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# Buying Decision Coordination and Monopoly Pricing of Network Goods\*

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## Abstract

We analyse how consumer heterogeneity affects buying behaviour and the monopoly pricing of a network good and its usage. Under perfect information, sufficiently high heterogeneity yields a unique equilibrium, and the unit price is increasing in heterogeneity. Under incomplete information, we have a global game. The unit price is independent of heterogeneity, and it tends to be higher than the perfect information price, because the monopoly biases its tariff structure to incorporate the uncertainty over usage revenues. Under incomplete information, profits are decreasing in uncertainty. Consumer surplus increases in uncertainty, only if the level of uncertainty is high initially.

**Keywords:** Coordination, network externalities, heterogeneity, information, equilibrium uniqueness, global games.

**JEL classification:** D42, D82, L14.

## 1 Introduction

The product has a social dimension when its usage involves interaction between consumers. This is captured as network externalities. Because the consumer's utility from a social good depends on how many other consumers buy the good, expectations on other consumers' actions determine whether the consumer actually buys the good or not. Heterogeneity between consumer types may facilitate coordination, but under incomplete information

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it translates to uncertainty over other people's actions. In this paper, we analyse how expectations and uncertainty about buying decisions among consumers affects consumers' strategies and a monopolist's pricing strategy.

It is a well-known fact that network externalities cause multiple equilibria, because they induce a coordination game with strategic action complementarities à la Bulow et al. (1985). Indeterminacy caused by multiplicity of equilibria has been incorporated in the theory in the forms of de facto standards and bandwagon strategy profiles by the seminal papers by Arthur (1989), David (1985), Farrell and Saloner (1985, 1986), who study technology adoption, and Katz and Shapiro (1985) who examine brand competition and compatibility between networks. Those papers and subsequent literature accepting indeterminacy as a characteristic of economic networks, suggest that market structures in network industries are determined by random exogenous events. Multiplicity causes agents' strategies to be equilibrium-specific, hence in order to derive the optimal actions, the analysis must focus on one equilibrium at a time. Since the arguments that select the equilibrium are inevitably exogenous to the model, we do not learn much on the agents' behaviour at the end. This underlines the benefits from reaching a unique equilibrium, as the results become determinate.

Herrendorf et al. (2000) have shown that a coordination game with strategic action complementarities can have a unique equilibrium under perfect information, if consumer types are sufficiently dispersed. Under incomplete information, uniqueness presupposes a *possibility* of sufficient heterogeneity (Carlsson and van Damme 1993, Morris and Shin 2003, Mason and Valentinyi 2003). We leverage the recent theory to obtain a unique equilibrium in a network model. We show that the solutions and results under perfect and incomplete information differ. The requirement for uniqueness under perfect information implies that consumers make their buying decisions dominantly on non-network-specific attributes, and endogenous pricing is an insufficient mean to eliminate multiplicity. For a model of network effects this constraint is troublesome. Under incomplete information, uniqueness can be reached even if network attributes are driving decision making. The assumption about incomplete information is also supported by the observation that, with a launch of a new device, people are not usually able to tell how much utility the device yields to other people. This informational asymmetry is aggravated the more drastic innovation the new device is.

We offer two real world examples. The first example is online gaming. Sony PlayStation, Microsoft Xbox and Nokia N-Gage consoles all have standalone and interaction usage features. Players can play alone or against other players on the same console. In addition, console manufacturers run platforms<sup>1</sup> that offer services and content, and allow people to play and talk with other people over the Internet. It is also usual that people swap games

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<sup>1</sup>Sony Central Station and Xbox Live. N-Gage allows playing over the air directly.

with their friends. A consumer who considers buying a console, takes into account how many games a particular console has in supply and what is the quality of the console. Another point he bears in mind is whether his friends have the same brand so that he can play with, and against them.

The second example is mobile telecommunications. Mobile phones can be also used nowadays for checking latest news and e-mails, or to listen to the radio and music, and even to watch television. These features create intrinsic value for a phone. The main value driver, of course, is the possibility to talk with friends and send them messages.

We model a market consisting of consumers and a monopoly firm. The firm sells a novel device that constitutes an efficient medium for interaction. Like with many network goods, the total utility from the good consists of usage utility from interaction with other people, and of intrinsic utility from standalone services used independently of other consumers.<sup>2</sup> Everybody agrees that all pre-innovation interaction can be mediated by the new device, and the quality of interaction is improved so that the product generates positive network externalities. Since interaction can take place only after people have acquired the goods, we separate the usage stage from the acquisition stage. The monopolist sets a two-part tariff for the product and its usage, and is not able to commit to a tariff structure in the first stage.

We analyse two informational cases. We first assume that all information is perfect. Heterogeneity between consumers is directly observed, and achievable network benefits are perfectly taken into account in the buying decisions. Under incomplete information, consumer heterogeneity induces uncertainty about buying decisions, and consumers must take expectations about achievable network benefits.

We assume that consumers are homogeneous with respect to interaction utility, whereas the consumer types are horizontally differentiated according to their perception of the intrinsic value à la Hotelling (1929). Under incomplete information, consumers derive their types through noisy signals of the intrinsic value. Differentiation captures the idea of consumer satisfaction with product's technical performance.<sup>3</sup>

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<sup>2</sup>Intrinsic utility may include status-enhancing type of utility, and any non-direct benefits from belonging to a network including higher-order interaction benefits, which comprise utility from interaction taking place between one's friends' friends, between friends' friends' friends, and so forth.

<sup>3</sup>Why is intrinsic utility subjected to differentiation while usage is not? Usage utility is directly associated with the people who interact, or more precisely, with the social relation the interacting parties have. The device is a mere medium, which does not influence the value of the social relation. We assume that each consumer has equally valuable social relations and the improvement in interaction efficiency is identical for all. In contrast, how different consumers perceive the novel features of the device, the capacity to use it, and the attitude towards new technology differ between people. Intrinsic utility does not even have to be positive in relation to older generation products. As an example, "mobility" is the principal improvement of mobile telecommunications with respect to fixed line telephony. Being able to call and to be called independent of time and place is an objectively measured

We derive the optimal two-part tariff structure for the monopoly, and analyse the effects of a marginal change in consumer heterogeneity. The optimal unit price is increasing in heterogeneity under perfect information. If information is incomplete, the price is independent of heterogeneity. The monopolist sets a higher unit price, compared to the price under perfect information, in the first period in order to incorporate the possibility of a wide perception of a low quality of its goods and subsequent low usage profits. There is no such bias under perfect information, as the monopolist is able to perfectly capture the usage utility. We also discuss the possibility that the second period is characterised by perfect competition. In that case, the bias for a higher first period price is aggravated.

The effect of a marginal change in heterogeneity on profits and consumer surplus is ambiguous under perfect information, but the firm's expected profits increase as uncertainty (heterogeneity) is reduced under incomplete information. The effect of a marginal change in uncertainty on expected consumer surplus is positive, if the change is aligned with the absolute level. That is, if uncertainty is high in the first place, then further uncertainty is of good. Similarly, the expected consumer surplus increases, when there is little uncertainty and we further reduce it.

Most of the related network externalities literature is concerned with the question about socially optimal standardisation. Shy (2001) and Baake and Boom (2001) find that the intensity of price competition is reduced with compatible networks compared to incompatible networks. Bental and Spiegel (1995) derive that the largest network is also the most expensive one, if people's willingness to pay increases in the size of the network. Cabral et al. (1999) and Mason (2000) prove that the Coase conjecture fails in its strongest form with durable goods inducing network externalities. Our model is also related to the telecommunications network competition literature set out by Armstrong (1998) and Laffont et al. (1998a, 1998b).

Our model is an application of the global games theory that has been successfully applied in a number of macroeconomic and financial problems. Morris and Shin (1998) and Heinemann et al. (2004) analyse a model of speculative currency attacks. Englmaier and Reisinger (2003) apply global games to an economic development framework. Morris and Shin (2004) and Rochet and Vives (2004) study solvent but illiquid financial institutions. Myatt and Wallace (2002) analyse public goods provision with global games techniques. Chwe (1998) finds that goods with social externalities advertise

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improvement (you can always keep the phone switched off whenever you wish!) However, mobile phones tend to be small in size and their use can therefore be very difficult for example, for elderly people. The size factor is positive for most consumers, but it can also be negative. Alternatively, some people believe that mobile phones emit radiation harmful to the brain. Other people fear that third parties can secretly monitor the user. Whether it is due to the fear of brain tumors or malicious surveillance, some people may be reluctant to carry a mobile phone, even if they get one for free.

"more on more expensive popular [TV] shows because viewers of popular shows know that many other people are also watching" supporting global games predictions. Our paper, together with Argenziano (2004), are the first applications of global games to network economics. These models are also the first to endogenise the payoffs with a pricing problem. Argenziano (2004) studies a platform competition with pure membership externalities. Our model differs from her model in that we study a market with membership and usage decisions. In addition, we analyse a monopolist that sets a two-part tariff, whereas Argenziano (2004) analyses a Bertrand duopoly with linear prices. She shows that network sizes are socially too balanced. Endogenous strategic pricing aggravates the social inefficiency, as the firm with higher expected quality sets the highest price.

In section 2, we explain the model structure. In section 3, we link network externalities to interaction on a network of social relations. We analyse the benchmark case of perfect information in section 4. In section 5, we analyse the incomplete information case. The results from the two information regimes are compared in section 6. We conclude in section 7.

## 2 Players, actions, and timing

There is a continuum  $I = 1$  of consumers in the market. The fundamental intrinsic value of the product  $\theta$  is drawn from a uniform distribution  $F(\theta)$ . Consumer types  $x$  are i.i.d. around the fundamental according to a conditional uniform distribution  $G(x | \theta)$ .

Timing is summarised in Figure (1). The first period problem for consumer  $i \in I$  is to choose action  $a_i \in \{B, N\}$ , where  $B =$  buy the device and  $N =$  do not buy. If the consumer bought the device, then he needs to decide how much he uses it in the second period. Those consumers, who did not buy, collect the reservation utility of zero and make no further decisions.

The firm sets a unit price in the first period, and in the second period, it charges a usage fee. The unit cost for manufacturing one device is  $c_f$ , and the firm incurs a unit cost  $c_a \in [0, 1]$  for interaction mediation. Fixed costs are zero. We assume no discounting. The purchase of the device is a sunk cost to consumers, but it enables subsequent interaction usage. The firm is not able to commit to a tariff structure in the first period, thus it sets the usage charge after the consumers have made their purchasing decisions.

The new device can be used in interaction only if both parties have bought the device. For example, let consumer  $i$  have a need to interact with  $j$ . If both  $i$  and  $j$  have bought the device, then they use it. If either one does not have the product, then they use conventional ways to interact. We assume an *exogenous social network structure*, so that from consumer  $i$ 's point of view, interaction with  $j$  is not the same as interaction with  $k$ , and the social relations are unequal in terms of interaction utility. Consequently,

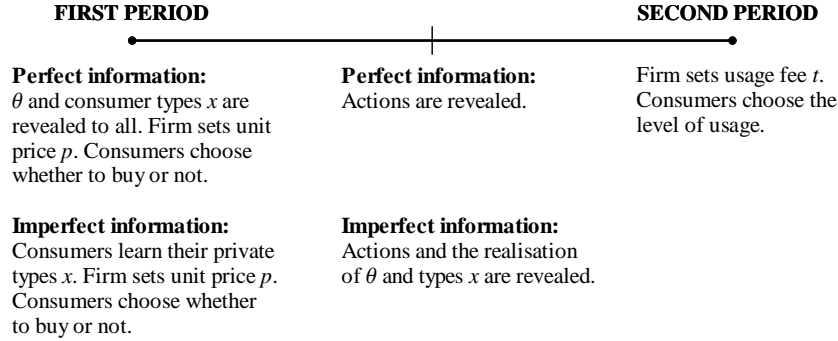


Figure 1: Time Line.

inability to interact with  $j$  cannot be compensated by interaction with  $k$ . We assume that each consumer is interested in interacting with the whole population, so a consumer decides how large a fraction of the population he wants to interact for a given price.<sup>4</sup> Only the person paying for the usage gets utility. "Reception" is cost-less and yields zero utility. In a given social relation, both consumers may pay for usage and get utility. Since the underlying social network is exogenous, the interaction needs are independent of who buys the device. As a result, the consumers are symmetric with respect to usage demand in the second period.

The second period is deterministic. In the first period, the model is exposed to two informational regimes. First, information is perfect to all. Second, information is incomplete, so that consumers observe noisy signals of  $\theta$  which correspond to consumer types. The realisations of the signals are private information, but the prior  $F(\theta)$  and the posterior  $G(x | \theta)$  are common knowledge. The firm observes nothing and resorts to the prior. Such informational asymmetry follows from an assumption that consumers know their needs better than the firm.

The expected net utility of the consumer of type  $x$  from buying ( $a = B$ ), when he expects a fraction  $q$  of the population to buy, is

$$u(x, q, B) = x + \lambda(\alpha, t, q) - p,$$

where  $p$  is the unit price. The intrinsic value is given by  $x$ , and the network benefits from efficient communication is given by  $\lambda(\alpha, t, q)$  that is increasing in  $q$ .  $\alpha$  is the fraction of population the consumer interacts with, and  $t$  is the usage fee per social relation. If  $x - p > 0$ , then  $a = B$  is a strictly

<sup>4</sup>In Sääskilahti (2005a p.78-82), we analyse a version of the model where each consumer is interested in interaction with only a sub-set of total population. We show that the "global" and "local" interaction models coincide when all consumers have the same number of contacts, inducing network symmetry. See Sääskilahti (2005b) for an analysis on asymmetric social relations.

dominating strategy. Action  $N$  is a strictly dominating strategy when  $x + \lambda(\alpha, t, q = 1) - p < 0$  for all  $(\alpha, t)$ . Action  $B$  is the best response, if the usage utility is higher than the price net of intrinsic utility,  $\lambda(\alpha, t, q) > p - x$ .<sup>5</sup>

### 3 Second period

Let  $\alpha_i \in [0, 1]$  be the marginal person consumer  $i$  wants to interact with for a given usage price.  $\alpha_i = 1$  means that  $i$  wants to interact with the whole population. The exogenous social network guarantees  $\alpha_i = \alpha$  for all  $i \in I$ . We sacrifice some generality and assume that the marginal utility is linear, but any function with decreasing marginal utility yields qualitatively the same results. If the fraction  $q \in [0, 1]$  of consumers has played  $a = B$  in the first period, then by the law of large numbers,  $q$  is also the probability that a particular person has bought the product. The net marginal utility from interaction with the social contact indexed  $\alpha$  is  $\frac{\partial \lambda(\alpha, t, q)}{\partial \alpha} = q(1 - \alpha - t)$ , which yields the expected net usage utility

$$\lambda(\alpha, t, q) = q \left( \alpha - \frac{1}{2} \alpha^2 - \alpha t \right).$$

Because only the proportion  $q$  of the population has bought the product, the consumer cannot use it with more people. Hence, the consumer's second period objective is

$$\max_{\alpha} \{ \lambda(\alpha, t, q) \}, \text{ s.t. } \alpha \in [0, q].$$

The optimal level of usage is

$$\alpha^*(t, q) = \min \{ 1 - t, q \}. \quad (1)$$

The firm's second period problem is to maximise usage profits  $\Pi_2 = q\alpha^*(t, q)(t - c_a)$  by setting the usage fee  $t$ , where the per consumer demand is given by equation (1).

The firm always charges a usage fee such that the consumers are maintained at the elastic part of the demand. To see this, assume that consumers are constrained in their usage, i.e.  $1 - t > q$ . Then, the firm could increase its fee up till point  $t = 1 - q$  without triggering a decrease in demand. By a similar argument, the firm charges a fee less than one. As a result we can write the firm's second period problem as

$$\max_t \{ q\alpha^*(t, q)(t - c_a) \}, \text{ s.t. } t \in [1 - q, 1].$$

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<sup>5</sup>The payoff function is in line with the utility specification of Katz and Shapiro (1985), where consumers are differentiated in terms of intrinsic utility, and utility depending on the network size is the same for all buyers. De Palma and Leruth (1996) analyse the polar case, where buyers have different valuations for the network benefits.



The optimal usage fee is

$$t^* = \max \left\{ \frac{1}{2} (1 + c_a), 1 - q \right\}, \quad (2)$$

with the interior solution  $t^* = \frac{1}{2} (1 + c_a)$  satisfying second order conditions,  $\frac{\partial^2 \Pi_2}{\partial t^2} = -2q < 0$ . When the optimal usage fee (2) is plugged back into the second period profits, we get

$$\Pi_2^*(c_a, q) = \begin{cases} q\pi_2^{**}(c_a), & q \geq \frac{1}{2}(1 - c_a) \\ q\pi_2^*(c_a, q), & q < \frac{1}{2}(1 - c_a) \end{cases},$$

where  $\pi_2^{**}(c_a) = \frac{1}{4}(1 - c_a)^2$  and  $\pi_2^*(c_a, q) = q(1 - c_a - q)$ . Double star indicates that the monopolist is at the interior solution and single star that the monopolist is at the corner solution, where it is capacity constrained. Naturally, we have  $\pi_2^{**}(c_a) \geq \pi_2^*(c_a, q)$ .

Because the firm keeps consumers at the elastic part of the demand,  $\alpha^*(t^*, q) = 1 - t^*$  and  $\lambda^*(\alpha^*(t^*, q), t^*, q) = \frac{1}{2}q(1 - t^*)^2$  hold. Substituting the fee (2) in the expected indirect usage utility, we get

$$\lambda^*(q) = \begin{cases} \frac{1}{8}q(1 - c_a)^2, & \text{if } q \geq \frac{1}{2}(1 - c_a) \\ \frac{1}{2}q^3, & \text{if } q < \frac{1}{2}(1 - c_a) \end{cases}.$$

## 4 First period with perfect information

### 4.1 Equilibrium

The fundamental intrinsic utility  $\theta$  is drawn from the uniform distribution  $F(\theta)$  over the support  $[-M, M]$ . When  $\theta$  is the realisation, consumers obtain i.i.d. private values  $x$  according to the conditional uniform distribution  $G(x | \theta)$  over  $[\theta - \epsilon, \theta + \epsilon]$ . All types  $x$  and the fundamental  $\theta$  are perfectly observed by the consumers and the firm at the beginning of the first period. With zero reservation utility, the payoff gain from  $a = B$  versus  $a = N$  is

$$v(x, q, p) = x + \lambda^*(q) - p. \quad (3)$$

Denote by  $\Gamma$  the coordination game of perfect information with  $I = 1$  consumers, pure actions  $a \in \{B, N\}$ , payoff (3), and parameterised by price  $p$ . The payoff function (3) is continuous in its arguments, even at the cut-off point  $q = \frac{1}{2}(1 - c_a)$  where  $\lambda^*(q)$  changes its shape. It is also differentiable, except at  $q = \frac{1}{2}(1 - c_a)$ . The payoff presents strictly increasing differences in  $x$ . Actions are strategic complements, because the payoff gain (3) is strictly higher when a larger proportion of population choose  $a = B$ . Since the action set  $a \in \{B, N\}$  is a compact subset of  $\mathbb{R}$ , the complementarity and continuity properties of (3) imply that  $\Gamma$  is supermodular (see e.g. Vives 2001 ch.2). Supermodularity guarantees the existence of a Nash equilibrium

(NE) with a smallest and a largest element unless it is unique. Because the actions are strategic complements, the maximal equilibrium element is Pareto dominating.

When the consumer expects proportion  $\mathbb{E}(q) = q^e$  of people to play  $B$ , he is indifferent between buying and not when his type is

$$\bar{x}(q^e, p) = p - \lambda^*(q^e). \quad (4)$$

The corresponding demand schedule is

$$q(p, q^e) = \begin{cases} 0, & \text{if } \bar{x}(q^e, p) > \theta + \epsilon \\ 1 - G(\bar{x}(q^e, p) | \theta), & \text{if } \theta - \epsilon \leq \bar{x}(q^e, p) \leq \theta + \epsilon \\ 1, & \text{if } \bar{x}(q^e, p) < \theta - \epsilon. \end{cases} \quad (5)$$

The marginal type defined by (4) is unique for a given pair  $(q^e, p)$ , because the payoff (3) is continuous and strictly increasing in  $x$ . More "optimistic" expectations reduce the marginal type,  $\frac{\partial \bar{x}(q^e, p)}{\partial q^e} < 0$ . This captures the correspondence between efficient coordination and the Pareto-dominant maximal NE. Expectations on the number of consumers who buy are fulfilled in the equilibrium,  $q_i^e = q$  for all  $i \in I$ , but with multiplicity of equilibria it is undetermined on which level of  $q$  expectations converge.

**Lemma 1** *The action profile  $a^*(x)$  is a Nash equilibrium of  $\Gamma$  if*

$$\begin{cases} a^*(x) = B, & \text{if } x \geq \bar{x}(q, p) \\ a^*(x) = N, & \text{if } x < \bar{x}(q, p) \end{cases},$$

*with fulfilled expectations  $\mathbb{E}_i(q) = q$ , and  $\bar{x}(q, p) = p - \lambda^*(q)$  for all  $i \in I$ .*

We allow negative unit prices, but we rule out, throughout our analysis, prohibitively negative states  $\theta$  and prohibitively high production costs  $c_f$ , in order to exclude cases where the firm chooses to remain inactive.<sup>6</sup>

We have to distinguish between two cases: (i) (relatively) high network externalities and (ii) low network externalities. Network externalities are high, if they dominate the posterior distribution  $G(x | \theta)$  in the sense  $v(\theta - \epsilon, q = 1, p) > v(\theta + \epsilon, q = 0, p) \Leftrightarrow \epsilon < \frac{1}{16}(1 - c_a)^2$ . When network externalities are high, the price  $\underline{p} = \theta - \epsilon + \frac{1}{8}(1 - c_a)^2$ , which leaves the lowest type indifferent between actions, when everybody else buys, exceeds the highest type's intrinsic valuation  $\theta + \epsilon - \underline{p} < 0$ .

<sup>6</sup>Let  $p^*(\theta, c_f)$  and  $q^*(\theta, c_f)$  be the optimal price and quantity respectively for state  $\theta$  and costs  $c_f$ . A prohibiting state-cost pair  $(\theta^-, c_f^+)$ , for which first period losses outweigh second period profits, is defined implicitly by

$$\begin{aligned} 0 &\leq \pi_2^*(c_a, q^*(\theta^-, c_f^+)) < c_f^+ - p^*(\theta^-, c_f^+), & \text{if } q^*(\theta^-, c_f^+) < \frac{1}{2}(1 - c_a) \\ 0 &\leq \pi_2^{**}(c_a) < c_f^+ - p^*(\theta^-, c_f^+), & \text{if } q^*(\theta^-, c_f^+) \geq \frac{1}{2}(1 - c_a) \end{aligned}$$

**Definition 2** *Network externalities are high (low) relative to intrinsic value when  $\epsilon \stackrel{(<)}{(>)} \frac{1}{16} (1 - c_a)^2$ .*

The optimal monopoly price is defined as  $p^* = \arg \max \{\Pi(p)\}$ , where

$$\Pi(p) = \begin{cases} q(p)(p - c_f) + q(p)\pi_2^{**}(c_a), & \text{if } q(p^*) \geq \frac{1}{2}(1 - c_a) \\ q(p)(p - c_f) + q(p)\pi_2^*(c_a, q(p)), & \text{if } q(p^*) < \frac{1}{2}(1 - c_a) \end{cases} \quad (6)$$

Endogenous pricing turns out to be an insufficient remedy to the multiplicity of equilibria problem. If network externalities are high, we have always multiple equilibria in the coordination game  $\Gamma$ , and the firm cannot influence the equilibrium selection. On the other hand, low network externalities mean that the consumer distribution  $G(x | \theta)$  is sufficiently dispersed, so that the price can induce the consumers to play both actions as strictly dominating strategies simultaneously, ruling out the multiplicity of equilibria.

**Lemma 3** *With endogenous pricing, a unique equilibrium is feasible only with low network externalities.*

*Proof.* In the appendix. ■

Lemma 3 indicates that uniqueness relates to the driver in consumers' decision making. If the product is mainly used for interaction purposes, there emerges multiplicity of equilibria and demand is indeterminate. Opposing, if the product is mainly used standalone, with interaction usage being merely a complementary feature, there is a unique equilibrium.

We close this section by characterising the optimal unit price under low network externalities in cases where the equilibrium is unique. We need to distinguish between various possible states of  $\theta$ , so that there are two cases to consider: (i)  $q(p^*) \geq \frac{1}{2}(1 - c_a)$ , and (ii)  $q(p^*) < \frac{1}{2}(1 - c_a)$ .

(i) Assume first that  $q(p^*) \in [\frac{1}{2}(1 - c_a), 1]$  holds in the second period. We get demand from equation (5) by setting  $q^e = q$ ,

$$q = \frac{\theta + \epsilon - p}{2\epsilon - \frac{1}{8}(1 - c_a)^2}. \quad (7)$$

The term  $-\frac{1}{8}(1 - c_a)^2$  captures the second period usage utility.

The differentiation of the monopoly's profits (6) with respect to  $p$  gives the optimal unit price detailed in Proposition 4.<sup>7</sup>

**Proposition 4** *The optimal unit price under low network externalities and  $q(p^*) \in [\frac{1}{2}(1 - c_a), 1]$  is*

$$p^* = \frac{1}{2}(\theta + \epsilon + c_f - \pi_2^{**}). \quad (8)$$

<sup>7</sup>The second order condition is satisfied due to our assumption on low network externalities,  $\frac{\partial^2 \Pi(p)}{\partial p^2} = -\frac{1}{\epsilon - \frac{1}{16}(1 - c_a)^2} < 0$ .

**Proof.** Obtained directly by differentiating  $\Pi(p) = q(p)(p - c_f) + q(p)\pi_2^{**}(c_a)$ , where  $q(p)$  is given by equation (7), with respect to  $p$ . ■

(ii) Assume next that  $q(p^*) \in [0, \frac{1}{2}(1 - c_a)[$  holds in the second period and that  $p(q) = \theta + \epsilon - q(2\epsilon - \frac{1}{2}q^2) > \theta - \epsilon + \frac{1}{8}(1 - c_a)^2$ . It is more convenient to solve for the indirect demand  $p(q)$  from equation (5), where  $q^e = q$  holds in equilibrium, and let the firm choose optimal quantity  $q^*$ .

**Proposition 5** *The optimal quantity under low network externalities and  $q(p^*) \in [0, \frac{1}{2}(1 - c_a)[$  is given by the first order condition<sup>8</sup>*

$$2q^3 - 3q^2 - 2[2\epsilon - (1 - c_a)]q + \theta + \epsilon - c_f = 0. \quad (9)$$

**Proof.** Obtained directly by differentiating  $\Pi(q) = q(p(q) - c_f) + q\pi_2^*(c_a, q)$ , where  $p(q) = \theta + \epsilon - q(2\epsilon - \frac{1}{2}q^2)$ , with respect to  $q$ . ■

## 4.2 Comparative statics

We discuss here the characteristics of the equilibrium in the cases where it is unique. Assume first  $q(p^*) \geq \frac{1}{2}(1 - c_a)$ . The optimal unit price is given by equation (8). From demand (7), we see that in this case the state must be bounded from below  $\theta \geq c_f - \epsilon - \frac{1}{4}(1 - c_a)^2 + 2(1 - c_a)\left[\epsilon - \frac{1}{16}(1 - c_a)^2\right]$ . The monopoly increases the unit price as heterogeneity increases in order to capture a higher share of surplus from the high consumer types.

**Proposition 6** *The optimal unit price is increasing in the heterogeneity of consumers, if the fundamental intrinsic value is high  $\theta \geq c_f - \epsilon - \frac{1}{4}(1 - c_a)^2 + 2(1 - c_a)\left[\epsilon - \frac{1}{16}(1 - c_a)^2\right]$ .*

**Proof.** Differentiation of the equation (8) gives  $\frac{\partial p^*}{\partial \epsilon} = \frac{1}{2}$ . ■

When the optimal price is plugged back into (7), we get

$$q^* = \frac{\theta + \epsilon - c_f + \frac{1}{4}(1 - c_a)^2}{4\left[\epsilon - \frac{1}{16}(1 - c_a)^2\right]}. \quad (10)$$

The differentiation of (10) with respect to  $\epsilon$  gives

$$\frac{\partial q^*}{\partial \epsilon} \begin{matrix} > \\ < \end{matrix} 0 \Leftrightarrow \theta \begin{matrix} < \\ > \end{matrix} c_f - \frac{5}{16}(1 - c_a)^2.$$

The above rule defines a minimum state above which demand is decreasing in consumer heterogeneity. Since higher values of  $\theta$  are associated with both higher demand and higher price elasticity, the marginal decrease in demand with respect to  $\epsilon$  is larger for higher values of  $\theta$ .

<sup>8</sup>The second order condition requires  $3q(q - 1) < 2\epsilon - (1 - c_a)$ .

When an increase in heterogeneity increases demand, the higher demand in turn increases the value of the good inducing a positive reinforcement effect on demand. Because the firm compensates this by increasing the price, we obtain the mixed result that for low values of  $\theta$  demand increases, and for high values of  $\theta$ , it decreases with marginal changes in  $\epsilon$ . A marginal increase in  $\epsilon$  has a stronger effect on demand the less heterogeneous consumers are (that is the closer  $\epsilon$  is to  $\frac{1}{16}(1 - c_a)^2$ ), because in that case the network externalities are more important and drive conformity in buying decisions.

Profits are

$$\Pi(p^*, t^*) = \frac{\left[\theta + \epsilon - c_f + \frac{1}{4}(1 - c_a)^2\right]^2}{8\left[\epsilon - \frac{1}{16}(1 - c_a)^2\right]}.$$

Consumer surplus is

$$\begin{aligned} S &= \frac{1}{2\epsilon} \int_{x=\bar{x}(q(p^*), p^*)}^{\theta+\epsilon} \left[ x + \frac{1}{8}(1 - c_a)^2 q(p^*) - p^* \right] dx \\ &= \left\{ \frac{\theta + \epsilon - c_f + \frac{1}{4}(1 - c_a)^2}{4\left[\epsilon - \frac{1}{16}(1 - c_a)^2\right]} \right\}^2 \epsilon. \end{aligned}$$

A marginal change in  $\epsilon$  has an ambiguous effect on profits and consumer surplus. There is a tendency for them to move in the same direction for marginal changes in  $\epsilon$ . Profits increase when  $\frac{\partial p^*}{\partial \epsilon}$  and  $\frac{\partial q(p^*)}{\partial \epsilon}$  are both positive, obviously. In the cases where demand decreases as  $\epsilon$  increases, profits tend to decrease when  $\epsilon$  is close to its minimum, as network externalities cause a stronger demand effect, and profits tend to increase when  $\epsilon$  is large.

Assume next that the value of  $\theta$  is relatively low enough compared to production costs so that the firm is constrained in the second period,  $q(p^*) < \frac{1}{2}(1 - c_a)$ . The resulting optimal price  $p(q^*)$  and the second period usage fee  $t^*$  are higher in this case due to lower demand in the first period.

**Proposition 7** *Demand is increasing (decreasing) in consumer heterogeneity, if the equilibrium demand is relatively low (high) and constrains the second period usage.*

**Proof.** *Totally differentiating the first order condition (9) gives*

$$\begin{aligned} \frac{dq^*}{d\epsilon} &> 0 \Leftrightarrow q^* < \min \left\{ \frac{1}{4}, \frac{1}{2}(1 - c_a) \right\} \\ \frac{dq^*}{d\epsilon} &< 0 \Leftrightarrow \frac{1}{4} < q^* < \frac{1}{2}(1 - c_a). \end{aligned}$$

■

As a result, a positive marginal change in  $\epsilon$  causes similar effects on demand, as in the case where the firm is not constrained in the second period. The monopoly compensates the broader type distribution and the increase in the value of usage utility (the reinforced demand effect) with a higher unit price. Comparative statics for profits and consumer surplus are computatively more complicated but present analogous tendencies.

## 5 First period with incomplete information

With incomplete information, the game remains otherwise unchanged, except that consumers and the monopolist do not observe directly  $\theta$  until at the end of period one. Consumers' first period observations of  $\theta$  are blurred by noise, whereas the firm resorts to the prior in its first period estimates. The consumers know that the firm is uninformed, which removes all possible information about  $\theta$  that might otherwise be inferred from prices  $(p, t)$ .

The value of  $\theta$  is drawn from the uniform distribution  $F(\theta)$  with support  $[-M, M]$ . The consumers draw i.i.d. signals  $x$  from the uniform conditional distribution  $G(x | \theta)$  with the support  $[\theta - \epsilon, \theta + \epsilon]$ . The consumer who observes signal  $x$  gets an expected payoff gain (3) from action  $a = B$  versus  $a = N$ , identical to the payoff gain under perfect information. Now, uncertainty over  $\theta$  corresponds to horizontal differentiation, similarly to the perfect information case, and the first period game is a global game with private values. The payoff (3) is continuous and increasing in  $(x, q)$ . In addition, it satisfies the "strict Laplacian state monotonicity" condition (see Morris and Shin 2003). Namely, there exists a unique  $\tilde{x}$  that solves  $\int_{q=0}^1 v(\tilde{x}, q, p) dq = 0$ .<sup>9</sup> In sum, the payoff (3) satisfies all the conditions on strategic complementarities and continuity that global games require for equilibrium uniqueness. The remaining condition we need to impose is on the dominance regions. For all expectations, some consumers must play  $a = B$  as a strictly dominating strategy at the same time as another group plays  $a = N$  as a strictly dominating strategy. Because the support of the prior is bounded and price is chosen endogenously, the existence of the dominance regions is not trivially satisfied.

**Condition 8** *Dominance regions of strictly dominating strategies exist if*

$$M > \max \left\{ c_f - \frac{1}{8} (1 - c_a)^2 - 2\epsilon, -\frac{1}{3}c_f + \frac{1}{6} (1 - c_a)^2 - \frac{2}{3}\epsilon \right\}.$$

**Proof.** *In the appendix.* ■

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<sup>9</sup>Define  $z(x) = \int_{q=0}^1 v(x, q, p) dq$ . Integration gives  $z(x) = x - p + \frac{1}{128} (1 - c_a)^2 [8 - (1 - c_a)^2]$  where  $\frac{1}{128} (1 - c_a)^2 [8 - (1 - c_a)^2] \geq 0$ . Hence,  $\frac{\partial z(x)}{\partial x} > 0$  for all  $x$ , and there exists a unique  $\tilde{x}$  that solves  $z(\tilde{x}) = 0$ . If  $p > (\leq) \frac{1}{128} (1 - c_a)^2 [8 - (1 - c_a)^2]$ , then  $\tilde{x}$  is positive (negative).

Condition 8 requires the prior to be sufficiently uninformative. When  $M$  is sufficiently large, the heterogeneity  $\epsilon$  can afford to go to zero at the limit.

Denote by  $\Gamma_{II}$  the incomplete information game parameterised by price  $p$  with  $I = 1$  consumers, pure actions  $a \in \{B, N\}$ , payoff (3), where  $\theta$  is distributed according to  $F(\theta)$ , signals are i.i.d. according to  $G(x | \theta)$ , and where the distributions satisfy Condition 8. The game  $\Gamma_{II}$  is supermodular, and the global games theory predicts that it has a unique Bayesian switching equilibrium solvable by the iterated deletion of strictly dominated strategies.

**Lemma 9** *The game  $\Gamma_{II}$  has a unique switching strategy equilibrium  $\Gamma_{II}^*$  that survives the iterated deletion of strictly dominated strategies*

$$\begin{aligned} a^*(x) &= N, \text{ if } x < \tilde{x} \\ a^*(x) &= B, \text{ if } x > \tilde{x}, \end{aligned}$$

where  $\tilde{x}$  is the unique solution to  $\int_{q=0}^1 v(\tilde{x}, q, p) dq = 0$ .

**Proof.** *The proof follows straightforwardly from the proof of Proposition 2.1 in Morris and Shin (2003).<sup>10</sup> ■*

We obtain from Lemma 9 that the consumer's expectations about the fraction of people who play  $a = B$  follows a uniform distribution on the unit interval. Hence, the marginal signal  $\tilde{x}$ , which acts as the cut-off rule in the consumers' equilibrium strategy, is given by

$$\int_{q=0}^{\frac{1}{2}(1-c_a)} \left( \tilde{x} + \frac{1}{2}q^3 - p \right) dq + \int_{q=\frac{1}{2}(1-c_a)}^1 \left[ \tilde{x} + \frac{1}{8}(1-c_a)^2 q - p \right] dq = 0,$$

where we have taken into account the cut-off point  $q = \frac{1}{2}(1-c_a)$ , at which the firm reaches the interior optimal usage fee. Integration gives

$$\tilde{x} = p + \tau(c_a),$$

where  $\tau(c_a) = \frac{1}{128}(1-c_a)^2 \left[ (1-c_a)^2 - 8 \right]$  captures the consumers' expectations on second period usage utility. When  $\theta$  is the realisation of the fundamental, the proportion of consumers who get signals higher than  $\tilde{x}$  is  $q = 1 - G(\tilde{x} | \theta)$ , giving the first period demand

$$q(\theta, p) = \begin{cases} 1, & \text{if } \theta > \tilde{x} + \epsilon \\ \frac{\theta + \epsilon - \tau(c_a) - p}{2\epsilon}, & \text{if } \tilde{x} - \epsilon \leq \theta \leq \tilde{x} + \epsilon \\ 0, & \text{if } \theta < \tilde{x} - \epsilon. \end{cases} \quad (11)$$

Having defined the demand, we can turn to the pricing problem. Define a cut-off state  $\hat{\theta}$  as  $q(\hat{\theta}, p) = \frac{1}{2}(1-c_a)$ . Whenever the true state is higher

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<sup>10</sup>The proof to Lemma 9 is detailed in Sääskilähti (2005a pp. 73-76) as the "Proof to Proposition 4".

than  $\hat{\theta}$ , the firm is not constrained in the second period. Firm's first period problem is to choose unit price  $p^*$  that maximises expected profits (12).

$$\begin{aligned} \mathbb{E}(\Pi) = & \frac{1}{2M} \left\{ \int_{\theta=\tilde{x}-\epsilon}^{\tilde{x}+\epsilon} q(\theta, p) (p - c_f) d\theta + \int_{\theta=\tilde{x}+\epsilon}^M (p - c_f) d\theta + \right. \\ & + \int_{\theta=\tilde{x}-\epsilon}^{\hat{\theta}} q(\theta, p) \pi_2^*(c_a, q) d\theta + \int_{\theta=\hat{\theta}}^{\tilde{x}+\epsilon} q(\theta, p) \pi_2^{**}(c_a) d\theta + \\ & \left. + \int_{\theta=\tilde{x}+\epsilon}^M \pi_2^{**}(c_a) d\theta \right\}. \end{aligned} \quad (12)$$

The first two integrals in (12) are the first period profits. The last three integrals capture the internalised effects on the second period profits. A low unit price facilitates coordination between consumers, but it erodes the first period profits. Proposition 10 presents the optimal tariff structure.

**Proposition 10** *When the true state is  $\theta^*$ , the optimal tariff structure is*

$$\begin{aligned} t^* &= \max \left\{ \frac{1}{2} (1 + c_a), 1 - q(\theta^*) \right\} \\ p^* &= \frac{1}{2} (M + c_f) - \frac{1}{2} \tau(c_a) - \frac{1}{8} (1 - c_a)^2. \end{aligned} \quad (13)$$

*Proof.* In the appendix. ■

The optimal prices are increasing in the usage cost,

$$\frac{dp^*}{dc_a} = \frac{1}{16} (1 - c_a) \left[ \frac{1}{4} (1 - c_a)^2 + 3 \right] \geq 0,$$

and

$$\frac{dt^*}{dc_a} = \begin{cases} \frac{1}{2}, & q(\theta^*) \geq \frac{1}{2} (1 - c_a) \\ \frac{1-c_a}{4\epsilon} \left[ \frac{5}{8} - \frac{1}{32} (1 - c_a)^2 \right], & q(\theta^*) < \frac{1}{2} (1 - c_a) \end{cases}$$

which is positive for  $0 \leq c_a < 1$ , and zero if the firm is at the corner solution and  $c_a = 1$ . The optimal unit price is increasing in production costs,  $\frac{\partial p^*}{\partial c_f} > 0$ . The usage fee is independent of production cost, as long as the firm is not constrained when setting  $t^*$ . If the realised demand binds the optimal usage fee, then the usage fee increases in production costs  $\frac{\partial t^*}{\partial c_f} = \frac{1}{4\epsilon} > 0$ .

## 5.1 Role of uncertainty on profits and consumer surplus

Demand increases (decreases) in the precision of signals only if the state  $\theta$  is higher (lower) than the marginal signal. Why? When the precision of the signal is high, it tells the consumer that other people observe signals very close to the one he has observed. If the realisation of  $\theta$  is below the marginal signal, and if signals are relatively accurate, then the consumer infers that most people do not buy. So, if  $\theta < \tilde{x}$  and we decrease the precision of signals ( $d\epsilon > 0$ ), then a larger proportion of people may observe signals



that are above the marginal signal. This increases demand directly, and also indirectly as consumers infer that a larger proportion of people buy, which yields higher network externalities. Opposing, if  $\theta > \tilde{x}$  and signals are relatively precise, a reduction in the precision of signals causes a larger proportion of people drawing signals that fall below the marginal signal, which reduces demand both directly and indirectly.

**Lemma 11** *A decrease in the precision of signals decreases (increases) demand when the true  $\theta$  is above (below) the marginal signal,  $\frac{\partial q(\theta, p)}{\partial \epsilon} \begin{smallmatrix} < \\ > \end{smallmatrix} 0 \iff \theta \begin{smallmatrix} > \\ < \end{smallmatrix} p + \tau(c_a), \tilde{x} - \epsilon \leq \theta \leq \tilde{x} + \epsilon.$*

**Proof.** *Proof follows directly from equation (11). ■*

The optimal unit price (13) is independent of uncertainty over signals, because the prior and signals are uniformly distributed. Resulting demand is linear, which renders first period profits neutral with respect to  $\epsilon$ . If the firm reaches the interior solution  $t^* = \frac{1}{2}(1 + c_a)$ , also the usage fee is independent of uncertainty. However, if the firm is held at the corner solution, the optimal usage fee depends on the precision of signals, and we have  $\frac{\partial t^*}{\partial \epsilon} \Big|_{q(\theta^*, p^*) < \frac{1}{2}(1 - c_a)} = -\frac{\partial q(\theta^*, p^*)}{\partial \epsilon}$ , and the constrained optimal usage fee is higher than the interior solution. Because the firm is constrained with low values of  $\theta$ , it is likely that  $\frac{\partial q(\theta^*, p^*)}{\partial \epsilon} > 0$  holds. Then, if uncertainty is increased, the first period demand increases. This relaxes the capacity constraint in the second period. Consequently, the firm decreases its usage fee in order to increase its second period sales.

Because there is the possibility that the true demand is low and the firm cannot charge the unconstrained optimal usage fee, expected second period profits are not independent of the precision of signals. Subsequently, the expected total profits are positively correlated with the precision of signals.

**Proposition 12** *Increase in the precision of signals increases firm's profits*

$$\frac{\partial \mathbb{E}(\Pi(p^*, t^*))}{\partial \epsilon} = -\frac{(1 - c_a)^4}{192M} \leq 0.$$

**Proof.** *In the appendix. ■*

If  $\epsilon$  is increased marginally, demand increases (decreases) in states that are below (above) the marginal signal. The firm's profits are zero for  $\theta < \tilde{x} - \epsilon$ . Therefore states above the marginal signal have a larger weight in expected profits, and the negative effect on demand is dominating. The negative effect on profits comes mainly from high states where the consumers are confident on high sales, thus on high interaction utility, and where the demand is decreasing (both directly and indirectly) in uncertainty. As a result, the firm always benefits from more accurate information about the

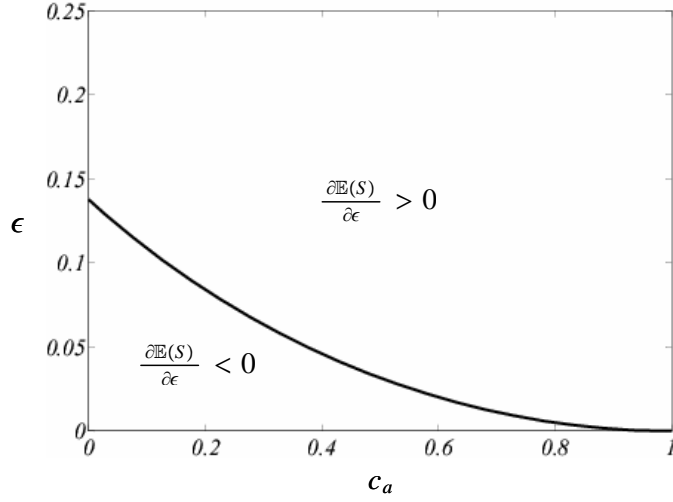


Figure 2: The sign of  $\frac{\partial \mathbb{E}(S)}{\partial \epsilon}$ .

value of its good. Only if  $c_a = 1$ , so that second period usage is prevented by high costs, expected profits are independent of uncertainty.

Expected consumer surplus is

$$\begin{aligned} \mathbb{E}(S) = & \frac{1}{4M\epsilon} \left\{ \int_{\theta=\tilde{x}(p^*)-\epsilon}^{\hat{\theta}(p^*)} \int_{x=\tilde{x}(p^*)}^{\theta+\epsilon} \left[ x + \frac{1}{2}q(\theta, p^*)^3 - p^* \right] dx d\theta + \right. & (14) \\ & + \int_{\theta=\hat{\theta}(p^*)}^{\tilde{x}(p^*)+\epsilon} \int_{x=\tilde{x}(p^*)}^{\theta+\epsilon} \left[ x + \frac{1}{8}(1-c_a)^2 q(\theta, p^*) - p^* \right] dx d\theta + \\ & \left. + \int_{\theta=\tilde{x}(p^*)+\epsilon}^M \int_{x=\theta-\epsilon}^{\theta+\epsilon} \left[ x + \frac{1}{8}(1-c_a)^2 - p^* \right] dx d\theta \right\}. \end{aligned}$$

To see the effect of a change in signals' precision, we differentiate (14) with respect to  $\epsilon$ . The sign of  $\frac{\partial \mathbb{E}(S)}{\partial \epsilon}$  depends only on  $c_a$  and  $\epsilon$ , and we denote the solution to  $\frac{\partial \mathbb{E}(S)}{\partial \epsilon} = 0$  by  $\bar{\epsilon}(c_a)$ . We have plotted the curve  $\epsilon = \bar{\epsilon}(c_a)$  in Figure (2). Above the curve, the derivative  $\frac{\partial \mathbb{E}(S)}{\partial \epsilon}$  is positive, and below it is negative. Hence, the minimum for  $\mathbb{E}(S)$  with respect to  $\epsilon$  is given by  $\bar{\epsilon}(c_a)$ .

**Proposition 13** *A decrease in the signals' precision ( $d\epsilon > 0$ ), induces:*

(i) *for relatively precise signals  $\epsilon < \bar{\epsilon}(c_a)$ , a decrease in expected consumer surplus.*

(ii) *for relatively imprecise signals  $\epsilon > \bar{\epsilon}(c_a)$ , an increase in expected consumer surplus.*

**Proof.** *Obtained straightforwardly by differentiating consumer surplus (14) with respect to  $\epsilon$ . ■*

Unlike with profits, the absolute magnitude of  $\epsilon$  plays a role in whether consumer surplus increases or decreases for marginal changes in the precision. When the signals are precise ( $\epsilon < \bar{\epsilon}(c_a)$ ), the expected consumer surplus decreases, if uncertainty is marginally increased. When signals are less accurate ( $\epsilon > \bar{\epsilon}(c_a)$ ), consumer surplus is positively affected by a marginal increase in uncertainty. Expected surplus is affected via two effects. For a given  $\theta$ , there is a change in the expected demand for the product. There is also a change in the expectation of the intrinsic value  $\theta$ .

The negative effect on surplus is foremost associated with the very high states  $\theta > \tilde{x} + \epsilon$ , where expected consumer surplus unambiguously decreases as  $\epsilon$  increases. This is the segment, where consumers are confident on high sales and subsequent high usage utility. The negative effect is stronger the smaller  $\epsilon$  is, which shows up in that the total effect turns negative in the area  $\epsilon < \bar{\epsilon}(c_a)$ . For lower values of  $\theta$ , there is a mixture of positive and negative effects which sum up to the result illustrated in Figure (2).

If the signals are precise ( $\epsilon < \bar{\epsilon}(c_a)$ ), so that the consumers are relatively homogeneous, a consumer benefits from the knowledge that other people are like him, and network externalities have important role in decision-making. In this case, further information ( $d\epsilon < 0$ ) on that perceived network externalities, which give the same utility for everyone, are driving everybody's decisions increases expected surplus. And if we reduce the precision of signals, the little extra uncertainty about the other people hurts. However, when the precision of signals drops to a relatively low level ( $\epsilon > \bar{\epsilon}(c_a)$ ), higher uncertainty is of good. Why? Low precision is analogous to high heterogeneity between consumers. If signals are imprecise, the consumer knows that there is a large variance in the perception of the true intrinsic value of the product within the population, and knows that other people know that everybody is equally uninformed. This reduces the expected value of interaction utility. The consumer then bases his buying decision on the intrinsic value, rather than on the expected behaviour of other people. In this case, the consumer benefits from further knowledge ( $d\epsilon > 0$ ) about the fact that he can base his decision on his private value.

## 6 Comparison

In this section, we discuss the differences between perfect and incomplete information. We consider only the perfect information case where network externalities are sufficiently low to guarantee a unique equilibrium, and the firm is not constrained in the second period. Equations (8) and (13) give the optimal unit prices under perfect and incomplete information respectively.

The term  $-\frac{1}{8}(1 - c_a)^2$ , present in both price equations, is the effect from second period profits. The firm takes into account that a high first period price reduces second period profits. This effect is eliminated, if we introduce

perfect competition in the second period, so that the usage fee is  $t = c_a$ . The optimal first period monopoly price under incomplete information, when the second period is characterised by perfect competition, is

$$p_C^* = \frac{1}{2}M + \frac{1}{2}c_f - \frac{1}{2}\tau_C(c_a),$$

where  $\tau_C(c_a) \leq 0$ . Derivation of  $p_C^*$  is in the appendix.

Prices (8) and (13) diverge in two respects. First, because the firm must take expectations on the distributions, the price tends to be higher under incomplete information, as  $M$  replaces  $\theta + \epsilon$  in the price function. Under incomplete information, the unit price is independent of heterogeneity  $\epsilon$ , which is in contrast to the perfect information case, in which the firm increases the price for a marginal increase in heterogeneity.

The second difference is the term  $-\frac{1}{2}\tau(c_a) \geq 0$ , which captures the firm's accurate perception on what are *consumers' expectations* on the second period usage utility under incomplete information. Because under perfect information, all players, including the firm, observe perfectly how much usage utility consumers get in the second period, the firm neutralises the effect by incorporating the usage utility fully in the unit price. Consumers' expectations are "fixed" under incomplete information, so there is a potential gap between expected and actual usage utility. This gap induces a bias: the firm prices high in the first period, before consumers learn the true state, at the expense of uncertain second period profits. Thus, when the firm is uncertain about second period sales, it adjusts the unit price upwards compared to the perfect information price. This effect is aggravated, when the second period is characterised by perfect competition. We have  $\tau_C(c_a) \leq \tau(c_a)$  indicating that the monopoly does not have any incentives to insure second period profits by setting a low first period price. Demand, however, is higher for the (second period) competition case than for the two-period monopoly,  $q(p_C^*) > q(p^*)$ , because the monopoly limits supply in the second period. The following remark summarises the differences between unit prices.

**Remark 14** *The unit price tends to be higher under incomplete information than under perfect information. The unit price with second period perfect competition is higher than the two-period monopoly unit price under incomplete information.*

We cannot tell unambiguously whether demand is higher under perfect or incomplete information. The numerical simulations we have carried out tend to result in higher demand under perfect information.

The term determining real heterogeneity between consumers,  $\epsilon$ , has an important coordination role for a unique equilibrium under perfect information. This role is taken away if consumers' valuations are private information. Even the smallest amount of uncertainty is sufficient to result in

a unique equilibrium, whereas we had an explicit rule for minimum heterogeneity under perfect information.

The firm's preferences over heterogeneity can differ under perfect and incomplete information. Assume high network externalities and perfect information. If the market sentiment is pessimistic, so that coordination is biased against the Pareto-efficient NE, the firm may prefer higher heterogeneity between consumers, which would support a larger unique equilibrium. If we maintain high network externalities, but impose incomplete information, coordination is unaffected by the real heterogeneity between consumers. Moreover, we know that the firm's profits increase as the precision of signals increases. So, it prefers little heterogeneity.

## 7 Concluding remarks

We have analysed a market for network goods. A monopolist launches a device that enables efficient interaction between people. The consumers face a coordination problem in whether to switch to using the new device or to stick with the prevailing interaction systems. This kind of coordination game has multiple Nash equilibria under perfect information and homogenous consumers. We have carried out a comprehensive analysis on the conditions for equilibrium uniqueness. The benefit of uniqueness is that the subsequent comparative statics are determinate, and do not depend on argumentation on the likelihood of out-of-model factors. Uniqueness of equilibrium, under perfect information, requires high *real* consumer heterogeneity. Adversely, this means that we must limit the role of network externalities in consumers' decision making. Under incomplete information, uniqueness does not hinge on the real heterogeneity. Instead, we need to raise a *possibility* that the fundamental value of the product can be very low or very high. As a result, the set of buying decision criteria is less restricted, and the decisions can be driven by network-intense attributes.

The results from perfect information case differ from the results from the incomplete information case. The unit price is independent of consumer heterogeneity under incomplete information, but it is increasing in heterogeneity under perfect information. We showed that under incomplete information, the monopolist biases its tariff structure in favour of the unit price, at the expense of lower usage demand in the second period. Under perfect information, such bias did not exist as the monopolist is able to perfectly incorporate second period usage utility in its first period price. The unit price therefore tends to be lower under perfect information.

A marginal change in consumer heterogeneity has an ambiguous effect on profits and consumer surplus under perfect information. Under incomplete information, the expected profits increase as the precision of signals improves (i.e. heterogeneity between consumers is reduced). The effect on

expected consumer surplus depends on the absolute level of signal precision. Consumer surplus increases, if the marginal change in the precision is in-line with the way consumers make their buying decisions. If signals are precise (imprecise), further improvement (reduction) in accuracy raises surplus. In this sense, better agreement on the factor that drives decision making among the consumers is of good, not the precision of signals as such.

In the model, the consumers had to invest up-front to the device in order to benefit from it in the usage stage, while the monopolist was not able to commit to prices in the first period, and it set the usage fee after consumers had made their buying decisions. The separation of stages invites further research. First, the case with credible commitment to prices in the first period could potentially result in a different balance between the unit price and the usage fee. Secondly, the monopolist could be allowed to sell the device in the second period to those who opted for not buying in the first period. This option might cause some consumers to wait in the first period.

The incomplete information case offers a number of interesting extensions. We have used fairly specific distributions and utility functions that could be generalised. The simplicity of the NE in the coordination game also allows analysis on strategic investments that has been previously made difficult by the multiplicity problem. We have assumed that consumers know their needs better than the firm. It would be interesting to allow the firm to observe something more than nothing. It could then use prices to manipulate consumers' perceptions of the value of the good. This modification would give information on when the firm has incentives to reveal information about its goods, and how this improvement in the precision of the public signal affects consumer surplus.

Perfect information is a strong condition. For ordinary consumable goods, this condition is not (necessarily) problematic, but in problems of coordination, even a marginal difference between perfect and almost perfect information can produce strikingly different outcomes (see also Morris 2002). Under perfect information, the coordination failure is a possible scenario. If the main selling argument is based on networking benefits, the perfect information variant is in trouble in explaining which equilibrium is the most probable one. Under incomplete information, there is no coordination failure. For a novel, technologically advanced, product, incomplete information regime characterises the real world more accurately. Just think about how we are more capable of saying how much utility a fax machine or e-mail yields to other people today than, say, twenty years ago. As the product matures, information becomes more accessible. Thus, coordination failure seems to be less of a problem, compared to what the earlier network externalities literature has proposed.

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## 9 Appendix

**Proof of Lemma 3.** We establish in (A i) - (A iv) that the unit price is bounded, and any price in the feasible range induces multiple equilibria in the coordination game  $\Gamma$  when network externalities are high. In (B v) - (B vi), we establish that under low network externalities, endogenous pricing can induce multiple equilibria, if the unit price lies below  $\underline{p} = \theta - \epsilon + \frac{1}{8}(1 - c_a)^2$ . In (B vii), we prove that the unit price never exceeds the highest type's intrinsic valuation under low network externalities, and that a price  $p \in [\underline{p}, \theta + \epsilon]$  induces a unique equilibrium in  $\Gamma$ . For parts (A i) - (A iv),  $\epsilon < \frac{1}{16}(1 - c_a)^2$  holds. For parts (B v) - (B vii),  $\epsilon > \frac{1}{16}(1 - c_a)^2$  holds.

**(A i)** Due to the continuity of the payoff (3) and the fact that it is increasing in  $x$  and  $q$ , there is a lower limit price  $\underline{\underline{p}} \geq \theta - \epsilon$  for which the lowest type has always a (weakly) dominant strategy to buy  $v(\theta - \epsilon, q, \underline{\underline{p}}) \geq 0$  for all  $q \in [0, 1]$ . A price  $p \leq \underline{\underline{p}}$  induces a unique all buy equilibrium, but the monopolist never decreases the price below  $\underline{\underline{p}}$  as that neither increases demand nor induces a different outcome in  $\Gamma$ .

**(A ii)** For a price  $p \in ]\underline{\underline{p}}, \theta + \epsilon]$ ,  $q = 1$  is an equilibrium, because the lowest type gets  $v(\theta - \epsilon, q = 1, p) > 0$  due to high network externalities. If consumers expect  $q^e < 1$ , the lowest type does not buy, because the price exceeds  $\underline{\underline{p}}$  defined in part (A i), thus  $v(\theta - \epsilon, q, p) < 0$ . At the same time, the highest type has a dominant strategy to buy  $v(\theta + \epsilon, q, p) \geq 0$ , and therefore there exists an equilibrium with  $q \in ]0, 1[$ .

**(A iii)** For a price  $p \in [\theta + \epsilon, \bar{p}]$ , where  $\bar{p} = \theta - \epsilon + \frac{1}{8}(1 - c_a)^2$ , the lowest type gets  $v(\theta - \epsilon, q = 1, p) > 0$ , and  $q = 1$  is an equilibrium, due to continuity of the payoff (3) and the fact that it is increasing in  $x$  and  $q$ . If

all consumers expect that no-one buys, then the highest type gets negative utility  $v(\theta + \epsilon, q = 0, p) < 0$ , and no-one will buy in equilibrium. Hence, for price  $p \in [\theta + \epsilon, \underline{p}]$  there exist at least two equilibria  $q = 0$  and  $q = 1$ .

**(A iv)** When the firm expects that coordination reaches the maximal equilibrium, we have a minimum price that the firm will ever charge  $\underline{p} = \theta - \epsilon + \frac{1}{8}(1 - c_a)^2$ , which corresponds to the price that leaves the lowest type indifferent between actions when everybody else buys. Because the payoff (3) is continuous and increasing in  $x$  and  $q$ , there is an upper boundary price  $\bar{p} = \theta + \epsilon + \lambda^*(q) \geq \underline{p}$ , above which it becomes a dominant strategy for everyone not to buy. For a price  $p \in [\underline{p}, \bar{p}]$ , no-one will buy if consumers expect that no-one will buy, because even the highest type gets negative utility  $v(\theta + \epsilon, q = 0, p) < 0$ . If consumers expect the maximal equilibrium, we have a non-zero equilibrium. Hence, any price  $p \in [\underline{p}, \bar{p}]$  supports simultaneously two equilibria  $q = 0$  and  $q \in ]0, 1[$ .

**(B v)** Part (A i) applies under low network externalities as well.

**(B vi)** Assume that  $p \in [\underline{p}, \underline{p}]$ , where  $\underline{p} = \theta - \epsilon + \frac{1}{8}(1 - c_a)^2$ . The highest type has a strictly dominant strategy to buy due to the assumption on low network externalities, so the case  $q = 0$  is ruled out as an equilibrium. The demand must satisfy

$$q = 1 - \frac{\bar{x}(q, p) - \theta + \epsilon}{2\epsilon},$$

where  $\bar{x}(q, p) = p - \lambda^*(q)$ . In order to check if there exists a consistent equilibrium with  $q < 1$ , plug the upper limit  $\underline{p}$  in the above equation because the demand is decreasing in  $p$ . We obtain

$$q = 1 - \frac{\frac{1}{8}(1 - c_a)^2 - \lambda^*(q)}{2\epsilon}.$$

If the firm is unconstrained in the second period, the demand can be written as

$$2(1 - q) \left[ \epsilon - \frac{1}{16}(1 - c_a)^2 \right] = 0,$$

which holds only if  $q = 1$ . Thus there is no equilibrium with  $q \in ]\frac{1}{2}(1 - c_a), 1[$  and  $p < \underline{p}$ .

If the firm is constrained in the second period, we write the demand as

$$q = 1 - \frac{\frac{1}{8}(1 - c_a)^2 - \frac{1}{2}q^3}{2\epsilon}.$$

The above equation has two solutions which satisfy  $q \in ]0, \frac{1}{2}(1 - c_a)[$ , when the heterogeneity parameter  $\epsilon$  is close to its minimum value  $\frac{1}{16}(1 - c_a)^2$ . As a result, there can be multiple equilibria with  $p \in [\underline{p}, \underline{p}]$  under low network externalities. In particular, the equilibria include  $q^e = q = 1$  and  $q^e = q < 1$ , but  $q^e = 0$  is ruled out.

**(B vii)** Next we prove that the optimal price does not exceed the highest type's valuation,  $p^* \leq \theta + \epsilon$ , and that the equilibrium is unique for any price  $p \in [\underline{p}, \theta + \epsilon]$ , where  $\underline{p} = \theta - \epsilon + \frac{1}{8}(1 - c_a)^2$ , under low network externalities. Start by assuming  $q(p^*) \geq \frac{1}{2}(1 - c_a)$ . In this case, the demand corresponding to fulfilled expectations is given by

$$q(p) = \begin{cases} \frac{\theta + \epsilon - p}{2[\epsilon - \frac{1}{16}(1 - c_a)^2]}, & \text{if } \underline{p} \leq p \leq \theta + \epsilon - (1 - c_a) \left[ \epsilon - \frac{1}{16}(1 - c_a)^2 \right] \\ 1, & \text{if } p < \underline{p} \end{cases}$$

We have  $\frac{\partial q(p)}{\partial p} \leq 0$ , so the highest feasible price is obtained with  $q = \frac{1}{2}(1 - c_a)$ . This price is  $p = \theta + \epsilon - (1 - c_a) \left[ \epsilon - \frac{1}{16}(1 - c_a)^2 \right] \leq \theta + \epsilon$  with equality at  $c_a = 1$ .

Assume next that  $q(p^*) < \frac{1}{2}(1 - c_a)$ . It is more convenient to solve for the inverse demand function

$$p(q) = \theta + \epsilon - q \left( 2\epsilon - \frac{1}{2}q^2 \right), \quad (15)$$

which can be increasing in  $q \in [0, \frac{1}{2}(1 - c_a)[$ . The firm maximises profits by choosing  $q^* \in [0, \frac{1}{2}(1 - c_a)[$ . Since we are interested in the possibility of the case  $p(q^*) > \theta + \epsilon$ , the term in parenthesis in (15) should be negative. So, we require that  $\epsilon < \frac{1}{4}q^2$  holds. As we combine this condition with the initial assumption on low network externalities, we obtain a range within the heterogeneity parameter must strictly be  $\frac{1}{16}(1 - c_a)^2 < \epsilon < \frac{1}{4}q^2$ . This condition is the least binding when  $q$  is at its maximum. The assumption  $q^* < \frac{1}{2}(1 - c_a)$  gives the maximal consistent level. Once this level is plugged into the condition, we end up with  $\frac{1}{16}(1 - c_a)^2 < \epsilon < \frac{1}{16}(1 - c_a)^2$ , which cannot hold. If we force  $\epsilon < \frac{1}{4}q^2$  to hold, we violate  $\frac{1}{16}(1 - c_a)^2 < \epsilon$ , and vice versa. Consequently, the term in the parenthesis in (15) is always positive, hence the price remains bounded from above  $p(q) \leq \theta + \epsilon$ . Note that the price is continuous in  $q \in [0, 1]$ . If we plug  $q = \frac{1}{2}(1 - c_a)$  in (15), we get  $p = \theta + \epsilon - (1 - c_a) \left[ \epsilon - \frac{1}{16}(1 - c_a)^2 \right]$ .

For price  $p \in [\underline{p}, \theta + \epsilon]$ , the lowest type gets zero payoff at maximum

$$v(\theta - \epsilon, q, p) \leq 0 \quad \forall q \in [0, 1]. \quad (16)$$

At the same time, the highest type always gets at least zero payoff

$$v(\theta + \epsilon, q, p) \geq 0 \quad \forall q \in [0, 1]. \quad (17)$$

Inequalities (16) and (17) establish (weak) dominance regions which together with increasing differences  $\frac{\partial v(x, q, p)}{\partial x} > 0$  guarantee equilibrium uniqueness in  $\Gamma$ . At the boundaries  $p \in \{\underline{p}, \theta + \epsilon\}$  everybody may play the same action, but indeterminacy is restricted to the marginal (the lowest or the highest)

type only, and we can ignore them. As a result, the equilibrium is unique.

■

**Proof of Condition 8.** The lower and upper dominance regions must coexist for all consumer expectations.

(i) Start with the conditions for the upper dominance region,

$$\exists \bar{\theta} \in ]-M, M[ \text{ so that } v(x, q, p) > 0 \text{ for all } q \in [0, 1] \text{ and } x \geq \bar{\theta}.$$

Assume that consumers are "optimistic" and expect  $q^e = 1$ , thus they expect second period usage utility  $\lambda^*(q^e) = \frac{1}{8}(1 - c_a)^2$ . The consumer who observes  $x$  and has expectations  $q^e = 1$ , gets expected payoff gain  $v(x, q, p) = x + \frac{1}{8}(1 - c_a)^2 - p$ . Because  $v(x, q, p)$  is strictly increasing in  $x$ , we get the marginal type  $\bar{x}_{q^e=1} = p - \frac{1}{8}(1 - c_a)^2$ , who is indifferent between buying and not buying, and the true demand schedule

$$q(p) = \begin{cases} 0, & \text{if } \bar{x}_{q^e=1} > \theta + \epsilon \\ \frac{\theta + \epsilon + \frac{1}{8}(1 - c_a)^2 - p}{2\epsilon}, & \text{if } \theta - \epsilon \leq \bar{x}_{q^e=1} \leq \theta + \epsilon \\ 1, & \text{if } \bar{x}_{q^e=1} < \theta - \epsilon. \end{cases} \quad (18)$$

Demand (18) corresponds to the most optimistic expectations, thus it supports the highest monopoly price. Define the cut-off state  $\hat{\theta}_{q^e=1}$  below which the monopoly is constrained in the second period. The monopoly's expected profits with expectations  $q^e = 1$  are

$$\begin{aligned} \mathbb{E}(\Pi) &= \frac{1}{2M} \left\{ \int_{\bar{x}_{q^e=1} - \epsilon}^{\hat{\theta}_{q^e=1}} [q(p)(p - c_f) + q(p)\pi_2^*] d\theta + \right. & (19) \\ &+ \int_{\hat{\theta}_{q^e=1}}^{\bar{x}_{q^e=1} + \epsilon} [q(p)(p - c_f) + q(p)\pi_2^{**}] d\theta + \\ &\left. + \int_{\bar{x}_{q^e=1} + \epsilon}^M (p - c_f + \pi_2^{**}) d\theta \right\} \end{aligned}$$

The optimisation of (19) gives the price

$$p_{q^e=1}^* = \frac{1}{2} \left[ M + c_f - \frac{1}{8}(1 - c_a)^2 \right].$$

The second order conditions are satisfied,  $\frac{\partial^2 \mathbb{E}(\Pi)}{\partial p^2} = -\frac{1}{M} < 0$ . Given the price  $p_{q^e=1}^*$ , the highest type must have a strictly dominant strategy to buy, even if no-one else buys,  $M + \epsilon - p_{q^e=1}^* > 0$ , where we have used  $\lambda^*(q^e = 0) = 0$ , which gives the following condition on the bandwidths of  $F(\theta)$  and  $G(x | \theta)$

$$M + 2\epsilon > c_f - \frac{1}{8}(1 - c_a)^2. \quad (20)$$

(ii) A similar line of reasoning must apply to the lower dominance region,

$$\exists \underline{\theta} \in ]-M, M[ \text{ so that } v(x, q, p) < 0 \text{ for all } q \in [0, 1] \text{ and } x \leq \underline{\theta}.$$

Now, we look for the optimal price corresponding to the most "pessimistic" expectations  $q^e = 0$ . This price is the lowest price the firm will ever set. We skip the derivation of the true demand schedule corresponding to expectations  $q^e = 0$ , and the calculation of the respective optimal price. The procedures are identical to those explained in part (i). Given the optimal price  $p_{q^e=0}^* = \frac{1}{2} \left[ M + c_f - \frac{1}{4} (1 - c_a)^2 \right]$  corresponding to the most pessimistic expectations, the lowest type must have a strictly dominant strategy not to buy, even if everybody else buys,  $-M - \epsilon + \frac{1}{8} (1 - c_a)^2 - p_{q^e=0}^* < 0$ , where we have applied  $\lambda^*(q^e = 1) = \frac{1}{8} (1 - c_a)^2$ . As a result, the following requirement for the distribution bandwidths is obtained

$$-M - \frac{2}{3}\epsilon < \frac{1}{3} \left[ c_f - \frac{1}{2} (1 - c_a)^2 \right]. \quad (21)$$

The requirements (20) and (21) are satisfied simultaneously, when we expand the support of  $F(\theta)$  by increasing  $M$  sufficiently. ■

**Proof of Proposition 10.** The optimal usage fee  $t^*$  is derived in Section 3. To obtain the optimal unit price, write the expected profits (12) as

$$\begin{aligned} \mathbb{E}(\Pi) = & \frac{1}{2M} \left\{ \int_{\tilde{x}-\epsilon}^{\tilde{x}+\epsilon} q(\theta, p) (p - c_f) d\theta + \int_{\tilde{x}+\epsilon}^M (p - c_f) d\theta + \right. \\ & + \int_{\tilde{x}-\epsilon}^{\hat{\theta}} q(\theta, p)^2 [1 - c_a - q(\theta, p)] d\theta + \\ & \left. + \int_{\hat{\theta}}^{\tilde{x}+\epsilon} \frac{1}{4} (1 - c_a)^2 q(\theta, p) d\theta + \int_{\tilde{x}+\epsilon}^M \frac{1}{4} (1 - c_a)^2 d\theta \right\}. \end{aligned}$$

The maximisation of the above expression with respect to  $p$  gives

$$p^* = \frac{1}{2} (M + c_f) - \frac{1}{2} \tau(c_a) - \frac{1}{8} (1 - c_a)^2.$$

The second order condition for local maximum is satisfied  $\frac{\partial^2 \mathbb{E}(\Pi)}{\partial p^2} = -2 < 0$ . Because the first period profits maximisation problem is unconstrained,  $p^*$  gives the global maximum. ■

**Proof of Proposition 12.** Write the profit function with the optimal price structure as

$$\begin{aligned} \mathbb{E}[\Pi(p^*, t^*)] = & \frac{1}{2M} \left[ \int_{\tilde{x}(p^*)-\epsilon}^{\tilde{x}(p^*)+\epsilon} q(\theta, p^*) (p^* - c_f) d\theta + \right. \\ & + \int_{\tilde{x}(p^*)+\epsilon}^M (p^* - c_f) d\theta + \\ & + \int_{\tilde{x}(p^*)-\epsilon}^{\hat{\theta}(p^*)} q(\theta, p^*)^2 [1 - c_a - q(\theta, p^*)] d\theta + \\ & \left. + \int_{\hat{\theta}(p^*)}^{\tilde{x}(p^*)+\epsilon} \frac{1}{4} (1 - c_a)^2 q(\theta, p^*) d\theta + \int_{\tilde{x}(p^*)+\epsilon}^M \frac{1}{4} (1 - c_a)^2 d\theta \right]. \end{aligned} \quad (22)$$

To see the effect of an increase in the precision of signals, differentiate (22) with respect to  $\epsilon$ . By applying the envelope theorem, the reported result is obtained

$$\frac{\partial \mathbb{E} [\Pi (p^*, t^*)]}{\partial \epsilon} = -\frac{(1 - c_a)^4}{192M} < 0.$$

■

**Vertical separation: perfect competition in the second period.**

The introduction of competition in the second period does not change the solution process, and the coordination game still satisfies global game conditions. In the second period, the usage fee is  $t = c_a$ , which gives the indirect usage utility

$$\lambda^*(q) = \begin{cases} \frac{1}{2}(1 - c_a)^2 q, & q \geq 1 - c_a \\ q(q - \frac{1}{2}q^2 - c_a q), & q < 1 - c_a \end{cases}.$$

In calculating the marginal type, we need to take into account the cut-off point  $q = 1 - c_a$ . The marginal type is  $\tilde{x} = p + \tau_C(c_a)$ , where  $\tau_C(c_a) = \frac{1}{24}(1 - c_a)^2 [(1 - c_a)^2 - 6] \leq 0$ .

The firm does not take into account whether consumers are constrained in the second period or not, and it maximises expected profits

$$\mathbb{E} [\Pi_1(p)] = \frac{p - c_f}{M} (M - p - \tau_C(c_a)).$$

The optimal monopoly price equals

$$p_C^* = \frac{1}{2} (M + c_f - \tau_C(c_a)).$$

The second order conditions are satisfied,  $\frac{\partial^2 \mathbb{E}(\Pi_1(p))}{\partial p^2} = -\frac{1}{M} < 0$ . If we plug the optimal price back to the demand function, we get

$$q(p_C^*) = \frac{\theta^* + \epsilon - \frac{1}{2}M - \frac{1}{2}c_f - \frac{1}{2}\tau_C(c_a)}{2\epsilon},$$

where  $\theta^*$  is the realisation of the state. If we compare  $q(p_C^*)$  with the monopoly demand of the main model

$$q(p^*) = \frac{\theta^* + \epsilon - \frac{1}{2}M - \frac{1}{2}c_f - \frac{1}{2}\tau(c_a) + \frac{1}{8}(1 - c_a)^2}{2\epsilon},$$

we see that demand is higher with competition, because the monopoly restricts demand in the second period, which reduces usage utility. ■