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29. October 2013

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Abstract – The hedge fund represents a unique investment opportunity for the institutional and private investors in the diffusion-type financial systems. The main objective of this condensed article is to research the hedge fund’s optimal investment portfolio strategies selection in the global capital markets with the nonlinearities. We provide a definition for the hedge fund, describe the hedge fund’s organization structures and characteristics, discuss the hedge fund’s optimal investment portfolio strategies and review the appropriate hedge fund’s risk assessment models for investing in the global capital markets in time of high volatilities. We analyze the advanced techniques for the hedge fund’s optimal investment portfolio strategies replication, based on both the Stratonovich – Kalman - Bucy filtering algorithm and the particle filtering algorithm. We developed the software program with the embedded Stratonovich – Kalman - Bucy filtering algorithm and the particle filtering algorithm, aiming to track and replicate the hedge funds optimal investment portfolio strategies in the practical cases of the non-Gaussian non-linear chaotic distributions.

JEL: G00, G01, G11, G12, G 20, G23, C10, C58, C60 ; LCC: HF 4001-4280.7, HF 5681.H43 ; PACS numbers: 89.65.Gh, 89.65.-s, 89.75.Fb.

Keywords: hedge fund, investment portfolio, investment strategy, global tactical asset allocation investment strategy, investment decision making, return on investments, value at risk, arbitrage pricing theory, Sharpe ratio, separation theorem, Sortino ratio, Sterling ratio, Calmar ratio, Gini coefficient, value at risk (VaR), Ledenyov investment portfolio theorem, stability of investment portfolio, Kolmogorov chaos theory, Sharkovsky chaos theory, Lyapunov stability criteria, bifurcation diagram, nonlinearities, stochastic volatility, stochastic probability, Markov chain, Bayesian estimation, Bayesian filters, Wiener filtering theory, Stratonovich optimal non-linear filtering theory, Stratonovich – Kalman – Bucy filtering algorithm, Hodrick-Prescott filter, Hirose - Kamada filter, particle filtering methods, particle filters, multivariate filters, Gaussian linear distribution, non-Gaussian nonlinear distribution, Monte-Carlo simulation, Brownian motion, diffusion process, econophysics, econometrics, global capital markets.
Introduction


Making the initial research on the financial systems, the European and American scientists came up with the understanding that the financial systems can be classified as the diffusion-type financial systems and can be accurately described in the frames of the econophysics theory in Bachelier (1900), Shiryaev (1998a, b), Bernanke (1979), Ledenyov D O, Ledenyov V O (2013f, g). The general understanding that the Brownian-like motion can accurately characterize the properties of the diffusion-type financial system has been proposed in the frames of the speculation theory in Bachelier (1900). Sometime later, the role of the Brownian motion in the stock market in the diffusion-type financial system has been researched in Osborne (1959). The investments in the diffusion-type financial system have been comprehensively researched in Shiryaev (1961, 1963, 1964, 1965, 1967, 1978, 1998a, b, 2002, 2008a, b, 2010), Grigelionis, Shiryaev (1966), Graversen, Peskir, Shiryaev (2001), Kallsen, Shiryaev (2001, 2002), Jacod, Shiryaev (2003), Peskir, Shiryaev (2006), Feinberg, Shiryaev (2006), du Toit, Peskir, Shiryaev (2007), Eberlein, Papapantoleon, Shiryaev (2008, 2009), Shiryaev, Zryumov (2009), Shiryaev, Novikov (2009), Gapeev, Shiryaev (2010), Karatzas, Shiryaev, Shkolnikov (2011), Shiryaev, Zhitlukhin (2012), Zhitlukhin, Shiryaev (2012), Feinberg, Mandava, Shiryaev (2013). The post-earnings announcement drift in the stock returns in the diffusion-type financial system has been documented in Ball, Brown (1968). The investments and monetary policy decisions in the diffusion-type financial system have also been researched in Bernanke (1979, 2002, 2004, 2007, 2009a, b, c, d, e, 2010a, b, 2012a, b, 2013a, b, c, d), Bernanke, Blinder (1992), Bernanke, Gertler (1995), Bernanke, Reinhart (2004), Bernanke, Reinhart, Sack (2004), Bernanke, Blanchard, Summers, Weber (2013). The diffusion of the interest and information among various investors in the diffusion-type financial system has been considered in Shiller, Pound (1989). The problem on the stopping of Brownian motion without anticipation as close as possible to its ultimate maximum in the diffusion-type financial system has been analyzed in Graversen, Peskir, Shiryaev (2001). The macroeconomic forecasting problem, using the diffusion indexes in the diffusion-type financial system has been investigated in Stock, Watson (2002). The quickest detection of drift change for the Brownian motion in the
generalized Bayesian and mini-max settings in the diffusion-type financial system has been analyzed in Feinberg, Shiryaev (2006). The research topic on the prediction the last zero of Brownian motion with the drift in the diffusion-type financial system has been investigated in du Toit, Peskir, Shiryaev (2007). The Bayesian quickest detection problems for some diffusion processes in the diffusion-type financial system have been explained in Gapeev, Shiryaev (2010). The interesting research idea that the financial systems can be accurately characterized in the frames of the diffusion theory has been also commented in Bernanke (1979), Shiryaev (1998a, b), Ledenyov D O, Ledenyov V O (2013f, g). Xiaohong Chen, Hansen, Carrasco (2009) suggested that the drift and diffusion coefficients, which describe the diffusion-type financial system, may also have the nonlinear time dependences: “Nonlinearities in the drift and diffusion coefficients influence temporal dependence in scalar diffusion models.” The one-sided Tanaka equation with the drift in the diffusion-type financial system has been researched in Karatzas, Shiryaev, Shkolnikov (2011). The optimal stopping problems for a Brownian motion with a disorder on a finite interval in the diffusion-type financial system have been researched in Shiryaev, Zhitlukhin (2012). The Bayesian disorder detection problems on the filtered probability spaces the diffusion-type financial system have been considered in Zhitlukhin, Shiryaev (2012). The solutions of Kolmogorov’s equations for nonhomogeneous jump Markov processes in the diffusion-type financial system have been obtained in Feinberg, Mandava, Shiryaev (2013).

At present time, the problem on the optimal investment portfolio strategies selection by the hedge funds in the diffusion-type financial system represents a subject of our strong research interest. Therefore, this research article aims to discover the hedge fund optimal investment portfolio strategies in the process of investment in the global capital markets in presence of the nonlinearities. Moreover, we analyze the advanced techniques for the hedge fund’s optimal investment portfolio strategies replication, based on the Stratonovich – Kalman - Bucy filtering algorithm. We focus on the development of software program with the embedded Stratonovich – Kalman - Bucy filtering algorithm and particle filtering algorithm with the purpose of the hedge fund’s optimal investment portfolio strategies tracking and replication in the practical cases of the non-Gaussian non-linear chaotic distributions. This research logically continues a cycle of our innovative research publications on the nonlinearities in the finances in Ledenyov V O, Ledenyov D O (2012a, b), Ledenyov D O, Ledenyov V O (2012c, d), Ledenyov D O, Ledenyov V O (2013a, b, c, d, e, f, g), which are written, using the knowledge base on the nonlinearities in the microwave superconductivity in Ledenyov D O, Ledenyov V O (2012e).
Theoretical framework for hedge fund investment portfolio allocation in global capital markets in presence of nonlinearities

Let us review the milestones of development of the investment portfolio theories in the finances in the XX-XXI centuries. The Modern Portfolio Theory (MPT) in Markowitz (1952, 1956, 1959, 1987) is based on a fundamental concept that the price changes by the different interrelated assets must be taken to the account in the process of the investment portfolio building. Mitra (2009) explains: “Markowitz proposed a portfolio’s risk is equal to the variance of the portfolio’s returns. If we define the weighted expected return of a portfolio \( R_p \) as

\[
R_p = \sum_{i=1}^{N} w_i \mu_i,
\]

then the portfolio’s variance \( \sigma_p^2 \)

\[
\sigma_p^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} \sigma_{ij} w_i w_j,
\]

where

- \( N \) is the number of assets in a portfolio;
- \( i, j \) are the asset indices and \( i, j \in \{1, \ldots, N\} \);
- \( w_i \) is the asset weight, subject to the constraints:
  \[ 0 \leq w_i \leq 1, \]
  \[ \sum_{i=1}^{N} w_i = 1; \]
- \( \sigma_{ij} \) is the covariance of asset \( i \) with asset \( j \);
- \( \mu_i \) is the expected return for asset \( i \).

The Efficient Frontier (EF) in Markowitz (1952) illustrates the MPT’s ideas graphically as shown in Fig 1 in Mitra (2009). More information on the efficient frontier can also be found in Shiryaev (1998a, b), Hull (2005-2006, 2010, 2012), Ledenyov D O, Ledenyov V O (2013a). Mitra (2009) writes: “MPT also introduces the idea of an efficient frontier. For a given set of funds or assets available to invest in, an upper concave boundary exists on the maximum portfolio returns possible as risk or variance increases. Furthermore this concave relation between risk and return incorporates the theory of expected utility concavely increasing with risk.”
Engle (2003, 2006) states: “Markowitz (1952) and Tobin (1958) associated risk with the variance in the value of a portfolio.” The Tobin’s mutual fund theorem in Tobin (1958) says that the investment portfolio’s assets allocation problem can be viewed as a decision to allocate between a riskless asset and a risky portfolio. Continuing the research on the investments, Mandelbrot (1963) investigated the variation of certain speculative prices. The Mandelbrot’s research proposals and the stable Paretoian hypothesis were discussed in Fama (1963).

Hassine, Roncalli (2013) summarize some important research findings in the investment portfolio theory, made by various authors over the recent decades: “The market portfolio concept has a long history and dates back to the seminal work of Markowitz (1952). In that paper, Markowitz defines precisely what portfolio selection means: “the investor does (or should) consider expected return a desirable thing and variance of return an undesirable thing”. Indeed, Markowitz shows that an efficient portfolio is a portfolio that maximizes the expected return for a given level of risk (corresponding to the variance of return). Markowitz concludes that there is not only one optimal portfolio, but a set of optimal portfolios called the efficient frontier (represented by the solid blue curve in Figure 2). By studying liquidity preference, Tobin (1958) shows that the efficient frontier becomes a straight line in the presence of a risk-free asset. If we consider a combination of an optimized portfolio and the risk-free asset, we obtain a straight line (represented by the dashed black line in Figure 2). But one straight line dominates all the other straight line and the efficient frontier. It is called the Capital Market Line (CML), which corresponds to the green dashed line in Figure 2. In this case, optimal portfolios correspond to a
combination of the risk free asset and one particular efficient portfolio named the tangency portfolio. Sharpe (1964) summarizes the results of Markowitz and Tobin as follows: “the process of investment choice can be broken down into two phases: first, the choice of a unique optimum combination of risky assets; and second, a separate choice concerning the allocation of funds between such a combination and a single riskless asset”. This two-step procedure is today known as the **Separation Theorem**.

![Efficient frontier and tangency portfolio](image)

**Fig. 2. Efficient frontier and tangency portfolio (after Hassine, Roncalli (2013)).**

The **Capital Asset Pricing Model (CAPM)** theory in Sharpe (1964), Lintner (1965) and Mossin (1966) was introduced to accurately determine the expected returns of the selected assets in an investment portfolio. The CAPM main idea is that the assets that correlate perfectly with the market fluctuations as a whole have more risk and thus require a higher return in compensation. The CAPM provided a theoretical framework for the understanding: Why can the different expected returns be obtained across the numerous asset classes? The applications of the CAPM theory were further described in Sharpe (1965, 1966, 1968, 1992, 1994) and in Sharpe, Alexander, Bailey (1999). The dynamic consumption CAPM (CCAPM) theory extends the static CAPM theory in Merton (1973) by providing a theoretical framework to evaluate the market portfolio dynamically. Engle (2003, 2006) explains: “Sharpe (1964) developed the implications, when all investors follow the same objectives with the same information. This theory is called the **Capital Asset Pricing Model or CAPM**, and shows that there is a natural relation between
expected returns and variance”. *Mitra (2009)* states: “The CAPM model is applied generally in finance to determine a theoretically appropriate return of an asset. It presumes that investors must be compensated for investing in a risky asset in 2 ways 1) time value of money and 2) risk itself. The time value of money is accounted for by the risk-free rate $R_f$ whereas the return from risk arises from $\beta(R_m - R_f)$. The term $(R_m - R_f)$ represents the expected risk premium, which is the return obtained above the risk-free rate for investing in a risky asset. The beta term can be considered the “sensitivity” of the asset’s risk to market risk (both measured by variance). Consequently more “sensitive” assets ought to produce higher returns by CAPM.”

$$R_a = R_f + \hat{\beta}(R_m - R_f) + \epsilon,$$

where

- $R_a$ is expected return of an asset;
- $R_f$ is the risk-free rate of return;
- $R_m$ is the expected market return;
- $\epsilon$ is the error term;
- $\hat{\beta} = \frac{\sigma_{am}}{\sigma_{mm}}$;
- $\sigma_{am}$ is the market and asset’s covariance;
- $\sigma_{mm}$ is the market’s variance.

*Mitra (2009)* notes: “*Capocci and Hubner (2004)* state that in the 1980s CAPM and its variants (e.g. *Jensen’s measure*) were applied to hedge fund risk measurement. The CAPM theory and its practical applications were further researched in *Fama, French (2004)*.


$$S = \frac{R_p - R_f}{\sigma_p}$$

where $\sigma_p$ is the portfolio return’s standard deviation.

The *Sharpe ratio* can be interpreted as “*(Return - Risk-free rate)/risk*” since *Sharpe* considers standard deviation to be a risk measure. The *Sharpe ratio* provides a portfolio risk
measure in terms of the quality of the portfolio’s return at its given level of risk. A discussion on the Sharpe ratio can be found at Sharpe’s website (www.stanford.edu/wfsharpe/).”

In the investment portfolio analysis, the investment portfolio that maximizes the Sharpe ratio is also the tangency portfolio on the efficient frontier from the mutual fund theorem in Sharpe, Alexander, Bailey (1999). The maximum Sharpe ratio investment portfolio is situated on the efficient frontier in Fig. 2. Hassine, Roncalli (2013) continue to explain: “One of the difficulties faced when computing the tangency portfolio is that of precisely defining the vector of expected returns of the risky assets and the corresponding covariance matrix of returns. In 1964, Sharpe developed the CAPM theory and highlighted the relationship between the risk premium of the asset (the difference between the expected return and the risk-free rate) and its beta (the systematic risk with respect to the tangency portfolio). Assuming that the market is at equilibrium, he showed that the prices of assets are such that the tangency portfolio is the market portfolio, which is composed of all risky assets in proportion to their market capitalization. That is why we use the terms, tangency portfolio and market portfolio indiscriminately nowadays.”

Mitra (2009) writes: “Fung and Hsieh in (2000b) and (1999b) use a modified version of the Sharpe ratio to rank hedge fund performance so to specifically cater for hedge fund return distributions. This is simply the Sharpe ratio without subtracting the risk free rate from the numerator:

\[
\text{Modified Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p}.
\]

“Jensen (1968) introduced a measure of risk-adjusted performance, the so-called “Jensen’s alpha,” which is essentially the intercept of a regression of excess returns on risk factors, such as the Fama-French three factors,” in Economic Sciences Prize Committee of the Royal Swedish Academy of Sciences (2013). Mitra (2009) also explains the true meanings of the Jensen’s Alpha and Treynor ratio: “Based on CAPM, Jensen formulated a portfolio risk measure to quantify portfolio returns above that predicted by CAPM called \( \alpha \):

\[
\alpha = R_p - \left[ R_f + \beta_p \left( R_m - R_f \right) \right].
\]

One can interpret \( \alpha \) as a measure of “excess returns” or portfolio manager’s investment ability or i.e. “beating the market”.

The Treynor ratio is a lesser well known portfolio ratio measure, similar to the Sharpe ratio, but assesses portfolio performance on a CAPM model basis:
Like the Sharpe ratio, the Treynor ratio can be interpreted as the “quality” of portfolio return for the given level of risk but risk measured on a CAPM theory basis.”

In addition to the single factor CAPM theoretical model, Fama, French (1993) proposed the **Fama, French Three Factor Model**, suggesting to consider the two new factors: 1) the book-to-market value and 2) the price-earnings ratio for the listed companies, aiming to predict the expected returns. “New factors – in particular the book-to-market value and the price-earnings ratio – have been demonstrated to add significantly to the prior understanding of returns based on the standard CAPM,” stated in Economic Sciences Prize Committee of the Royal Swedish Academy of Sciences (2013). Mitra (2009) comments on the Three Factor Model by Fama, French: “The CAPM model is a single factor model that compares a portfolio with the market as a whole. Fama and French modified this model in (1993) to take into account 2 empirical observations about asset classes that tend to have higher returns:

- small sized companies;
- value stocks (companies with high book to market value).

Having a higher return implies a higher risk premium associated with them. The 3 factor model accounts for these higher premiums with the following equation:

$$ R_{\alpha} = R_f + \beta_{p1}(R_m - R_f) + \beta_{p2}SMB + \beta_{p3}HML + \varepsilon, $$

where

- $SMB$ is the difference in return for small and large sized companies;
- $HML$ is the difference in return for high book to market value and low book to market value companies;
- $\beta_{p1}, \beta_{p2}, \beta_{p3}$ are regression gradients (slopes).

Essentially the three factor model is a multiple linear regression equation. Jagadeesh and Titman in (1993) modify the CAPM model by adding a momentum to account for return. Fung and Hsieh in [2004] apply both these models to long/short equity Hedge Funds, giving regression results.”

Let us discuss the **Sharpe’s Asset Class Factor Model**. Mitra (2009) writes: “Sharpe in (1992) invented an asset factor model for risk measurement of Mutual Funds but Fung and Hsieh in (1997) have applied it to Hedge Funds. This model essentially suggests that most
Mutual Fund performances can be replicated by a small number of major asset classes e.g. large capitalisation growth stocks, large capitalization value stocks, small capitalisation stocks etc... . Using Fung and Hsieh (1997) notation Sharpe’s model is:

\[ R_p = \sum_k w_k F_k + \epsilon, \]

subject to

1. \( w_k = \sum_j x_j \lambda_j; \)
2. \( \epsilon = \sum_j x_j \epsilon_j; \)

where

- \( j \) is the asset class;
- \( k \) is the total number of asset classes;
- \( x_j \) is the weighting of asset class \( j \);
- \( \lambda_j \) is the factor loading for asset \( j \) (change in fund return/change in asset \( j \) return);
- \( \epsilon_j \) is the error term for asset \( j \)

Thus Hedge Fund return is a weighted average of a small number of asset classes, rather than a weighted average of a large number of individual asset returns as in MPT.”

In the practical case of the risk management, the risk can be mitigated, going from the principles of diversification, hedging and risk measurements, by the financial practitioners. The actual risk management concept is reflected in the Economic Capital and Credit Modeling theories, and the risk and return are taken to the account during the calculation of the Cost of Capital in Ideas At Work (2006), Ledenyov D O, Ledenyov V O (2012d):

1. **Cost of Capital** is calculated using the Weighted Average Cost of Capital (WACC) model, which includes the following financial variables and ratios: Levered Beta, Debt/Total Capitalization, Tax Rate, Unlevered Beta, Targeted Capital Structure, Risk Free Rate, Market Risk Premium, Spread over Risk Free Rate. The Weighted Average Cost of Capital (WACC) is the weighted average of the marginal costs of all sources of capital. The formula for estimating WACC is as follows in Schnoor (2006):

\[
WACC = K_d (1-T) D / V + K_e E / V + K_p P / V
\]

where:

- \( K_d \) = the pre-Tax Cost of Debt;
• **T** = the Marginal Tax Rate of the entity being valued;

• **D/V** = the Long-term target Net Debt to Total Capitalization;

• **K_p** = the market-determined Cost of Traditional Preferred Stock;

• **P/V** = the Long-term target Market Value of Preferred Stock to Total Capitalization.

2. **Cost of Equity** is calculated using the Capital Asset Pricing Model (CAPM), which includes the following financial variables and ratios: 

\[ \text{Cost of Equity} = \text{Risk Free Rate} + \beta \times \text{Market Risk Premium} \]

where:

• **Ke** = the market-determined Cost of Equity Capital;

• **R_f** = the Risk Free Rate;

• **β** = the company’s beta. The beta is a measure of stock price volatility relative to the overall benchmark market index. In other words, the beta is the price volatility of a financial instrument relative to the price volatility of a market or index as a whole. Beta is most commonly used with respect to equities. A high-beta instrument is riskier than a low-beta instrument. If a stock moves up or down in the same proportion as the overall market, it has a Beta of 1.0. A stock with Beta of 1.2 is considered riskier than the overall market. Higgins (2007) states that the beta can also be considered as an angle of incline:

\[ \beta = \frac{P_{jm}Y_1}{Y_m} \]

where:

• **P_{jm}** is the non-diversified risk.

There are many categories of risk, which have to be considered by the hedge funds and other financial institutions in the frames of the **Basel III capital requirements in Basel Committee**
Market Risk; 2) Credit Risk; 3) Operational Risk; 4) Rollover Risk; 5) Transaction Risk; 6) Foreign exchange risk; 7) Interest rates risk; 8) Liquidity risk; 9) Reputation risk; 10) Emerging markets risk; 11) Environmental risk; 12) Geopolitical risk. Let us consider the appropriate modern approaches to model the volatility and evaluate the 

**Market Risk.** The Autoregressive Conditional Heteroskedasticity (ARCH) model in Engle (1982a, 2003) is used in the field of statistical modeling of volatility in Barone-Adesi, Giannopoulos, Vosper (1999); McNeil, Frey (2000). The ARCH enables to model the financial and economic variables, such as the interest rates and equity prices, by performing the Monte Carlo simulation, using the stochastic differential equations (SDE). The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) performs the modeling over the big window of sequential events, using the weighted averages and giving more weight to the recent events and less weight to the distant events in Bollerslev (1986). Engle (2003) emphasized that the GARCH model presents the theory of dynamic volatilities. The GARCH volatility is proportional to the Value at Risk (VaR). Manganelli, Engle (2001) write: “The most prominent of these risks in trading is market risk, since it reflects the potential economic loss caused by the decrease in the market value of a portfolio. Value at Risk (VaR) has become the standard measure that financial analysts use to quantify this risk. It is defined as the maximum potential loss in value of a portfolio of financial instruments with a given probability over a certain horizon. In simpler words, it is a number that indicates how much a financial institution can lose with probability $q$ over a given time horizon. The great popularity that this instrument has achieved among financial practitioners is essentially due to its conceptual simplicity: VaR reduces the (market) risk associated with any portfolio to just one number, that is the loss associated with a given probability.” Manganelli, Engle (2001) continue to explain: “While VaR is a very easy and intuitive concept, its measurement is a very challenging statistical problem. Although the existing models for calculating VaR employ different methodologies, they all follow a common general structure, which can be summarized in three points:

1) Mark-to-market the portfolio,
2) Estimate the distribution of portfolio returns,
3) Compute the $VaR$ of the portfolio.

The main differences among $VaR$ methods are related to point 2, that is the way they address the problem of how to estimate the possible changes in the value of the portfolio. CAViaR models skip the estimation of the distribution issue, as they allow computing directly the quantile of the distribution. We will classify the existing models into three broad categories:
1) **Parametric:** *(RiskMetrics and GARCH)*: the variance is computed using an *Exponentially Weighted Moving Average*, which correspond to an *Integrated GARCH model*:

\[
\sigma_t^2 = \lambda \sigma_{t-1}^2 + (1 - \lambda) y_{t-1}^2
\]

with \( \lambda \) usually set equal to 0.94 or 0.97. *RiskMetrics* also assumes that standardized residuals are normally distributed);

2) **Nonparametric** *(Historical Simulation and Hybrid model: Historical Simulation)* is based on the concept of *rolling windows*. First, one needs to choose a window of observations, that generally ranges from 6 months to two years. Then, portfolio returns within this window are sorted in ascending order and the \( \theta \)-quantile of interest is given by the return that leaves \( \theta \) % of the observations on its left side and \( (1-\theta) \)% on its right side. If such a number falls between two consecutive returns, then some interpolation rule is applied. To compute the *VaR* the following day, the whole window is moved forward by one observation and the entire procedure is repeated.);

3) **Semiparametric:** *(Extreme Value Theory, CAViaR and quasi-maximum likelihood GARCH)*: (EVT seems to be a very general approach to tail estimation. The main strength is that the use of a *GEV* distribution to parameterize the tail doesn’t seem to be a very restrictive assumption, as it covers most of the commonly used distributions. On the other hand, there are several problems that need to be considered. The *Conditional Autoregressive Value at Risk*, or CAViaR model was introduced by *Engle and Manganelli* (1999). The basic intuition is to model directly the evolution of the quantile over time, rather than the whole distribution of portfolio returns. *Engle and Manganelli* (1999), at the end of section 9, suggest computing the *VaR* of a portfolio by first fitting a *QML GARCH* and then multiplying the empirical quantile of the standardized residuals by the square root of the estimated variance. This estimation method is a mix of a *GARCH* fitted to portfolio returns and historical simulation applied to the standardized residuals.)*

*Mitra* (2009) adds: “*VaR* (value at risk) was invented by *JP Morgan* in 1994 as a general risk management tool and has now become the industry standard for risk. It has become a popular and important risk measure primarily because of the *Basel Committee*, who standardize international banking regulations and practices. *Gupta and Liang* in (2005) applied *VaR* to *Hedge Funds*, specifically for assessing a *Hedge Fund’s* sufficient capital adequacy.

*VaR* tells us in monetary terms how much one’s portfolio can expect to lose, for a given cumulative probability and for a given time horizon. For example, for a cumulative probability
of 99% over a period of 1 day, the VaR amount would tell us the amount by which one would expect the portfolio to lose e.g. $100.

VaR can be calculated by simulation using historical data or some mathematical formula. VaR can also be calculated by the “variance-covariance method” (also known as the delta-normal method) but makes unrealistic assumptions about portfolio returns e.g. returns are normally distributed.”


We have learned that there is a dependence of the expected return on investments on the various risk factors. However, the perfectly optimized investment portfolio from the risk point of view can be inherently unstable from the stability point of view in Fig. 3. Therefore, aiming to optimize the investment portfolio and make it stable, Ledenyov D O, Ledenyov V O (2013a) proposed the Ledenyov investment portfolio theorem: “The investment portfolio is stable in the case, when any pair of randomly selected assets from the investment portfolio is stable,
satisfying the Lyapunov stability criteria; namely the two randomly selected assets must have the two close trajectories at the start and continue to have the two close trajectories always.”

The Ledenyov investment portfolio theorem was formulated, using the important research results in the science of chaos in Kolmogorov (1931, 1938, 1940, 1941, 1959, 1985, 1986), Kolmogorov, Petrovsky, Piskunov (1937), Alexandrov, Khinchin (1953), and in Sharkovsky (1964, 1965), Sharkovsky, Maistrenko, Romanenko (1986). In addition, Ledenyov D O, Ledenyov V O (2013a) suggested a quite interesting theoretical proposition: “We propose to use the dynamic regimes modeling on the bifurcation diagram, based on the dynamic chaos theory, with the purpose to make the accurate characterization of the dynamic properties of the combining risky investments in the investment portfolio, namely to precisely characterize the stability of investment portfolio.” For example, Shiryaev (1998a, b) reviewed the nonlinear chaotic models, highlighting a well known fact that the diffusion-type financial systems can be characterized as the chaotic diffusion-type financial systems or the deterministic nonlinear diffusion-type financial systems. Shiryaev (1998a, b) considers the nonlinear dynamic diffusion-type financial system, described by the logistic equation

\[ x_n = \lambda x_{n-1} (1 - x_{n-1}), \quad n \geq 1, \quad 0 < x_0 < 1, \]

where the nonlinear dynamic diffusion-type financial system has a number of the stable and unstable states at the increase of parameter \( \lambda \), resulting in the transition to the chaos state at the parameter \( \lambda = 3.6 \). Shiryaev (1998) notes that the below expression is true in the case of all the parabolic systems, where \( F = 4.669201 \) is the Feigenbaum number

\[ \frac{\lambda_k - \lambda_{k-1}}{\lambda_{k+1} - \lambda_k} \rightarrow F, \quad k \rightarrow \infty. \]
Hedge fund definition, organization structures and characteristics, optimal investment portfolio strategies and risk assessment models for investing in global capital markets in presence of nonlinearities

The hedge fund can be described as an unregulated or loosely regulated fund which can freely use various active investment strategies to achieve positive absolute returns in Mitra (2009). “According to Fung (1999a), the first ever Hedge Fund was formed by Albert Wislow Jones in 1949, so called as the main investment strategy was to take hedged equity investments.” By hedging (the act of removing risk in some investment by taking an investment in another (typically related) investment) Winslow was able to eliminate some market risks” as stated in Mitra (2009).

Let us discuss the problem of the investment returns computing by the hedge funds. Freed, McMillan (2011) state: “At the most general level then, hedge fund returns comprise some idiosyncratic returns, some known and measurable returns, and some other “stuff” that in a linear regression of hedge fund returns and risk factors appears as statistical noise.” Freed, McMillan (2011) write: “For a single hedge fund, we may describe this more formally as

\[ R^f = \alpha^f + B^f X_T + \epsilon^f, \]

where

\[ B^f = \left[ \beta_1^f, \beta_2^f, \ldots, \beta_n^f \right], \]

\[ X_T = \left[ X_T^1, X_T^2, \ldots, X_T^n \right]. \]

Takahashi, Yamamoto (2008) write the following formula to evaluate the hedge fund return

\[ R_i = \alpha_i + \sum_k \beta_{ik} F_k, \]

where

- \( R_i \) is the return of fund \( i \);
- \( F_k \) is the return of factor \( k \);
- \( \beta_{ik} \) is the exposure of fund \( i \) to factor \( k \);
• $\alpha_i$ is the rest of return $R_i$.

In the case of the portfolio of hedge funds, the return of the portfolio of hedge funds is

$$\sum_{i=1}^{n} w_i R_i = \sum_{i=1}^{n} \alpha_i + \sum (w_i \beta_{1k} + \ldots + w_i \beta_{nk}) F_k,$$

where

• $w_i$ is the weight on fund $i$.

Gibson, Wang (2010) write the formula for the hedge fund portfolio return as

$$r_{i,t} = \alpha_{i,0} + \alpha_{i,1} z_{t-1} + \beta'_{i,0} f_t + \beta'_{i,1} (f_t \otimes z_{t-1}) + \epsilon_{i,t},$$

$$f_t = \alpha_f + A_f z_{t-1} + \epsilon_{f,t},$$

$$z_t = \alpha_z + A_z z_{t-1} + \epsilon_{z,t},$$

where

• $r_{i,t}$ is the return of hedge fund $i$ in excess of riskless rate in month $t$;

• $z_t$ is the vector of $M$ business cycle variables observed at the end of month $t$;

• $f_t$ is a vector of $K$ zero-cost benchmarks;

• $\beta_{k,0}$ is the fixed component of fund risk loadings;

• $\beta_{k,1}$ is the variable component of fund risk loadings;

• $\epsilon_{i,t}$ is fund-specific event, which is assumed to be uncorrelated across hedge funds and over time, and normally distributed with mean zero and variance $\Psi_i$.

Gibson, Wang (2010) note that the problem of the optimal hedge fund investment portfolios formation can be solved by the optimization of the investment portfolio, namely each investor forms his portfolio by maximizing the conditional expected value of a quadratic utility function

$$U(W_t, R_{p,t+1}, a_t, b_t) = a_t + W_t R_{p,t+1} - \frac{b_t}{2} W_t^2 R_{p,t+1}^2,$$

where

• $W_t$ denotes the time $t$ invested wealth;

• $b_t$ reflects the absolute risk aversion parameter;

• $R_{p,t+1}$ is the realized excess return on the optimal of hedge funds computed as
\[ R_{p,t+1} = 1 + r_{ft} + w_t r_{t+1} \]

where

- \( r_{ft} \) being the risk-free interest rate;
- \( r_{t+1} \) denoting the vector of excess fund returns;
- \( w_t \) denoting the vector of optimal hedge fund allocations.

The optimization problem reduces to the equation

\[
 w_t^* = \arg \max_{w_t \geq 0} \left\{ w_t' \mu_t - \frac{1}{2 \left( 1 / \gamma_t - r_{ft} \right)} w_t' \Lambda_t^{-1} w_t \right\},
\]

where

- \( \gamma_t = (b_t W_t) / (1 - b_t W_t) \) is the relative risk-aversion parameter,
- \( \Lambda_t = [\Sigma_t + \mu_t \mu_t']^{-1} \), with \( \mu_t \) and \( \Sigma_t \) being respectively mean vector and variance matrix of future hedge fund returns;
- the possibility of leveraging and short selling is excluded when forming optimal hedge funds’ portfolios.


**Fig. 4.** Daily hedge fund returns (after Boyson, Stahel, Stulz (2008)).
Let us discuss the hedge fund organization structures.

Mitra (2009) writes: “Hedge Funds typically prefer to concentrate their efforts on the key activity of maximizing investment return, so non-essential operations are outsourced e.g. “back office” functions. Actual trading transactions too are outsourced to “Prime Brokers”. Prime brokers are banks or securities firms, offering brokerage and other financial services to large institutional clients e.g. Pension Funds. It is also worth noting that Hedge Funds typically reside “offshore” to take advantage of more favourable tax treatments and regulations.”

Let us review the various hedge fund organization structures in details in Figs. 6 – 11 in Cao, Ogden, Tiu (2011):
Fig. 6. Traditional investment bank model (after Cao, Ogden, Tiu (2011)).

Fig. 7. Inside-only hedge fund model (after Cao, Ogden, Tiu (2011)).
Fig. 8. Straddling hedge fund model (after Cao, Ogden, Tiu (2011)).

Fig. 9. Straddling “feeder” fund of funds model (after Cao, Ogden, Tiu (2011)).
Fig. 10. Stand-alone outside hedge fund model (after Cao, Ogden, Tiu (2011)).

Fig. 11. Outside "feeder" fund of funds model (after Cao, Ogden, Tiu (2011)).
Going to the discussion on *the possible investment portfolio strategies by the hedge funds*, let us clearly identify a main difference between the *hedge funds investment strategies* and the *mutual funds investment strategies*: “*Hedge funds generally using dynamic and leveraged trading strategies*, which is in contrast to *mutual funds* that typically engage in *buy-and-hold strategies,*” as clarified in Mitra (2009).

*Boyson, Stahel, Stulz (2008)* reviewed the eight possible investment strategies by the hedge funds:

1. **“Convertible Arbitrage:** *Convertible Arbitrage* involves taking long positions in convertible securities and hedging those positions by selling short the underlying common stock. A manager will, in an effort to capitalize on relative pricing inefficiencies, purchase long positions in convertible securities, generally convertible bonds, convertible preferred stock or warrants, and hedge a portion of the equity risk by selling short the underlying common stock. Timing may be linked to a specific event relative to the underlying company, or a belief that a relative mispricing exists between the corresponding securities. Convertible securities and warrants are priced as a function of the price of the underlying stock, expected future volatility of returns, risk free interest rates, call provisions, supply and demand for specific issues and, in the case of convertible bonds, the issue-specific corporate/Treasury yield spread. Thus, there is ample room for relative misvaluations.

2. **Distressed Securities:** *Distressed Securities* managers invest in, and may sell short, the securities of companies where the security's price has been, or is expected to be, affected by a distressed situation. *Distressed Securities* managers invest primarily in securities and other obligations of companies that are encountering significant financial or business difficulties, including companies which (i) may be engaged in debt restructuring or other capital transactions of a similar nature while outside the jurisdiction of *Federal bankruptcy law*, (ii) are subject to the provisions of *Federal bankruptcy law* or (iii) are experiencing poor operating results as a result of unfavorable operating conditions, over-leveraged capital structure, catastrophic events, extraordinary write-offs or special competitive or product obsolescence problems. Managers will seek profit opportunities arising from inefficiencies in the market for such securities and other obligations. Negative events, and the subsequent announcement of a proposed restructuring or reorganization to address the problem, may create a severe market imbalance as some holders attempt to sell their positions at a time when few investors are willing to purchase the securities or other obligations of the troubled company. If manager believes that a
market imbalance exists and the securities and other obligations of the troubled company may be purchased at prices below the value of such securities or other obligations under a reorganization or liquidation analysis, the manager may purchase the securities or other obligations of the company. Profits in this sector result from the market's lack of understanding of the true value of the deeply discounted securities. Results are generally not dependent on the direction of the markets, and have a low to moderate expected volatility.

3. **Equity Hedge**: Equity Hedge, also known as long/short equity, combines core long holdings of equities with short sales of stock or stock index options. *Equity hedge portfolios* may be anywhere from net long to net short depending on market conditions. *Equity hedge managers* generally increase net long exposure in bull markets and decrease net long exposure or even are net short in a bear market. Generally, the short exposure is intended to generate an ongoing positive return in addition to acting as a hedge against a general stock market decline. Stock index put options are also often used as a hedge against market risk. Profits are made when long positions appreciate and stocks sold short depreciate. Conversely, losses are incurred when long positions depreciate and/or the value of stocks sold short appreciates. *Equity hedge managers* source of return is similar to that of traditional stock pickers on the upside, but they use short selling and hedging to attempt to outperform the market on the downside.

4. **Equity Market Neutral**: Equity Market Neutral strategies strive to generate consistent returns in both up and down markets by selecting positions with a total net exposure of zero. *Trading Managers* will hold a large number of long equity positions and an equal, or close to equal, dollar amount of offsetting short positions for a total net exposure close to zero. A zero net exposure is referred to as "dollar neutrality" and is a common characteristic of all equity market neutral managers. By taking long and short positions in equal amounts, the equity market neutral manager seeks to neutralize the effect that a systematic change will have on values of the stock market as a whole. Some, but not all, equity market neutral managers will extend the concept of neutrality to risk factors or characteristics such as beta, industry, sector, investment style and market capitalization. In all equity market neutral portfolios stocks expected to outperform the market are held long, and stocks expected to underperform the market are sold short. Returns are derived from the long/short spread, or the amount by which long positions outperform short positions.
5. **Event Driven**: Event Driven investment strategies or "corporate life cycle investing" involves investments in opportunities created by significant transactional events, such as spin-offs, mergers and acquisitions, industry consolidations, liquidations, reorganizations, bankruptcies, recapitalizations and share buybacks and other extraordinary corporate transactions. Event Driven trading involves attempting to predict the outcome of a particular transaction as well as the optimal time at which to commit capital to it. The uncertainty about the outcome of these events creates investment opportunities for managers who can correctly anticipate their outcomes. As such, Event Driven trading embraces merger arbitrage, distressed securities, value-with-a-catalyst, and special situations investing. Some Event Driven Trading managers will utilize a core strategy and others will opportunistically make investments across the different types of events. Dedicated merger arbitrage and distressed securities managers are not included in the Event Driven index. Instruments include long and short common and preferred stocks, as well as debt securities, warrants, stubs, and options. Trading Managers may also utilize derivatives such as index put options or put option spreads, to leverage returns and to hedge out interest rate and/or market risk. The success or failure of this type of strategy usually depends on whether the Trading Manager accurately predicts the outcome and timing of the transactional event. Event Driven Trading Managers do not rely on market direction for results; however, major market declines, which would cause transactions to be repriced or break, may have a negative impact on the strategy.

6. **Macro**: Macro strategies attempt to identify extreme price valuations in stock markets, interest rates, foreign exchange rates and physical commodities, and make leveraged bets on the anticipated price movements in these markets. To identify extreme price valuations, Trading Managers generally employ a top-down global approach that concentrates on forecasting how global macroeconomic and political events affect the valuations of financial instruments. These approaches may be systematic trend following models, or discretionary. The strategy has a broad investment mandate, with the ability to hold positions in practically any market with any instrument. Profits are made by correctly anticipating price movements in global markets and having the flexibility to use any suitable investment approach to take advantage of extreme price valuations. Trading Managers may use a focused approach or diversify across approaches. Often, they will pursue a number of base strategies to augment their selective large directional bets.

7. **Merger Arbitrage**: Merger Arbitrage, also known as risk arbitrage, involves investing in securities of companies that are the subject of some form of extraordinary corporate
transaction, including acquisition or merger proposals, exchange offers, cash tender
offers and leveraged buy-outs. These transactions will generally involve the exchange of
securities for cash, other securities or a combination of cash and other securities.
Typically, a manager purchases the stock of a company being acquired or merging with
another company, and sells short the stock of the acquiring company. A manager engaged
in merger arbitrage transactions will derive profit (or loss) by realizing the price
differential between the price of the securities purchased and the value ultimately realized
when the deal is consummated. The success of this strategy usually is dependent upon the
proposed merger, tender offer or exchange offer being consummated. When a tender or
exchange offer or a proposal for a merger is publicly announced, the offer price or the
value of the securities of the acquiring company to be received is typically greater than
the current market price of the securities of the target company. Normally, the stock of an
acquisition target appreciates while the acquiring company's stock decreases in value. If a
manager determines that it is probable that the transaction will be consummated, it may
purchase shares of the target company and in most instances, sell short the stock of the
acquiring company. Managers may employ the use of equity options as a low risk
alternative to the outright purchase or sale of common stock. Many managers will hedge
against market risk by purchasing S&P put options or put option spreads.

8. **Relative Value Arbitrage:** Relative Value Arbitrage is a multiple investment strategy
approach. The overall emphasis is on making "spread trades" which derive returns from
the relationship between two related securities rather than from the direction of the
market. Generally, Trading Managers will take offsetting long and short positions in
similar or related securities when their values, which are mathematically or historically
interrelated, are temporarily distorted. Profits are derived when the skewed relationship
between the securities returns to normal. In addition, relative value managers will decide
which relative value strategies offer the best opportunities at any given time and weight
that strategy accordingly in their overall portfolio. Relative value strategies may include
forms of fixed income arbitrage, including mortgage-backed arbitrage, merger arbitrage,
convertible arbitrage, statistical arbitrage, pairs trading, options and warrants trading,
capital structure arbitrage, index rebalancing arbitrage and structured discount
convertibles (which are more commonly known as Regulation D securities) arbitrage.”

Gibson, Wang (2010) created more detailed list the eleven possible investment strategies
by the hedge funds:
1. **Convertible Arbitrage**: This strategy is identified by hedge investing in the convertible securities of a company. A typical investment is to be long the convertible bond and short the common stock of the same company. Positions are designed to generate profits from the fixed income security as well as the short sale of stock, while protecting principal from market moves.

2. **Dedicated Short Bias**: Dedicated short sellers were once a robust category of hedge funds before the long bull market rendered the strategy difficult to implement. A new category, short biased, has emerged. The strategy is to maintain net short as opposed to pure short exposure. Short bias managers take short positions in mostly equities and derivatives. The short bias of a manager’s portfolio must be constantly greater than zero to be classified in this category.

3. **Emerging Markets**: This strategy involves equity or fixed income investing in emerging markets around the world. Because many emerging markets do not allow short selling, nor offer viable futures or other derivative products with which to hedge, emerging market investing often employs a long-only strategy.

4. **Equity Market Neutral**: This investment strategy is designed to exploit equity market inefficiencies and usually involves being simultaneously long and short matched equity portfolios of the same size within a country. Market neutral portfolios are designed to be either beta or currency neutral, or both. Well designed portfolios typically control for industry, sector, market capitalization, and other exposures. Leverage is often applied to enhance returns.

5. **Event-Driven**: This strategy is defined as equity-oriented investing designed to capture price movement generated by an anticipated corporate event. There are four popular subcategories in event-driven strategies: risk arbitrage, distressed securities, Regulation D and high yield investing.

6. **Fixed Income Arbitrage**: The fixed income arbitrageur aims to profit from price anomalies between related interest rate securities. Most managers trade globally with a goal of generating steady returns with low volatility. This category includes interest rate swap arbitrage, US and non-US government bond arbitrage, forward yield curve arbitrage, and mortgage-backed securities arbitrage. The mortgage-backed market is primarily US based, over-the-counter and particularly complex.

7. **Global Macro**: Global macro managers carry long and short positions in any of the world’s major capital or derivative markets. These positions reflect their views on overall market direction as influence by major economic trends and/or events. The portfolios of
these funds can include stocks, bonds, currencies, and commodities in the form of cash or derivatives instruments. Most funds invest globally in both developed and emerging markets.

8. **Long/Short Equity Hedge**: This directional strategy involves equity-oriented investing on both the long and short sides of the market. The objective is not to be market neutral. Managers have the ability to shift from value to growth, from small to medium to large capitalization stocks, and from a net long position to a net short position. Managers may use futures and options to hedge. The focus may be regional, such as long/short US or European equity, or sector specific, such as long and short technology or healthcare stocks. Long/short equity funds tend to build and hold portfolios that are substantially more concentrated than those of traditional stock funds.

9. **Managed Futures**: This strategy invests in listed financial and commodity futures markets and currency markets around the world. The managers are usually referred to as Commodity Trading Advisors, or CTAs. Trading disciplines are generally systematic or discretionary. Systematic traders tend to use price and market specific information (often technical) to make trading decisions, while discretionary managers use a judgmental approach.

10. **Multi-Strategy**: The funds in this category are characterized by their ability to dynamically allocate capital among strategies falling within several traditional hedge-fund disciplines. The use of many strategies, and the ability to reallocate capital between them in response to market opportunities, means that such funds are not easily assigned to any traditional category.

11. **Fund of Funds**: Just as the name implies, this is a hedge fund that invests in other hedge funds. Diversification can be across styles by including funds with different strategies, or can be within a single strategy but spread among various hedge funds employing that strategy.”

The possible **hedge fund investment strategies** are summarized in Fig. 12 in Gilroy, Lukas (2005), in Tab. 1 in Sabrina Khanniche (2009), and in Tab. 2 in Piluso, Amerise (2011).
**Fig. 12.** Hedge funds investment strategies (after Gilroy, Lukas (2005)).

**Tab. 1.** Hedge fund strategies description (after Sabrina Khanniche (2009)).

<table>
<thead>
<tr>
<th>Directional strategies</th>
<th>Arbitrage strategies</th>
<th>Specific situation strategies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity hedge Macro</td>
<td>Convertible arbitrage</td>
<td>Distressed securities</td>
</tr>
<tr>
<td></td>
<td>Equity market neutral</td>
<td>Event driven</td>
</tr>
<tr>
<td></td>
<td>Relative value arbitrage</td>
<td>Merger arbitrage</td>
</tr>
</tbody>
</table>
Tab. 2. Hedge fund investment strategies (after Piluso, Amerise (2011)).

<table>
<thead>
<tr>
<th>Variabile</th>
<th>Modalità</th>
<th>Frequenze</th>
<th>%</th>
</tr>
</thead>
<tbody>
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<td>28</td>
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<td></td>
<td>Bottom-Up</td>
<td>83</td>
<td>10,533</td>
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<tr>
<td></td>
<td>CTA / Managed Futures</td>
<td>35</td>
<td>4,442</td>
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<td></td>
<td>Distressed Debt</td>
<td>7</td>
<td>0,888</td>
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<tr>
<td></td>
<td>Diversified Debt</td>
<td>9</td>
<td>1,142</td>
</tr>
<tr>
<td></td>
<td>Dual Approach</td>
<td>11</td>
<td>1,396</td>
</tr>
<tr>
<td></td>
<td>Event Driven</td>
<td>30</td>
<td>3,807</td>
</tr>
<tr>
<td></td>
<td>Fixed Income</td>
<td>46</td>
<td>5,838</td>
</tr>
<tr>
<td></td>
<td>Long / Short Equities</td>
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<td>50,000</td>
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<tr>
<td></td>
<td>Macro</td>
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<td>5,076</td>
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<td></td>
<td>Multi-Strategy</td>
<td>25</td>
<td>3,173</td>
</tr>
<tr>
<td></td>
<td>Others</td>
<td>30</td>
<td>3,807</td>
</tr>
<tr>
<td></td>
<td>Relative Value</td>
<td>32</td>
<td>4,061</td>
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<tr>
<td></td>
<td>Top-Down</td>
<td>18</td>
<td>2,284</td>
</tr>
</tbody>
</table>

Discussing the possible hedge fund investment strategies, Fung, Hsieh (2006) summarize some interesting observations about the hedge funds:

1. “The high attrition rates in hedge funds are comparable to those of young firms. Hedge fund returns also contain substantial idiosyncratic risk, typical of small undiversified firms.

2. Beyond having low correlation to standard asset classes, hedge funds form a heterogeneous group that use many different strategies delivering returns.

3. Hedge funds can become the transmission mechanism of systemic risk because they borrow from and trade with regulated financial institutions, such as prime brokers and investment banks.

4. The risk hedge funds pose to market integrity has shifted to that of a convergence of leveraged opinions among funds that individually may easily operate unnoticed.

5. The identification of systemic risk factors inherent in hedge fund strategies is the key input to important questions such as optimal contract design between buyers and sellers of hedge fund products.”

Papademos (2007) write: “The positive contribution of hedge funds to the efficiency and liquidity of global financial markets is widely recognized, but there are also concerns that in times of stress their activities may create risks to financial stability.”
Finally, let us present some information on the hedge fund indexes providers in Tab. 3.

<table>
<thead>
<tr>
<th>Index-Anbieter</th>
<th>Gründung</th>
<th>Start der Datensätze</th>
<th>Fondsanzahl im Index</th>
<th>Anzahl der Indizes</th>
<th>Homepage</th>
</tr>
</thead>
<tbody>
<tr>
<td>CISDM / MAR</td>
<td>1990</td>
<td>1990</td>
<td>1500</td>
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<td>osdmg.org</td>
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<td>Credit Suisse/Tremont LLC</td>
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<td>1994</td>
<td>900</td>
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<td>Dow Jones Hedge Fund Indexes, Inc.</td>
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<td>6</td>
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<td>Edhec Alternative Indizes</td>
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<td>EurekaHedge</td>
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<td>1998</td>
<td>6586</td>
<td>200</td>
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<td>Feni Alternative Assets GmbH</td>
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<td>2002</td>
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<td>7</td>
<td>fen-alfa.de</td>
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<td>FTSE International Ltd.</td>
<td>2004</td>
<td>1998</td>
<td>40</td>
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<td>ftse.co.uk</td>
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<td>Hedge Fund Intelligence</td>
<td>1998</td>
<td>2000</td>
<td>3378</td>
<td>43</td>
<td>hedgefundintelligence.com</td>
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<td>Hedge Fund Research, Inc.</td>
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<td>1990</td>
<td>1800</td>
<td>37</td>
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<td>InvestForce / Altvest</td>
<td>1993</td>
<td>1993</td>
<td>2304</td>
<td>14</td>
<td>investforce.com</td>
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<tr>
<td>MSCI Hedge Fund Indexes</td>
<td>2003</td>
<td>2002</td>
<td>2050</td>
<td>190</td>
<td>msci.com</td>
</tr>
<tr>
<td>Standard &amp; Poor's</td>
<td>2002</td>
<td>1988</td>
<td>41</td>
<td>13</td>
<td>spglobal.com</td>
</tr>
</tbody>
</table>

**Tab. 3.** Hedge fund indexes providers (after Heidorn, Hoppe, Kaiser (2006)).

Tracking and replication of hedge fund optimal investment portfolio strategies in global capital markets in presence of nonlinearities, applying Bayesian filters: 1. Stratanovich – Kalman – Bucy filters for Gaussian linear investment returns distribution and 2. Particle filters for non-Gaussian non-linear investment returns distribution

First of all, let us formulate the problem of the hedge fund investment portfolio tracking and replication in the finances.

Takahashi, Yamamoto (2008) explain: “In recent years, investment banks and investment companies have released hedge fund replication products, which provide investors access to hedge fund returns at lower costs. In addition, these products avoid some shortcomings of hedge funds that will be discussed later. Some replication products mimic a simple trading strategy of hedge funds while others attempt to infer the actual investment positions of hedge funds and take similar positions.” Takahashi, Yamamoto (2008) continue with the notion: “The development of such techniques has proven to be a challenging task. Currently, the biggest banks such as Goldman Sachs, Merrill Lynch, and JP Morgan, and some large investment companies such as Partners Group, have launched such products. (See, for example, [15].) Some of these institutions developed cloning technique collaborating with the pioneers in hedge fund research such as William Fung, David Hsieh, and Narayan Naik. (See also [15].) They and other researchers have proposed various methods, but these techniques are still work-in-progress.”

Takahashi, Yamamoto (2008) add: “Since hedge fund returns cannot be replicated perfectly, a number of different methods have surfaced. These methods are classified into three approaches:

1. **Rule-based approach**: the rule based approach mimics typical hedge fund investment strategies, which access alternative risk premium. Dynamic trading strategies can be replicated by using listed index options. If an index option is not listed, we can replicate its payoff through a delta-hedging strategy of the underlying asset;

2. **Factor-based approach**: the factor-based hedge fund clone providers try to replicate hedge fund accessibility to alternative risk premium and control exposures to risk factors; and

3. **Distribution replication approach**: the distribution replicating methodology aims to replicate the joint distribution of an investor’s portfolio and hedge fund returns. Unlike the factor-based approach, the distribution replication approach does not aim to replicate the target hedge fund returns on a month-to-month basis. Instead, this method aims to generate returns that have the same distribution pattern as the hedge fund returns.

These approaches aim to replicate different aspects of hedge fund returns.”
Roncalli, Weisang (2008) write: “Even though, HF returns' characteristics make them an attractive investment, investing in hedge funds is limited for many investors due to regulatory or minimum size constraints, in particular for retail and institutional investors. Hedge funds as an investment vehicle have also suffered from several criticisms: lack of transparency of the management's strategy making it difficult to conduct risk assessment for investors; poor liquidity, particularly relevant in period of stress; and the problem of a fair pricing of their management fees. It is probably the declining average performance of the hedge fund industry coupled with a number of interrogations on the levels of fees [17] which led many major investors [4, page 5] to seek means of capturing hedge fund investments strategies and performance without investing directly into these alternative investment vehicles. Hence, the idea of replicating hedge funds' portfolios, already common in the context of equity portfolios, gained momentum.” Roncalli, Weisang (2008) add: “…the academic interest in replication is foremost to assess performance particularly with the goal of assessing the quality of management and understand the structure of risk behind specific hedge funds, replication as a process to create investment vehicles will have better chances of succeeding if it aims at replicating an aggregate of funds, where the idiosyncratic management styles the talent are averaged out, letting instead emerging investment decisions made on a macro scale.”

![Fig. 13. Hedge fund performance in 1994-2008 in Roncalli, Weisang (2008).](image-url)
Thus, let us summarize the above statements by saying that the tracking and replication problem can be solved by applying the concept of time-series filtering in the finances. At present time, the concept of the time-series filtering in the finances attracts a considerable attention of researchers in Javaheri, Lautier, Galli (2002). The theories and practical techniques towards the analogue and digital signals processing and filtering have been early researched in Ledenyov D O, Ledenyov V O (2013g), Ledenyov D O, Ledenyov V O (2012e), Wanhammar (1999). The filtering of the time series in the finances is usually performed with the application of the Stratonovich – Kalman – Bucy filtering algorithm in Stratonovich (1959a, b, 1960a, b), Kalman, Koepcke (1958, 1959), Kalman, Bertram (1958, 1959), Kalman (1960a, b, 1963), Kalman, Bucy (1961), which was developed within the optimal non-linear filtering theory in Stratonovich (1959a, b, 1960a, b). Presently, the investment banks, mutual funds, commodity trading advisor (CTA) funds, other hedge funds conduct the advanced research projects to develop the software programs for the tracking and replication of hedge fund optimal investment portfolio strategies in global capital markets in presence of nonlinearities, applying the so called Bayesian filters in Javaheri, Lautier, Galli (2002), Roncalli, Weisang (2008), Takahashi, Yamamoto (2008), Ledenyov D O, Ledenyov V O (2013g):

1. Stratanovich – Kalman – Bucy filters for Gaussian linear returns distribution, and
2. Particle filters for non-Gaussian non-linear returns distribution.

The Stratanovich – Kalman – Bucy filters have been well described in Ledenyov D O, Ledenyov V O (2013g): “The Kalman filter, also known as Linear Quadratic Estimation (LQE), is an algorithm that uses a series of measurements observed over time, containing noise (random variations) and other inaccuracies, and produces estimates of unknown variables that tend to be more precise than those based on a single measurement alone. More formally, the Kalman filter operates recursively on streams of noisy input data to produce a statistically optimal estimate of the underlying system state.”

The particle filters have been accurately characterized in Roncalli, Weisang (2008): “Particle filtering methods are techniques to implement recursive Bayesian filters using Monte-Carlo simulations. The key idea is to represent the posterior density function by a set of random samples with associated weights and to compute estimates based on these samples and weights [7, 20, 25, 26, 27, 28].”

Roncalli, Weisang (2008) write that the generic procedure for the hedge fund investment portfolio strategies replication can therefore be decomposed into the two stages:
The generic procedure for the tracking problem can be defined as in Roncalli, Weisang (2008):

\[ r_k^{HF} = \sum_{i=1}^{m} W^{(i)}_k r_k^{(i)} + \varepsilon_k, \]

\[ r_k^{Clone} = \sum_{i=1}^{m} \hat{W}^{(i)}_k r_k^{(i)}. \]

The generic procedure for the tracking problem can be defined as in Roncalli, Weisang (2008):

\[
\begin{align*}
    x_k &= f(t_k, x_{k-1}, V_k) \\
    z_k &= h(t_k, x_k, \eta_k)
\end{align*}
\]

However, the problem is that there the nonlinearities in the distribution of hedge fund returns. Roncalli, Weisang (2008) write: “Indeed, the distributions of HF returns are well known to exhibit skewness and excess kurtosis, and nonlinear effects have been documented in HF returns ever since the seminal paper of Fung and Hsieh in 1997.” Therefore, the particle filters have to be used to capture the nonlinearities in the hedge fund returns instead of the Stratanovich – Kalman – Bucy filters. Takahashi, Yamamoto (2008) write: “Although exposure estimation by Kalman filter was the best in this example, it can be insufficient for some cases. The state space model that Kalman filter uses is for the case of normal white noise. Therefore, this model cannot capture drastic exposure changes. [21] estimated the exposure changes of mutual funds for non-Gaussian white noise cases using the Monte Carlo filter. Similar research should be done for hedge funds using Monte Carlo and other non-linear filtering methods to catch drastic exposure changes appropriately.”

Roncalli, Weisang (2008) propose the more advanced procedure for the hedge fund investment portfolio strategies replication to capture the nonlinear returns, which can be decomposed into the following two stages:

\[
r_k^{HF} = \sum_{i=1}^{m} W^{(i)}_k r_k^{(i)} + \sum_{i=m_1+1}^{m_1+m_2} W^{(i)}_k r_k^{(i)} + \eta_k,
\]

\[
r_k^{Clone}(d) = \left(1 - \sum_{i=1}^{m} \hat{W}^{(i)}_{k+d+1|k+d} \right) r_k^{(0)} + \sum_{i=1}^{m} \hat{W}^{(i)}_{k+d+1|k+d} r_k^{(i)}.\]
Roncalli, Weisang (2008) conclude by making the following statements: “From the academics' point of view, introducing particle filters opens a door for a better understanding of HF returns and the underlying risks of the HF strategies,” and “… particles filters are one of the main avenues toward a better monitoring of for now unaccounted risks, as they are contained in the higher moments of the returns' distribution.”

We completed the research objectives by providing the accurate characterization of the hedge fund’s optimal investment portfolio strategies selection techniques and by developing the software program with both:

1) the embedded Stratonovich – Kalman – Bucy filtering algorithm, and

2) the embedded particle filtering algorithm,

aiming to track and replicate the optimal investment portfolio strategies by the high performing hedge funds in the practical cases of the non-Gaussian non-linear chaotic investment returns distributions in the diffusion-type financial systems in the near real time settings. Our software program can be potentially used by the investment banks, mutual funds, and central banks.

We think that the high performing hedge funds represent the unique investment opportunities for the institutional and private investors in the diffusion-type financial systems in Europe, Asia and North America. In the beginning of our research, we provided a definition for the hedge fund, described the hedge fund’s organization structures and characteristics, discussed the hedge fund’s optimal investment portfolio strategies and reviewed the appropriate hedge fund’s risk assessment models for investing in the global capital markets in time of high volatilities. In the course of research, we analyzed the advanced techniques for the hedge fund’s optimal investment portfolio strategies tracking and replication, based on both the various types of the Stratonovich – Kalman – Bucy filters and the particles filters. We would like to emphasis that the Stratonovich – Kalman – Bucy filtering algorithms and the particle filtering algorithms can be effectively applied to solve the following complicated econophysical problems in the finances: 1) the dynamic system state estimation and prediction problems by means of the time - series filtering and interpolation, and 2) the dynamic system state tracking and replication problems by means of the time - series filtering and interpolation. We completed our research objectives by providing the information review on the accurate characterization of the hedge fund’s optimal investment portfolio strategies selection techniques and by developing the software program with 1) the embedded Stratonovich – Kalman – Bucy filtering algorithm and 2) the embedded particle filtering algorithm to track and replicate the optimal investment portfolio strategies by the high performing hedge funds in the practical cases of the non-Gaussian non-linear chaotic investment returns distributions in the diffusion-type financial systems in the near real time settings. In our opinion, more research is necessary to improve and adapt the software program to the new 64 bit operating systems. We would like to conclude using the statements in Weber (2007): “The international financial system is undergoing a sustained process of structural change characterized by features such as the rapid growth of the hedge fund industry and credit risk transfer markets. In general, this development should generate positive effects for the efficiency of the financial markets. As the financial system is becoming more complex and less transparent, however, it is becoming a growing challenge for central banks to make an adequate assessment of the potential risks to financial stability.”
Acknowledgement

We gratefully acknowledge the insightful thoughtful scientific discussions in the finances and economics and considerable research interest by Dr. Ben Shalom Bernanke, Chairman of the Board of Governors of the Federal Reserve System, encouraging us to think wisely in the course of our advanced research and to make everything possible to complete the research program timely by giving us access to the electronic copies of his most recent research articles, analytic research reports, informative slide presentations, minutes of research discussions on the various financial topics and strategic economic issues within the scope of our research interest as well as an electronic copy of his Ph. D. Thesis: “Long-term commitments, dynamic optimization, and the business cycle,” supervised by Stanley Fischer, Professor of Economics, Massachusetts Institute of Technology, USA. It is a real privilege for the authors to thank Axel A. Weber, President, Deutsche Bundesbank for an electronic copy of his interesting research article: “Hedge funds: A central bank perspective,” which presents an insightful scientific opinion on the role of the hedge funds in the stability of the international financial system from the central bank’s perspective. The authors would like to acknowledge Dr. Thomas J. Jordan, Chairman of the Governing Board, Swiss National Bank, Zurich, Switzerland and Dr. Jean-Pierre Danthine, Vice Chairman of the Governing Board, Berne, Switzerland for their kind support of the research on the hedge fund optimal investment portfolio strategies in the global capital markets in presence of the nonlinearities by providing us with an unrestricted access to the collection of the research papers and analytic reports at the Swiss National Bank’s servers in Zurich, Switzerland: “Applying the Hirose-Kamada filter to Swiss data: Output gap and exchange rate pass-through estimates,” written by Franziska Bignasca and Enzo Rossi; and "On the predictability of stock prices: A case for high and low prices," written by Massimiliano Caporin, Angelo Ranaldo and Paolo Santucci de Magistris. We are very grateful to Prof. Robert F. Engle III, Department of Finance, New York University, New York, USA for his visionary statements, interesting discussions and comprehensive research data on the Stratonovich – Kalman – Bucy filtering algorithm, in particular, we would like to mention our thoughtful discussion on the derivation of the probability density function as a sum of its predictive or conditional densities in the case the state-space model in Engle (2006). The knowledge base on the hedge funds industry, created and accumulated by David A. Hsieh, Professor, Duke University, represents a comprehensive source of objective information on the topic of research interest, that is why our business meetings with the leading professors in the finances in the atmosphere of friendship and partnership at the Duke University, Raleigh, USA in 2005 are appreciated. It is a real privilege for the first author to
deliver his special personal thanks to Prof. Janina E. Mazierska, Personal Chair, Electrical and Computer Engineering Department, James Cook University, Townsville and former IEEE Director, who helped us to cultivate the logical scientific thinking to tackle the complex scientific problems on the nonlinearities in the microwave superconductivity, applying the interdisciplinary scientific knowledge together with the advanced computer modeling skills in 2000-2013. A total number of invitations to present the lectures on the nonlinearities in the finances is much more bigger than the available time in our busy work schedules in the nearest years to come, that is why we are able to deliver the invited lectures at the most innovative universities only. Lecturing at the top ranked universities worldwide, we prefer to create the open discussion forum atmosphere and to take as many original questions as possible during the questions and answers (Q&A) sessions after the presentations. Usually, it may take one or two hours to answer all the original questions, posed by the international students, Ph. D. researchers, professors and financiers at the regular Q&A sessions, allowing the meeting participants to let us know their questions and opinions, frequently delivered as the short lectures despite of the moderator’s repeated requests to formulate the short questions only. We let the participants to express their opinions in details, because we do believe that the multiple research inputs by the young scientists provide us with an unique opportunity to think differently on the complex scientific problems, which can only be solved by applying the multidisciplinary knowledge and the scientific intuition, coming with the years of research experience. In our global world, we make the research using our mobile office: the Apple’s MacBookPro laptops, iPAD Air tablets and iPhone 5S smart-phones, to obtain and analyze the research data from the cloud computing network servers via the satellite or mobile wireless Internet links. The authors always try to be connected with our research collaborators and partners despite of such annoying obstacles as the very long distances and the different time zones. We frequently apply the integrative creative thinking techniques in the financial engineering problems optimal solution finding process, hence we sincere thank Profs. Roger L. Martin and John C. Hull from the Rotman School of Management, University of Toronto, Canada for the numerous thoughtful long-hours scientific discussions. Finally, we highly value the established business relationships with the Mathworks senior management team and would like to thank the Mathworks for a license to perform the computer modeling with the parallel computing techniques and a kind permission to use the software library with the different implementations of the Stratonovich – Kalman - Bucy filtering algorithm in the Matlab at the Mathworks servers in the USA.

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