists in a specification of the economy and its initial equilibrium, together with a social welfare function and its welfare weights, aimed at verifying if social welfare improvements can be obtained by marginal tax variations.

From conditions (3) we get

$$\lambda = -\frac{\frac{\partial W}{\partial t_i}}{\frac{\partial T}{\partial t_i}} = -\frac{\frac{\partial W}{\partial t_j}}{\frac{\partial T}{\partial t_j}} \qquad \text{for any } i,j = 1,...I \text{ and } i \neq j \tag{14}$$

which implies that, when both $\frac{\partial T}{\partial t_i}$ and $\frac{\partial T}{\partial t_j}$ are not equal to zero, a sufficient condition for a marginal commodity tax reform to be welfare improving is that the following holds for at least a pair of goods

$$-\frac{\frac{\partial W}{\partial t_i}}{\frac{\partial T}{\partial t_i}} \neq -\frac{\frac{\partial W}{\partial t_j}}{\frac{\partial T}{\partial t_j}} \qquad i,j = 1,...I \ (i \neq j). \tag{15}$$

More specifically, $-\frac{\partial W}{\partial t_i} \Big/ \frac{\partial T}{\partial t_i} \geq -\frac{\partial W}{\partial t_j} \Big/ \frac{\partial T}{\partial t_j}$ would mean that the marginal social cost of taxing commodity *i* (i.e. the welfare loss caused by a marginal increase in t_i relative to the corresponding gain caused in the tax revenue) is greater than the marginal social cost of taxing commodity *j*, implying that we could increase social welfare without reducing the tax revenue (or, alternatively, increase the tax revenue without reducing the social welfare) by increasing t_j and decreasing $t_{i..}$

For an exhaustive and updated survey of the existing literature on marginal commodity tax reforms we refer to Santoro (2007) who also stresses the related theoretical limitations and implementation issues. Here, instead, we are more interested in showing how easily we can replicate the Ahamad and Stern's idea of marginal tax reforms to a pricing context characterized by multi-product monopoly (Coady, 2006).

As a matter of fact, condition (5) implies

$$\mu = -\frac{\frac{\partial W}{\partial p_i}}{\frac{\partial \Pi}{\partial p_i}} = -\frac{\frac{\partial W}{\partial p_j}}{\frac{\partial \Pi}{\partial p_j}} \quad \text{for any } i,j = 1,...I \text{ and } i \neq j . \tag{16}$$

Therefore. the sufficient condition for a welfare improving marginal price reform is

$$\frac{\frac{\partial W}{\partial p_i}}{\frac{\partial \Pi}{\partial p_i}} \neq \frac{\frac{\partial W}{\partial p_j}}{\frac{\partial \Pi}{\partial p_j}} \qquad i,j = 1,...I \ (i \neq j). \tag{17}$$

which implies that, as long as $-\frac{\partial W}{\partial p_i} \Big/ \frac{\partial \Pi}{\partial p_i} \geq -\frac{\partial W}{\partial p_j} \Big/ \frac{\partial \Pi}{\partial p_j}$, we could increase social welfare without reducing profits (or, alternatively, increase profits without reducing the social welfare) by increasing p_j and decreasing $p_{i.}$

2.6 The inverse optimum problem

It could be argued that the existence of social welfare improvements depends on the social welfare function that has been chosen at first. Even if the initial equilibrium taxes and/or prices admit welfare improvements for a specific social welfare function it is still possible that those taxes and/or prices are an optimum for another social welfare function. Under this respect one could evaluate a tax system from a different perspective aimed at finding out "*whether there is a set of* *value judgements* [i.e. a set of welfare weights] *under which, given the model of the economy, the initial state of affairs would be deemed as optimum. That is the inverse optimum problem. The value judgements may then be used in a number of ways. One might infer that these are indeed the value judgements of the government and use them in appraising other decisions. Or if the computed value judgements were seen as objectionable, then they could be employed to criticise the existing state of affairs. in the sense that it could be seen as optimum only with respect to disagreeable values*" (Ahamad and Stern, 1984, p. 259).

Almost contemporaneously, but independently, both Ahamad and Stern (1984) and Ross (1984) proposed an easy procedure to extract social welfare weights from the existing tax and price structure, respectively. For an optimal indirect taxation problem, we can compact the system of I equations reported in the second line of (10) in the following way

$$\boldsymbol{\beta}'\cdot\mathbf{Q} = \lambda\mathbf{T}' \tag{18}$$

where $\boldsymbol{\beta}'$, $\mathbf{Q}$ and $\mathbf{T}'$ are, respectively, the transpose of the (Hx1) vector of social welfare weights with hth element β_h, the (HxI) consumption matrix with hith element $q_{i,h}$, and the transpose of the (Ix1) vector with ith element $\frac{\partial T}{\partial t_i^*}$. The inverse optimum problem (IOP) consists in finding out the vector $\boldsymbol{\beta}$ that satisfies (18). When $I=H$, the solution of the IOP is

$$\boldsymbol{\beta}' = \mathbf{T}'\cdot\mathbf{Q}^{-1} \tag{19}$$

where $\mathbf{Q}^{-1}$ is the inverse of $\mathbf{Q}$ and, as in Ahmad and Stern (1984), we conveniently set $\lambda = 1$. The IOP of an equivalent optimal public pricing problem can be easily defined along the same lines in terms of finding out the vector $\boldsymbol{\beta}$ that satisfies $\boldsymbol{\beta}'\cdot\mathbf{Q} = \lambda\boldsymbol{\Pi}'$, where $\boldsymbol{\Pi}'$ is the transpose of the (Ix1) vector with $\frac{\partial \Pi}{\partial p_i^*}$ being the ith element. The solution in this case is

$$\boldsymbol{\beta}' = \prod{}'\cdot\mathbf{Q}^{-1} \tag{19'}$$

2.7 Welfare improving and poverty reducing marginal price reforms

Ahamad and Stern (1984) recognise that their theory of marginal commodity tax reforms relies on specific, and possibly controversial, social welfare functions. For this reason they suggest also an alternative approach aimed at discovering possible Pareto-improving tax reforms. Even if the Paretian approach avoids normative value judgements, it is nonetheless of little practical importance since it would require that no household is negatively affected by the reform. On this motivation Mayshar and Yitzhaki (1995) generalize Yitzhaki and Slemrod (1991) and propose an intermediate approach based on the Daltonian principle. According to this principle, a tax reform improves social welfare if, given a prior social ranking of households, it redistributes from high-ranking to low-ranking households (let say from a rich to a poor), without reverting the initial ranking. As a matter of fact, assume that the only information on the social welfare function is that, for any pair of households $h,k = 1,...H \quad (h \neq k)$ $\beta_h \geq \beta_k$ whenever $y_h \leq y_k$. We can define a marginal increase in welfare as a positive weighted sum of variations of equivalent income, that is

$$\mathrm{d}W = \sum_{h=1}^{H} \beta_h \, \mathrm{d}y_h \geq 0 \tag{20}$$

where $\mathrm{d}y_h$ is the variation of equivalent income of household *h*, that is the variation of income that in terms of household's *h* utility is equivalent to the variation of prices. Since we can rewrite (20) as

$$\sum_{h=1}^{H} \beta_h \, \mathrm{d}y_h = \sum_{h=i}^{H-1} \left[\left(\beta_h - \beta_{h+1}\right) \sum_{k=1}^{h} \mathrm{d}y_k \right] + \beta_H \sum_{k=1}^{H} \mathrm{d}y_k \geq 0 \tag{20'}$$

and since $\beta_H \geq 0$ and $\beta_h \geq \beta_{h-1}$, then a sufficient condition for a marginal increase in welfare is $\sum_{k=1}^{h} \mathrm{d}y_k \geq 0$ for any *k*=1,…*H*. Of course this framework applies to both a commodity tax and a public monopoly context since the variation of prices causing an equivalent variation $\mathrm{d}y_h$ to household h can be due to both tax and pricing policy.

Within this stream of literature some scholars have extended the analysis even further by considering marginal indirect tax and pricing reforms as a possible poverty-reducing instrument (see, for instance, Yitzhaki and Slemrod, 1991, Makdissi and Wodon, 2002 and Liberati, 2003) in a framework that can also include higher order classes of ethical judgments that the Daltonian principle used in Mayshar and Yitzhaki (1995) (see also Duclos, Makdissi and Wodon, 2008).

We follow most of this literature and define $f(y)$ as the density function of income $y \in [0, y^{\max}]$, $\mathbf{p}^R$ as a (1xI) vector of reference prices that can be used to asses the consumers' welfare in the presence of prices' variations, $y^E(y,\mathbf{p},\mathbf{p}^R)$ as the equivalent income function that, for any level of y, provides consumers who face the reference prices $\mathbf{p}^R$ with the same level of utility that they would yield if they faced $\mathbf{p}$, i.e.

$$v(y^E(y,\mathbf{p},\mathbf{p}^R),\mathbf{p}^R) \equiv v(y,\mathbf{p})\,. \tag{21}$$

We still consider a setting where H households consume I goods but, for expositional reasons, we refer only to price variations. Indeed, when prices are under the direct control of the social planner, the results obtained for the impact on poverty and social welfare of balanced budget marginal price reforms are similar to that for the impact of indirect tax reforms (Makdissi and Wodon, 2007).

Consider first the standard problem of a social planner aiming at increasing welfare. We deal with welfare indices defined by utilitarian social welfare functions

$$U = \int_0^{y^{\max}} u(y^E(y,\mathbf{p},\mathbf{p}^R))f(y)dy \tag{22}$$

such that $U \in \Omega^s$ (s=1,2,…), where the classes Ω^s is defined as

$$\Omega^s = \left\{U \middle| u(y^E) \in C^s(\infty), (-1)^{i+1} u^{(i)}(y^E) \geq 0 \; for \, i = 1,2,...s \right\}. \qquad (23)$$

where $C^s(\infty)$ is the set of continuous function that are s-time differentiable over $\mathbb{R}_+$. Within this framework we can give a specific normative interpretation to every class Ω^s. First of all, for any $s\geq1$, social welfare indices are Paretian, that is they weakly increase (i.e. $u^{(i)}(y^E)\geq o$) when an individual's income increases, and obey to symmetry (or anonymity) axiom, that is interchanging any two individuals' incomes does not modify the social welfare indices. Moreover, when $s\geq2$, social welfare indices are concave and respect the Daltonian principle of transfer. When $s\geq3$, social welfare increases if, provided that the variance of the distribution does not increase, an adverse Daltonian transfer in the upper part of the distribution is accompanied by a beneficial Daltonian transfer within the lower part of the distribution. Further interpretations for higher classes of welfare indices are also possible (see Fishburn and Willig, 1984) but are not discussed here.

A very similar setting can be used for the case of a social planner whose task is to reduce poverty. As a matter of fact, a poverty index can be thought as a social welfare index censored at a poverty line (Duclos and Makdissi, 2004), hence we can express poverty indices as

$$P(z) = \int_0^{y^{\max}} \psi(y^E(y,\mathbf{p},\mathbf{p}^R), z) f(y) dy \qquad (22')$$

where $P(z)$ is an additive poverty index, z is the poverty line that, for the sake of convenience, is assumed to be defined in the equivalent income space, and $\psi(y^E(y,\mathbf{p},\mathbf{p}^R), z)$ is the contribution to total poverty of a consumer with an equivalent income $y^E\leq z$. We consider the classes of poverty indices $P(z) \in \Psi^s$ defined as

$$\Psi^s(z) = \left\{ P(z) \middle| \begin{array}{c} \psi(y^E, z) = 0 \text{ if } y > z, \psi(y,z) \in \hat{C}^s(z), \\ (-1)^i \psi^{(i)}(y^E, z) \geq 0 \text{ for } i = 0,1,...,s \\ \psi^{(i)}(z,z) = 0 \text{ for } i = 0,1,...,s-2 \text{ when } s \geq 2 \end{array} \right\} \qquad (23')$$

where $\hat{C}^s(z)$ is the set of function that are s-time piecewise differentiable over $[0,z]$ with respect of y^E and the normative meaning of classes $\Psi^s(z)$ is basically the same of Ω^s. A fundamental instrument to compare alternative distributions of incomes in terms of poverty indices of classes $\Psi^s(z)$ is the stochastic dominance curve (see, for instance, Davidson and Duclos, 2000) that, when $\mathbf{p}=\mathbf{p}^R$, can be written as

$$D^s(z) = \frac{1}{(s-1)!}\int_0^z [z-y]^{(s-1)} f(y)dy. \tag{24}$$

It is possible to show (see Duclos and Makdissi, 2004) that, for any $U \in \Omega^s$, a sufficient condition for U_b-$U_a \geq 0$, that is for improving welfare by moving from the relative density functions f_a to f_b, is

$$\begin{aligned} &D_a^s(y) - D_b^s(y) \geq 0 \ \text{ for any } y \in \left[0, y^{\max}\right] \\ &\text{and, if } s \geq 3, \\ &D_a^s(y^{\max}) - D_b^s(y^{\max}) \geq 0 \ \text{ for any } i \in \{2,\dots,s-1\}. \end{aligned} \tag{25}$$

Analogously, for any $z \in \left[0, z^+\right]$ and any $P(z) \in \Psi^s$, a sufficient and necessary condition for $P_b(z)$-$P_a(z) \leq 0$, that is for reducing poverty by moving from the relative density functions f_a to f_b, is

$$D_a^s(y) - D_b^s(y) \geq 0 \ \text{ for any } y \leq z^+. \tag{25'}$$

However, when we want to assess how a given income distribution is affected by a marginal price reform we need to consider how stochastic dominance curves are affected by such reforms and deal with normalized consumption dominance curves which are defined as

$$\overline{CD}_k^s(z) = H\frac{CD_k^s(z)}{q_k(\mathbf{p})} \tag{26}$$

where $CD_k^s(z) = \frac{\partial D^s(z)}{\partial p_k}$ is the consumption dominance curve of good k (k=1, …, I), and $\frac{q_k(\mathbf{p})}{H}$ is the normalizing factor which is the reciprocal of the average consumption of that good.

The impact on the stochastic dominance curve of a marginal price reform that keeps the firm's profit constant is, by definition,

$$dD^s(z) = \sum_{i=1}^{I} CD_i^s(z) dp_i \tag{27}$$

that, in the simpler case of a marginal price reform that decreases the price of good i, increases the price of good j, can be shown (Makdissi and Wodon, 2007) to be equal to

$$dD^s(z) = \left[\overline{CD}_i^s(z) - \gamma \overline{CD}_j^s(z)\right] \frac{q_i(\mathbf{p})}{H} dp_1 \tag{27'}$$

where $\gamma = -\frac{dp_i}{dp_j} \frac{q_j(\mathbf{p})}{q_i(\mathbf{p})}$.

By (25) and (27') we can show that, for any $U \in \Omega^s$,

$$\begin{aligned} &\overline{CD}_i^s(y) - \gamma \overline{CD}_j^s(y) \geq 0 \text{ for any } y \in \left[0, y^{\max}\right] \\ &\text{and, if } s \geq 3, \\ &\overline{CD}_i^\sigma(y^{\max}) - \overline{CD}_j^\sigma(y^{\max}) \geq 0 \text{ for any } \sigma \in \{2, \ldots, s-1\}. \end{aligned} \tag{28}$$

is a sufficient condition for increasing U when we marginally decreases the price of good i and increases the price of good j in order to keep the firm's profit constant.

Similarly, by (25') and (27') we can show that, for any $P(z) \in \Psi^s$,

$$\overline{CD}_i^s(y) - \gamma \overline{CD}_j^s(y) \geq 0 \text{ for any } y \in \left[0, z^+\right] \tag{28'}$$

is a necessary and sufficient condition for increasing $P(z)$ when we marginally decreases the price of good i and increases the price of good j in order to keep the firm's profit constant.

In the next section we will see how Makdissi and Wodon (2007) extend these results to the context where the social planner has not prices under her direct control and she has to rely on price cap regulations.

3 The normative analysis of price cap regulation: allocative efficiency, distributional and poverty issues

Price cap is a regulatory instrument typically used to control the dynamic of prices in utility markets which are characterized by some degree of market power. If the regulated market is a multi-product monopoly and the regulator is a benevolent social welfare maximizer, her objective can be still represented as the problem we have already outlined in (4). The regulator's possibility of solving that maximization problem depends greatly on her knowledge of demand and cost functions. In fact, almost any form of regulation is characterized by asymmetric information where the less informed part is supposed to be the regulator who cannot directly observe either some behaviour by the firm - usually the level of effort put to reduce costs - or the realisation of some stochastic parameter generally regarding the structure of cost and/or demand. On the other hand, the regulated firm knows these parameters but does not have incentives to truthfully report them or to behave in accordance with the regulator's wishes.

Price cap regulation represents a useful instrument which is easy to implement and allows to bypass the regulatory problems due to asymmetric information. As a matter of fact, price cap is a non-Bayesian regulatory instrument in the sense that the regulator can implement and enforce the contract with no need of having prior information - even in probabilistic terms – on the unobservable parameters of the problem. In fact, in multi period contexts price cap regulation can be designed as a routine that allows to enforce socially efficient prices (at least in the long run).

Moreover, price cap regulation belongs to fixed-price contracts (i.e. the regulated firm has no chance to affect the cap on its prices) that always guarantee productive efficiency because the firm is residual claimant of any possible gain due to its effort of reducing costs.

3.1 Price cap regulation and Ramsey-Boiteux prices: the Vogelsang and Finsinger mechanism

Vogelsang and Finsinger (1979) first highlighted that a Laspeyres-type price cap can be structured as an incentive mechanism which enforces the use of Ramsey-Boiteux prices by a multiproduct monopolist.

Suppose the regulatory maximization problem is that defined in (4) where the welfare function is defined as the simple sum the quasi-linear indirect utility functions of the H individuals purchasing the I goods and, therefore, (6) and (7) apply. Let $\mathbf{p}^t$ be the I-dimensional vector of market prices at time t, where $t = 0, \ldots, \infty$ and assume the regulated monopolist myopically maximises its profits $\Pi(\mathbf{p}^t) = \sum_i p_i^t \, q_i(\mathbf{p}^t) - c(\mathbf{q}(\mathbf{p}^t))$ in each period of time t, where $c(\mathbf{q}(\mathbf{p}^t))$ is the cost function at period t when the firm fixes a vector of prices $\mathbf{p}^t$ and sells the corresponding vector of quantities $\mathbf{q}(\mathbf{p}^t)$. The cost function has the same properties discussed in section 2 for the single period case and it is also assumed to show decreasing ray average cost, that is $c(\lambda\mathbf{q})\leq\lambda\, c(\mathbf{q})$ for any $\lambda \geq 1$. Both cost and demand functions are supposed to be stable over time while myopia implies that the regulated firm does not maximise any discounted flow of future profits, disregarding the effects that its choice at any time t may have on the problem it has to face in the subsequent periods. The regulator does not know neither the demand functions nor the cost function. Nevertheless, in any period t, the regulator can observe both the total cost which has been realised by the firm in the previous period and the corresponding vector of sold quantities $\mathbf{q}(\mathbf{p}^{t-1})$.

Within this framework, Vogelsang and Finsinger (1979) suggest a bright sequential mechanism, or algorithm, built on a price constraint just exploiting the regulator's capacity to observe those previous period's realisations. Suppose that t=1 is the period of time when the mechanism is implemented for the first time. Then, the regulatory constraint requires that the vector of prices chosen by the firm at any period t must satisfy the following inequality:

$$\mathbf{q}(\mathbf{p}^{t-1})\cdot\mathbf{p}^{t}-c(\mathbf{q}(\mathbf{p}^{t-1}))\leq 0. \tag{29}$$

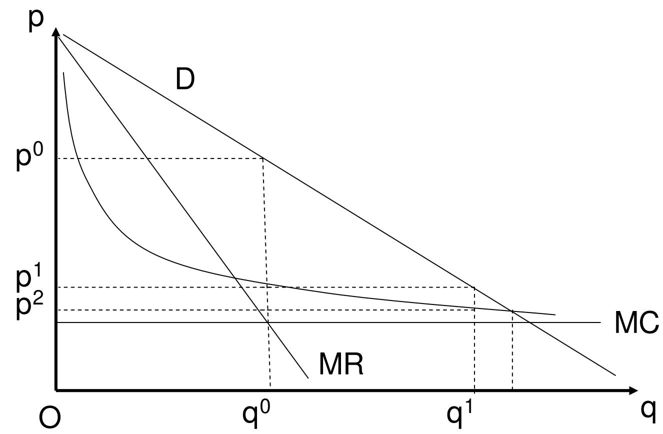
In words, the pseudo-revenue given by multiplying the previous period's vector of quantities by the current vector of prices cannot exceed the total cost occurred to the firm at time t-1. Then, if we start with positive profit at t=0, (29) requires that $\mathbf{p}^1$ cannot be equal to $\mathbf{p}^0$ and, in general, $\mathbf{p}^t$ cannot be equal to $\mathbf{p}^{t-1}$ until the zero profit contingency takes place. Furthermore, positive profit at t=0 and decreasing ray average cost causes (29) to induce $\Pi^t \geq 0$ for any subsequent period. Indeed, as prices go down, profits tendency to decrease is partially balanced by the assumption of decreasing average cost. As a matter of fact, as prices go down, quantities go up and decreasing average cost assures that unit costs go down. Under the above assumptions, it can be also shown that, whenever Π^{t-1} is positive, (29) guarantees $W(\mathbf{p}^t) \geq W(\mathbf{p}^{t-1})$ and the sequence of the price vector $\{\mathbf{p}^t\}$ converges to a long run stationary equilibrium where social welfare is maximized under the $\overline{\Pi}$=0 constraint.

Here we provide a graphical intuition of these results for the simpler single product case. When I=1, the constraint (29) becomes $q(p^{t-1})\cdot p^{t}-c(q(p^{t-1}))\leq 0$ which implies

$$p^t \leq \frac{c(q(p^{t-1}))}{q(p^{t-1})} \tag{29'}$$

that is the price chosen by the firm at time t cannot be higher that the average costs at time t-1. The assumption of decreasing (ray) average cost implies the following figure:

Figure 1



Let t=1 be the first period when the price cap (29') comes into force and be p^0 and q^0 the profit maximizing price and quantity pair: then, according to (29'), p^1 is the highest level of price – equal to the firm's average costs at time 0 – that the firm will charge at time 1 and q^1 will be the corresponding level of quantity that will be produced and supplied. Given q^1, p^2 is the highest level of price – equal to the firm's average costs at time 1 – that the firm will charge at time 2 and so on till the stationary point where average costs and demand cross each other. This converging process depends on the assumption of decreasing average costs. Indeed, under increasing (ray) average costs either the process converges to second best prices with profits and losses following each other in a hog cycle or, if the average cost curve is steeper than the demand curve (in absolute terms) the process does not converge and some further steps must be added to the basic regulatory algorithm consisting in the regulator imposing (29) whenever firm's profits where positive in the previous period (see the flow chart II at figure 8, p. 169 of Vogelsang and Finsinger, 1979).

The price cap formula proposed by Vogelsang and Finsinger (1979) has some similarity with the *RPI-X* price cap first introduced in 1984 for regulating British

Telecom (Littlechild, 1983) and then adopted in many other markets and countries (OECD, 2000). The *RPI-X* constraint is a limit over the increase of a Laspeyres price index, that is

$$\frac{\mathbf{p}^t \cdot \mathbf{q}(\mathbf{p}^{t-1})}{\mathbf{p}^{t-1} \cdot \mathbf{q}(\mathbf{p}^{t-1})} \leq \frac{RPI^t}{RPI^{t-1}} - X \tag{30}$$

where $RPI^{\,t}$ is the retail price index at period t and X is an exogenous adjustment factor aimed at inducing productivity improvements over time. This formula can be rewritten, and it is usually presented, as a *RPI-X* threshold to a weighted average of the prices' changes over time

$$\sum_{i=1}^{I} \frac{p_i^t}{p_i^{t-1}} \cdot \frac{p_i^{t-1} \cdot q_i(\mathbf{p}^{t-1})}{\mathbf{p}^{t-1} \cdot \mathbf{q}(\mathbf{p}^{t-1})} \leq \frac{RPI^t}{RPI^{t-1}} - X \tag{30'}$$

where the weights are the firm's revenue shares calculated at period t-1. This *RPI-X* is essentially similar to the V-F mechanism given in (29) that can be rewritten as

$$\mathbf{p}^t \cdot \mathbf{q}(\mathbf{p}^{t-1}) - c(\mathbf{q}(\mathbf{p}^{t-1})) \leq 0 = \mathbf{p}^{t-1} \cdot \mathbf{q}(\mathbf{p}^{t-1}) - c(\mathbf{q}(\mathbf{p}^{t-1})) - \Pi(\mathbf{p}^{t-1})$$

in order to obtain

$$\frac{\mathbf{p}^t \cdot \mathbf{q}(\mathbf{p}^{t-1})}{\mathbf{p}^{t-1} \cdot \mathbf{q}(\mathbf{p}^{t-1})} \leq 1 - \frac{\Pi(\mathbf{p}^{t-1})}{\mathbf{p}^{t-1} \cdot \mathbf{q}(\mathbf{p}^{t-1})} \tag{31}$$

Indeed, if we allow for an inflationary element in costs, (31) is the same as the tariff basket *RPI-X* approach with X varying from period to period according to the size of profits (see also Bradley and Price, 1988). The Laspeyres type price caps' property of converging towards Ramsey-Boiteux prices is also showed by Brennan (1989) for a further simplified version of (31) where the second term on the right-hand side is set equal to zero.

3.2 Distributional issues of RPI-X regulation and the Generalized Price Cap

As we have seen is section 2.3, there may exist some possible adverse distributional effects of Ramsey-Boiteux prices since they entail higher mark-ups on those goods with lower demand elasticity which, in turn, often represent a large share of low-income consumers' expenditures. There have been a number of papers (see, for instance, Hancock and Waddams Price, 1995 and 1998) that have questioned the desirability of the so-called process of tariff re-balancing undertaken by many price capped utilities. This process has entailed a sharp rise in the price of items with low price elasticity and a decrease in the price of items whose demand is more sensitive to price changes with a largely documented regressive effect.

This widespread concern led Oftel (the former regulator of the telecommunications industry in the UK) to modify in 1997 the *RPI-X* formula that had been used since 1984 to regulate the prices set by British Telecom. Basically, Oftel shifted from a typical Laspeyres type price cap as in (30') to a new price cap formula where different weights were chosen for price changes of the different goods included in the regulated bundle. These weights were no longer the revenue shares for the previous period but the shares of total revenues accruing to the regulated firm only from those consumers who are in the first eight deciles of total expenditure in telecommunications services. This new price cap formula implied that a stricter control was placed on the prices of the goods that make up a large share of the typical bill of low-consumption customers. Formally, indicating by $\breve{q}_i$ the quantity of good *i* purchased by consumers who are in the first eight deciles of total expenditure in telecommunications services, the price cap formula adopted by Oftel can be approximated by the following:

$$\sum_{i=1}^{I} \frac{p_i^t}{p_i^{t-1}} \frac{p_i^{t-1} \breve{q}_i^{t-1}}{\sum_{j=i}^{I} p_j^{t-1} \breve{q}_j^{t-1}} \leq RPI - X \ . \tag{32}$$

It is straightforward to show that, if $\partial W \big/ \partial p_i^{t-1} = \breve{q}_i^{t-1}$ for some specification of $W(\mathbf{p}, \mathbf{y})$, (32) can be related to the generalized price cap (GPC) formula proposed by Iozzi, Poritz and Valentini (2002). For the simplified case of *RPI*=1 and *X*=0, the GPC can be written as

$$\sum_{i=1}^{I} \frac{p_i^t}{p_i^{t-1}} \frac{p_i^{t-1} \dfrac{\partial W}{\partial p_i^{t-1}}}{\sum_{j=i}^{I} p_j^{t-1} \dfrac{\partial W}{\partial p_j^{t-1}}} \leq 1 \tag{33}$$

When the regulator knows her preferences, she can attach a value to any $\partial W \big/ \partial p_i^{t-1}$ and implement (33). Moreover, if the regulator can observe the prices set by the firm in the previous and current periods, she is able to check whether the firm is complying with (33) and enforce it.

Iozzi, Poritz and Valentini (2002) prove that, if the social welfare function $W(\mathbf{p}, \mathbf{y})$ is quasi-convex and under the other hypotheses we have already discussed in 3.1, the application of a GPC like (33) to a multi-product monopolist may guarantee that social welfare does not decrease over time. To show this point it is convenient to rewrite the GPC as

$$\sum_{i=1}^{I} p_i^t \cdot \frac{\partial W\left(\mathbf{p}^{t-1}\right)}{\partial p_i} \geq \sum_{i=1}^{I} p_i^{t-1} \cdot \frac{\partial W\left(\mathbf{p}^{t-1}\right)}{\partial p_i} \tag{33'}$$

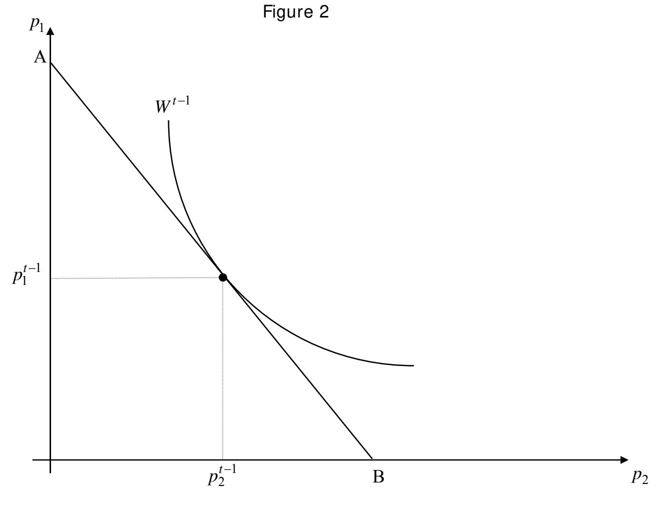
where the direction of the inequality is simply due the negative values of the derivatives of the welfare functions. From (33') it must be

$$\sum_{i=1}^{I} \left(p_i^t - p_i^{t-1}\right) \cdot \frac{\partial W\left(\mathbf{p}^{t-1}\right)}{\partial p_i} \geq 0$$

which, since we are assuming that $W(\boldsymbol{p})$ is quasi-convex and strictly decreasing in prices, implies $W(\mathbf{p}^t) \geq W(\mathbf{p}^{t-1})$, that is social welfare is monotonically non-decreasing in time.

In Figure 2 we can see this result in graphical terms. Let W^{t-1} be the iso-welfare curve going through the price vector $\mathbf{p}^{t-1}$. By totally differentiating $W(\cdot)$, it is straightforward to show that the slope of W^{t-1} at $\mathbf{p}^{t-1}$ is $-\frac{\partial W / \partial p_1^{t-1}}{\partial W / \partial p_2^{t-1}}$. Note, from (33'), that this is also the slope of the GPC constraint imposed on the firm at time *t*. As the prices set by the firm at time *t*-1 satisfy as an equality the GPC constraint at time *t*, the tangent to W^{t-1} at $\mathbf{p}^{t-1}$ and the GPC constraint at time *t* are actually the same line. Thus, the GPC restricts the set of feasible prices for the firm at time *t* to those on (or below) a line tangent to the iso-welfare curve going through the prices set at time *t*-1. Because of the quasi-convexity of the welfare function, the GPC constraint never lies above the iso-welfare line at *t*-1. Thus, it cannot happen that a vector of prices selected by the firm at time *t* and satisfying the GPC constraint reduces social welfare.

Figure 2



Moreover, Iozzi Poritz and Valentini (2002) show that, under the GPC, the sequence of prices chosen by a regulated firm that maximizes profits in each period t converges to a price vector which respects the allocative optimum conditions defined in the second line of (5). In other terms, when the regulated firm faces a constraint as in (33), the only long run equilibrium is such that the firm chooses the price vector which maximizes social welfare, given that the firm obtains a specified amount of profits in equilibrium. Here we provide a *heuristic* argument of this sequence convergence, mainly based on graphical interpretation, while we refer the interested reader to the original paper for a more rigorous proof (see Proposition 2 of Iozzi, Poritz and Valentini, 2002, p. 102).

First of all, it must be noted that the price vector $\mathbf{p}^*$, coming as the result of the maximization of social welfare given a constraint on the minimum profit level, can also be obtained as the solution to the dual problem of maximizing firm's profits under a constraint of a minimum level of welfare. Note also that the GPC can be seen as a linear approximation of the constraint on the welfare when this is fixed at the level $W(\mathbf{p}^{t-1})$. In a two-goods case (see again Figure 2) this observation implies that in any period the GPC can be seen as the line tangent to the iso-welfare contour at the prices set in the previous period. Therefore, in any period t, the regulated monopolist chooses its optimal price vector $\mathbf{p}^t$ such that the upper contour set $\Pi(\mathbf{p}^t)$ is tangent to the GPC constraint. Since the GPC corresponds to the slope of the welfare function at $\mathbf{p}^{t-1}$ prices, two alternative possibilities can occur. The first possibility is that $\mathbf{p}^t$ is not equal to $\mathbf{p}^{t-1}$ as it is illustrated in Figure 3. Therefore, the GPC constraint at time t+1 (the line A'B') is different from the GPC constraint at time t (the line AB), implying that the process of convergence is not finished yet and the level of social welfare is still increasing over time.

Figure 3

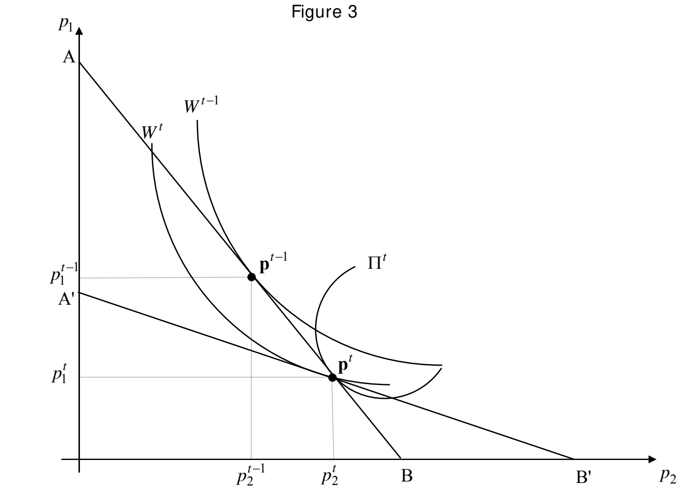
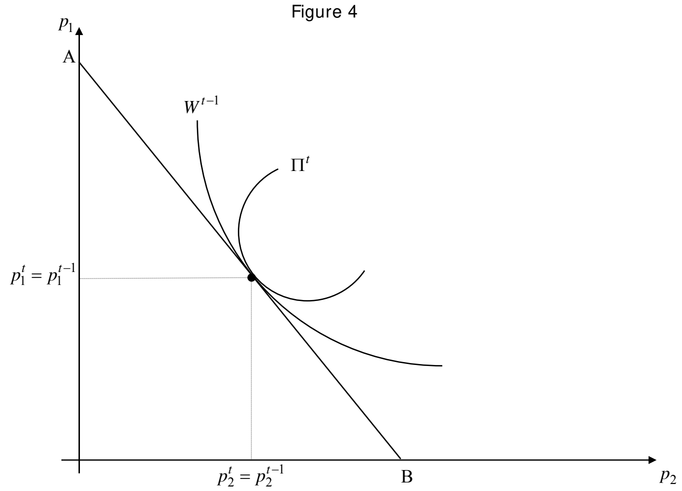


Figure 4



The second possibility, instead, is that $\boldsymbol{p}^t$ is equal to $\boldsymbol{p}^{t\text{-}1}$ which implies that the GPC will not move in the following period (i.e. the convergence is concluded) and the iso-profit and the iso-welfare are tangent to each other at $\boldsymbol{p}^t$, which is exactly what the constrained welfare maximisation requires. This alternative situation is illustrated in Figure 4.

Iozzi, Poritz and Valentini (2002) provide a description of the properties of price cap regulatory schemes under very general hypothesis on the structure of the regulator's preferences. Their result then can be interpreted as a generalisation of Vogelsang and Finsinger (1979) and Brennan (1989) where the convergence to Ramsey-Boiteux prices is optimal as long as the welfare function is strictly utilitarian (i.e. when it is an un-weighted sum of the individuals' welfare).

Since the only restriction on the welfare function which it is required in Iozzi, Poritz and Valentini (2002) is that it is quasi-convex, we can assert that the GPC is able to guarantee a long run equilibrium with optimal prices for almost *any* welfare function; hence, when the welfare function is strictly utilitarian and consumers have quasi-linear preferences, $\frac{\partial W}{\partial p_i} = -q_i$, for all i = 1, .. , I, and the GPC simply takes the form of the Laspeyres-type price cap studied by Brennan (1989).

Similarly, the GPC can be accommodated to provide a specification which is suitable for the case of distributionally weighted utilitarian preferences. Indeed, when the regulator's preferences can be represented by the following welfare function $W(\mathbf{p};\mathbf{y}) = \sum_h S_h(\mathbf{p}) \cdot u'(y_h)$, we can show that the GPC defined by (33') can guarantee the convergence of the prices set by the regulated firm to the optimal prices as defined by condition (11'), provided that the (33') takes the following characterization

$$\sum_{i=1}^{I} p_i^t \cdot \tilde{q}_i^{t-1} \leq \sum_{i=1}^{I} p_i^{t-1} \cdot \tilde{q}_i^{t-1} \tag{34}$$

where,

$$\tilde{q}_i^{t-1} = R_i^{t-1} q_i^{t-1} = \sum_{h=1}^{H} q_i^h(\mathbf{p}) \cdot u'(y^h) \tag{35}$$

with R_i^{t-1} which is the distributional characteristic of the good i at time t as it is defined in (12). Then $\tilde{q}_i^{t-1}$ is an adjusted measure of the aggregate consumption of good i at time t-1, which entails that the quantities consumed by each individual are adjusted using the marginal social utility of income of that individual.

It is quite easy to prove that when the GPC takes the form (32), social welfare can never decrease in time, and the sequence of price vectors $\{\boldsymbol{p}^t\}$ which come as the solution of the firm's maximization problem converges to a unique vector which satisfies the first order conditions of problem (4) for the special case when, as in Feldstein (1972), $W(\mathbf{p};\mathbf{y}) = \sum_h S_h(\mathbf{p}) \cdot u'(y_h)$. Indeed, $\sum_h S_h(\mathbf{p}) \cdot u'(y_h)$ is a strictly decreasing function in prices which respects the required properties of continuously differentiability and quasi-convexity. Moreover, the constraint in (34) is identical to the GPC that has been defined in (33') since, from (35) and from the fact that $\dfrac{\partial S_h(\mathbf{p})}{\partial p_i} = -q_{i,h}(p_i)$, we have $\dfrac{\partial W(\mathbf{p}^{t-1})}{\partial p_i} = -\tilde{q}_i^{t-1}$.

It is straightforward to see that $\breve{q}_i^{t-1} = \tilde{q}_i^{t-1}$, that is the specification of the GPC given in (32) as representation of the price cap formula implemented by Oftel in 1997 is equal to the specification given in (34), as long as the social welfare weights (that in the case of $W(\mathbf{p};\mathbf{y}) = \sum_h S_h(\mathbf{p}) \cdot u'(y_h)$ are the social marginal utility of income, $u'(y_h)$) are one for households belonging to the first eight deciles of expenditure, and zero otherwise.

3.3 Uncovering social welfare weights under price cap regulation

Valentini (2006) extends the analysis of Iozzi, Poritz and Valentini (2002) and explores the possibility of adapting the framework suggested by Ross (1984) in order to detect the implicit welfare weights of a regulator who is implementing a GPC. Since in Ross (1984) prices are directly chosen by the regulator, they exactly reveal the regulator's preferences over consumers, and the strategy of inverting a "generic" Ramsey formula with potentially diverse welfare weights may be usefully followed. Under price cap regulation, instead, the observed prices might be not optimal since they eventually converge to the second best in the long run. However, if we assume that the regulator's maximization problem is given as in (4), that is,

$$\begin{aligned}&\max_{\mathbf{p}} \quad W(\mathbf{p},\mathbf{y})\\&\text{s.t. } \Pi(\mathbf{p}) \geq \overline{\Pi}\end{aligned}$$

and the price cap rule is given by

$$\sum_{i=1}^{I} \frac{p_i^t}{p_i^{t-1}} \frac{p_i^{t-1}\Phi_i^t}{\sum_{j=i}^{I} p_j^{t-1}\Phi_j^t} \leq 1 \tag{36}$$

we can try to get information on $W(\mathbf{p},\mathbf{y})$ at any time t by simply observing the vector $\mathbf{\Phi}$ of the weights used in (36). Indeed, the price cap rule defined in (36) has the same properties of a the Iozzi, Poritz and Valentini's GPC defined by (33) as long as

$$\Phi_i^t = \frac{\partial W}{\partial p_i^{t-1}} \quad \text{for any } i=1,\ldots I. \tag{37}$$

We can adapt (9') to the present context by rewriting it as

$$\frac{\partial W}{\partial p_i^{t-1}} = -\sum_{h=1}^{H} \beta_h q_{i,h}^{t-1} \tag{38}$$

where $q_{i,h}^{t-1}$ is the quantity of good i consumed by consumer h at time t-1 and $\beta_h = \frac{\partial W}{\partial v_h}\alpha_h$ is again the marginal social utility of income (that is the social welfare weight) of consumer h. Therefore, as long as at any time t we can observe the weight Φ_i^t and the quantities of good i consumed by each consumer h=1,…H at time t-1, (38) allows to uncover β_h, conditional on all the other β_k (k=1,…,h-

1,h+1,….H),. More precisely, we can write the system of I equations given by (38) as

$$\mathbf{W}^t = \mathbf{Q}^t \cdot \boldsymbol{\beta} \tag{39}$$

where $\mathbf{W}^t$ is the $(I \times 1)$ vector whose i^{th} element is $\frac{\partial W}{\partial p_i^{t-1}}$, $\mathbf{Q}^t$ is the (IxH) non-singular matrix whose i,h^{th} element is $q_{i,h}^t$ and $\boldsymbol{\beta}$ is the (Hx1) vector of social welfare weights whose h^{th} element is β_h.

In this framework, the inverse optimum problem (IOP) consists in finding out the vector $\boldsymbol{\beta}$ that satisfies (39). When $I=H$, the solution of the IOP problem is

$$\boldsymbol{\beta} = \mathbf{Q}^{-1} \cdot \mathbf{W}^t \tag{40}$$

where $\mathbf{Q}^{-1}$ is the inverse of $\mathbf{Q}^t$.

3.4 Welfare improving and poverty reducing marginal price cap reforms

We can extend the analysis on welfare improving and poverty reducing marginal price reforms to the case where the social planner has not prices under her direct control and she relies on price cap regulation (Makdissi and Wodon, 2007). Let us assume, as usual, that the regulated monopolist chooses **p** in order to maximize its profit Π given a static version of price cap (see, for instance, Armstrong and Vickers, 1991 and 1993) which is given by

$$\sum_{i=1}^{I} \omega_i p_i \leq \overline{p}\,. \tag{41}$$

where the regulator's choice of ω_i reflects her social preferences. For instance, when the weights ω_i are set equal to the realized quantities (i.e. ω_i=q_i for any i=1, …, I), then the profit maximization problem yields to Ramsey-Boiteux prices that imply a regulator who aims to maximize a strictly (i.e. unweighted) utilitarian social welfare function. Given (41), the first order conditions of the monopolist's problem are

$$v\omega_i = q_i(\mathbf{p}) + \sum_{j=1}^{I}(p_j - c_j)\frac{\partial q_j(\mathbf{p})}{\partial p_j} \quad (i = 1,...,I) \tag{42}$$

where v is the Lagrange multiplier.

In this setting we define a marginal price cap reform as a reform that affects the weights of two prices in the price cap basket by increasing ω_i and decreasing ω_j in such a way that $d\omega_i = -d\omega_j$. However, differently from the case of direct price reform, where just the prices of two goods change, any $d\omega_i$ may imply that the regulated firm adjusts its whole price structure and changes all prices. As a matter of fact, by totally differentiating the set of equations given by (42), we get that, for any i=1, …, I,

$$vd\omega_i = \sum_{j=1}^{I}\left[\frac{\partial q_i(\mathbf{p})}{\partial p_j} + \left(1 - \sum_{k=1}^{I}\frac{\partial c_j}{\partial q_k}\frac{\partial q_k(\mathbf{p})}{\partial p_j}\right)\frac{\partial q_j(\mathbf{p})}{\partial p_i} + (p_j - c_j)\frac{\partial^2 q_j(\mathbf{p})}{\partial p_i \partial p_j}\right]dp_j \tag{43}$$

We know from section 2.7 (see eq. (27)) that the impact on the stochastic dominance curve of a marginal change of prices is given by

$$dD^s(z) = \sum_{i=1}^{I} CD_i^s(z)dp_i \,. \tag{44}$$

In the present setting we can rewrite (44) as

$$dD^s(z) = \sum_{h=1}^{I} CD_h^s(z)\frac{\partial p_h}{\partial \omega_i}d\omega_i + \sum_{h=1}^{I} CD_h^s(z)\frac{\partial p_h}{\partial \omega_j}d\omega_j \tag{45}$$

that, taking into account that $\omega_i = -d\omega_j$, becomes

$$dD^s(z) = \sum_{h=1}^{I} CD_h^s(z)\left(\frac{\partial p_h}{\partial \omega_i} - \frac{\partial p_h}{\partial \omega_j}\right)d\omega_i \tag{45'}$$

Equation (45') allows to identify the sufficient condition for a marginal price cap reform being welfare improving for any welfare index $U \in \Omega^s$, namely

$$\sum_{h=1}^{I} CD_h^s(y)\left(\frac{\partial p_h}{\partial \omega_i} - \frac{\partial p_h}{\partial \omega_j}\right) \le 0 \text{ for any } y \in \left[0, y^{\max}\right]$$
$$\text{and, if } s \ge 3, \qquad (46)$$
$$\sum_{h=1}^{I} CD_h^\sigma(y^{\max})\left(\frac{\partial p_h}{\partial \omega_i} - \frac{\partial p_h}{\partial \omega_j}\right) \le 0 \quad \text{for any } \sigma \in \{2,...,s-1\}$$

and the necessary and sufficient condition for a marginal price cap reform being poverty reducing for any poverty index $P(z) \in \Psi^s$, namely

$$\sum_{h=1}^{I} CD_h^s(y)\left(\frac{\partial p_h}{\partial \omega_i} - \frac{\partial p_h}{\partial \omega_j}\right) \le 0 \text{ for any } y \in \left[0, z^+\right]. \qquad (46')$$

The main difference between the conditions implied by (46) and (46') and those obtained in the case of marginal price reforms [i.e. (28) and (28')] is that the former relies on *CD* curves of all goods while the latter on only $\overline{CD}_i^s$ and $\overline{CD}_j^s$. This is due to the effect of cross-price elasticities in (43). In fact, if we assume that the cross-price elasticities of goods are zero, (43) can be written as

$$vd\omega_i = \left[2\frac{\partial q_i(\mathbf{p})}{\partial p_i} - \frac{\partial c_i}{\partial q_i}\left(\frac{\partial q_i(\mathbf{p})}{\partial p_i}\right)^2 + (p_i - c_i)\frac{\partial^2 q_i(\mathbf{p})}{\partial p_i^{\,2}}\right]dp_j, \quad (i = 1,...,I)$$

(46) and (46') become

$$CD_i^s(y)\frac{\partial p_i}{\partial \omega_i} - CD_j^s(y)\frac{\partial p_j}{\partial \omega_j} \le 0 \text{ for any } y \in \left[0, y^{\max}\right]$$
$$\text{and, if } s \ge 3, \qquad (47)$$
$$CD_i^\sigma(y^{\max})\frac{\partial p_i}{\partial \omega_i} - CD_j^\sigma(y^{\max})\frac{\partial p_j}{\partial \omega_j} \le 0 \text{ for any } \sigma \in \{2,...,s-1\}$$

and

$$CD_i^s(y)\frac{\partial p_i}{\partial \omega_i} - CD_j^s(y)\frac{\partial p_j}{\partial \omega_j} \le 0 \text{ for any } y \in \left[0, z^+\right] \qquad (47')$$

respectively.

4 Conclusion

This paper provides a unified vision of several results that appeared in the three streams of literature that, almost independently from each others, have analyzed a number of welfare properties arising under indirect taxation, public pricing and price cap regulation.

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