



Munich Personal RePEc Archive

Heterogeneous Consumers, Demand Regimes, Monetary Policy and Equilibrium Determinacy

Di Bartolomeo, Giovanni and Rossi, Lorenza

Universita Cattolica del Sacro Cuore, Milano

5 September 2005

Online at <https://mpra.ub.uni-muenchen.de/5100/>

MPRA Paper No. 5100, posted 01 Oct 2007 UTC

Heterogeneous Consumers, Demand Regimes, Monetary Policy and Equilibrium Determinacy^{*}

Giovanni Di Bartolomeo
University of Teramo and University of Crete

Lorenza Rossi
Università Cattolica del Sacro Cuore, Milano

2007

Forthcoming *Rivista di Politica Economica*

Abstract

This paper investigates the effects of monetary policy in presence of heterogeneous consumers. We study the effectiveness (quantitative effects) of monetary policy and equilibrium determinacy properties of a New Keynesian DSGE model where a fraction of households cannot smooth consumption. We show that two-demand regimes can emerge (according to the “slope” of IS curve) and that the main unconventional results, stressed by recent literature, only hold in the unconventional case of an IS curve positively sloped.

Keywords: Heterogeneous consumers, liquidity constraints, determinacy, demand regimes.
JEL codes: E61, E63.

1. Introduction

Following a strand of recent literature, this paper aims to study the effectiveness and the stability properties of different monetary policy rules in a New Keynesian DSGE model augmented by a fraction of agents who do not smooth consumption (i.e. spenders). We aim to show that the main unconventional results derived by recent literature only hold under some particular circumstances.

^{*} The authors are grateful to two anonymous referees, N. Acocella, P. Claeys, A. Colciago, V. Di Simone, N. Giammarioli, R. Maggi, F. Mattesini, M. Manzo, G. Piga, G. Rodano, D.H. Romer, M. Tancioni, P. Tirelli and SIE and MMF 2006 Conference participants for useful discussions on earlier drafts. Giovanni Di Bartolomeo acknowledges financial support by MIUR (PRIN 2005) and the European Union (Marie Curie Programme). Lorenza Rossi acknowledges her stay at IRE (Lugano, Switzerland). This research project has been supported by a Marie Curie Transfer of Knowledge Fellowship of the European Community's Sixth Framework Programme under contract number MTKD-CT-014288.

Agents who do not smooth consumption can be interpreted in various ways. One can view their behavior as resulting from consumers who face binding borrowing constraints. Alternatively, myopic deviations from the assumption of fully rational expectations should be assumed (rule-of-thumb), i.e. consumers naively extrapolate their current income into the future, or weigh their current income too heavily when looking ahead to their future income because current income is the most salient piece of information available.¹

Whatever the reason why some agents do not smooth consumption, their analytical modeling is however similar and for this reason we will generically refer to rule-of-thumb consumers to include both categories of non-smoothing consumers.

Campbell and Mankiw (1989, 1990, 1991) provide compelling evidence for the empirical existence of heterogeneous consumers. If we specifically refer only to households who *can* smooth consumption (*savers*) and agents whose current consumption equals current income (*spenders*), spenders' behavior is quantitatively important, with about one-fourth of income accruing to them in the United States (see Fuhrer, 2000).²

Spenders have been extensively used to analyze fiscal policy issues; they play a crucial role in breaking the Ricardian equivalence, for this reason, savers and spenders are often referred as Ricardian and non-Ricardian consumers.³ More recently, Amato and Laubach (2003), Gali *et al.* (2004) and Bilbiie (2004)⁴ have introduced spenders (modeled as rule-of-thumb consumers) in a New Keynesian framework to study monetary policy. They show that the presence of spenders' behavior may alter dramatically the properties of the standard model and overturn some of the conventional results.

¹ See Mankiw (2000) and references therein.

² Muscatelli *et al.* (2006) find an even larger proportion. They suggest that about 37% of consumers are spenders, whilst 84% of total consumption in steady state is given by optimizing consumers; spenders account for about 59% of total employment. Additional evidence is provided by Jappelli (1990), Shea (1995), Parker (1999), Souleles (1999) and Fuhrer and Rudebusch (2004).

³ See e.g. Mankiw (2000) and Muscatelli *et al.* (2006) and references therein. Christiano *et al.* (2005) investigate the effects of a rule-of-thumb behavior in firms' decisions

⁴ We share some results with Bilbiie (2004, 2005). Unluckily both of us recognize our common interests at a late stage of our research agenda.

In particular, Amato and Laubach (2003) study optimal monetary policy and find that monetary policy effectiveness increases in the fraction of spenders. Galí *et al.* (2004) explain how the Taylor principle becomes a sufficient but non necessary condition for stability when monetary policy is set according to a Taylor rule. By contrast, in the case of forward-looking interest rate rules, Galí *et al.* (2004) show that the conditions for a unique equilibrium are somewhat different from the usual ones. Bilbiie (2004) shows that the difference between the economic performance of the pre- and post-Volker's era can be explained in terms of a New Keynesian model augmented with rule-of-thumb consumers.

By using a simplified version of Galí *et al.* (2004) we are able to study the effectiveness and the stability properties of different monetary policy rules under rule-of-thumb consumers from an analytical point of view rather than considering calibrations/simulations as in previous works.⁵ We find that some important and unconventional results derived by aforementioned authors only hold under some particular circumstances related to the demand regimes, defined according to the response of aggregate demand to nominal interest rate movements.

Specifically, we assert that models with rule-of-thumb consumers are associated with two different demand regimes. In a *standard regime* an increase of the interest rate, everything equal, reduces inflation and output as usual. By contrast in an *inverse regime* the contrary occurs. In short, an increase in the nominal interest rate may increase output since deflation and falls in markups make real wages to rise, thus inducing spenders to consume more.

We find that many of the results obtained by the recent literature, as e.g. the impact of Taylor rules in Galí *et al.* (2004), only hold in the inverse regime. Thus rule-of-thumb consumers can explain some empirical puzzle, but the explanation comes at the cost of an additional puzzle, i.e. a positively sloped IS curve, which summarizes the positive relation between aggregate demand and the interest rate.

The rest of the paper is organized as follows. Next section outlines our basic framework and describes the two demand regimes implied by the presence of spenders and their implications for monetary policy. Section 3 investigates the

⁵ An exception is Bilbiie (2005).

stability of the model under different monetary policy rules and discusses the monetary policy transmission mechanism. Section 4 concludes.

2. The Basic Framework and Demand Regimes⁶

We consider a simple New Keynesian model augmented by non-Ricardian consumers (Gali *et al.*, 2004). In order to simplify the analysis and draw attention to the demand-side effects of spenders' behavior, we do not consider any capital accumulation process. We assume a continuum of infinitely-lived heterogeneous agents normalized to one. Savers are a fraction $1-\lambda$, they consume and accumulate wealth as in the standard setup. The remaining fraction agents λ is instead composed by spenders who do not own any asset, cannot smooth consumption and thus consume all their current disposable income.

By solving the inter-temporal optimization problems of savers and spenders, aggregating and then log-linearizing,⁷ we obtain the following description of the demand side of the economy:

$$(1) \quad c_t = -(1 - \lambda \zeta_N)(i_t - E_t \pi_{t+1}) + E_t c_{t+1} - \lambda \zeta_N E_t \Delta \omega_{t+1},$$

$$(2) \quad \omega_t = y_t + \nu n_t,$$

Equation (1) is the aggregate consumption, it represents a modified version of the standard consumption Euler equation, where c_t is consumption, i_t is the nominal interest rate, π_t is the inflation rate. $\zeta^N = (1 + \nu)\kappa(1 + \kappa)^{-1}$ is the steady state share of spenders' consumption, where the parameter ν is the inverse of the Frisch labor supply elasticity. Our Euler equation differs from the standard one in which the last term of the right hand side of equation (1) is absent. This is due to

⁶ A large part of the model is rather standard (see e.g. Rotemberg and Woodford, 1997; or Woodford, 2003). Thus here the model is only described in its main equations. The demand side of the economy is derived in more detail in Appendix A since it plays a crucial role. A technical appendix with a full-model derivation is available upon request:

⁷ Notice that Gali *et al.* (2004) log-linearized the saver and spender consumption around the steady state aggregate consumption. By contrast, we log-linearized the Saver and Spender consumption around their steady state levels and then we aggregate them. Although both procedures are correct and equivalent (leading to the same final results), the procedure followed by Gali *et al.* (2004) hides some crucial features of the transmission mechanism.

the presence of savers, which establish a link between the demand for goods and the real wage ω_t (see equation (2)). The variables y_t and n_t are respectively aggregate output and employment.

The supply side of the economy is represented by a standard forward-looking Phillips curve, a labor demand and a markup equation:⁸

$$(3) \quad \pi_t = \beta E_t \pi_{t+1} + kx_t + u_t,$$

$$(4) \quad \omega_t = mc_t + (y_t - n_t)$$

$$(5) \quad mc_t = (1 + \nu)x_t$$

where $x_t = y_t - a_t$ is the output gap with respect to the flexible-price output, which coincides with the exogenous technology shock, a_t . The variable u_t is $AR(1)$ process representing the standard exogenous cost-push shock. Equation (4) is firms' aggregate labor demand. Equation (5) is the equation for real marginal costs.

By considering the log-linearized production function $x_t = n_t$, aggregate consumption can be written as

$$(6) \quad c_t = -(1 - \lambda\zeta_N)(i_t - E_t \pi_{t+1}) + E_t c_{t+1} - \lambda\zeta_N(1 + \nu)E_t \Delta x_{t+1} - \lambda\zeta_N \Delta a_{t+1}.$$

Current consumption depends on real interest rate (because of the Euler intertemporal substitution effect), expected future consumption, and on the output-gap expected growth.

After some more algebra, equation (4) can finally be re-written as:

$$(7) \quad x_t = E_t x_{t+1} - \Omega(i_t - E_t \pi_{t+1}) + \Omega \Delta a_{t+1},$$

where $\Omega = \frac{1 - \lambda\zeta_N}{1 - (1 + \nu)\lambda\zeta_N}$ is the *income monetary multiplier*, i.e. the semi-elasticity of real output to the real interest rate.⁹

Equation (7) is similar to the standard one proposed by the New-Keynesian literature, the existence of spenders however affects the impact of interest rate policy on aggregate demand from both a quantitative and a qualitative point of

⁸ The production function is $Y_t = A_t N_t$, the labor demand simply equates the real wage to the marginal productivity of labor.

⁹ It should be noticed that neither the share of spenders' consumption nor the Frisch elasticity depends on the fraction of spenders (see Appendix B).

view. According to the sign of the income multiplier, equation (7) individuates two different regimes:

1. A standard regime holds if the income monetary multiplier is positive. Such a regime is dominated by the hypothesis of life-cycle permanent income and thus by the consumption smoothing theory (see e.g. Clarida, *et al.* 1999).
2. An inverse regime holds if the income monetary multiplier is negative and it is dominated by the liquidity-constraint effect. An increase in real interest rates is expansionary and interest rate cuts imply contractions since a large part of consumers cannot access to financial markets and saving.

Aggregate consumption (6) negatively depends on real interest rates but positively on the current output by the aggregate income elasticity of consumption. Thus if the income elasticity of consumption is low, the monetary multiplier will be negative. By contrast, if the elasticity is high, the multiplier is positive. The intuition of the result can be found in the labor-market dynamics. Consider an increase of the interest rate that reduces the savers' demand for consumption goods, which prefer to consume more tomorrow, it also shifts left the labor demand curve pressing the wage downward (equation (4)). Given that savers decide to decrease consumption demand for leisure increases and therefore savers labor supply decreases, (see equation (2)).¹⁰ The net effect of labor and demand supply on the real wage depends on the extent of the inverse Frisch elasticity and the spenders' fraction. If the elasticity and the savers' fraction are large, the net effect is negative: a wage reduction follows. The wage reduction further stimulates demand falls because also non-Ricardian decreases their consumption and further falls in the real wage, so that the standard regime holds. A reverse result holds when the proportion of savers is low: a real wage increase follows, so that spenders, which are sensible to real wage movements, increase their

¹⁰ Note that employment of spenders does not rise in a demand-driven boom because we have assumed a logarithmic functional form for the consumer's instantaneous utility (see also Galí *et al.*, 2004; or Muscatelli *et al.*, 2006). A different form (e.g. constant relative risk aversion) eliminates the inelasticity of spenders' labor supply, but does not affect our main conclusions. Although this inelasticity is a drawback of the model, the logarithmic functional form greatly helps to simplify the exposition.

consumption. Given that the proportion of spenders is high an increase in the interest rate increase the aggregate output, and the IS-curve is positive sloped (the inverse regime holds).

Formally, the two regimes¹¹ depend on a threshold value of λ , the traditional regime holds for:¹²

$$(8) \quad \lambda < \lambda^* = \frac{1}{\zeta_N(1+\nu)} = \frac{\kappa(1+\kappa)}{(\kappa+\theta)^2},$$

otherwise we are in the liquidity-constrained regime. The parameter $\theta = (\eta - 1)\eta^{-1} \in (0,1)$ indicates the inverse of the firms markup, where η is the elasticity of substitution across differentiated products.

For relatively low values of θ and high values of κ , the threshold value is greater than one ($\lambda^* > 1$). In such a case, only the standard regime occurs since $\lambda \in [0,1]$.

In other terms the inverse Frisch elasticity is smaller than one. For relatively high values of θ and low values of κ , the liquidity-constrained regime can emerge. In addition, if θ is greater than 0.5, λ^* is always smaller than one. Thus, in such a case, the liquidity-constraint regime always holds for a value of λ sufficiently great.

We can now discuss on the effectiveness of monetary policy. In the standard regime the effectiveness, which is measured by the size of the income monetary multiplier, is increasing in the fraction of spenders. By contrast, in the liquidity-constrained regime, its effectiveness is decreasing in λ . The effectiveness of monetary policy is represented in the figure 1, where the absolute value of income monetary multiplier, $|\Omega|$, vis-à-vis the fraction of spenders, λ , is plotted.

Figure 1

Notice that without non-Ricardian consumers, as $\lambda \rightarrow 0$, $\Omega \rightarrow -1$, thus elasticity of real output to the real interest rate is minus one, i.e. as in the standard case with logarithmic utility. In such a case, a positive correlation between expected

¹¹ The inverse Frisch elasticity and the steady state fraction of spenders depend on the deep parameters: the intermediate sector elasticity of substitution and the labor disutility coefficient.

¹² The following condition implies that income elasticity of consumption is smaller than one. It is obtained by considering that the steady state value of N is $\theta(\kappa+\theta)^{-1}$ (see the Appendix B).

consumption growth and real interest rate is found. As long as λ increases the effectiveness of monetary policy raises till $\Omega = +\infty$ (the income elasticity of consumption is equal to one). For $\lambda > \lambda^*$ the regime shifts to the liquidity-constrained one (where there is a negative correlation between expected consumption growth and real interest rate), an interest cut affects positively real output and the effectiveness of monetary policy is decreasing in the fraction of spenders, i.e. λ .¹³

Summarizing, if the IS curve is positive sloped monetary policy effectiveness increases in the fraction of spenders (as Amato and Laubach, 2003). A reverse result is obtained if the IS curve is negative sloped.

3. Taylor Principle and Determinacy¹⁴

3.1. Monetary policy based on Taylor rules

A description of monetary authority behavior completes the model above-presented. Monetary policy can be introduced by an exogenous rule, which relates the interest rate to the other variables, or by an endogenous one, directly derived by the solution of an optimization problem, e.g. welfare maximization. One fundamental property which is requested for the monetary authority behavior is to support rational expectation equilibrium determinacy.

Let us start by considering an exogenous Taylor rule as the following:¹⁵

$$(9) \quad i_t = \alpha_1 \pi_t + \alpha_2 x_t,$$

where α_1 and α_2 are both positive.

In the standard regime determinacy requires an *active* policy rule:

$$(10) \quad a_1 > 1 - \frac{1-\beta}{k} a_2.$$

¹³ Of course, for values of θ and κ implying $\lambda^* > 1$, only the first (decreasing) part of the figure is economically relevant.

¹⁴ Determinacy conditions are derived in Appendix C.

¹⁵ John Taylor has proposed that U.S. monetary policy in recent years can be described by an interest-rate feedback rule as that considered here. See, among others, Taylor (1993) or Woodford (2004).

The above determinacy condition has a simple usual interpretation. A feedback rule satisfies the Taylor principle if in the event of a sustained increase in the inflation rate by one percentage point, the nominal interest rate will eventually be raised by more than one percentage point. Each percentage point of permanent increase in the inflation rate implies an increase in the long-run average output gap of $(1-\beta)k^{-1}$ percent. An exogenous Taylor rule thus conforms to the Taylor principle if and only if its coefficients satisfy $a_1 + (1-\beta)k^{-1}a_2 > 1$ (see, among others, Woodford, 2004).

In the liquidity-constrained regime, Ω is negative. Hence, to simplify the exposition, we redefine it as $\bar{\Omega} = -\Omega$, which is a positive measure of monetary policy effectiveness. Determinacy thus requires

$$(11) \quad a_1 > \max \left\{ 1 - \frac{1-\beta}{k} a_2, \left(\frac{2}{\bar{\Omega}} - a_2 \right) \frac{1+\beta}{k} - 1 \right\} \text{ or}$$

$$(12) \quad a_1 < \min \left\{ 1 - \frac{1-\beta}{k} a_2, \left(\frac{2}{\bar{\Omega}} - a_2 \right) \frac{1+\beta}{k} - 1 \right\} \text{ when } a_1 < \frac{1-\beta}{\bar{\Omega}k} - \frac{a_2}{k}.$$

By inspecting equation (11) we find that a rule satisfying the Taylor Principle can be not sufficient to assure determinacy and a more aggressive rule, i.e. a rule that strongly react to current inflation, may be requested. By inspecting equation (12), a passive policy as indicated by condition (12) implies determinacy, but if both values between brackets or the r.h.s. of the last term are negative the equilibrium is always indeterminate. This occurs if the central bank places a high weight to the output stabilization.

In the standard regime, the Taylor principle is thus the necessary and sufficient condition for determinacy. By contrast, in the liquidity-constrained regime, we have to consider different cases. More in detail, determinacy may be related to the monetary policy effectiveness as follows.¹⁶

- a) For a relative high effectiveness of monetary policy, i.e. $\bar{\Omega} > \frac{1+3\beta}{k + \beta a_2}$, the

Taylor principle is a necessary and sufficient condition for determinacy.

¹⁶ See appendix C.

b) For relative medium effectiveness, i.e. $\bar{\Omega} \in \left(\frac{1+\beta}{k+\beta a_2}, \frac{1+3\beta}{k+\beta a_2} \right)$, the

Taylor principle is a sufficient condition for determinacy but not necessary since also a non-aggressive (with respect to inflation) state contingent policy implies determinacy.

c) Finally, if monetary policy has a relatively low efficacy (i.e. $\bar{\Omega} < \frac{1+\beta}{k+\beta a_2}$),

the Taylor principle is neither a necessary nor a sufficient condition for determinacy; in fact, a more aggressive policy than the one satisfying the Taylor principle leads to determinacy. This is however a sufficient condition only, since all the other rules that does not satisfy it lead to determinacy in a sort of inverse Taylor principle.

The economic intuition of our results will be clearer after describing the case on an endogenous-Taylor rule and the monetary policy transmission mechanism in the liquidity-constrained regime.

The effects of the rule-of-thumb consumer on the equilibrium determinacy are depicted in the following figure.¹⁷

Figure 2

The figure replicates and amends the discussion of Gali *et al.* (2004). They show the relationship between the fraction of rule-of-thumb consumers and the inverse of the Taylor rule inflation coefficient, $1/\alpha_1$ (or α_1 as in panel (b), (d)) which guarantees determinacy. Figure 2 shows only the inverse regime, because, as stated by Gali *et al.* (2004) important results only apply in this regime. Panel (a) and (c) represent the sufficient conditions for determinacy in case of the two monetary policy regimes: a very aggressive and a passive monetary policy, respectively (cf. figure 1 in Gali *et al.*, 2004). Panel (b) and (d) represent the same conditions in terms of the inflation coefficient instead of the inverse inflation coefficient as panel (a) and (c). The reason why we consider both representations will be clear soon.

¹⁷ Our parameterization is similar to Gali *et al.* (2006), we impose an equilibrium steady state for employment of about 1/3. We have used: $\beta = 0.99$; $\eta = 6$, $\kappa = 1.7$. Results do not change under other realistic values for parameters.

Panel (a) reproduces the Gali *et al.*'s results: as the fraction of rule-of-thumb consumers increases, a more aggressive monetary policy is required in order to guarantee the stability. It is however clear from panel (b) that the coefficients required for stability grows exponentially with the fraction of rule-of-thumb consumers, making this case true for a narrow realistic parameterization.¹⁸ By contrast, stability also emerges in the case of a non aggressive policy and requires more plausible values of the parameters, e.g. they can be consistent with the coefficients estimated for the case of the pre-Volker policies.

Although the Taylor rules of the above-described kind are frequently used, monetary policy rules consistent with loss minimization are often presented as forward-looking. Formally, in such a case, the central bank should follow an *optimal path* for the nominal interest rate satisfying:

$$(13) \quad i_t = \alpha_3 E\pi_{t+1}.$$

where the coefficient α_3 is determined by the monetary policy regime where the central bank act and the parameters of central bank loss. Equation (11) is usually derived from the solution of an optimization problem¹⁹ and thus represents a sort of endogenous (forward-looking) Taylor rule.

In the standard regime, determinacy requires:

$$(14) \quad a_3 \in \left(1, 1 + 2 \frac{(1-\beta)}{k\Omega} \right).$$

Equation (12) is standard and nests the Taylor principle: monetary policy should respond more than one-for-one to increases in inflation, and should also not be too aggressive as noticed by Bernanke and Woodford (1997).

¹⁸ In other words, in the neighborhood of the lower value of λ^* , we need an interest rate rule that is more aggressive than the Taylor prescription. However, when the value of λ is lower than λ^* the standard Taylor principle holds, while when the value of λ is greater than λ^* the degree of aggressiveness of the central bank increases. In the latter case, also small increases in λ with respect to λ^* imply a monetary policy unrealistically aggressive with respect to inflation.

¹⁹ More in detail, equation (11) is derived from the so-called *flexible inflation targeting approach* (Svensson, 1999, 2003) under different monetary policy regimes (i.e. discretion, commitment or timeless perspective). It can be also seen as the results of utility-based welfare maximization (Woodford, 2003: Ch. 6). However, to generalize our results to such a case one should show that the central bank's loss parameters (and thus α_3) are independent of the spenders' fraction. An analysis of the utility based welfare criterion is beyond the scope of the present paper thus we stick us to the interpretation of equation (11) as an optimal policy derived from an exogenous loss as e.g. Evans and Honkapohja (2005), i.e. *flexible inflation targeting approach*.

In the liquidity-constrained regime, stability requires:

$$(15) \quad a_3 \in \left(1 - \frac{2(\beta+1)}{\bar{\Omega}k}, 1 \right).$$

Monetary policy has now to be conducted by a sort of inverted Taylor Principle. The central bank should respond less than one-to-one to increases in inflation. However, too much passive monetary policies may also lead to indeterminacy. In particular, if monetary policy has relatively high effectiveness, $\bar{\Omega} > 2\frac{\beta+1}{k}$, indeterminacy may also derives from a weak (positive) reaction to expected inflation of the nominal interest rate, i.e. $a_3 < 1 - \frac{2(\beta+1)}{\bar{\Omega}k}$.

The rationale of the inverse Taylor principle is as follows. A positive non-fundamental shock in the expectations reduces the real interest rate; in the liquidity-constrained regime, if monetary policy is passive, it does not lead to the self-fulfillment of expectation since output falls. By contrast if monetary policy is set according to the Taylor principle, the real interest rate will increase as well as output and expectations will be self fulfilled.

Figure 2 synthesizes the above results in the parameter space. Panel (a) ((b)) refers to a relatively low (high) fraction of non-Ricardian consumers. In the standard regime, (white area) the Taylor Principle always holds. In the liquidity constraint regime we must distinguish between two type of monetary policy effectiveness: a relatively low effectiveness (dark area) and a relatively high one (light area). In the dark area, although an inverted Taylor principle holds, monetary policy leads to determinacy. By contrast, in the light area, even if an inverted Taylor principle still holds a very non-aggressive (with respect to inflation) monetary policy leads to indeterminacy.

Figure 3

In the standard regime, if the policy rule is not active, a non-fundamental increase in expected inflation generates an increase in the current output gap and, by the current Phillips curve, inflation increases, validating the initial non-fundamental expectation. The Taylor principle is needed to guarantee determinacy since an active rule generates a fall in output gap and thus in actual inflation, contradicting initial expectations. By contrast, in a liquidity-constrained regime, if the policy

rule is active, a non-fundamental increase in expected inflation generates an increase in the current output gap and an increase in inflation (by the Phillips curve), validating the initial non-fundamental expectation. Thus, in such a regime, the Taylor principle leads to indeterminacy, instead a passive policy rule is requested. If the central bank follows a passive policy rule, a non-fundamental increase in expected inflation is associated with a fall in the real interest rate, a fall in the output gap, and deflation, contradicting the initial expectation that are hence not self-fulfilling.

Our result is consistent with Galí *et al.* (2004). It is worth noticing that, by comparing our results to those of Galí *et al.* (2004), the introduction of capital accumulation does not qualitatively affect the determinacy requirements. However, differently from Galí *et al.* (2004), our model stresses the key element of the determinacy analysis: the demand regimes (i.e., the IS curve slope).

3.2. Model calibration

By considering the model (1)-(5) and the Taylor rule (9), (for the sake of simplicity, we assume $a_2 = 0$), we simulate the effects of a unit standard deviation cost push and unit standard deviation technology shock in the two different demand regimes. Table 1 shows the parameters used.²⁰

Table 1

The impulse response functions (IRFs) to these shocks are represented in figure 4.

Figure 4

Panel (a1) and (b1) refer to the standard regime (the cases of a cost push shock and a technology shock, respectively); panels (a2)-(b2) and (a3)-(b3) refer instead to the inverse regime.

As shown in figure 4 (a1) and (b1), in the standard regime the IRFs correspond to the ones of the standard New Keynesian model. The nominal interest rate follows the Taylor principle and implement an interest rate rule with $a_1 > 1$.

²⁰ Parameters are similar to Galí *et al.* (2004). Labor disutility is chosen to have a steady state value for N equal to about 1/3. In Galí *et al.* (2004) the coefficient of relative risk aversion is equal to one. This is compatible with a log specification of the households' utility function, as the one we use to show the main results. Nevertheless, our results survive to more complex specifications, including utility with consumption habits, details are available upon request.

By contrast, in the inverse regime, i.e., with a positive sloped IS, in order to guarantee determinacy, two interest rate rule can be implemented:

- a very passive monetary policy;
- a rule which should be even more aggressive than what established by the Taylor principle.

Figure 4 (a2)-(a3) and (b2)-(b3) describe IRFs with a Central Bank which follows the two different rules. As shown in (a3) after a cost-push shock hits the economy the nominal interest rate can increase less than the increase in the inflation rate. In fact, with a fraction of rule-of-thumb consumer equal to 0.73, in order to guarantee determinacy, it is necessary $a_1 < 0.008$ or $a_1 > 2.04$. In particular, it is worth noticing that with a very aggressive monetary policy, as in (a2), households expect that inflation decreases on impact instead of increasing, this means that monetary policy should decrease the nominal interest rate instead of increasing it in order to not generate self-full filling deflation.

A similar mechanism holds in response of a positive technology shock. In the case of a very aggressive monetary policy, as in (b2), monetary policy should be very accommodating, households expect that inflation increases on impact instead of decreasing; this means that monetary policy should increase the nominal interest rate instead of decreasing it in order to not generate self-full filling inflation.

Otherwise, as shown in (b3) after a positive technology shock hits the economy monetary policy is accommodating but the nominal interest rate can decrease less than the decrease in the inflation rate.

4. Conclusions

This paper introduces consumers' heterogeneity into a DSGE New Keynesian model and considers the quantitative effects of monetary policy (effectiveness) and studies the equilibrium determinacy properties. By assuming that a fraction of consumers cannot smooth consumption, we show that two-demand regimes can emerge (according to the slope of the IS curve) and that some unconventional results on monetary policy effectiveness and equilibrium determinacy, recently

stressed by macroeconomic literature, are only associated to the unconventional regime where the IS curve is positively sloped.

By considering the liquidity-constrained agents, we find that if the slope of the IS is negative, monetary policy effectiveness increases in the fraction of spenders. The rationale of the result is as follows, although a smaller fraction of savers reduces the effects of interest rate policy on the inter-temporal allocation of consumption, the greater fraction of spenders increases the effects of monetary policy by the variations in spenders' consumption. By contrast, in the liquidity-constrained regime, the reverse effect holds.

Regarding determinacy of the rational expectation equilibrium, as long as the IS slope is conventional, standard results hold. Otherwise determinacy may be guaranteed by a passive monetary policy and the standard Taylor principle can be denied. More in details, in the liquidity-constrained regime, results on determinacy can be summarized as follows.

1. If monetary policy is set according to a standard Taylor rule, the Taylor principle is only a sufficient condition for determinacy when monetary policy is relatively effective whereas a more aggressive central bank is needed if the monetary policy has a (relative) low effectiveness. However, irrespectively of the policy effectiveness, determinacy can also be achieved by a (relative) non-aggressive policy, which clearly does not satisfy the Taylor principle.
2. If the central bank supports an (optimal) dynamic relationship between output and expected inflation, determinacy requires a non aggressive central bank (i.e. a monetary policy that does not satisfy the Taylor principle) but not too much, since, in such a case, a non-fundamental increase in expected inflation needs an higher interest rate to be not self-fulfilling.

Finally, we want to stress that if the liquidity-constrained regime matters, determinacy needs to be studied with attention by policymakers, who must take into account of the regime where they are since a good policy for a regime can be explosive in the another one. A possible additional factor explaining the explosion of bubbles in emerging markets could be related to attempt of managing the

monetary policy according to rules designed for developed financial markets in economy where the financial market were not fully developed. This provocative reflection however is rather preliminary and need of more empirical verifications.

Appendix A – The demand side

Representative consumers are indexed by R (Ricardian) and N (non-Ricardian), they maximize the following utility functions:²¹

$$(a.1) \quad E_t \sum_{i=0}^{\infty} \beta^i u(C_{t+i}^j, N_{t+i}^j, \phi^j) \quad j \in \{R, N\}$$

where $\beta \in (0,1)$ is the discount factor, C_t represents household consumption at time t , while N_t is labor. ϕ^j is a binary variable such that when $j = R$, $\phi^R = 1$ and when $j = N$, $\phi^N = 0$. We assume the following logarithmic instantaneous utilities, $u(\cdot) = \ln C_{t+i}^j + \kappa \ln(1 - N_{t+i}^j)$ with $\chi > 0$ and $\kappa > 0$. By solving their optimization problems, consumers face the budget constraints:

$$C_t^j = \frac{W_t}{P_t} N_t^j + \phi^j \left[\Pi_t^j - \frac{B_t^j - (1+i_{t-1})B_{t-1}^j}{P_t} \right],$$

where W_t is the nominal wage at time t , and Π_t is profit sharing. Note that real wages are the only source of fluctuations of non-Ricardian disposable income, and therefore, they are subject to a static budget constraint, while savers (Ricardian consumers) are the only ones facing a dynamic constraint.

By solving the Ricardian and non-Ricardian representative consumers' maximization problems, we obtain the following first-order conditions:

$$(a.2) \quad C_t^R = [\beta(1+i_t)P_t]^{-1} E_t [P_{t+1}C_{t+1}^R]$$

$$(a.3) \quad C_t^N = \frac{W_t}{P_t} N_t^N$$

²¹ To compare our results to the ones of Gali *et al.* (2004) and Bilbiie (2004, 2005) we do not consider real money balances. The introduction of real money balances will in fact imply possible additional channels of monetary transmission by the liquidity effects. The study of these effects is beyond the scope of this paper.

$$(a.4) \quad W_t P_t^{-1} = \kappa C_t^j (1 - N_t^j)^{-1} \quad j \in \{R, N\}$$

Equations (a.2) and (a.3) are the optimal consumption for Ricardian (i.e. inter-temporal stochastic consumption Euler equation) and non-Ricardian consumers (who consume the whole labor income). Equation (a.4) is; the optimal condition for the labor supply. From equations (a.3) and (a.4), it is easy to find that non-Ricardian consumers supply a fixed quantity of labor, i.e. $N_t^N = \frac{1}{1+\kappa}$.

The aggregate consumption and employment are

$$(a.5) \quad C_t = (1 - \lambda) C_t^R + \lambda C_t^N$$

$$(a.6) \quad N_t = (1 - \lambda) N_t^R + \lambda N_t^N$$

From equations (a.4) and (a.6), we obtain the wage aggregate supply:

$$(a.7) \quad C_t = \frac{1}{\kappa} \frac{W_t}{P_t} (1 - N_t)$$

By log-linearizing equation (a.7) we obtain equation (2), recall that $Y_t = C_t$ in equilibrium. By log-linearizing equations (a.2) and (a.3) we find:²²

$$(a.8) \quad c_t = (1 - \lambda) \zeta_R c_t^R + \lambda \zeta_N c_t^N$$

$$(a.9) \quad c_t^R = -(i_t - E_t \pi_{t+1}) + E_t c_{t+1}^R$$

$$(a.10) \quad c_t^N = w_t - p_t$$

Solving equation (a.8) for c_t^R and using equations (a.9) and (a.10) we obtain equation (1).

Appendix B – Demand Regimes

This appendix shows the independence between the income monetary multiplier and the fraction of rule-of-thumb consumers. We need to relate the fraction of steady state fraction of non-Ricardian consumption and the inverse Frisch elasticity only to deep parameters.

²² Note that, differently from Galí *et al* (2004), saver and spender consumption have been log-linearized around their steady state levels and not around the steady state level of aggregate consumption. The different log-linearization does not affect the result on the demand regimes.

Regarding the former, from the demand side of the economy, i.e. equations (a.3) and (a.8), we obtain $\zeta^N = C^N C^{-1} = (1+\nu)\kappa(1+\kappa)^{-1}$, recall that Ricardian consumers supply a fixed amount of labor.

To find the steady state value of the employment, we introduce the supply side of the economy, but since it is rather standard we will briefly discuss it (a technical appendix is available upon request). As usual, we consider an economy composed by a continuum of firms (indexed by $z \in [0,1]$) producing differentiated intermediate goods with a constant return to scale technology $Y_t(z) = A_t N_t(z)$. Intermediate goods are used as inputs by a perfectly competitive final goods firm. In such a context, under flexible prices, firms real marginal costs, are constant and equal to the inverse of firm mark-up:

$$(b.1) \quad \theta = (\eta - 1)\eta^{-1}.$$

Moreover, given the constant return to scale technology and the aggregate nature of shocks, real marginal costs are the same across the symmetric intermediate good producing firms. Accordingly, from the cost minimization, real marginal cost is:

$$(b.2) \quad MC_t = A_t W_t P_t^{-1}.$$

By equating equations (a.8) and (b.2), we obtain that in the steady state:

$$(b.3) \quad N = \theta(\kappa + \theta)^{-1},$$

which is independent of the fraction of spenders.

By numerical analysis it is found that for combinations of relatively low values of θ and high values of κ only the standard regime holds. By contrast, the liquidity-constraint regime is then more likely to be observed for relative high values of θ and λ , and relative low values of κ .

Appendix C – Determinacy

Determinacy is studied by augmenting the log-linearized dynamic system (3)-(6) with a simple feedback rule (9), we obtain:²³

$$(c.1) \quad \begin{bmatrix} 1 & \Omega \\ 0 & \beta \end{bmatrix} E_t \begin{bmatrix} y_{t+1} \\ \pi_{t+1} \end{bmatrix} = \begin{bmatrix} 1 + \Omega a_2 & \Omega a_1 \\ -k & 1 \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \end{bmatrix}$$

Stability depends on the eigen-structure of the following matrix:

$$(c.2) \quad M = \begin{bmatrix} 1 & \Omega \\ 0 & \beta \end{bmatrix}^{-1} \begin{bmatrix} 1 + \Omega a_2 & \Omega a_1 \\ -k & 1 \end{bmatrix} = \begin{bmatrix} 1 + \Omega(a_2 + k\beta^{-1}) & \Omega(a_1 - \beta^{-1}) \\ -k\beta^{-1} & \beta^{-1} \end{bmatrix}$$

By indicating with $D(\cdot)$ and $T(\cdot)$ the determinant and trace operators, we have:

$$(c.3) \quad \begin{cases} D(M) = \beta^{-1} + \Omega(a_2 + ka_1)\beta^{-1} \\ T(M) = 1 + a_2\Omega + (1 + k\Omega)\beta^{-1} \end{cases}$$

The eigen-structure of matrix M is studied as in Woodford (2003: Appendices to Chapter 4). Since the analysis of the standard regime does not differs from Woodford (2003), we only consider the liquidity-constrained regime, determinacy requires either: *i*) $D(M) > 1$, i.e. $a_1 < [(1 - \beta)\bar{\Omega}^{-1} - a_2]k^{-1}$, $D(M) \pm T(M) + 1 > 0$ or *ii*) $D(M_1) \pm T(M_1) + 1 < 0$.

Being:

$$(c.4) \quad D(M) + T(M) + 1 = \{2(1 + \beta) - \bar{\Omega}[(1 + \beta)a_2 + (1 + a_1)k]\} \beta^{-1}$$

$$(c.5) \quad D(M) - T(M) + 1 = -\bar{\Omega}[(1 - \beta)a_2 + k(a_1 - 1)] \beta^{-1}$$

from equations (c.4) and (c.5) we derive conditions (11) and (12), respectively.

By considering a rule (13), in a similar manner as above, the dynamic system is governed by the following matrix:

$$(c.6) \quad M = \frac{1}{\beta} \begin{bmatrix} \beta - \Omega(a_3 - 1)k & \Omega(a_3 - 1) \\ -k & 1 \end{bmatrix}$$

²³ In order to investigate the stability properties we do not need to look at the stochastic part that is thus omitted for the sake of brevity. We assume stationary disturbance processes.

It is easy to verify that $D(M) = \beta^{-1} > 1$, stability requires $D(M_1) \pm T(M_1) > -1$ and $D(M_1) \pm T(M_1) < -1$. $D(M_1) \pm T(M_1) < -1$ is never satisfied; by contrast, $D(M_1) \pm T(M_1) > -1$ requires condition (15).

Regarding the relationship between determinacy and effectiveness of monetary policy under a standard Taylor rule, determinacy requires (11) and (12), but since $1 - \frac{1-\beta}{k}a_2 > \left(\frac{2}{\bar{\Omega}} - a_2\right)\frac{1+\beta}{k} - 1$ if and only if $\bar{\Omega} > \frac{1+\beta}{k+\beta a_2}$, the following statements hold.

1. For $\bar{\Omega} > \frac{1+\beta}{k+\beta a_2}$ determinacy requires: 1a) $a_1 > 1 - \frac{1-\beta}{k}a_2$ or 1b)

$$a_1 < \left(\frac{2}{\bar{\Omega}} - a_2\right)\frac{1+\beta}{k} - 1 \text{ if } a_1 < \frac{1-\beta}{\bar{\Omega}k} - \frac{a_2}{k}.$$

2. For $\bar{\Omega} < \frac{1+\beta}{k+\beta a_2}$ determinacy requires: 2a) $a_1 > \left(\frac{2}{\bar{\Omega}} - a_2\right)\frac{1+\beta}{k} - 1$ or 2b)

$$a_1 < 1 - \frac{1-\beta}{k}a_2 \text{ if } a_1 < \frac{1-\beta}{\bar{\Omega}k} - \frac{a_2}{k}.$$

From conditions 1a) and 2a) follow that a standard Taylor principle holds for a relatively high effectiveness a more aggressive principle should be used for a relatively low degrees of effectiveness. In addition, note that

$$(c.7) \quad \frac{1-\beta}{\bar{\Omega}k} - \frac{a_2}{k} > \left(\frac{2}{\bar{\Omega}} - a_2\right)\frac{1+\beta}{k} - 1 \text{ for } \bar{\Omega} < \frac{1+3\beta}{k+\beta a_2}$$

$$(c.8) \quad \frac{1-\beta}{\bar{\Omega}k} - \frac{a_2}{k} > 1 - \frac{1-\beta}{k}a_2 \text{ for } \bar{\Omega} > \frac{1-\beta}{k+\beta a_2}.$$

Thus condition 1b is binding if $\bar{\Omega} < \frac{1+3\beta}{k+\beta a_2}$ and condition 2b is binding if

$\bar{\Omega} > \frac{1-\beta}{k+\beta a_2}$. By putting all together, condition 1b is binding if

$$\bar{\Omega} \in \left(\frac{1-\beta}{k+\beta a_2}, \frac{1+3\beta}{k+\beta a_2}\right) \text{ and condition 2b is always binding.}$$

Summarizing (as described in the main text):

1. For relative high effectiveness, i.e. $\bar{\Omega} > \frac{1+\beta}{k+\beta a_2}$ determinacy requires $a_1 > 1 - \frac{1-\beta}{k} a_2$ (Taylor principle).
2. For relative medium effectiveness, i.e. $\bar{\Omega} \in \left(\frac{1+\beta}{k+\beta a_2}, \frac{1+3\beta}{k+\beta a_2} \right)$, requires $a_1 > 1 - \frac{1-\beta}{k} a_2$ (Taylor principle) or $a_1 < \left(\frac{2}{\bar{\Omega}} - a_2 \right) \frac{1+\beta}{k} - 1$ (strong inverse Taylor principle, a very non-aggressive (to inflation) policy implies the equilibrium determinacy).
3. For relative low effectiveness of monetary policy, i.e. $\bar{\Omega} < \frac{1+\beta}{k+\beta a_2}$, determinacy requires: $a_1 > \left(\frac{2}{\bar{\Omega}} - a_2 \right) \frac{1+\beta}{k} - 1$ (strong Taylor principle, a more aggressive policy is requested to obtain the equilibrium determinacy) or $a_1 < 1 - \frac{1-\beta}{k} a_2$ (inverse Taylor principle).

References

- Amato, J. and T. Laubach (2003), Rule-of-Thumb Behavior and Monetary Policy, *European Economic Review*, 47: 791-831.
- Bernanke, B. and M. Woodford (1997), Inflation Forecasts and Monetary Policy, *Journal of Money, Credit and Banking*, 24: 653-684.
- Bilbiie, F.O. (2004), The Great Inflation, Limited Asset Markets Participation and Aggregate Demand: FED Policy was Better than you Think, *ECB Working Paper* No. 408.
- Bilbiie, F.O. (2005), Limited Asset Markets Participation, Monetary Policy and (Inverted) Keynesian Logic, *Nuffield College Working Paper* No. 9.
- Campbell, J.Y. and N.G. Mankiw (1989), Consumption, Income, and Interest Rates: Reinterpreting the Time Series Evidence, *NBER Macroeconomics Annual*, 185-216.
- Campbell, J.Y. and N.G. Mankiw (1990), Permanent Income, Current Income, and Consumption, *Journal of Business and Economic Statistics*, 8: 265-279.
- Campbell, J.Y. and N.G. Mankiw (1991), The Response of Consumption to Income: a Cross-country Investigation, *European Economic Review*, 35: 723-767.
- Christiano, L. J., M. Eichenbaum and C. Evans (2005), Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy, *Journal of Political Economy*, 113: 1-45
- Clarida R., J. Galí, and M. Gertler (1999), The Science of Monetary Policy, *Journal of Economic Literature*, 37: 1661-1707.
- Evans G. and S. Honkapohja (2005), Monetary Policy, Expectations and Commitment, *Scandinavian Journal of Economics* (forthcoming).
- Fuhrer, J.C. (2000), Habit Formation in Consumption and Its Implications for Monetary-Policy Models, *American Economic Review*, 90: 367-90.
- Fuhrer, J.C. and G.D. Rudebusch (2004), Estimating the Euler Equation for Output, *Journal of Monetary Economics*, 51: 1133-1153.
- Galí, J., D. López-Salido, J. Vallés (2004), Rule-of-Thumb Consumers and the Design of Interest Rate Rules, *Journal of Money Credit and Banking*, 36: 739-763.
- Jappelli, T. (1990) Who is Credit Constrained in the US Economy?, *Quarterly Journal of Economics*, 219-234.
- Mankiw, G.N. (2000), The Saver-Spenders Theory of Fiscal Policy, *American Economic Review*, 90: 120-125.

- Muscattelli V.A., P. Tirelli, and C. Trecroci (2006), Fiscal and Monetary Policy Interactions in a New Keynesian Model with Liquidity Constraints, available at SSRN, <http://ssrn.com/abstract=880084>.
- Parker, J. (1999), The Response of Household Consumption to Predictable Changes in Social Security Taxes, *American Economic Review*, 89: 959-973.
- Shea, J. (1995), Union Contracts and the Life-Cycle/Permanent-Income Hypothesis, *American Economic Review*, 85: 186-200.
- Souleles, N.S. (1999), The Response of Household Consumption to Income Tax Refunds, *American Economic Review*, 89: 947-958.
- Svensson L.E.O. (1999), Inflation Targeting as a Monetary Policy Rule, *Journal of Monetary Economics*, 43: 607-654.
- Svensson L.E.O. (2003), What is Wrong with Taylor Rules? Using Judgment in Monetary Policy through Targeting Rules, *Journal of Economic Literature*, 41: 426-477.
- Taylor, J.B. (1993), Discretion Versus Policy Rules in Practice, *Carnegie Rochester Conference Series on Public Policy*, 39: 195-214.
- Woodford, M. (2003), *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press, Princeton.
- Woodford, M. (2004), The Taylor Rule and Optimal Monetary Policy, *American Economic Review*, 91: 232-237.

Table 1 – Model calibration

Parameters	Values	Description of the Parameters
β	0.99	Depreciation rate
η	6	Elasticity of substitution among intermediate goods
ν	0.5	Inverse of the Frisch labor supply elasticity
φ	0.75	Fraction of firms that leave their price unchanged
κ	1.7	Value of leisure relative to consumption

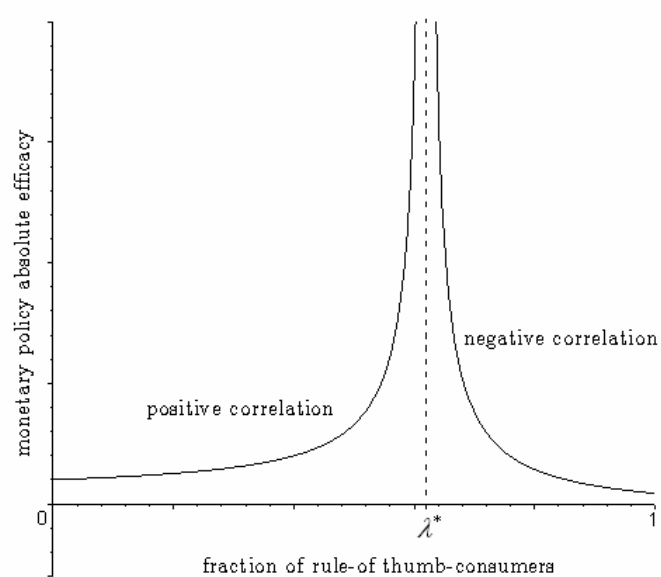


Figure 1

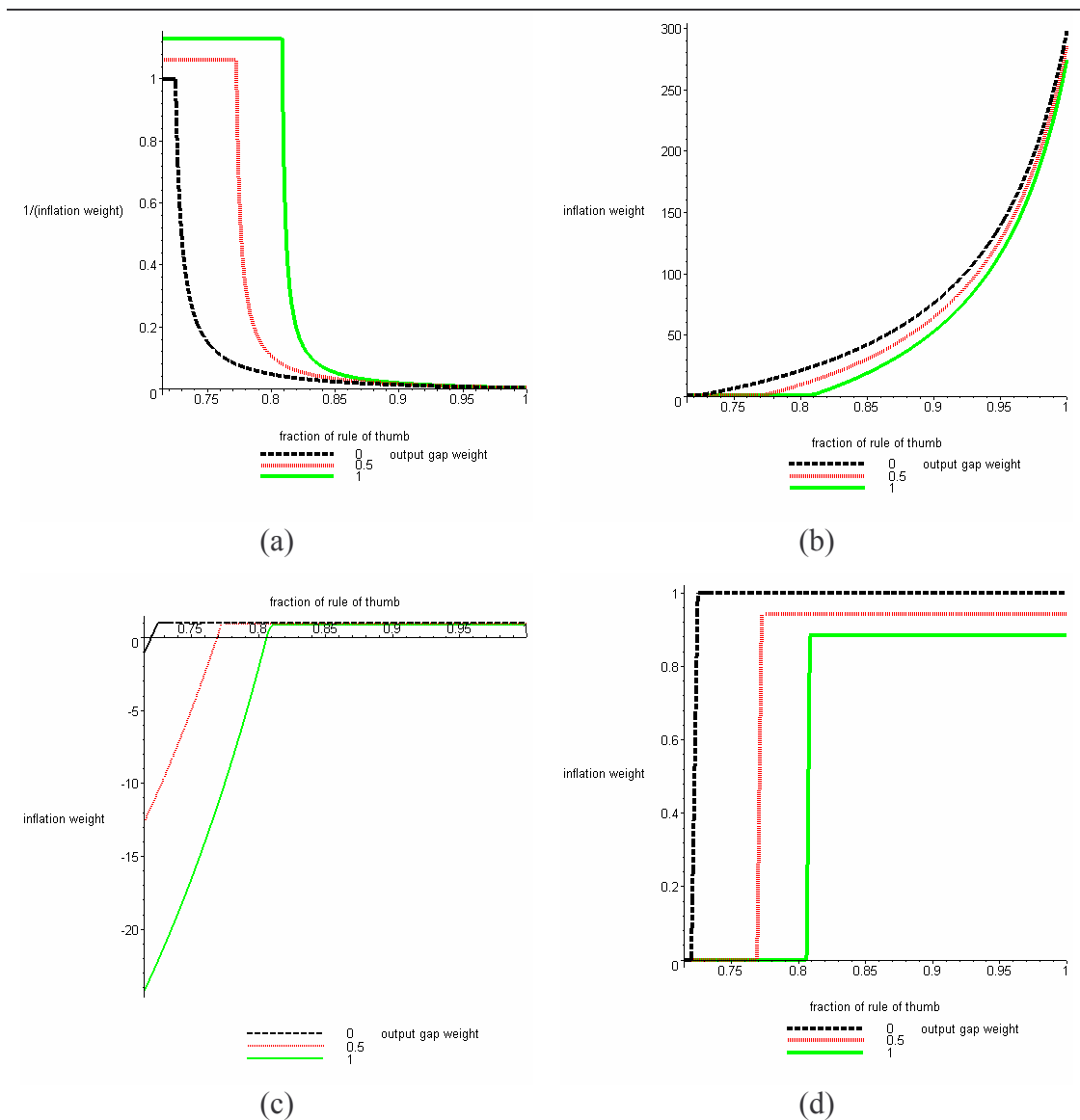


Figure 2

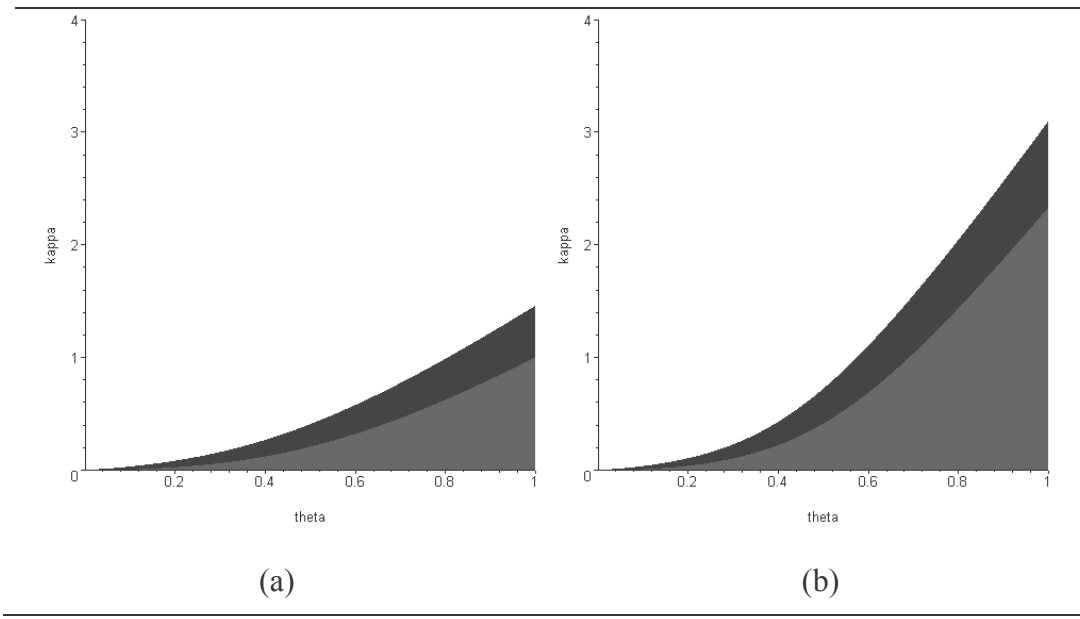


Figure 3

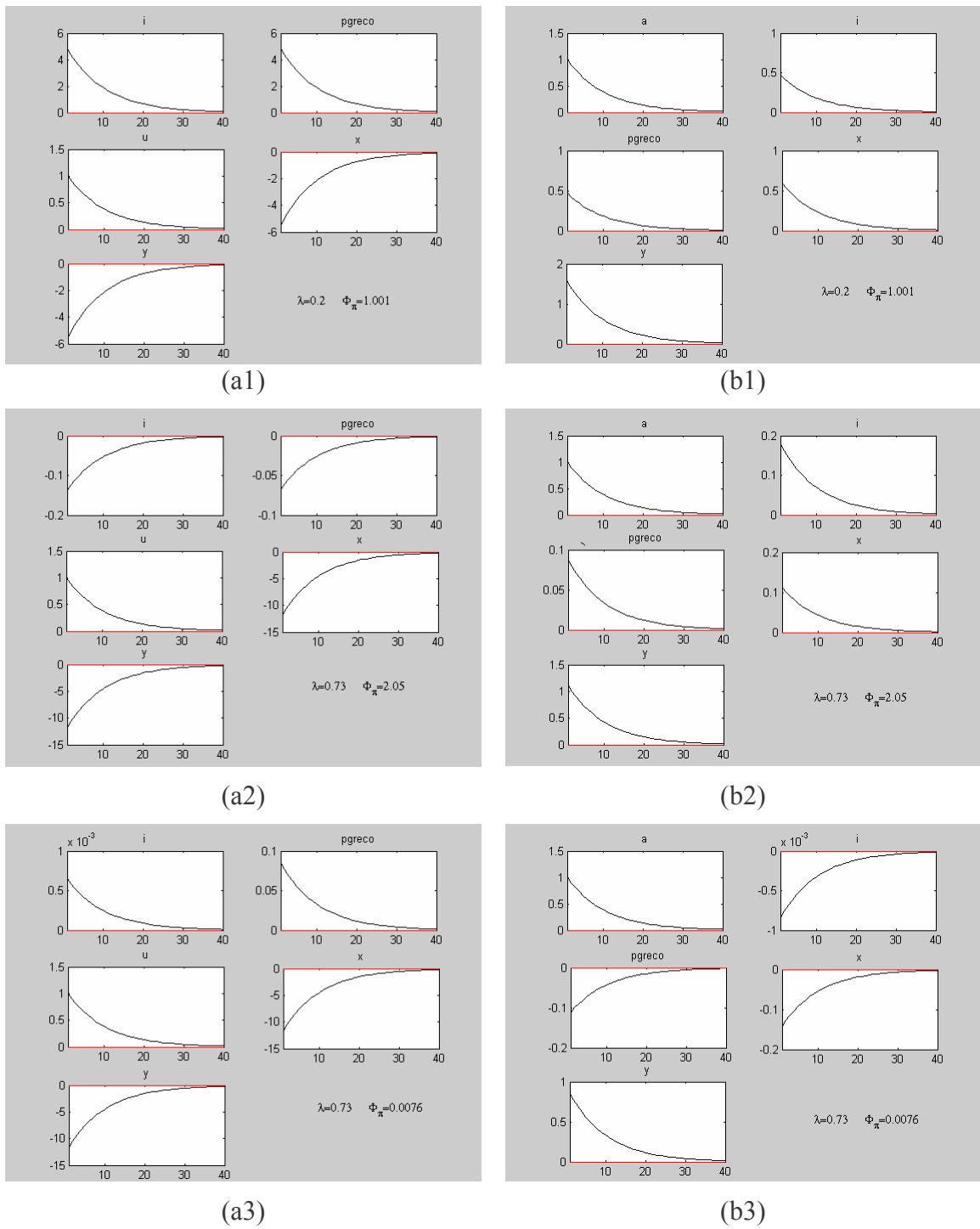


Figure 4