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Abstract

Disparities in economic conditions among different ethnic, racial, or religious groups continue to be serious concerns in most economies. Relative standings of different groups are rather persistent, although some groups initially in disadvantaged positions successfully caught up with then-advantaged groups. Two obstacles, costly skill investment and negative stereotypes or discrimination in the labor market, seem to distort investment and sectoral choices, give rise to skill and labor market segregations by ethnicity, and slow down the progress of disadvantaged groups.

How do these obstacles affect skill investment and sectoral choices of different groups and the dynamics of their economic outcomes and inter-group inequality? Is affirmative action necessary to significantly improve conditions of subordinate groups, or redistributive policies sufficient? In order to tackle these questions, this paper develops a dynamic model of statistical discrimination and examines how initial economic standings of groups and initial institutionalized discrimination affect subsequent dynamics and long-run outcomes.

Keywords: ethnic inequality; statistical discrimination; labor market segregation; skill investment.

JEL Classification Numbers: J15, J24, J31, J62, J70, O17

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1 Introduction

Disparities in economic conditions among different ethnic, racial, or religious groups continue to be serious concerns in most economies. Relative standings of different groups are rather persistent (Borjas, 1994; Darity, Dietrich, and Guilkey, 2001),[1] although some groups initially in disadvantaged positions successfully caught up with then-dominant groups. Two obstacles seem to slow down the progress of subordinate groups. One is costly skill investment: the quality of public schools is low in many countries, thus people expend on supplementary study materials and tutoring, or attend high-quality but costly private schools (Baker et al., 2001; Bray and Kwok, 2003).[2] The other is negative stereotypes or discrimination in the labor market, which compels many individuals of subordinate groups to invest less in skill, or to choose occupations or sectors where performance is less affected by such handicaps but earnings tend to be lower, such as informal-sector jobs and neighborhood jobs (Telles, 1993; Bayard et al., 1999; van de Walle and Gunewardena, 2001).[3] Even skilled people of the groups often avoid mainstream jobs and sectors and run small businesses instead.

How do these obstacles affect skill investment and sectoral choices of different groups and the dynamics of their economic outcomes and inter-group inequality? Is affirmative action necessary to significantly improve conditions of disadvantaged groups, or redistributive policies sufficient? In order to tackle these questions, this paper develops a dynamic model of statistical discrimination and examines how initial economic standings of groups and initial institutionalized discrimination affect subsequent dynamics.

The model. The analysis is based on a discrete-time small-open OLG model. There exists a continuum of two-period-lived individuals who belong to one of two ethnic (racial, religious) groups. In childhood, an individual receives a transfer from her parent and spends it on assets and skill investment needed to become a skilled worker. No credit market exists for skill investment, so she cannot invest if the transfer is not enough. Since she can spend wealth on assets too, she invests in skill only if it is affordable *and* profitable. In adulthood, she chooses a sector to work (detailed next), obtains income from assets and work, and spends it on consumption and a transfer to her single child.

[1]Borjas (1994) shows that wages of a U.S. worker in 1940 and 1980 are significantly related to the average wage of immigrants of the worker's ethnic group in 1910, after individual characteristics are controlled for (blacks are not in the data). Darity, Dietrich, and Guilkey (2001) find that the occupational status of a U.S. worker in 1980 and 1990 is significantly related to human capital endowments and the degree of favorable or unfavorable treatment in the labor market in the period between 1880 and 1910 of his/her group.

[2]Baker et al. (2001) find that about 40% of seven and eight graders in a large sample from 41 developed and developing countries participate weekly in private supplementary tutoring, such as tutoring sessions and cram schools, to study mathematics. Further, at the national level, they find that the average participation rate is significantly negatively related to the percentage of public expenditure on education in GNP. Bray and Kwok (2003) briefly review existing studies, which show that the use of private tutoring is extensive even among primary school students in developing countries.

[3]Telles (1993) finds, in Brazilian metropolitan areas, that minorities (except Asians) are overrepresented in low-wage informal-sector jobs in which being minorities has less negative effects on earnings. For the Vietnam economy, van de Walle and Gunewardena (2001) show that, compared to the majority Kinh, returns to education are lower but returns to land are higher for minorities, suggesting that minorities choose to exert more efforts on farming in which performance is less affected by their disadvantaged positions. For the U.S. economy, Bayard et al. (1999) find that greater racial and ethnic wage disparities for men than for women can be explained largely by more severe occupational and industry-level segregation among men.

The economy is composed of *up to* two sectors, the primary sector with advanced technology and the secondary sector with backward technology. They respectively correspond to formal/modern and informal/traditional sectors in developing economies, while in advanced economies, typical secondary-sector jobs are neighborhood jobs at small businesses. Skilled and unskilled workers are perfectly substitutable.

In real economy, labor and product markets of the primary sector tend to be ethnically more mixed than the secondary sector. In the integrated primary sector, a *subordinate group*, who are typically a minority but could be a majority in a historically disadvantaged position, are prone to face disadvantages in production or suffer greater disutility of work, because prevalent language, customs, taste, and culture are different from theirs, or they face taste-based discrimination. Hence, the effect of skill investment on human capital in the primary sector, where skilled workers have comparative advantages, is assumed to be smaller for the subordinate group, while the investment raises human capital in the secondary sector equally for both groups. Main implications of the model, however, *remain intact without this assumption* (although the dynamics are affected): it is imposed for analytical simplicity as well as for reality.

In the primary sector, due to complex production processes and organizational structures, evaluating each worker's contribution to output tends to be difficult. Accurate evaluations are particularly difficult at least initially, if a worker and her evaluators belong to different groups due to the above inter-group differences.[4] Qualifications of a job applicant too tend to be assessed less precisely when interviewers are from other groups. Hence, the wage is assumed to depend partly on her human capital and partly on its signal, the average human capital (average wage) of her group in the sector (in the spirit of classic models of statistical discrimination by Aigner and Cain, 1977, and Lundberg and Startz, 1983), *and* the signal's importance decreases with the group's share in the sector's skilled workers.[5] In the secondary sector, typically, each worker's contribution is easy to measure, thus wage equals human capital.[6]

Wealth in the initial period is unequally distributed, and the inequality is transmitted intergenerationally through transfers. Hence, generally, individuals are heterogeneous in accessibility to skill investment, and those without enough wealth do not invest even if it is profitable. Their descendants, however, may become accessible if enough wealth is accumulated. (The opposite is true for offspring of non-poor agents.)

An important property of the model is that skill investment and sectoral choices of individuals within *and* across groups could be *interrelated*, because a worker's wage in the primary sector depends on her group's

[4]See footnotes 15 and 16 in Section 3 for evidence consistent with this and the next claim.

[5]As with many papers in the literature, it is implicitly supposed that education *level* is not a good signal, which implies that the model is concerned with an economy where the quality of public schools is low or varies greatly across schools and thus many people expend on supplementary study materials and tutoring or attend private schools, which, as mentioned at the beginning, is the case in many countries. Skill investment of the model may be interpreted as spendings on these activities.

[6]The wage equations can be derived from profit maximization problems of firms that hire workers *and* physical capital for production (see footnote 19). Further, productivity growth can be incorporated without affecting results qualitatively, as long as the cost of skill investment is assumed to grow proportionately.

average human capital in the sector (termed the group's *reputation*) and the reputation's importance in the wage (termed *the degree of prejudice* toward the group), which decreases with the group's share in the sector's skilled workers. Hence, the dynamics of wealth and economic positions of people too could be interrelated. The paper examines how the initial distribution of wealth within and across groups affects the dynamics of skill investment, sectoral choices, intra and intergroup disparities, and the steady-state outcome.

Main results. First, sectoral choices and skill investment may not be socially optimal. Even if unskilled workers are less productive in the primary sector,[7] they may choose the sector due to a positive effect from skilled workers through the reputation. Individuals may not carry out productive investment due to the negative effect from unskilled workers. For a similar reason, it is possible that all skilled workers of a group choose the secondary sector and all unskilled workers choose the *primary* sector, even if the *former* have comparative advantages and are more productive in the primary sector. This result may explain the fact that skilled people of subordinate groups often avoid primary-sector jobs and run small businesses instead.

Second, multiple equilibria could exist regarding skill investment and sectoral choices of skilled workers: both the non-poor of a group invest (skilled workers of a group choose the primary sector) and do not could be equilibria. Within a group, the source of multiplicity is strategic complementarity: to take the investment as an example, as more people invest and get skilled, prejudice toward the group eases, primary-sector wages reflect individual human capital more closely, and the return to investment rises. Across groups, strategic substitutability is at work: as more people of a group invest, prejudice toward the other group intensifies and their return to investment falls. Hence, if the latter effect is strong for both groups, *either group* invest (choose the primary sector) and the other do not are equilibria; if the former effect too is strong for both, both groups invest (choose the sector) is also an equilibrium.

Third, the dynamics and long-run outcomes of groups, particularly of the subordinate group, depend greatly on initial conditions and could be quite different from a "prejudice-free" economy. Since *good* (*bad*) *reputation tends to beget good* (*bad*) *reputation*, a group starting with a good (bad) initial condition, i.e. a high (low) fraction of them can afford skill investment initially, tend to be in a good (bad) position in the long run, according with empirical evidence such as Borjas (1994) cited above.

The mechanism can be explained intuitively based on an economy in which workers always choose the primary sector and offspring of unskilled workers of the dominant group become accessible to skill investment over time (analyzed in Section 5.1.1). If the subordinate group's initial condition is good and thus a relatively large fraction of them are skilled initially, the wage of the group's unskilled workers is relatively high because

[7] Individuals, particularly the minority, could be less productive in the primary sector if the quality of formal institutions and thus the sector's productivity are low, if non-statistical discrimination exists, or if the disutility of work is greater in the sector (human capital may be measured *net* of the disutility).

of the group's good reputation. Further, as the dominant group increasingly become skilled, the reputation becomes more important, which has a positive effect on the wage. As a result, unskilled workers of the subordinate group accumulate wealth over time, and some of their offspring come to afford investment at some point. The number of the group's skilled workers and the reputation start to rise, and the improved reputation stimulates the upward mobility of the unskilled further. In the long run, all workers are skilled. If the initial condition is bad, a similar mechanism affects the group's skilled workers negatively, and the group are totally unskilled in the end.[8]

The dynamics of sectoral choices and the degree of labor market segregation too could be affected greatly by initial conditions. An intuitive explanation can be made based on an economy similar to the previous one except that unskilled workers of the subordinate group choose the secondary sector while the reputation is unimportant (analyzed in Section 5.1.2). As the dominant group increasingly become skilled and thus reputation become more important to the subordinate group, unskilled workers of the latter group increasingly choose the primary sector *inefficiently*, deteriorating their reputation. If the group's initial condition is good, however, the reputation stays high enough that the shift to the primary sector continues, and the labor market becomes ethnically integrated eventually (and the dynamics become similar to the previous economy). By contrast, if the initial condition is bad, the downward mobility of the group's skilled workers starts at some point, which worsens the reputation and increases its importance (deepens the prejudice) further. Hence, the group's unskilled workers increasingly choose the *secondary* sector. Eventually, all of the subordinate group are unskilled and in the secondary sector, thus *the labor market is segregated completely by ethnicity.* The inefficient sectoral choices make the dynamics sensitive to the initial condition.

Fourth, when multiple equilibria exist regarding skill investment or sectoral choices of skilled workers, which is the case when prejudice is severe or the efficacy of skill investment is low, *given initial conditions*, the *initial selection of equilibrium* could affect the dynamics greatly.[9] When multiple equilibrium choices exist for the subordinate group, it is possible that, if the group's non-poor *happen to* (*not to*) invest [or choose the primary sector] initially, the number of the group's skilled workers grows (falls) over time and the group are totally skilled (unskilled) eventually (Sections 5.2.1 and 6.2). When multiple equilibria exist for both groups, the long-run outcome of the dominant group too is sensitive to the initial selection (Section 5.2.2). The dominant group staring with a *much better* condition than the subordinate group could end up with the *smaller* fraction of skilled workers, if they (the subordinate group) *happen not to* (*to*) invest [choose

[8]When institutionalized disadvantages of the subordinate group and advantages of the dominant group are small, the dynamics of the dominant group too could be affected greatly by initial conditions (Section 5.1.3). If both groups start with bad conditions, *both* could end up without skilled workers: a bad impression each group has about the other group affects the skilled wage negatively, which causes the downward mobility of skilled workers and the impression deteriorates further.

[9]It is assumed that the initial coordination among individuals of a group continues for subsequent periods: if the group's non-poor happen to invest initially, they invest subsequently too. The assumption would be reasonable since children tend to mimic parental behaviors in real society. Kim and Loury (2009) make a similar assumption in their model (see footnote 29).

the primary sector] initially. The result suggests that, if the initial selection is affected by institutionalized discrimination limiting a group's access to investment or skilled jobs in the primary sector, the discrimination could have a lasting impact on their well-beings well after its abolishment, consistent with the finding of the persistent effect of initial discrimination by Darity, Dietrich, and Guilkey (2001) cited above. Income or wealth redistribution does little to change the situation, while affirmative action treating them favorably in skill investment or primary-sector employment, such as tuition or wage subsidies, can be very effective.

Organization of the paper. Section 2 reviews the related literature and details contributions of the paper in the literature. Section 3 presents and analyzes the model's static part. Section 4 presents the full-fledged model and Section 5 analyzes the dynamics. Section 6 examines a general case by lifting one assumption that excludes situations of severe prejudice and low relative efficacy of skill investment in the primary sector. Section 7 concludes. Appendix contains proofs of lemmas and propositions.

2 Related Literature

Literature on statistical discrimination. Models of statistical discrimination examine the situation where employers cannot observe workers' skills and thus use two kinds of signals, race and a signal imperfectly correlated with individual skill, such as a test and an on-the-job monitoring, to screen workers (see Fang and Moro, 2010, for a survey). The first type of models such as Coate and Loury (1993) explain skill and earnings disparities among groups with equal endowment based on multiple equilibria. Employers assign individuals to two kinds of jobs, jobs requiring skill investment for good performance and those not, based on the signals. Since one's return to investment increases with investments by others of her race, multiple equilibria with different shares of skilled workers could exist. The second type of models, by contrast, assume that the non-race signal is nosier for the subordinate group to explain the disparities. Lundberg and Startz (1983), drawing on Phelps (1972) and Aigner and Cain (1977), develop a model where wage equals expected marginal productivity conditional on the signals. The return to investment is lower for the subordinate group due to the noisier signal and thus they invest less even if groups' endowment is identical.

Recent major progress in the literature are twofold. One is the extension to a *dynamic* setting. This is particularly important to the first type of models, where employers' self-confirming beliefs about groups' skill levels select an equilibrium, because a static model does not explain how such beliefs are formed. Kim and Loury (2009) develop a continuous-time OLG model in which employers' beliefs are formed based on objective information on groups' present and future skill levels (reputations) and are updated with changing investments. If the initial reputation of a group is high (low), the group converges to the high (low) reputation steady state, while if it is intermediate, the group could converge to either steady state, i.e. self-confirming expectations determine the final state as in static models.

The other is the consideration of *inter-group interactions*. In the above models, different groups do not interact and thus behaviors and welfare of one group do not affect those of other groups.[10] Chaudhuri and Sethi (2008) present a static model of the first type in which the investment cost depends on both individual ability and the fraction of skilled peers, which equals a weighted average of the fractions in one's own group and in the overall population and the constant weight on own group is interpreted as the degree of segregation. In a special case, they show that, in an economy where inter-group inequality exists under complete segregation, complete integration eliminates inequality and raises (lowers) shares of skilled workers of both groups, if the fraction of the initially disadvantaged group is low (high). Lundberg and Startz (2007) construct a random search model with a second-type element where searchers observe imperfect signals of potential partners' abilities. In a one-sided search model where homogenous white searchers observe more accurate signals of whites than of blacks, there could exist an equilibrium where they trade only with whites with good signal, even if both groups have identical ability distribution. In a two-sided search model where searchers are heterogenous in ability and race (and signals observed by black searchers reveal abilities of both races equally), they numerically show that there could exist an equilibrium of racially segregated transactions where high ability whites (blacks) accept only whites (blacks) with good signal.

Contributions of the paper in the literature. This paper shares with the second type of models such as Lundberg and Startz (1983) the feature that the importance of own group's average human capital (reputation) in wage is different among groups (footnote 17). The existing works assume that the importance is *constant* and greater for a subordinate group, while, in this paper, it decreases with the share of own group in primary-sector skilled workers. Unlike these works, the model is *dynamic* and inter-group disparities could change over time, thus making the reputation's importance depend on the endogenous variable would be crucial. Such formulation yields a different kind of inter-group interactions from works such as Lundberg and Startz (2007) and Chaudhuri and Sethi (2008), and the interactions generate a different type of multiple equilibria from works such as Coate and Loury (1993).

Further, the paper models *sectoral choices* between the primary and the secondary sectors, where reputation could affect wage only in the former, and the credit constraint in skill investment, both of which are not considered in other works but are important real-economy elements, as stated in the introduction. The credit constraint generates upward and downward mobilities of lineages through intergenerational transmis-

[10] Hence, the models cannot provide economic rationales for institutionalized discrimination used to be enforced by dominant groups in many countries. Moro and Norman (2004) construct a static general equilibrium model of the Coate and Loury type, in which productivities of two types of jobs are interrelated. When the two jobs are complementary, an increased share of skilled workers has a negative (positive) effect on the wage of good (bad) jobs, and thus the return to investment of a dominant group is negatively affected by investment of the disadvantaged group, giving dominant groups an incentive for the discrimination.

sion of wealth and thus the interesting group dynamics described in the introduction, whereas modeling the sectoral choice allows the paper to examine the *dynamics of labor market segregation*.

Regarding several elements, the paper employs a simpler setting: there is no non-race signal, which implicitly supposes that one's contribution to production cannot be observed initially but is fully revealed later; the investment cost is homogeneous; and the generational structure is simpler than the dynamic model of Kim and Loury (2009). However, because of the simpler setting, it can consider the above-mentioned new elements and examine how *transitional dynamics* as well as steady states depend on the initial condition using phase diagrams. Further, it can identify conditions under which multiple equilibria exist, the dynamics are different from a "prejudice-free" economy, inter-group disparities are eradicated in the long run, etc.

Other related studies. Studies that examine the dynamics of inter-group inequality based on models without statistical discrimination too are closely related.[11] Lundberg and Startz (1998), based on Loury (1977) and the 'ethnic capital' model of Borjas (1992), examine a dynamic two-group economy in which human capital is the engine of growth and there exist spillovers from coworkers in production and from elder neighbors and, for the minority, from elders of the majority in skill development. Individuals are *assumed* to be segregated by ethnicity both in the workplace and in residence. There are no spillovers from the minority to the majority and inter-group inequality disappears in the long run. Using a version of the model with heterogeneous innate ability and without the third spillover, they examine the effect of workplace desegregation, i.e. allowing the minority to move to majority-dominated jobs by paying a mobility cost, on the dynamics. They examine the effect of *one-time* workplace desegregation, while this paper examines the *dynamics* of labor market segregation in an economy where workers can freely choose sectors.

The modeling of skill investment and intergenerational transmission of wealth draws on Galor and Zeira (1993) and Yuki (2008), in which, as in this paper, skill investment is constrained by intergenerational transfers motivated by impure altruism.

3 Static Model

This section presents and analyzes the static part of the model. The dynamic part is presented in the next section. Consider a small open economy (interest rate r is exogenous) populated by a continuum of individuals who belong to one of two ethnic (racial, religious) groups. Results in this section can be applied to traits that are not intergenerationally transmitted, such as gender and native region, as well.

[11] Bowles, Loury, and Sethi (2012), building on Loury (1977), construct a discrete-time OLG model with two groups, where the cost of skill investment is modeled as in Chaudhuri and Sethi (2008). They prove that, when the degree of segregation is sufficiently high, the long-run group equality cannnot be attained even with very small initial inequality. In a special case, they show a dynamic version of the result of Chaudhuri and Sethi (2008) mentioned earlier. Yuki (2009) examines the dynamics of disparities between educated and uneducated workers in a one-group and one-sector economy where innate ability is heterogeneous and wage is determined as in this paper (education is the signal).

Individuals decide whether or not to invest in skill. The cost of skill investment c_h must be self-financed, so they must have enough wealth. Skilled and unskilled workers are perfectly substitutable.

There exist *up to* two sectors, the primary sector with advanced technology and the secondary sector with backward technology. They respectively correspond to formal/modern and informal/traditional sectors in developing economies, while in advanced economies, typical secondary-sector jobs would be neighborhood jobs at small businesses. In real economy, labor and product markets of the primary sector tend to be ethnically more mixed than the other sector (Aslund and Skans, 2010), probably due to differences in needed skills, scales of operations, and enforcement of law.[12]

Two assumptions are made based on the fact. First, skill investment raises human capital from h_u (u is for unskilled) to h_s (s is for skilled) in the secondary sector, while, in the primary sector, it raises human capital of ethnic (racial, religious) group i from $A_{ui}h_u$ to $A_{si}h_s$, where the relative human capital A_{ki} ($k = u, s$) is weakly greater for the dominant group (*group 1*) than the subordinate group (*group 2*):

$$A_{k1} \geq A_{k2}. \tag{1}$$

Given skill, human capital in the sector is lower for the subordinate group, who are typically the minority but could be the majority in a historically disadvantaged position.[13] The assumption captures the fact that, in the integrated primary sector, they are prone to face disadvantages in production or suffer greater disutility of work (human capital may be measured *net* of the disutility), because prevalent language, customs, taste, and culture are different from theirs. Further, if taste-based discrimination exists, they are not assigned relevant tasks and end up in lower productivity.[14] Note that $A_{ki} < 1$ is possible if the quality of formal institutions and thus the sector's productivity are low (as explained in footnote 19 below, A_{ki} increases with the sector's relative productivity), if the discrimination exists, or if the disutility is greater in the sector.

As is made clear later, the assumption is imposed for analytical simplicity as well as for reality, and main implications *remain intact without it*. By contrast, the next assumption is crucial. In the primary sector, due to complex production processes and organizational structures, evaluating each worker's contribution to output tends to be difficult. Accurate evaluations are particularly difficult at least initially, if a worker and her evaluators belong to different groups due to the above-mentioned inter-group differences (Giuliano,

[12]Primary-sector firms need workers with highly specialized skills and scales of operations tend to be large. Thus, to assign jobs to workers with appropriate skills efficiently, labor markets tend to be anonymous and ethnically integrated. Further, the sector tends to be regulated by laws prohibiting overt employment discrimination. By contrast, in the secondary sector with the contrasting features, employment is largely through personal connections and thus labor markets are more segregated. Also, products of the primary sector are supplied to national markets, while those of the secondary sector, especially services, are mainly for local markets of particular groups. For the Swedish economy, Aslund and Skans (2010) find that the tendency for a minority worker to work with people of his/her group is stronger in smaller establishments.

[13]Skill and human capital are *different*: skill is ability and has two levels, while human capital is the contribution of skill to output and workers of given skill can be different in human capital levels depending on sectoral choices and ethnicity.

[14]Taste-based discrimination seems to affect labor market outcomes even in advanced nations. For the U.S., Charles and Guryan (2008) find that white-black wage gaps in a state are related to the degree of bias by whites in the left tail of the bias distribution in the state, consistent with the model of Becker (1971).

Levine, and Leonard, 2011; Pinkerson, 2006).[15] Qualifications of a job applicant too tend to be assessed less precisely when interviewers are from the other group (Stoll, Raphael, and Holzer, 2004; Fryer, Pager, and Spenkuch, 2011).[16] Hence, the wage is assumed to depend partly on her human capital and partly on its signal, the average human capital (average wage) of her group in the sector (in the spirit of classic models by Aigner and Cain, 1977, and Lundberg and Startz, 1983),[17] *and* the signal's importance decreases with the group's share in the sector's skilled workers.

As with many papers in the literature, it is implicitly supposed that observable education *level* is not a good signal, implying that the model is concerned with an economy where the quality of public schools is low or varies greatly across schools and thus many people expend on supplementary study materials and tutoring or attend private schools, which, as mentioned in the introduction, is the case in many countries (see footnote 2 for details). Skill investment of the model may be interpreted as spendings on these activities.

The wage of an individual with skill level k ($k = u, s$) of group i is given by:

$$(1 - s_i) A_{ki} h_k + s_i E[A_i h_i], \tag{2}$$

where $s_i \in [0,1]$ measures the importance of the average human capital, $E[A_i h_i]$, and decreases with the share:[18]

$$s_i = s\left(\frac{p_{si} H_i N_i}{p_{si} H_i N_i + p_{sj} H_j N_j}\right),\ j \neq i,\ s'(\cdot) < 0,\ s(1) = 0. \tag{3}$$

H_i is the fraction of skilled workers in group i, N_i is the group's population, and p_{si} is the probability that a skilled worker of the group chooses the primary sector. The size of s_i reflects the degree of the incomplete information and is named *the degree of prejudice* toward the group. If the sector's skilled workers are all from her group, $s(1) = 0$ for simplicity. The average human capital, termed the group's *reputation*, equals (p_{ui} is the probability for an unskilled worker):

$$E[A_{ki} h_k] = \frac{p_{si} H_i A_{si} h_s + p_{ui}(1 - H_i) A_{ui} h_u}{p_{si} H_i + p_{ui}(1 - H_i)}. \tag{4}$$

In the secondary sector, typically, each worker's contribution is easy to measure, thus wage equals human capital, h_k ($k = u, s$).

[15] Giuliano, Levine, and Leonard (2011) find, for a large U.S. retail firm, that employees generally have better outcomes, particularly in dismissals and promotions, when they are the same race as their supervisors. Further, Pinkerson (2006) finds that the effect of AFQT scores, a measure of skills not observed directly by employers, on wages increases with experience for black men but does not change for white men in the U.S., which could be explained by the fact that managers are overwhelmingly white. By contrast, Arcidiacono, Bayer, and Hizmo (2010) find similar effects of AFQT scores on wages of the two groups.

[16] Stoll, Raphael, and Holzer (2004) find that establishments where blacks are in charge of hiring are significantly more likely to employ blacks than those with white hiring agents, and this can be explained by the higher application rate of blacks and the higher hiring rate of black applicants in the former establishments in the U.S. Further, Fryer, Pager, and Spenkuch (2011) find that the black unemployed are willing to accept lower wages than whites who previously earned as much in New Jersey.

[17] Unlike this model, workers' contributions to output are never revealed and thus a worker's wage equals a weighted average of her group's average human capital and her non-race signal, and the importance of the race signal (corresponding to s_i in the equations below) is *constant* and is *assumed* to be greater for the subordinate group in their models.

[18] An interpretation of s_i is that evaluators cannot recognize her skill during the first s_i fraction of time but can identify it after that. Alternatively, $(1 - s_i) A_{ki} h_k$ may be construed as the amount of her contribution recognized precisely by them.

The wage equations can be derived from profit maximization problems of firms that hire workers *and* physical capital.[19] Further, productivity growth can be incorporated without affecting results qualitatively, as long as the cost of skill investment c_h is assumed to grow proportionately.

The following assumptions are imposed on A_{ki} $(k=u,s)$ and the function $s(\cdot)$.

Assumption 1 *(i)* $A_{si} \geq A_{ui}$

(ii) $A_{si} > 1$ *and* $A_{si}h_s - (1+r)c_h - \max\{A_{ui},1\}h_u \geq 0 \Leftrightarrow A_{si} \geq \frac{(1+r)c_h + \max\{A_{ui},1\}h_u}{h_s}$

(iii) $s(0) + \frac{h_s - h_u}{A_{si}h_s - A_{ui}h_u} \leq 1 \Leftrightarrow A_{si} \geq \frac{h_u}{h_s}A_{ui} + \frac{h_s - h_u}{(1-s(0))h_s}$.

The first assumption states that skilled workers have comparative advantages (weakly) in the primary sector, which would be justified from the fact that the sector adopts more advanced technology and thus workers' skills are more important. The second assumption states that skilled workers are more productive in the primary sector and the net *social* return to skill investment is non-negative. An old version of the paper (Yuki, 2012), which does not impose this assumption, shows that the result is different from the "prejudice-free" economy only when this condition holds. The last assumption means that the net private return to choosing the primary sector is weakly higher for skilled workers even when the degree of prejudice is severest, i.e. $s_i = s(0)$ (the assumption can be expressed as $[(1-s(0))A_{si} - 1]h_s \geq [(1-s(0))A_{ui} - 1]h_u$). The first two assumptions are maintained throughout the paper, while the last one is relaxed in Section 6.

3.1 Sectoral choices and skill investment

Since workers are freely mobile between the sectors, they choose the one(s) with higher earnings. The next lemma presents equilibrium values of p_{si} and p_{ui} for given p_{sj} $(j \neq i)$, when $H_i > 0$ and $p_{sj}H_j > 0$, in which case $s_i > 0$ holds from (3).[20] Only equilibria that are stable with respect to small perturbations to equilibrium p_{si} and p_{ui} are considered.[21]

Lemma 1 (Sectoral choices) *Suppose* $H_i > 0$ *and* $p_{sj}H_j > 0$ *for* $j \neq i$.

(i) When $A_{ui} \geq 1$, $p_{si} = p_{ui} = 1$.

(ii) When $A_{ui} < 1$, $p_{si} = 1$. $p_{ui} = 0$ *for* $s_i \leq \frac{(1-A_{ui})h_u}{A_{si}h_s - A_{ui}h_u}$, $p_{ui} = \frac{s_i A_{si}h_s + (1-s_i)A_{ui}h_u - h_u}{(1-A_{ui})h_u}\frac{H_i}{1-H_i} \in (0,1)$ *for* $s_i \in \left(\frac{(1-A_{ui})h_u}{A_{si}h_s - A_{ui}h_u}, \frac{1}{H_i}\frac{(1-A_{ui})h_u}{A_{si}h_s - A_{ui}h_u}\right)$, *and* $p_{ui} = 1$ *for higher* s_i.

When $(A_{si} \geq)\, A_{ui} \geq 1$, that is, when both types of workers weakly prefer the primary sector under $s_i = 0$, they do choose the sector. Intuitively, the reason is that, with $s_i > 0$ and $p_{si} > 0$, unskilled workers benefit

[19] Suppose that firms with an identical CRS technology hire both factors in each sector. Then, by normalizing the wage rate per human capital (which depends on total factor productivity and the interest rate) of the informal sector to 1, the same wage equations are obtained. The relative human capital in the formal sector A_{ki} increases with the sector's relative TFP.

[20] Clearly, when $H_i = 0$ or $p_{sj}H_j = 0$ for $j \neq i$ (thus $s_i = 0$), $p_{si} = 1$ (when $H_i > 0$) and $p_{ui} = 1 (= 0)$ if $A_{ui} > (<) 1$ and any $p_{ui} \in [0,1]$ if $A_{ui} = 1$.

[21] An equilibrium is defined to be *stable* regarding the perturbations if there exists a neighborhood of equilibrium p_{si} and p_{ui} such that, from any p_{si} and p_{ui} in the neighborhood, they have tendencies to return to equilibrium values in a simple dynamics in which p_{ki} increases (decreases) when the net return to choosing the formal sector for type k workers is positive (negative).

from the presence of skilled workers in the sector and thus strictly prefer the sector, and the net return from choosing the sector is higher for skilled workers from Assumption 1 (iii). When $(A_{si}>)\,1>A_{ui}$, skilled workers select the primary sector, while choices of unskilled workers depend on s_i: since the positive effect from skilled workers increases with s_i, they select the primary (secondary) sector when s_i is large (small), and when s_i is intermediate, they are indifferent between the sectors and $p_{ui}\in(0,1)$ is increasing in s_i.

Taking into account the dependence of wages on sectoral choices, an individual decides on skill investment. As detailed in the next section, she can spend wealth on assets too. Thus, she invests in skill only if it is financially accessible *and* profitable. Let F_i be the proportion of individuals who can afford the investment in group i. H_i cannot exceed F_i but may not equal F_i. Let p_{hi} be the probability that an individual with enough wealth does invest. To simplify the analysis, the following assumption is imposed on p_{hi}.

Assumption 2 *If individuals are indifferent among multiple values of p_{hi}, the highest value is realized.*

For example, when $p_{si}=p_{ui}=0$ and $h_s-(1+r)c_h-h_u=0$, $p_{hi}=1$ holds. The next lemma presents equilibrium $H_i=p_{hi}F_i$ for given H_j and p_{sj} $(j\neq i)$ when $F_i>0$. Only equilibria that are stable with respect to a small perturbation to equilibrium p_{hi} are considered.

Lemma 2 (Skill investment) *Suppose $F_i>0$.*

(i) When $h_s-(1+r)c_h\geq h_u$, $H_i=F_i$.

(ii) When $h_s-(1+r)c_h<h_u$, $H_i=F_i$ when $p_{sj}H_j=0$ for $j\neq i$. When $p_{sj}H_j>0$ (thus $s_i>0$),

(a) If $A_{ui}\geq 1$, $H_i=F_i$ if $s(0)\leq 1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$; otherwise, both $H_i=F_i$ and $H_i=0$ are equilibria ($H_i=0$ is the equilibrium) when $s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)<(\geq)1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$.

(b) If $A_{ui}<1$, $H_i=F_i$ (no stable equilibria exist) when $s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)<(\geq)1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$.

When $h_s-(1+r)c_h\geq h_u$, i.e. the investment is socially productive (and privately profitable) in the secondary sector, every individual with enough wealth invests in skill, because, in the case where choosing the primary sector is more profitable, the net private return to the investment is weakly higher than $h_s-(1+r)c_h-h_u$ from Assumption 1 (i) (when $p_{sj}H_j=0$ for $j\neq i$) and (iii) (when $p_{sj}H_j>0$).

By contrast, when $h_s-(1+r)c_h<h_u$, the decision depends on A_{ui}, s_i, and $s(0)$. When $A_{ui}\geq 1$ and $p_{sj}H_j>0$ for $j\neq i$, since all workers choose the primary sector from Lemma 1 (i), the net return equals $(1-s_i)[A_{si}h_s-A_{ui}h_u]-(1+r)c_h$ and decreases with s_i. Hence, if the value of s_i at $H_i=F_i$ is small enough that the net return is positive at $H_i=F_i$, i.e. $s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)<1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$, $H_i=F_i$ is an equilibrium, while if $s(0)>1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$ and thus the return is negative at $H_i=0$, $H_i=0$ is an equilibrium. Since $s(0)>s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)$, *both $H_i=F_i$ and $H_i=0$ are equilibria* when $s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)<1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}<s(0)$ due to *strategic complementarity*: as more people invest and become skilled workers, the degree of prejudice s_i falls and primary-sector wages reflect human capital more closely, raising the return. The result when

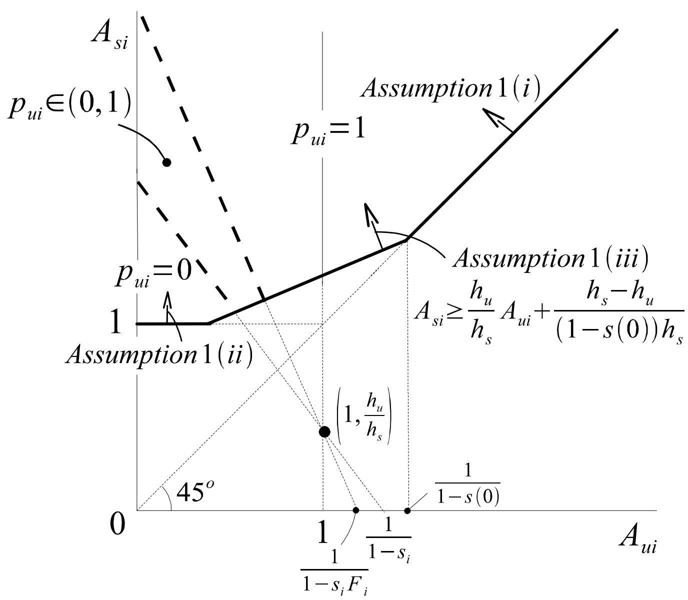

Figure 1: Sectoral choices of unskilled workers when $h_s-(1+r)c_h\geq h_u$ and $p_{sj}H_j>0$ for $j\neq i$

$A_{ui}<1$ and $p_{sj}H_j>0$ can be explained similarly. In this case, however, $H_i=0$ is not an equilibrium (since, given $H_i=0$, no unskilled workers choose the primary sector and thus the investment is profitable from Assumption 1 (ii), $A_{si}h_s-(1+r)c_h\geq h_s$), thus *no stable equilibria exist* if $s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)\geq 1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$, i.e. s_i is too high for the return to be positive at $H_i=F_i$.

By combining Lemmas 1 and 2, skill investment and sectoral choices of group i *given* choices by the other group are summarized in the following proposition.

Proposition 1 (Group i's investment and sectoral choices given choices by group j)

(i) When $h_s-(1+r)c_h\geq h_u$, $H_i=F_i$ and $p_{si}=1$.

(a) When $p_{sj}H_j>0$ for $j\neq i$, if $A_{ui}\geq 1$, $p_{ui}=1$; and if $A_{ui}<1$, $p_{ui}=0$ for $s_i\leq\frac{(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}$, $p_{ui}=\frac{s_iA_{si}h_s+(1-s_i)A_{ui}h_u-h_u}{(1-A_{ui})h_u}\frac{F_i}{1-F_i}\in(0,1)$ for $s_i\in\big(\frac{(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u},\frac{1}{F_i}\frac{(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}\big)$, and $p_{ui}=1$ for higher s_i.

(b) When $p_{sj}H_j=0$, $p_{ui}=1(=0)$ if $A_{ui}>(<)1$ and any $p_{ui}\in[0,1]$ if $A_{ui}=1$.

(ii) When $h_s-(1+r)c_h<h_u$, $p_{si}=1$ and, when $p_{sj}H_j=0$, $H_i=F_i$. When $p_{sj}H_j>0$,

(a) If $A_{ui}\geq 1$, $H_i=F_i$ if $s(0)\leq 1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$, otherwise, both $H_i=F_i$ and $H_i=0$ are equilibria ($H_i=0$ is the equilibrium) when $s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)<(\geq)1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$.

(b) If $A_{ui}<1$, $H_i=F_i$ (no stable equilibria exist) when $s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)<(\geq)1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$.

(c) p_{ui} is determined as in (i)(a) [(i)(b)] when $p_{sj}H_j>0$ and $H_i=F_i$ [in other cases].

Based on Proposition 1 (i), Figure 1 illustrates sectoral choices of unskilled workers when $h_s-(1+r)c_h\geq h_u$ and $p_{sj}H_j>0$ for $j\neq i$ on the (A_{ui},A_{si}) plane. (In this case, $H_i=F_i$ and $p_{si}=1$ always hold.) A_{ui} and A_{si} must satisfy Assumption 1 (i)−(iii), thus only the upper left region of the three bold solid lines is feasible.

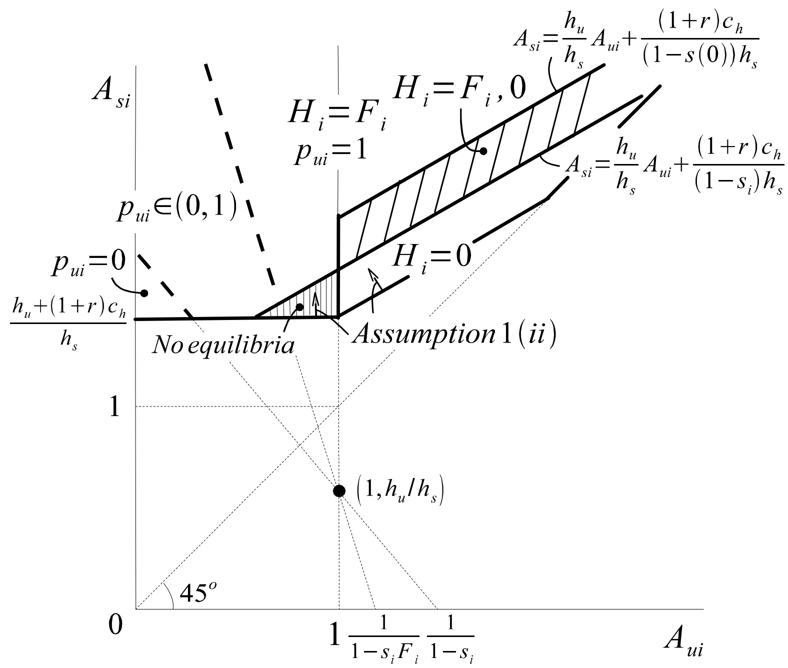

Figure 2: Skill investment and sectoral choices of unskilled workers when $h_s-(1+r)c_h<h_u$ and $p_{sj}H_j>0$

Their choices are determined by the two bold broken lines, and $p_{ui}=0$ $(=1)$ in the region at the left (right) side of the left (right) broken line and $p_{ui}\in(0,1)$ in the region between the lines. (When $p_{ui}\in(0,1)$, p_{ui} increases with A_{ui} and A_{si}.) $p_{ui}>0$ is possible with $A_{ui}<1$ because of the positive effect from skilled workers in the primary sector. Positions of the lines and thus the value of p_{ui} depend on F_i through s_i.

Figure 2 illustrates skill investment and sectoral choices of unskilled workers when $h_s-(1+r)c_h<h_u$ and $p_{sj}H_j>0$, based on Proposition 1 (ii).[22] ($p_{si}=1$ always holds.) When $A_{ui}\geq 1$, $p_{ui}=1$ holds, and $H_i=F_i$ $(H_i=0)$ is the only equilibrium in the region on or above $A_{si}=\frac{h_u}{h_s}A_{ui}+\frac{(1+r)c_h}{(1-s(0))h_s}$ (on or below $A_{si}=\frac{h_u}{h_s}A_{ui}+\frac{(1+r)c_h}{(1-s_i)h_s}$), while *both* $H_i=F_i$ *and* $H_i=0$ *are equilibria* between the two lines, the area with slanting lines. When $A_{ui}<1$, $H_i=F_i$ holds above $A_{si}=\frac{h_u}{h_s}A_{ui}+\frac{(1+r)c_h}{(1-s_i)h_s}$ and no equilibria exist on or below it (the area with vertical lines). Sectoral choices when $A_{ui}<1$ are as in Figure 1. Positions of $A_{si}=\frac{h_u}{h_s}A_{ui}+\frac{(1+r)c_h}{(1-s_i)h_s}$ and of the two broken lines and thus their choices depend on F_i through s_i.

Sectoral choices and skill investment may *not* be socially optimal when $s_i>0$. When $A_{ui}<1$, because of the positive effect from skilled workers, some or all of unskilled workers choose the less productive primary sector at the right side of the left broken line of Figures 1 and 2. When $A_{ui}\geq 1$, an individual may not carry out the socially productive investment in the region below $A_{si}=\frac{h_u}{h_s}A_{ui}+\frac{(1+r)c_h}{(1-s(0))h_s}$ due to the negative effect of unskilled workers on the private return to investment.

[22] The figure is drawn assuming $h_s-(1-s(0))(1+r)c_s\leq h_u$. When $h_s-(1-s(0))(1+r)c_s>h_u$, the bold solid line for Assumption 1 (ii) is located below the line for Assumption 1 (iii) when $A_{ui}>1$, as in Figure 1.

Investment and sectoral choices of the two groups are determined by applying the proposition to the groups simultaneously. Cases in which the investment is always profitable for both groups can be easily identified from the proposition (see Figure 2 too).

Corollary 1 (Cases in which the investment is always profitable for both groups) *$H_i = F_i$ for any i and F_i, when $h_s-(1+r)c_h \geq h_u$ and when $h_s-(1+r)c_h < h_u$ and $s(0) \leq 1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$ for any i.*

In other cases, the determination of H_i is not simple, which is examined in Proposition 2 of Section 5 with an additional assumption.

3.2 Wages

Wage levels depend on skill investment and sectoral choices. Denote the unskilled wage of group i by w_{ui} and the skilled wage *net of the investment cost* by w_{si}. Then, the wages when $H_i = F_i$ and $p_{sj}H_j > 0$ for $j \neq i$, i.e. $s_i > 0$, are:

if $p_{ui}=1$,
$$w_{ui} = (1-s_i)A_{ui}h_u + s_i[F_iA_{si}h_s + (1-F_i)A_{ui}h_u]$$
$$= A_{ui}h_u + s_iF_i(A_{si}h_s - A_{ui}h_u), \tag{5}$$
$$w_{si} = (1-s_i)A_{si}h_s + s_i[F_iA_{si}h_s + (1-F_i)A_{ui}h_u] - (1+r)c_h$$
$$= A_{si}h_s - s_i(1-F_i)(A_{si}h_s - A_{ui}h_u) - (1+r)c_h; \tag{6}$$

if $p_{ui} \in (0,1)$,
$$w_{ui} = (1-s_i)A_{ui}h_u + s_i\frac{F_iA_{si}h_s + p_{ui}(1-F_i)A_{ui}h_u}{F_i + p_{ui}(1-F_i)} = h_u, \tag{7}$$
$$w_{si} = (1-s_i)A_{si}h_s + s_i\frac{F_iA_{si}h_s + p_{ui}(1-F_i)A_{ui}h_u}{F_i + p_{ui}(1-F_i)} - (1+r)c_h$$
$$= h_u + (1-s_i)(A_{si}h_s - A_{ui}h_u) - (1+r)c_h; \tag{8}$$

and if $p_{ui} = 0$, $w_{ui} = h_u$ and $w_{si} = A_{si}h_s - (1+r)c_h$. When $H_i = 0$ or $s_i = 0$, $w_{ui} = \max\{A_{ui}, 1\}h_u$ and $w_{si} = A_{si}h_s - (1+r)c_h$ (when $H_i > 0$).

4 Dynamic model

Based on the previous section, this section presents the dynamic part of the model. Consider an OLG economy composed of a continuum of two-period-lived individuals. The distribution of wealth over the initial generation of each group is given, while distributions of subsequent generations are determined endogenously.

4.1 Lifetime of an individual

Childhood: In childhood, an individual receives a transfer from her parent (if she belongs to the initial generation, it is given) and spends it on two options, assets (yields interest rate r) and skill investment (costs c_h), to maximize future income. Consider an individual born into a lineage of group i in period $t-1$

who, in period t, receives b_{it} units of transfer and allocates it between asset a_{it} and skill investment v_{it}. When $H_{it}=F_{it}$, i.e. $p_{hit}=1$, the allocation is determined by b_{it}:

$$a_{it}=b_{it}, \qquad v_{it}=0, \quad \text{if} \quad b_{it}<c_h, \tag{9}$$

$$a_{it}=b_{it}-c_h, \quad v_{it}=c_h, \quad \text{if} \quad b_{it}\geq c_h. \tag{10}$$

By contrast, when $H_{it}=0$, i.e. $p_{hit}=0$, $a_{it}=b_{it}$ and $v_{it}=0$.

Adulthood: In adulthood, she chooses a sector to work, obtains income from assets and work, and spends it on consumption c_{it} and a transfer to her single child b_{it+1}. Her utility maximization problem is:

$$\max u_{it} = (c_{it})^{1-\gamma_b}(b_{it+1})^{\gamma_b}, \quad s.t. \quad c_{it}+b_{it+1} = w_{it}+(1+r)a_{it}, \tag{11}$$

where $\gamma_b \in (0,1)$ and w_{it} is her gross wage. By solving the problem, her consumption and transfer rules equal

$$c_{it} = (1-\gamma_b)\{w_{it}+(1+r)a_{it}\}, \tag{12}$$

$$b_{it+1} = \gamma_b\{w_{it}+(1+r)a_{it}\}. \tag{13}$$

Generational change: At the beginning of period $t+1$, current adults pass away, current children become adults, and new children are born into the economy. Since each adult has one child, the total (adult) population of the group is time-invariant and equals N_i.

4.2 Dynamics of individual transfers

The dynamic equation linking the received transfer b_{it} to the transfer given to the next generation b_{it+1} is derived from the transfer rule (13). For a current unskilled worker, it is obtained by substituting $w_{it}=w_{uit}$ and $a_{it}=b_{it}$ into (13):

$$b_{it+1} = \gamma_b\{w_{uit}+(1+r)b_{it}\}. \tag{14}$$

The assumption $\gamma_b(1+r) < 1$ is made so that the fixed point of the equation for given w_{uit}, $b^*(w_{uit}) \equiv \frac{\gamma_b}{1-\gamma_b(1+r)}w_{uit}$, exists. The fixed point becomes crucial in later analyses.

For a current skilled worker, who exists only when $H_{it}=F_{it}$, the dynamic equation is

$$b_{it+1} = \gamma_b\{w_{sit}+(1+r)b_{it}\}, \tag{15}$$

which is obtained by substituting $w_{it}=w_{sit}+(1+r)c_h$ and $a_{it}=b_{it}-c_h$ into (13).

The equations show that the dynamics of transfers within a lineage depend on the dynamics of wages and H_{it}, which in turn are determined by the time evolution of F_{it} and F_{jt} $(j\neq i)$.

4.3 Aggregate dynamics

The time evolution of F_{it} (the fraction of group i individuals who can afford skill investment) is determined by the dynamics of individual transfers. That is, the individual and aggregate dynamics are interrelated.

More specifically, when $H_{it} = F_{it}$, if offspring of some unskilled workers become accessible to the investment through wealth accumulation, $F_{it+1} > F_{it}$, while, if some of present skilled workers cannot leave enough transfers to cover the investment cost, $F_{it+1} < F_{it}$.

The former takes places iff there exist lineages satisfying $b_{it} < c_h$ and $b_{it+1} \geq c_h$. From (14), the following condition must hold for such lineages to exist:

$$b^*(w_{uit}) \equiv \tfrac{\gamma_b}{1-\gamma_b(1+r)} w_{uit} > c_h. \tag{16}$$

By contrast, the latter occurs iff lineages satisfying $b_{it} \geq c_h$ and $b_{it+1} < c_h$ exist. From (15), the necessary condition is

$$b^*(w_{sit}) \equiv \tfrac{\gamma_b}{1-\gamma_b(1+r)} w_{sit} < c_h. \tag{17}$$

Since $b^*(w_{sit}) \geq b^*(w_{uit})$, the above equations do not hold simultaneously. If (16) holds, $F_{it+1} \geq F_{it}$, while if (17) is true, $F_{it+1} \leq F_{it}$. ($F_{it+1} = F_{it}$ is possible depending on the distribution of transfers, but, if the condition continues to hold, F_{it} does change at some point.) When neither equations are satisfied, $F_{it+1} = F_{it}$. The dynamics when $H_{it} = 0$ depend on the relative value of $b^*(w_{uit})$ to c_h only.

Regarding the value of $b^*(w_{uit})$, the following is assumed.

Assumption 3 $h_u \leq \frac{1-\gamma_b(1+r)}{\gamma_b} c_h$

This implies that $b^*(w_{uit}) \leq c_h$ when $p_{ui,t} < 1$, that is, offspring of unskilled workers can never afford the investment if the unskilled wage stays at the level in the secondary sector, h_u. The assumption rules out the trivial case in which $h_u > \frac{1-\gamma_b(1+r)}{\gamma_b} c_h$ and thus F_{it} always increases.

Since the dynamics of individual transfers depend, through skill investment and wages, on the evolution of F_{jt} ($j \neq i$) as well, the dynamics of F_{it} and F_{jt} are interrelated. The next section examines the joint dynamics of the variables, thereby analyzing the dynamics of variables of interest.

5 Analyses

This section analyzes the time evolution of F_{it}, skill composition, sectoral choices, wages, and intergroup inequality by using phase diagrams. As can be inferred from Figures 1 and 2, various dynamics arise depending on values of exogenous variables such as A_{si} and A_{ui}, thus analyses focus on cases that are representative and yield clear-cut results.

For simplicity, the elasticity of s_i with respect to F_i (in absolute value) is assumed to be less than 1 in dynamic analyses, although all of main results hold without the assumption.

Assumption 4 $s(x)+s'(x)x(1-x) > 0$ *for any* $x \in [0,1) \Leftrightarrow \frac{\partial(s_i F_i)}{\partial F_i} > 0$ *always.*

5.1 When skill investment is always profitable

First, consider the case in which $H_i = F_i$ always holds for any group i. From Corollary 1, this is true when $h_s-(1+r)c_h \geq h_u$, or when $h_s-(1+r)c_h < h_u$ and $s(0)+\frac{(1+r)c_h}{A_{si}h_s - A_{ui}h_u} \leq 1$ for any i (see Figure 2). That is, skill investment is socially productive even in the secondary sector, the investment is highly productive in the primary sector, or the maximum degree of prejudice $s(0)$ is low.

5.1.1 When disadvantages of the subordinate group are moderate

Consider an economy where a dominant group (*group 1*) and a subordinate group (*group 2*) exist, i.e. $A_{k1} > A_{k2}$ $(k = u, s)$. Suppose that institutionalized disadvantages the latter group face are moderate enough (or the relative productivity of the primary sector is high enough, see footnote 19) that $A_{ki} > 1$ for any $k = u, s$ and $i = 1, 2$, i.e. all workers are more productive in the primary sector. Then, $p_{si} = p_{ui} = 1$ from Proposition 1, i.e. all workers choose the sector. Skill investment of those with enough wealth and sectoral choices are socially optimal from $A_{ki} > 1$ and the conditions of Corollary 1.

As for the dynamics of F_{1t}, it is assumed that institutionalized advantages of the dominant group are large enough that $A_{u1}h_u > \frac{1-\gamma_b(1+r)}{\gamma_b}c_h$, that is, even with the lowest wage, descendants of the group's unskilled workers can afford the investment eventually. Then, F_{1t} increases over time and $H_1^* = F_1^* = 1$ in the long run (superscript $*$ indicates the steady state value). In contrast, A_{u2} and A_{s2} are lower and the following is assumed: $A_{u2}h_u < \frac{1-\gamma_b(1+r)}{\gamma_b}c_h$, i.e. with the lowest wage, descendants of unskilled workers of the subordinate group remain unskilled; $\left[1-s\left(\frac{N_2}{N_1+N_2}\right)\right]A_{u2}h_u + s\left(\frac{N_2}{N_1+N_2}\right)A_{s2}h_s > \frac{1-\gamma_b(1+r)}{\gamma_b}c_h$, i.e. with the highest wage (from Assumption 4), they can afford the investment eventually; $A_{s2}h_s > \frac{c_h}{\gamma_b}$ and $(1-s(0))A_{s2}h_s + s(0)A_{u2}h_u < \frac{c_h}{\gamma_b}$, i.e. with the highest (lowest) wage, descendants of skilled workers of the group can (cannot) stay skilled. Then, $b^*(w_{u2}) = c_h$ and $b^*(w_{s2}) = c_h$ exist and equal:

$$A_{u2}h_u + s\left(\tfrac{F_2 N_2}{F_1 N_1 + F_2 N_2}\right) F_2 (A_{s2}h_s - A_{u2}h_u) = \frac{1-\gamma_b(1+r)}{\gamma_b}c_h, \tag{18}$$

$$A_{s2}h_s - s\left(\tfrac{F_2 N_2}{F_1 N_1 + F_2 N_2}\right)(1-F_2)(A_{s2}h_s - A_{u2}h_u) = \frac{c_h}{\gamma_b}, \tag{19}$$

which are obtained by plugging (5) and (6) with $s_2 = s\left(\frac{F_2 N_2}{F_1 N_1 + F_2 N_2}\right)$ into (16) (with $>$ replaced by $=$) and (17) (with $<$ replaced by $=$), respectively.[23]

[23]Since w_{u2} for given $\frac{F_2}{F_1}$ increases with F_2 and $b^*(w_{u2}) > (<) c_h$ at $(F_1, F_2) = (1,1)$ (as $F_2 \to 0$ on $\frac{F_2}{F_1} = 1$) from the assumptions on w_{u2} (see eq. 18), there exists $F_2 \in (0,1)$ on $\frac{F_2}{F_1} = 1$ satisfying $b^*(w_{u2}) = c_h$. $b^*(w_{s2}) = c_h$ exists for any $F_1 \in (0,1]$ $(b^*(w_{s2}) > c_h$ at $F_1 = 0)$ since, for $F_1 \neq 0$, w_{s2} increases with F_2 and $b^*(w_{s2}) < (>) c_h$ at $F_2 = 0$ $(=1)$ from the assumptions on w_{s2} (see eq. 19).

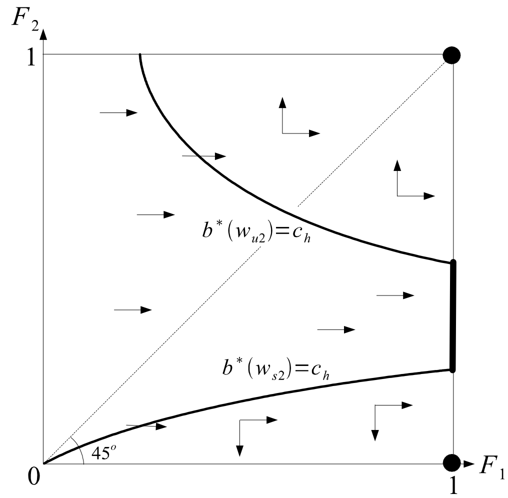

Figure 3: Dynamics when $H_{it}=F_{it}$ always and group 2's disadvantages are moderate

The dynamics of F_{1t} and F_{2t} can be analyzed graphically by placing $b^*(w_{u2})=c_h$ and $b^*(w_{s2})=c_h$ on the (F_1,F_2) plane (Figure 3).[24] $b^*(w_{u2})=c_h$ is negatively sloped and $b^*(w_{s2})=c_h$ is positively sloped from (18) and (19) (since s_2 increases (decreases) with F_1 (F_2) and s_2F_2 increases with F_2). The direction of motion of F_{2t} (F_{1t}) is represented by vertical (horizontal) arrows. Since w_{s2} decreases and w_{u2} increases with F_1, in the region at the right (left) side of $b^*(w_{s2})=c_h$, $b^*(w_{s2}) < (>) c_h$ and F_{2t} decreases (non-decreases) over time, while in the region at the right (left) side of $b^*(w_{u2})=c_h$, $b^*(w_{u2}) > (<) c_h$ and F_{2t} increases (non-increases).

Unlike the economy in which reputation does not affect wages, i.e. $s_{it}=0$ always, where F_{1t} increases and F_{2t} is constant over time, the long-run fate of the subordinate group could be very different depending on the level of F_2 in the initial period, $F_{2\,0}$.

When the initial distribution of wealth is such that a sufficiently large portion of them can afford the investment, to be accurate, when $b^*(w_{u2})>c_h$ at $(F_1,F_2)=(1,F_{2\,0})$, $H_2^*=F_2^*=1$ as well as $H_1^*=F_1^*=1$ in the long run. As an illustration, suppose that $F_{1\,0}$ is not so high that $b^*(w_{u2\,0})<c_h$ holds. Then, as $H_{1t}=F_{1t}$ grows over time, the influence of the dominant group in wage determination becomes stronger and wages of the subordinate group are affected more by their reputation, i.e. s_{2t} increases. As a result, the unskilled (skilled) wage of group 2 rises (falls) over time. Since $H_{2t}=F_{2\,0}$ is not low and thus their reputation (average human capital) is not bad, the wage of skilled workers stays high enough for their descendants to remain skilled, while the unskilled wage grows to the point that the investment becomes affordable to some of their

[24] $b^*(w_{u2}) = c_h$ intersects with $F_1 = 1$ at $F_2 \in (0,1)$ and with $F_2 = 1$ at $F_1 \in (0,1)$ from the two assumptions on w_{u2}. $b^*(w_{s2}) = c_h$ intersects with $F_1 = 1$ at $F_2 \in (0,1)$ from the assumptions on w_{s2}, does not intersect with $F_2 = 0$ from $(1-s(0))A_{s2}h_s + s(0)A_{u2}h_u < \frac{c_h}{\gamma_b}$, and not with $F_1 = 0$ from $A_{s2}h_s > \frac{c_h}{\gamma_b}$. (Thus, it approaches $(F_1,F_2) = (0,0)$.) $b^*(w_{u2}) = c_h$ and $b^*(w_{s2}) = c_h$ do not intersect from $w_{s2} > w_{u2}$ for $F_2 > 0$. In the figure, $b^*(w_{s2}) = c_s$ is always below the 45^0 line, but if $\left[1-s\left(\frac{N_2}{N_1+N_2}\right)\right]A_{s2}h_s + s\left(\frac{N_2}{N_1+N_2}\right)A_{u2}h_u < \frac{c_s}{\gamma_b}$, it crosses the line.

offspring at some point, i.e. $b^*(w_{u2t}) > c_h$. $H_{2t} = F_{2t}$ and the reputation start to rise, and the improved reputation further stimulates the upward mobility. In the long run, everyone becomes a skilled worker.

By contrast, when F_{20} is small enough that $b^*(w_{s2}) < c_h$ at $(F_1,F_2) = (1,F_{20})$, $(H_1^*,H_2^*) = (1,0)$ in the long run. Since group 2's initial reputation is low and its effect on wages strengthens over time, the wage of skilled workers falls to the point that their offspring become unable to afford the investment at some point. F_{2t} start to decrease and the deteriorated reputation spurs the downward mobility. In the long run, all of group 2 are unskilled. (When F_{20} is in the intermediate range, $(H_1^*,H_2^*) = (1,F_{20})$.)

As long as (F_{10},F_{20}) is located at the left side of the two loci, group 2's average skill and wage levels relative to group 1 fall at first. However, if F_{20} is sufficiently high, they start to rise at some point and both groups become totally skilled eventually. Otherwise, the relative levels continue to fall and, if F_{20} is low, the two groups are totally segregated by skill levels in the long run. The initial condition affects the long-run fate of the subordinate group through reputation: *good* (*bad*) *reputation begets good* (*bad*) *reputation.*

5.1.2 When disadvantages of the subordinate group are severe

Next consider an economy where institutionalized disadvantages of the subordinate group are severe enough (or the relative productivity of the primary sector is low enough) that $A_{u2} < 1 < A_{s2}$ holds, i.e. unskilled workers of the group are less productive (*net* of the disutility of work) in the sector. Then, $p_{si} = p_{u1} = 1$ $(i = 1,2)$, while $p_{u2} = 0$ for $s_2 \leq \frac{(1-A_{u2})h_u}{A_{s2}h_s - A_{u2}h_u}$, $p_{u2} = \frac{s_2 A_{s2}h_s + (1-s_2)A_{u2}h_u - h_u}{(1-A_{u2})h_u} \frac{F_2}{1-F_2} \in (0,1)$ for $s_2 \in \left(\frac{(1-A_{u2})h_u}{A_{s2}h_s - A_{u2}h_u}, \frac{1}{F_2}\frac{(1-A_{u2})h_u}{A_{s2}h_s - A_{u2}h_u}\right)$, and $p_{u2} = 1$ for higher s_2 from Proposition 1. Unlike the previous economy, sectoral choices of unskilled workers of group 2 are not socially optimal when $p_{u2} > 0$. The boundary between $p_{u2} = 0$ and $p_{u2} \in (0,1)$ and the one between $p_{u2} \in (0,1)$ and $p_{u2} = 1$ are given respectively by:

$$s\left(\frac{F_2N_2}{F_1N_1 + F_2N_2}\right) = \frac{(1-A_{u2})h_u}{A_{s2}h_s - A_{u2}h_u}, \tag{20}$$

$$s\left(\frac{F_2N_2}{F_1N_1 + F_2N_2}\right)F_2 = \frac{(1-A_{u2})h_u}{A_{s2}h_s - A_{u2}h_u}. \tag{21}$$

Assumptions related to the dynamics of F_{1t} and F_{2t} are same as the previous case except that $(1-s(0))A_{s2}h_s + s(0)A_{u2}h_u < \frac{c_h}{\gamma_b}$ is strengthened to $h_u + (1-s(0))(A_{s2}h_s - A_{u2}h_u) < \frac{c_h}{\gamma_b}$ (and $A_{u2}h_u < \frac{1-\gamma_b(1+r)}{\gamma_b}c_h$ now follows from Assumption 3). Thus, $b^*(w_{u2}) = c_h$ and $b^*(w_{s2}) = c_h$ when $p_{u2} = 1$ are given by (18) and (19), respectively. When $p_{u2} \in (0,1)$, $b^*(w_{s2}) = c_h$ equals

$$h_u + \left[1 - s\left(\frac{F_2N_2}{F_1N_1 + F_2N_2}\right)\right](A_{s2}h_s - A_{u2}h_u) = \frac{c_h}{\gamma_b}, \tag{22}$$

which is obtained by substituting (8) into (17) (with $<$ replaced by $=$).[25]

Figure 4 illustrates the dynamics of F_{1t}, F_{2t}, and p_{u2t} graphically. On the (F_1,F_2) plane, the dividing

[25] $b^*(w_{s2}) = c_h$ when $p_{u2} \in (0,1)$ exists since the LHS of (22) is strictly higher (lower) than the RHS at lowest (highest) s_2, i.e. at s_2 satisfying (20) $(s_2 = s(0))$, from $A_{s2}h_s > \frac{c_h}{\gamma_b}$ and $h_u + (1-s(0))(A_{s2}h_s - A_{u2}h_u) < \frac{c_h}{\gamma_b}$.

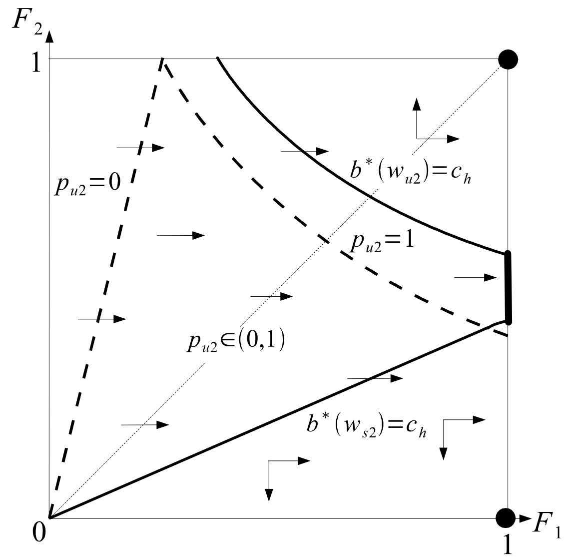

Figure 4: Dynamics when $H_{it} = F_{it}$ always and group 2's disadvantages are severe

line between $p_{u2} = 0$ and $p_{u2} \in (0,1)$ is a positively-sloped straight line that is located above the 45^0 line and approaches the origin ($p_{u2} = 0$ at $F_2 = 0$). The dividing line between $p_{u2} \in (0,1)$ and $p_{u2} = 1$ is a negatively-sloped curve, because the LHS of (21) depends positively on s_2F_2, like the LHS of the equation for $b^*(w_{u2}) = c_h$, (18).[26] The two lines are located at the left side of $b^*(w_{u2}) = c_h$ (from Assumption 3) and intersect at $F_2 = 1$. $b^*(w_{s2}) = c_h$ when $p_{u2} \in (0,1)$ is a positively-sloped straight line approaching the origin.[27]

The dynamics of F_{1t} and F_{2t} are as in the previous case: when F_{20} is large [small] enough that $b^*(w_{u2}) > c_h [b^*(w_{s2}) < c_h]$ at $(F_1,F_2) = (1,F_{20})$, F_{2t} starts to increase [decrease] eventually and $(F_1^*,F_2^*) = (H_1^*,H_2^*) = (1,1)[= (1,0)]$ in the long run.

What is new is that sectoral choices by unskilled workers of the subordinate group change over time. Suppose F_{10} is small enough that $p_{u20} = 0$, i.e. they choose the secondary sector initially. Then, as long as $p_{u2t} = 0$ is satisfied, group 2's wages equal human capital levels and are constant. After F_{1t} and thus s_{2t} become high enough that $p_{u2t} \in (0,1)$ holds, induced by the growth of s_{2t}, more and more of the group's unskilled workers choose the primary sector over time despite such choice is *inefficient*, i.e. $A_{u2} < 1$. This deteriorates their reputation and, together with the increasing importance of reputation (an increase in s_{2t}), lowers the skilled wage of the group, while the unskilled wage remains constant at h_u. That is, average earnings of the subordinate group *fall* (note $A_{u2} < 1$).

[26]Since $s(\frac{N_2}{N_1+N_2}) > \frac{(1-A_{u2})h_u}{A_{s2}h_s - A_{u2}h_u}$, i.e. $p_{u2} > 0$ on $\frac{F_2}{F_1} = 1$ and $p_{u2} = 1$ at $(F_1,F_2) = (1,1)$, from $\left[1 - s(\frac{N_2}{N_1+N_2})\right]A_{u2}h_u + s(\frac{N_2}{N_1+N_2})A_{s2}h_s > \frac{1-\gamma_b(1+r)}{\gamma_b}c_h$ and Assumption 3, the dividing line between $p_{u2} = 0$ and $p_{u2} \in (0,1)$ is above the 45^0 line and the one between $p_{u2} \in (0,1)$ and $p_{u2} = 1$ exists and intersects with the 45^0 line (and with $F_2 = 1$).

[27]Unlike the figure, $b^*(w_{s2}) = c_h$ when $p_{u2} \in (0,1)$ may be located above the 45^0 line or it may not intersect with the dividing line between $p_{u2} \in (0,1)$ and $p_{u2} = 1$, although main results are not affected.

After that, the dynamics of p_{u2t} and the wages differ greatly depending on the initial condition. When F_{20} is sufficiently high, $p_{u2t}=1$ is realized at some point and wage dynamics become qualitatively same as the previous economy. The labor market is integrated in the long run in the sense that all individuals work in the primary sector. By contrast, when F_{20} is small, the skilled wage of group 2 falls to the point that $b^*(w_{s2t}) < c_h$ and F_{2t} starts to decrease at some point. The fall of F_{2t}, like the growth of F_{1t}, raises s_{2t}, but it also worsens the group's reputation directly. While the positive effect on s_{2t} is stronger, p_{u2t} rises as before, but eventually the negative effect dominates and p_{u2t} starts to *fall*. In the long run, all of the subordinate group are unskilled and in the secondary sector, thus *the labor market is segregated completely by ethnicity*. Inefficient sectoral choices of the group's unskilled workers make the outcome sensitive to the initial condition and quite different from a "prejudice-free" economy: if their choices are optimal, i.e. $p_{u2t}=0$, F_{2t} is constant as under $s_{2t}=0$.

5.1.3 When there are no institutionalized disadvantages

Finally, consider an economy where the majority (group 1) and the minority (group 2) exist, none of whom face institutionalized disadvantages, i.e. $N_1 > N_2$ and $A_{k1}=A_{k2}\equiv A_k$ $(k=u,s)$. $A_u>1$ is assumed so that $p_{si}=p_{ui}=1$ $(i=1,2)$ holds from Proposition 1. As for the dynamics of F_{it}, suppose that A_k is not very high and thus the same assumptions as the ones for the subordinate group in the first economy (Figure 3) hold. Then, $b^*(w_{ui})=c_h$ and $b^*(w_{si})=c_h$ exist and are given by (18) and (19) with $A_{k1}=A_{k2}=A_k$. This economy may be seen as an approximation to an economy where both institutionalized disadvantages of the subordinate group and advantages of the dominant group are small.

Figure 5 illustrates the dynamics of F_{1t} and F_{2t}. Since $N_1 > N_2$ and thus $s_1 < s_2$, the region with $b^*(w_{u1})>c_h$ $(b^*(w_{s1})<c_h)$ is smaller than the region with $b^*(w_{u2})>c_h$ $(b^*(w_{s2})<c_h)$. Unlike Figures 3 and 4, $\left[1-s\left(\frac{N_1}{N_1+N_2}\right)\right]A_s h_s+s\left(\frac{N_1}{N_1+N_2}\right)A_u h_u<\frac{c_s}{\gamma_b}$ is assumed and thus $b^*(w_{si})=c_s$ intersects with the 45^0 line.[28]

Now, the long-run fate of the groups depends on both F_{10} and F_{20}. When (F_{10},F_{20}) is above $b^*(w_{u1})=c_h$ $(b^*(w_{u2})=c_h)$, F_{1t} (F_{2t}) increases over time and $H_1^*=1 (H_2^*=1)$ in the long run. Particularly, when both F_{10} and F_{20} are high, i.e. when $b^*(w_{ui})>c_h$ at $(F_1,F_2)=(F_{10},F_{20})$ for at least one group i and, for $j\neq i$, $b^*(w_{uj})>c_h$ at $F_i=1$ and $F_j=F_{j0}$, everyone is skilled in the long run. By contrast, when (F_{10},F_{20}) is at the left side of $b^*(w_{s1})=c_h$ (below $b^*(w_{s2})=c_h$), F_{1t} (F_{2t}) decreases over time and $H_1^*=0$ $(H_2^*=0)$. In particular, when (F_{10},F_{20}) satisfies both $b^*(w_{s1})<c_h$ and $b^*(w_{s2})<c_h$, it is possible that *nobody is skilled* in the long run: a bad impression that each group has about the other group affects the skilled wage negatively, thus F_{it} decreases and the impression deteriorates further. (In the area with chained lines, both F_{1t} and F_{2t} are

[28] A minor assumption, $b^*(w_{si})=c_h$ and $b^*(w_{uj})=c_h$ $(i\neq j)$ do not intersect, too is imposed. When F_2 is high (low), $b^*(w_{u2})=c_h$ is located at the left (right) side of $b^*(w_{u1})=c_h$ from Assumption 4.

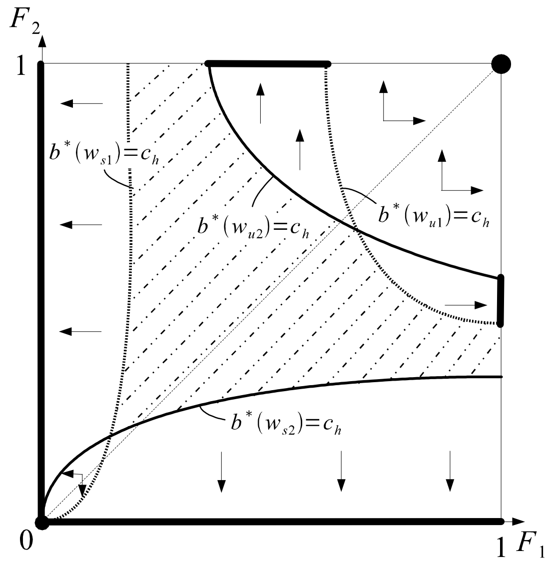

Figure 5: Dynamics when $H_{it} = F_{it}$ always and no institutionalized disadvantages exist

constant.) Long-run outcomes tend to be more sensitive to the initial conditions for the minority since their earnings are affected more by prejudice and reputation.

5.1.4 Summary and discussions

Analyses have shown that the dynamics and long-run outcomes of groups, particularly of the subordinate group, depend greatly on groups' initial conditions and could be quite different from a "prejudice-free" economy. Since good (bad) reputation tends to beget good (bad) reputation, a group starting with a good (bad) initial condition, i.e. a high (low) fraction of them can afford skill investment initially, tend to be in a good (bad) condition in the long run. In the first economy, if the initial condition of the subordinate group is good (bad), all of them are skilled (unskilled) in the long run. In the second economy, if the condition is good (bad), not only all of them are skilled (unskilled) but also are in the primary (secondary) sector, hence the labor market becomes ethnically integrated (segregated) eventually. In the third economy where institutionalized disadvantages do not exist, the dynamics and long-run outcomes of groups, particularly of the minority, tend to be affected greatly by initial conditions. The strong dependence on initial conditions arises because, unlike a "prejudice-free" economy in which the dynamics of an individual lineage are affected only by its initial condition, they are affected by initial conditions of the groups too owing to the dependence of primary-sector wages on group-level variables, reputation and the degree of prejudice.

Empirical findings support the strong dependence of economic outcomes of groups on their initial conditions: for example, Borjas (1994) finds that wages of a U.S. worker in 1940 and 1980 are significantly related to the average wage of immigrants of the worker's ethnic group in 1910 (blacks are not in the data), after individual characteristics are controlled for.

Note that the main implication that long-run outcomes of groups, particularly of the group whose earnings are affected more by prejudice and reputation, depend greatly on groups' initial conditions *remains unchanged* even if $A_{k1} = A_{k2}$ $(k = u, s)$ holds, although the dynamics are affected. That is, the implication holds even when neither group face disadvantages in production, suffer greater disutility of work, or face non-statistical discrimination in the primary sector. $A_{k1} > A_{k2}$ is assumed in the first two economies for analytical simplicity as well as for reality: when $A_{k1} = A_{k2}$, as in Figure 5, critical loci exist for both groups and thus analyses become complicated. The same is true for implications of later analyses.

5.2 When skill investment is not always profitable

Section 5.1 has examined the case in which $H_i = F_i$ always holds. Now consider the case in which $H_i = 0$ holds at least for one group (and equilibria are stable). From Proposition 1 (ii) (see Figure 2 too), this is true when $h_s - (1+r)c_h < h_u$ and, for such group i, $A_{ui} \geq 1$ and $s(0) + \frac{(1+r)c_h}{A_{si}h_s - A_{ui}h_u} > 1$. This is the case in which skill investment is unproductive in the secondary sector, and the investment is not very productive in the primary sector (at least for one group) or the maximum degree of prejudice is high.

Investment decisions of the two groups are interrelated, thus, depending on A_{si} and A_{ui}, equilibrium combinations of H_1 and H_2 are varied and multiple equilibria are possible. To limit possible combinations, the following is assumed.

Assumption 5 *When* $A_{k1} \geq A_{k2}$ $(k = u, s)$, $A_{s1}h_s - A_{u1}h_u \geq A_{s2}h_s - A_{u2}h_u$.

It states that the social return to investment in the primary sector is weakly higher for the dominant group, which would be reasonable because the subordinate group tend to have greater disadvantages in jobs requiring high interpersonal skills usually occupied by skilled workers (e.g. management jobs). The next proposition presents equilibrium (H_1, H_2) based on Proposition 1.

Proposition 2 (Equilibrium $(\mathbf{H}_1, \mathbf{H}_2)$ when the investment is not always profitable) *Assume* $h_s - (1+r)c_h < h_u$, $A_{k1} \geq A_{k2} \geq 1$ $(k = u, s)$, *and* $s(0) + \frac{(1+r)c_h}{A_{s2}h_s - A_{u2}h_u} > 1$.

(i) If $(1 - s(0))[A_{s1}h_s - A_{u1}h_u] \geq (1+r)c_h$, $H_1 = F_1$ *and both* $H_2 = F_2$ *and* $H_2 = 0$ *are equilibria* ($H_2 = 0$ *is the equilibrium) when* $s\left(\frac{F_2N_2}{F_1N_1 + F_2N_2}\right) < (\geq)\ 1 - \frac{(1+r)c_h}{A_{s2}h_s - A_{u2}h_u}$.

(ii) Otherwise, $(H_1, H_2) = (0, F_2), (F_1, 0)$, *and, when* $s\left(\frac{F_iN_i}{F_iN_i + F_jN_j}\right) < 1 - \frac{(1+r)c_h}{A_{si}h_s - A_{ui}h_u}$ *for any* $i, j = 1, 2$ $(j \neq i)$, $(H_1, H_2) = (F_1, F_2)$ *as well.*

Multiple equilibria exist unless $s(0) + \frac{(1+r)c_h}{A_{s1}h_s - A_{u1}h_u} \leq 1$ and $s\left(\frac{F_2N_2}{F_1N_1 + F_2N_2}\right) + \frac{(1+r)c_h}{A_{s2}h_s - A_{u2}h_u} \geq 1$.

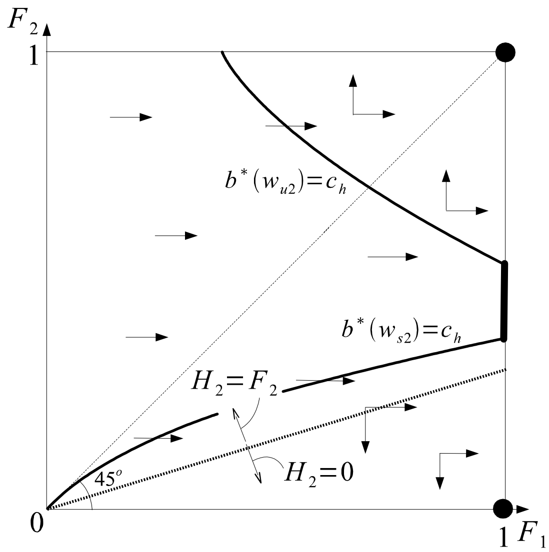

Figure 6: Dynamics when $H_{2t}=F_{2t}$ is selected in the region where both $H_{2t}=0$ and $H_{2t}=F_{2t}$ are equilibria

5.2.1 When investment is always profitable for the dominant group

If the investment is always (weakly) profitable for the dominant group, i.e. (A_{u1},A_{s1}) is in the region on or above $A_{s1}=\frac{h_u}{h_s}A_{u1}+\frac{(1+r)c_h}{(1-s(0))h_s}$ (and $A_{u1}\geq 1$) in Figure 2 (Proposition 2 (i)), $H_1=F_1$ is always true, while both $H_2=F_2$ and $H_2=0$ are equilibria ($H_2=0$ is the equilibrium) when $\frac{F_2N_2}{F_2N_2+F_1N_1}$ is strictly greater (smaller) than the value satisfying

$$s\left(\frac{F_2N_2}{F_1N_1+F_2N_2}\right)=1-\frac{(1+r)c_h}{A_{s2}h_s-A_{u2}h_u}. \tag{23}$$

As explained after Lemma 2, multiple equilibria arise due to strategic complementarity within the subordinate group: as more of them invest in skill and become skilled workers, s_2 decreases and the investment becomes more profitable. As for sectoral choices, since $A_{ui}\geq 1$, $p_{si}=1$ and $p_{ui}=1$ $(i=1,2)$ from Proposition 1.

Suppose that disadvantages of the subordinate group are not so severe (or the relative productivity of the primary sector is not so low) and advantages of the dominant group are not so small that assumptions related to the dynamics of F_{1t} and F_{2t} are same as the first economy in Section 5.1 (thus F_{1t} always increases), except that $(1-s(0))A_{s2}h_s+s(0)A_{u2}h_u<\frac{c_h}{\gamma_b}$ now follows from $A_{u2}h_u<\frac{1-\gamma_b(1+r)}{\gamma_b}c_h$. When multiple equilibria exist, assume that the initial coordination among the subordinate group continues for subsequent periods: for example, if $H_{20}=F_{20}$ happens to hold, then $H_{2t}=F_{2t}$ for any $t>0$. This assumption would be reasonable considering that children tend to mimic parental behaviors in real society.[29]

[29] Relatedly, in a dynamic model of statistical discrimination, Kim and Loury (2009) assume that, when there exist equilibrium paths to both good and bad steady states, an initial consensus on the final state shared by group members picks one path and the consensus is maintained over generations.

Then, if $s\big(\frac{F_{2\,0}N_2}{F_{1\,0}N_1+F_{2\,0}N_2}\big) < 1-\frac{(1+r)c_h}{A_{s2}h_s-A_{u2}h_u}$, i.e. both $H_{2\,0}=F_2$ and $H_{2\,0}=0$ are equilibria, *and* $H_{2\,0}=0$ *happens to* be realized initially, *the subordinate group never make productive investment*, F_{1t} rises and F_{2t} falls (since $H_{2t}=0$ and $A_{u2}h_u < \frac{1-\gamma_b(1+r)}{\gamma_b}c_h$) over time, and $H_1^*=F_1^*=1$ and $H_2^*=F_2^*=0$.

Otherwise (thus $H_{2\,0}=F_{2\,0}$ if $s\big(\frac{F_{2\,0}N_2}{F_{1\,0}N_1+F_{2\,0}N_2}\big)<1-\frac{(1+r)c_h}{A_{s2}h_s-A_{u2}h_u}$), the dynamics are as illustrated in Figure 6. The dividing line between $H_2=F_2$ and $H_2=0$ (eq. 23) is a positively-sloped straight line approaching the origin, and $H_2=0$ holds below the line. The dynamics of F_{2t} when $H_{2t}=F_{2t}$ are qualitatively same as the first economy of Section 5.1 (Figure 3), while when $H_{2t}=0$, F_{2t} decreases over time.

Hence, if $F_{2\,0}$ is not so small that $b^*(w_{s2})\geq c_h$ at $(F_1,F_2)=(1,F_{2\,0})$, *given the initial condition*, the long-run fate of the subordinate group is drastically different depending on which equilibrium *happens to* be realized initially: if $H_{2\,0}=F_{2\,0}$, $H_2^*=1$ (if $b^*(w_{u2})>c_h$ at $(F_1,F_2)=(1,F_{2\,0})$) or $H_2^*=F_{2\,0}$ (otherwise), whereas if $H_{2\,0}=0$, $H_2^*=F_2^*=0$.[30] The initial selection of good (bad) equilibrium brings the better (worse) long-run outcome than under $s_i=0$.

5.2.2 When investment is not always profitable for both groups

If the investment is not always profitable for the dominant group too, i.e. (A_{u1},A_{s1}) is in the region between $A_{s1}=\frac{h_u}{h_s}A_{u1}+\frac{(1+r)c_h}{(1-s(0))h_s}$ and the line for Assumption 1 (ii) in Figure 2 (Proposition 2 (i)), equilibria are $(H_1,H_2)=(0,F_2),(F_1,0)$, and, when $s\big(\frac{F_iN_i}{F_iN_i+F_jN_j}\big)<1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$ for any i,j, $(H_1,H_2)=(F_1,F_2)$ too. ($p_{si}=p_{ui}=1$.) $(H_1,H_2)=(0,F_2),(F_1,0)$ are equilibria since strategic substitutability is at work between the groups: as more individuals of one group invest, prejudice toward the other group intensifies and their return to investment falls. As in the previous economy, assumptions related to the dynamics of F_{it} are same as the first economy in Section 5.1, and the initial coordination is maintained when multiple equilibria exist.

Then, if only the subordinate (dominant) group *happen to* make productive investment initially, i.e. $H_{1\,0}=0$ and $H_{2\,0}=F_{2\,0}$ ($H_{1\,0}=F_{1\,0}$ and $H_{2\,0}=0$), F_{2t} is constant (falls) and F_{1t} rises and $H_1^*=0$ ($H_1^*=1$) and $H_2^*=F_{2\,0}$ ($H_2^*=0$). Since this type of equilibria exist for *any* $F_{1\,0}$ and $F_{2\,0}$, it is possible that the dominant group with a *much better* initial condition than the subordinate group, i.e. $F_{1\,0}>>F_{2\,0}$, end up with the *smaller* fraction of skilled workers, i.e. $H_1^*=0<H_2^*=F_{2\,0}$ ($F_1^*=1>F_2^*=F_{2\,0}$, though). If $s\big(\frac{F_{i0}N_i}{F_{i0}N_i+F_{j0}N_j}\big)<1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$ for any $i,j=1,2$ and $(H_{1\,0},H_{2\,0})=(F_{1\,0},F_{2\,0})$ happens to hold, the dynamics are similar to those illustrated in Figure 6.[31] Unlike the previous economy, the long-run outcome of *the*

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[30]When $A_{u2}<1$ and $s(0)+\frac{(1+r)c_h}{A_{s2}h_s-A_{u2}h_u}>1$ (the case not considered in the proposition or Corollary 1, see Figure 2), the dynamics are illustrated by a figure similar to Figure 6. Differences are that no stable equilibria exist, not $H_2=0$, in the region on or below $s\big(\frac{F_2N_2}{F_1N_1+F_2N_2}\big)=1-\frac{(1+r)c_h}{A_{s2}h_s-A_{u2}h_u}$, and in the region above the line, $p_{u2}<1$ is possible depending on F_1 and F_2 like Figure 4. Hence, if $s\big(\frac{F_{20}N_2}{F_{10}N_1+F_{20}N_2}\big)<1-\frac{(1+r)c_h}{A_{s2}h_s-A_{u2}h_u}$ and $s\big(\frac{F_{20}N_2}{N_1+F_{20}N_2}\big)\geq 1-\frac{(1+r)c_h}{A_{s2}h_s-A_{u2}h_u}$, F_{1t} rises and $H_{it}=F_{it}$ ($i=1,2$) at first, but after the economy crosses the line, the stable equilibrium fails to exist.

[31]Differences are that another positively-sloped straight line, $s\big(\frac{F_1N_1}{F_1N_1+F_2N_2}\big)=1-\frac{(1+r)c_h}{A_{s1}h_s-A_{u1}h_u}$, exists above the 45^0 line, and equilibrium $(H_1,H_2)=(F_1,F_2)$ exists only in the region between this line and $s\big(\frac{F_2N_2}{F_1N_1+F_2N_2}\big)=1-\frac{(1+r)c_h}{A_{s2}h_s-A_{u2}h_u}$ (the

dominant group too is sensitive to the initial selection of equilibrium *given* the initial condition.

5.2.3 Summary and discussions

To summarize, when skill investment is unproductive in the secondary sector, and the investment's productivity in the primary sector is low or the degree of prejudice is high, multiple equilibria could exist regarding skill investment, and *given* the initial distribution of wealth, the *initial selection of equilibrium* could affect the dynamics greatly. When the investment is profitable for the dominant group, it can be the case that, if the subordinate group with enough wealth *happen to* (*not to*) invest initially, F_{2t} increases (decreases) over time and all of the group are skilled (unskilled) in the long run. When the investment is not profitable with high s_i for the dominant group too, given the initial condition, the long-run outcome of *the dominant group too* is sensitive to the initial selection of equilibrium. The dominant group with a *much better* initial condition than the subordinate group could end up with the *smaller* fraction of skilled workers, if the former (latter) group happen not to (to) invest initially. As can been shown easily, a similar result holds for the majority and the minority when there are no institutionalized disadvantages, i.e. $A_{k1} = A_{k2}$ ($k = u, s$).

The results suggest that, in an economy where prejudice is severe ($s(0)$ is high) or the effectiveness of skill investment is low, if the initial selection is affected by institutionalized discrimination against a group that limits their access to investment opportunities, such discrimination could have a *lasting impact* on their well-beings well after its abolishment. Income or wealth redistribution raising F_i does little to change the situation, while affirmative action treating them favorably in skill investment, such as a tuition subsidy, could be very effective. To be successful, their c_h must be lowered so that, for any group, $(1-s(0))[A_{si}h_s - A_{ui}h_u] \geq (1+r)c_h$ holds and thus $H_i = F_i$ becomes the unique equilibrium (Corollary 1). The redistribution becomes effective only after such policy is implemented.

6 General case

So far Assumption 1 (iii), $A_{si} \geq \frac{h_u}{h_s} A_{ui} + \frac{h_s - h_u}{(1-s(0))h_s} \Leftrightarrow s(0) + \frac{h_s - h_u}{A_{si}h_s - A_{ui}h_u} \leq 1$, is imposed. This condition does not hold when $s(0)$ is high or the relative effectiveness of skill investment in the primary sector, $\frac{A_{si}h_s - A_{ui}h_u}{h_s - h_u}$, is low. So this is dropped now. Under the assumption, the net return to choosing the primary sector is weakly higher for skilled workers even when the degree of prejudice is severest ($s_i = s(0)$) and thus $p_{si} \geq p_{ui}$ always holds. Without it, it is possible that $p_{si} = 0$ and $p_{ui} = 1$ hold, i.e. all skilled workers choose the secondary sector and all unskilled workers choose the *primary* sector, even if the *former* have comparative advantages

dotted bold line). The economy is in this region and F_{1t} increases at first. If $s\left(\frac{F_{20}N_2}{N_1+F_{20}N_2}\right) \geq 1 - \frac{(1+r)c_h}{A_{s2}h_s - A_{u2}h_u}$, the economy crosses the dotted bold line eventually, after which the equilibrium switches to $(H_{1t}, H_{2t}) = (0, F_{2t})$ *or* $(F_{1t}, 0)$. If a group with smaller F_{it} switches to $H_{it} = 0$, as in the figure, F_{2t} continues to decline and $H_2^* = F_2^* = 0$ and $H_1^* = 1$ in the long run.

and are more productive in the primary sector. This result may explain the fact that skilled people of subordinate groups often avoid primary-sector jobs and run small businesses instead. Further, multiple equilibria could exist regarding *sectoral choices of skilled workers* as well as skill investment. Hence, the initial selection of equilibrium on the *sectoral choices* too could have lasting impacts on the dynamics.

6.1 Sectoral choices and skill investment

To analyze the model without Assumption 1 (iii), this subsection examines sectoral choices and skill investment when the assumption does *not* hold. Assumption 1 (iii) is replaced by:

Assumption 6 $s(0)+\frac{h_s-h_u}{A_{si}h_s-A_{ui}h_u}>1 \Leftrightarrow A_{si}<\frac{h_u}{h_s}A_{ui}+\frac{h_s-h_u}{(1-s(0))h_s}$.

As in Section 3.1, Assumptions 3 through 5 are not imposed in this subsection.

The following lemma on sectoral choices is parallel to Lemma 1 under the old assumption.

Lemma 3 (Sectoral choices under Assumption 6) *Suppose* $H_i>0$ *and* $p_{sj}H_j>0$, $j\neq i$.

(*i*) *When* $A_{ui}\geq 1$, $p_{ui}=1$. $p_{si}=1$ *if* $s(0)\leq\frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}$; *both* $p_{si}=1$ *and* $p_{si}=0$ *if* $s(0)>\frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}$ *and* $s\left(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\right)<\frac{(A_{si}-1)h_s}{(1-H_i)(A_{si}h_s-A_{ui}h_u)}$; *otherwise,* $p_{si}=0$.

(*ii*) *When* $A_{ui}<1$,

(*a*) *If* $(1-s_i)A_{si}h_s-h_s>(1-s_i)A_{ui}h_u-h_u$ *with* $p_{si}=1\Leftrightarrow s_i=s\left(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\right)<1-\frac{h_s-h_u}{A_{si}h_s-A_{ui}h_u}$, $p_{si}=1$ *and* p_{ui} *is determined as in Lemma 1* (*ii*).

(*b*) *Otherwise,* $p_{si}=p_{ui}=1$ *if* $s\left(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\right)<\frac{(A_{si}-1)h_s}{(1-H_i)(A_{si}h_s-A_{ui}h_u)}$, *or else, no stable equilibrium exists.*

When $A_{ui}\geq 1$, unskilled workers always choose the primary sector as before, while choices of skilled workers now depend on the net return to the primary sector: if it is positive with $p_{si}=1$, i.e. $s\left(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\right)<\frac{(A_{si}-1)h_s}{(1-H_i)(A_{si}h_s-A_{ui}h_u)}$, $p_{si}=1$ as before, whereas if it is negative with $p_{si}=0$, i.e. $s(0)>\frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}$, $p_{si}=0$.[32] That is, when $s(0)$ is sufficiently high, all skilled workers choose the secondary sector and all unskilled workers choose the *primary* sector, even if the *former* have comparative advantages and are more productive in the primary sector. Since $s\left(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\right)<s(0)$, *both* $p_{si}=1$ *and* $p_{si}=0$ *are equilibria* for some combinations of A_{si} and A_{ui} due to strategic complementarity among skilled workers (their net return increases with p_{si}). When $A_{ui}<1$ and the net return to the primary sector at $p_{si}=1$ is weakly lower for skilled workers, i.e. $s\left(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\right)\geq 1-\frac{h_s-h_u}{A_{si}h_s-A_{ui}h_u}$, if the return for the skilled (at $p_{si}=p_{ui}=1$) is positive, $p_{si}=p_{ui}=1$ holds, otherwise (thus $p_{si}<1$), *no stable equilibrium exists*: $p_{si}=0$ cannot be an equilibrium from $A_{si}>1>A_{ui}$, while an equilibrium with $p_{si}\in(0,1)$ is not stable due to strategic complementarity. (Choices are same as the corresponding case of Lemma 1 when $A_{ui}<1$ and the net return at $p_{si}=1$ is higher for the skilled.)

The next lemma corresponding to Lemma 2 under the old assumption presents equilibrium values of H_i.

[32] An equilibrium with $p_{si}\in(0,1)$ is not stable because the net return for the skilled increases with p_{si}.

Lemma 4 (Skill investment under Assumption 6) *Suppose* $F_i > 0$.

(*i*) *When* $h_s - (1+r)c_h \geq h_u$, $H_i = F_i$ *if* $p_{sj}H_j = 0$ *for* $j \neq i$. *If* $p_{sj}H_j > 0$,

(*a*) *When* $A_{ui} \geq 1$,

1. *If* $h_s - (1+r)c_h \geq A_{ui}h_u$, $H_i = F_i$.
2. *If* $h_s - (1+r)c_h < A_{ui}h_u$, $H_i = F_i$ *when* $s(0) \leq 1 - \frac{(1+r)c_h}{A_{si}h_s - A_{ui}h_u}$; *both* $H_i = F_i$ *and* $H_i = 0$ *are equilibria when* $s(0) > 1 - \frac{(1+r)c_h}{A_{si}h_s - A_{ui}h_u} > s\left(\frac{F_iN_i}{F_iN_i + p_{sj}H_jN_j}\right)$; *or else,* $H_i = 0$.

(*b*) *When* $A_{ui} < 1$, *if* $s\left(\frac{F_iN_i}{F_iN_i + p_{sj}H_jN_j}\right) < \min\left\{\max\left[\frac{(A_{si}-1)h_s}{(1-F_i)(A_{si}h_s - A_{ui}h_u)}, 1 - \frac{h_s - h_u}{A_{si}h_s - A_{ui}h_u}\right], 1 - \frac{(1+r)c_h}{A_{si}h_s - A_{ui}h_u}\right\}$, $H_i = F_i$, *otherwise, no stable equilibrium exists.*

(*ii*) *When* $h_s - (1+r)c_h < h_u$, *Lemma 2 (ii) applies (except case* $s(0) \leq 1 - \frac{(1+r)c_h}{A_{si}h_s - A_{ui}h_u}$ *does not exist).*

When $h_s - (1+r)c_h \geq h_u$, $H_i = F_i$ always under the old assumption, while *inefficient* $H_i = 0$ *can be an equilibrium and stable equilibria may not exist* under the new assumption. When $A_{ui} \geq 1$, $p_{si} = 0$ or 1 and $p_{ui} = 1$ from Lemma 3 (i). Hence, if the net return to investment is non-negative even under $p_{si} = 0$ and $p_{ui} = 1$ (the return is lower than under $p_{si} = p_{ui} = 1$), i.e. $h_s - (1+r)c_h \geq A_{ui}h_u$, $H_i = F_i$ holds; otherwise, when the net return with $H_i = 0$ is negative even under $p_{si} = p_{ui} = 1$, $H_i = 0$ holds, while when the return with $H_i = F_i$ is positive under $p_{si} = p_{ui} = 1$, $H_i = F_i$ holds (and both $H_i = 0$ and $H_i = F_i$ are equilibria when both conditions hold due to strategic complementarity). When $A_{ui} < 1$, no stable equilibria exist if stable p_{si} and p_{ui} do not exist or if stable H_i does not exist (otherwise, $H_i = F_i$). Stable H_i fails to exist when $s\left(\frac{F_iN_i}{F_iN_i + p_{sj}H_jN_j}\right) \geq 1 - \frac{(1+r)c_h}{A_{si}h_s - A_{ui}h_u}$ since the net return under $H_i = 0$ is *higher* than under $H_i > 0$ after the dependence of p_{ui} on H_i is taken into account: given $H_i = 0$, $p_{ui} = 0$ from $A_{ui} < 1$ and thus $H_i = 0$ is not an equilibrium from $A_{si}h_s - (1+r)c_h > h_u$ (Assumption 1 (ii)), whereas, given $H_i \in (0, F_i]$, the net return is non-positive from $s\left(\frac{F_iN_i}{F_iN_i + p_{sj}H_jN_j}\right) \geq 1 - \frac{(1+r)c_h}{A_{si}h_s - A_{ui}h_u}$ and thus $H_i \in (0, F_i]$ is not a stable equilibrium.[33]

Finally, investment and sectoral choices of group i for given choices by the other group under Assumption 6 are summarized as follows.

Proposition 3 (Investment and sectoral choices of group i under Assumption 6)

(*i*) *When* $h_s - (1+r)c_h \geq h_u$, *if* $p_{sj}H_j = 0$ *for* $j \neq i$, $H_i = F_i$, $p_{si} = 1$, *and Proposition 1 (i)(b) applies for* p_{ui}. *If* $p_{sj}H_j > 0$,

(*a*) *When* $A_{ui} \geq 1$, $p_{ui} = 1$.

1. *When* $h_s - (1+r)c_h \geq A_{ui}h_u$, $H_i = F_i$. $p_{si} = 1$ *if* $s(0) \leq \frac{(A_{si}-1)h_s}{A_{si}h_s - A_{ui}h_u}$; *otherwise, both* $p_{si} = 1$ *and* $p_{si} = 0$ *are equilibria if* $s\left(\frac{F_iN_i}{F_iN_i + p_{sj}H_jN_j}\right) < \frac{(A_{si}-1)h_s}{(1-F_i)(A_{si}h_s - A_{ui}h_u)}$, *or else* $p_{si} = 0$.
2. *When* $h_s - (1+r)c_h < A_{ui}h_u$, $p_{si} = 1$. *If* $s(0) \leq 1 - \frac{(1+r)c_h}{A_{si}h_s - A_{ui}h_u}$, $H_i = F_i$; *otherwise, both* $H_i = 0$ *and* $H_i = F_i$ *are equilibria if* $s\left(\frac{F_iN_i}{F_iN_i + p_{sj}H_jN_j}\right) < 1 - \frac{(1+r)c_h}{A_{si}h_s - A_{ui}h_u}$, *or else* $H_i = 0$.

[33] Note that $p_{ui} > 0$ must hold from $s\left(\frac{F_iN_i}{F_iN_i + p_{sj}H_jN_j}\right) \geq 1 - \frac{(1+r)c_h}{A_{si}h_s - A_{ui}h_u} > \frac{(1-A_{ui})h_u}{A_{si}h_s - A_{ui}h_u}$, Lemma 3 (ii), and Lemma 1 (ii), and $p_{si} = 1$ must hold from Lemma 3 (ii).

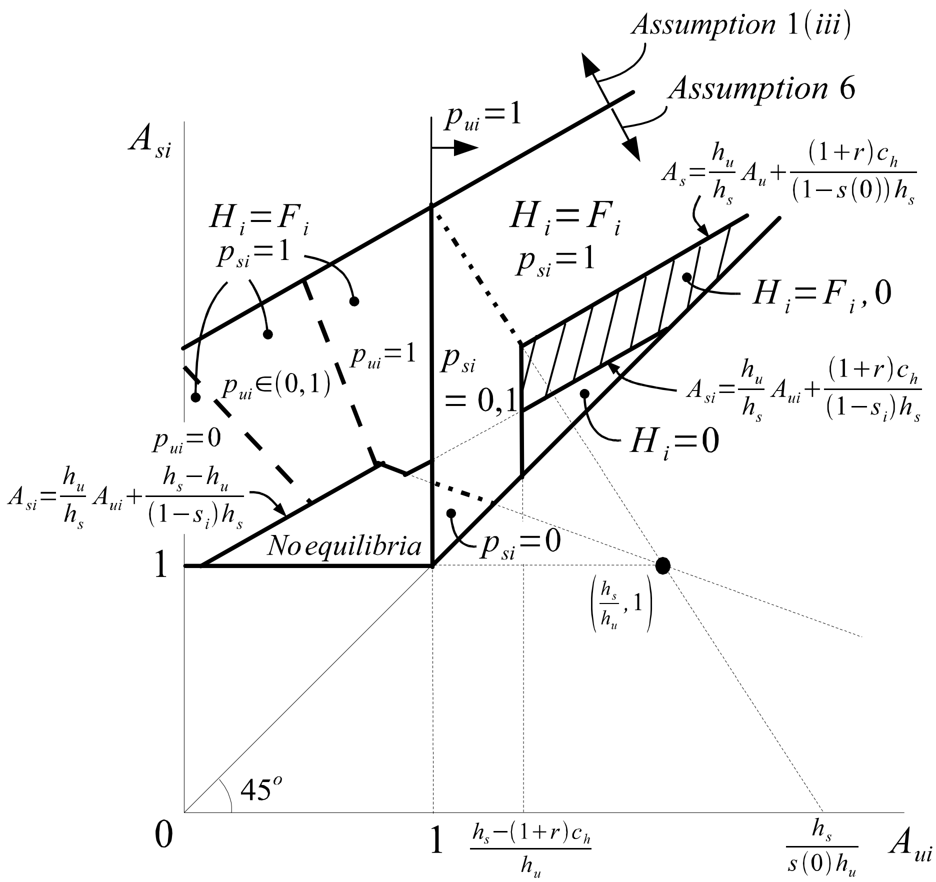

Figure 7: Investment and sectoral choices under Assumption 6 when $h_s-(1+r)c_h \geq h_u$ and $p_{sj}H_j>0$ $(j \neq i)$

(*b*) *When* $A_{ui}<1$,

1. *If* $s\left(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\right)<1-\frac{h_s-h_u}{A_{si}h_s-A_{ui}h_u}$, $H_i=F_i$, $p_{si}=1$, *and Proposition 1 (i)(a) applies for* p_{ui}.
2. *Otherwise, if* $s\left(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\right)<\min\left\{\frac{(A_{si}-1)h_s}{(1-F_i)(A_{si}h_s-A_{ui}h_u)},1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}\right\}$, $H_i=F_i$ *and* $p_{si}=p_{ui}=1$; *or else, no stable equilibrium exists.*

(*ii*) *When* $h_s-(1+r)c_h<h_u$, *Proposition 1 (ii) applies (no case* $s(0)\leq 1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$*).*

Figure 7 shows the choices when $h_s-(1+r)c_h \geq h_u$ and $p_{sj}H_j>0$ $(j \neq i)$. Although the figure looks complicated, the choices when $A_{ui}>\frac{h_s-(1+r)c_h}{h_u}$ are same as when $h_s-(1+r)c_h<h_u$ and $A_{ui}\geq 1$ under Assumption 1 (iii) illustrated in Figure 2, and the ones when $A_{ui}<1$ are similar to the corresponding case of Figure 2 (conditions determining the region of no stable equilibria are different, though). What is really new is when $A_{ui}\in\left[1,\frac{h_s-(1+r)c_h}{h_u}\right]$, in which $H_i=F_i$, $p_{ui}=1$, and, depending on A_{si} and A_{ui}, $p_{si}=0$, both $p_{si}=0$ and $p_{si}=1$, or $p_{si}=1$ (choices of skilled workers are inefficient when $p_{si}=0$). The choices when $h_s-(1+r)c_h<h_u$ are almost same as the corresponding case under Assumption 1 (iii) of Figure 2.[34]

[34]The only difference is that, when $A_{ui}\geq 1$, the region in which $H_i=F_i$ is the unique equilibrium does not exist. Note that $H_i=F_i$ is possible under Assumption 6 too: unlike Figure 2, if $h_s-(1-s(0))(1+r)c_s<h_u$ holds, the line dividing the regions satisfying Assumption 1 (iii) and Assumption 6 is located above the line for Assumption 1 (ii) in Figure 2.

6.2 Analyses

The dynamics are examined without imposing Assumption 1 (iii). Qualitatively new dynamics arise when $A_{ui} \in \left[1, \frac{h_s-(1+r)c_h}{h_u}\right]$ (thus $h_s-(1+r)c_h \geq h_u$) and $s(0) > \frac{(A_{si}-1)h_s}{A_{si}h_s - A_{ui}h_u}$ (thus Assumption 6) hold for at least one group i (the region below the upper dashed double-dotted line in Figure 7), and either $A_{uj} \geq 1$ or Assumption 1 (iii) holds for the other group j.[35] This is the case in which skill investment is socially productive, $s(0)$ is high, and A_{ui} is at an intermediate level for at least one group i.

Consider an economy in which the former set of conditions hold for the subordinate group (group 2), and either $A_{u1} > \frac{h_s-(1+r)c_h}{h_u}$ and $A_{s1} \geq \frac{h_u}{h_s}A_{u1} + \frac{(1+r)c_h}{(1-s(0))h_s}$ or $A_{u1} \geq 1$ and Assumption 1 (iii) hold for the dominant group (group 1). $H_i = F_i$ and $p_{ui} = 1$ for $i = 1, 2$, and $p_{s1} = 1$, while p_{s2} is 0 or 1 from Figures 7 and 1. The dividing line between the region $p_{s2} = 0, 1$ and the region $p_{s2} = 0$ is, from Proposition 3 (i)(a)1.:

$$s\left(\frac{F_2N_2}{F_1N_1+F_2N_2}\right)(1-F_2) = \frac{(A_{s2}-1)h_s}{A_{s2}h_s - A_{u2}h_u}. \tag{24}$$

Suppose that disadvantages of the subordinate group are moderate enough (or the relative productivity of the primary sector is high enough) and advantages of the dominant group are large enough that assumptions on the dynamics are same as the first economy in Section 5.1, implying that F_{1t} increases and, when $p_{s2} = 1$, $b^*(w_{s2}) = c_h$ (eq. 19) and $b^*(w_{u2}) = c_h$ (eq. 18) exist. Further, assume $h_s < \frac{c_h}{\gamma_b}$ and thus F_{2t} falls when $p_{s2t} = 0$. When multiple equilibria exist, initial coordination continues for subsequent periods as in Section 5.2.

Then, if $s\left(\frac{F_{2\,0}N_2}{F_{1\,0}N_1+F_{2\,0}N_2}\right)(1-F_{2\,0}) < \frac{(A_{s2}-1)h_s}{A_{s2}h_s - A_{u2}h_u}$ holds, i.e. both $p_{s2\,0} = 0$ and $p_{s2\,0} = 1$ are equilibria, *and* $p_{s2\,0} = 0$ *happens to* be realized, F_{1t} rises and F_{2t} falls over time and $H_1^* = 1$ and $H_2^* = 0$. Although skilled workers of the subordinate group are more productive in the *primary* sector, they choose the secondary sector to avoid the negative effect from their fellow unskilled workers. The sector's wage, however, is not high enough for their descendants to remain skilled and the group are totally unskilled in the long run.

Instead, if $p_{s2\,0} = 1$ *happens to* be realized under the *same* situation, the dynamics of F_{1t} and F_{2t} are as illustrated in Figure 8.[36]The skilled workers efficiently choose the primary sector and earn the higher wage than the previous case. In particular, if $F_{2\,0}$ is high enough that $b^*(w_{u2}) > c_h$ at $(F_1, F_2) = (1, F_{2\,0})$, F_{2t} starts to rise at some point and $H_1^* = H_2^* = 1$ in the long run. The group's unskilled workers benefit from the presence of the skilled workers in the primary sector, enabling the upward mobility of their descendants.

Given the initial condition, the long-run fate of the subordinate group differs greatly depending on the

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[35]The dynamics are similar to Section 5 in other cases. When $h_s-(1+r)c_h < h_u$, analyses of the corresponding case in Section 5 go through. When $h_s-(1+r)c_h \geq h_u$, if either $A_{ui} \geq 1$ and $s(0) \leq \frac{(A_{si}-1)h_s}{A_{si}h_s - A_{ui}h_u}$ (the region on or above the upper dashed double-dotted line in Figure 7) or $A_{ui} > \frac{h_s-(1+r)c_h}{h_u}$ is true for any i, analyses in Section 5 apply, while when Assumption 6 and $A_{ui} < 1$ hold for some i and thus stable equilibria fail to exist depending on F_i, analyses in footnote 30 of Section 5.2 apply.

[36]From (24), the dividing line (the dashed line) is positively sloped and approaches the origin (note $s(0) > \frac{(A_{s2}-1)h_s}{A_{s2}h_s - A_{u2}h_u}$). $(F_{1\,0}, F_{2\,0})$ is above the line since $p_{s2} = 1$ is possible only in the region above it. Shapes of $b^*(w_{s2}) = c_h$ and $b^*(w_{u2}) = c_h$ are as explained in Section 5.

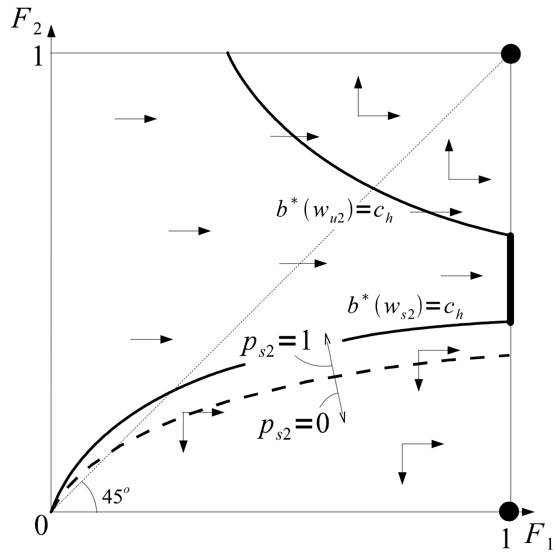

Figure 8: Dynamics when $p_{s2t}=1$ is selected in the region where both $p_{s2t}=0$ and $p_{s2t}=1$ are equilibria

initial selection of equilibrium on *sectoral choices* by the group's skilled workers. The same is true for *the dominant group too* when $A_{ui} \in \left[1, \frac{h_s-(1+r)c_h}{h_u}\right]$ and $s(0) > \frac{(A_{si}-1)h_s}{A_{si}h_s - A_{ui}h_u}$ hold for both groups. The result suggests that initial institutionalized discrimination against a group limiting their access to skilled jobs in the primary sector could have a lasting negative impact on their well-beings well after its abolishment. This is consistent with the finding by Darity, Dietrich, and Guilkey (2001) that the occupational status of a U.S. worker in 1980 and 1990 is significantly related to human capital endowments *and* the degree of favorable or unfavorable treatment in the labor market in the period between 1880 and 1910 of his/her group. Affirmative action treating them favorably in the sector, such as a wage subsidy to the sector's (particularly skilled) jobs that makes $p_{si}=1$ the unique equilibrium, could be very effective to change the situation.[37]

7 Conclusions

Disparities in economic conditions among different ethnic, racial, or religious groups continue to be serious concerns in most economies. Relative standings of different groups are rather persistent, although some groups initially in disadvantaged positions successfully caught up with then-advantaged groups. Two obstacles, costly skill investment and negative stereotypes or discrimination in the labor market, seem to distort investment and sectoral decisions and slow down the progress of the disadvantaged.

This paper has developed a dynamic model of statistical discrimination in which these obstacles affect skill

[37]A wage subsidy to all primary-sector jobs could lower incentives for skill investment and result in $H_i=0$, if the subsidy is too generous. A subsidy targeting skilled workers can be implemented, if workers' skill levels are revealed eventually (see footnote 18 for an interpretation of the wage function consistent with this).

investment and sectoral choices of individuals of two groups and examined how initial economic standings of the groups and initial institutionalized discrimination affect subsequent dynamics and long-run outcomes. The model economy has (up to) two sectors, the primary sector that is ethnically mixed and group reputation affects wages due to statistical discrimination, and the secondary sector with the contrasting features.

Main results are summarized as follows. First, sectoral choices and skill investment may not be socially optimal since choices of different individuals within and across groups could be interrelated. Second, multiple equilibria could exist regarding investment and sectoral choices of skilled workers: both the non-poor of a group invest (skilled workers choose the primary sector) and do not could be equilibria. Third, the dynamics and long-run outcomes of groups, particularly of the subordinate group, depend greatly on initial conditions and could be quite different from a "prejudice-free" economy. Since good (bad) reputation tends to beget good (bad) reputation, a group starting with a good (bad) initial condition tend to be in a good (bad) position in the long run. Fourth, when multiple equilibria exist, which is the case when the effect of stereotypes is strong or the efficacy of investment is low, given initial conditions, the initial selection of equilibrium could affect the dynamics and long-run outcomes greatly. If the initial selection is determined by institutionalized discrimination limiting a group's access to investment or skilled jobs in the primary sector, the discrimination could have a lasting impact on their welfare well after its abolishment. Income or wealth redistribution does little to change the situation, while affirmative action could have a large impact.

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Appendix Proofs of lemmas and propositions

Proof of Lemma 1. (i) If $p_{si}>0$, $p_{si}=p_{ui}=1$ is the only stable equilibrium because

$$(1-s_i)A_{si}h_s+s_iE[A_{ki}h_k]-h_s \geq (1-s(0))A_{si}h_s+s(0)E[A_{ki}h_k]-h_s \tag{25}$$

$$\geq (1-s(0))A_{ui}h_u+s(0)E[A_{ki}h_k]-h_u \tag{26}$$

$$\geq (1-s_i)A_{ui}h_u+s_iE[A_{ki}h_k]-h_u \tag{27}$$

$$> [(1-s_i)A_{ui}-1]h_u+s_iA_{ui}h_u=(A_{ui}-1)h_u \geq 0, \tag{28}$$

where the second inequality is from Assumption 1(iii) and the fourth inequality is from $p_{si}>0$. If $p_{si}=0$, $p_{ui}=0$ must hold from $(1-s_i)A_{si}h_s+s_iE[A_{ki}h_k]-h_s \geq (1-s_i)A_{ui}h_u+s_iE[A_{ki}h_k]-h_u$. However, $p_{si}=p_{ui}=0$ is not an equilibrium from $A_{si}>1$ (skilled workers deviate).

(ii) As shown in (i), $(1-s_i)A_{si}h_s+s_iE[A_{ki}h_k]-h_s \geq (1-s_i)A_{ui}h_u+s_iE[A_{ki}h_k]-h_u$. Thus, if $p_{si}=0$, $p_{ui}=0$ must hold, which is not an equilibrium, as shown in (i). If $p_{si}\in(0,1)$, (27) holds with $>$ and $p_{ui}=0$ must hold, which is not an equilibrium from $A_{si}>1$. Thus, if an equilibrium exists, $p_{si}=1$ and the net return to the primary sector for the unskilled is $(1-s_i)A_{ui}h_u+s_iE[A_{ki}h_k]-h_u=[(1-s_i)A_{ui}-1]h_u+s_i\frac{H_iA_{si}h_s+p_{ui}(1-H_i)A_{ui}h_u}{H_i+p_{ui}(1-H_i)}$, which decreases with p_{ui}. Hence, $p_{ui}=0$ if the return is non-positive with $p_{ui}=0$, i.e. $s_i\leq\frac{(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}$; $p_{ui}=1$ if it is non-negative with $p_{ui}=1$, i.e. $[(1-s_i)A_{ui}-1]h_u+s_i[H_iA_{si}h_s+(1-H_i)A_{ui}h_u]\geq 0\Leftrightarrow s_i\geq\frac{1}{H_i}\frac{(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}$; otherwise, $p_{ui}\in(0,1)$ and p_{ui} is determined from the zero return condition. Such p_{ui} and $p_{si}=1$ is an equilibrium when $p_{ui}=0$ from $A_{si}>1$ and when $p_{ui}>0$ from $(1-s_i)A_{si}h_s+s_iE[A_{ki}h_k]-h_s>(1-s_i)A_{ui}h_u+s_iE[A_{ki}h_k]-h_u$. If $p_{ui}=0$ $(=1)$ and the return for the unskilled is negative (positive), the equilibrium is stable since the one for the skilled is positive. Otherwise, it is stable since the return for the skilled is positive and the one for the unskilled decreases with p_{ui}. ■

Proof of Lemma 2. $H_i=F_i$ when $p_{sj}H_j=0$ for $j\neq i$ is obvious from Assumption 1 (ii).

(Existence/nonexistence of $H_i>0$ when $p_{sj}H_j>0$) (i) Given $H_i>0$, $p_{si}=1$ from Lemma 1. Thus, when $p_{ui}>0$, $(1-s_i)[A_{si}h_s-A_{ui}h_u]-(1+r)c_h>(1-s(0))[A_{si}h_s-A_{ui}h_u]-(1+r)c_h\geq h_s-h_u-(1+r)c_h\geq 0$ from Assumption 1 (iii) (thus $p_{hi}=1$). When $p_{ui}=0$, since $A_{si}>1$, $(1-s_i)A_{si}h_s+s_iE[A_{ki}h_k]-(1+r)c_h=A_{si}h_s-(1+r)c_h> h_s-(1+r)c_h\geq h_u$. Hence, $H_i=p_{hi}F_i=F_i$ is the only equilibrium with $H_i>0$, which is clearly stable.

(ii) Given $H_i>0$, $p_{si}=1$ from Lemma 1 and $s_i=s\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big)$. When $p_{ui}=0$, i.e. $s\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big)\leq\frac{(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}$ from Lemma 1 (ii) (occurs only in (b)), $A_{si}h_s-(1+r)c_h\geq h_u$ (from Assumption 1 (ii)) and thus $p_{hi}=1$ from Assumption 2. When $p_{ui}>0$, i.e. $s\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big)>\frac{(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}$, the net return is $\big[1-s\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big)\big](A_{si}h_s-A_{ui}h_u)-(1+r)c_h$ and $p_{hi}=1(=0)$ if $s\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big)\leq(>)\frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$ from Assumption 2. Note $\frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}\geq\frac{(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}$ from $A_{si}h_s-(1+r)c_h\geq h_u$. Hence, if $s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)> 1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$, the net return is negative for any $H_i\in(0,F_i]$ and no equilibrium with $H_i>0$ exists, while if $s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)<1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$, $H_i=F_i$ is the only stable equilibrium with $H_i>0$ since the return increases with H_i (if $s(0)>1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$, an equilibrium with $H_i\in(0,F_i)$ exists but is not stable). Similarly, if $s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)=1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$, $H_i=F_i$ is the only equilibrium with $H_i>0$ but is not stable.

(Existence/nonexistence of $H_i=0$ when $p_{sj}H_j>0$) Given $H_i=0$, if $A_{ui}<1$, the net return is $A_{si}h_s-(1+r)c_h-h_u\geq 0$ and $H_i=0$ is not an equilibrium (when the net return is 0, $p_{hi}=1$ from Assumption 2). Given $H_i=0$, if $A_{ui}\geq 1$, the net return is $max[(1-s_i)A_{si}h_s+s_iA_{ui}h_u,h_s]-(1+r)c_h-A_{ui}h_u=max\{(1-s(0))[A_{si}h_s-A_{ui}h_u],h_s-A_{ui}h_u\}-(1+r)c_h=(1-s(0))[A_{si}h_s-A_{ui}h_u]-(1+r)c_h\geq h_s-h_u-(1+r)c_h$ from Assumption 1 (iii) and $A_{ui}\geq 1$. Thus, in (i), $H_i=0$ is not an equilibrium when $A_{ui}\geq 1$ too (note Assumption 2). In (ii), the net return is negative (non-negative) when $s(0)>(\leq)1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$. Hence, $H_i=0$ is (is not) an equilibrium if $s(0)>(\leq)1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$ in (ii)(a). Equilibrium $H_i=0$ is stable in (ii)(a) since $s(0)>1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$ implies a negative net return with small $H_i > 0$ (since $A_{ui} \geq 1$, given $H_i > 0$, $p_{si} = p_{ui} = 1$ from Lemma 1 (i)). ■

Proof of Proposition 1. (i) $H_i = F_i$ from Lemma 2 (i), thus Lemma 1 applies with $H_i = F_i$. (ii)(a)/(b) The value of H_i is from Lemma 2 (ii)(a)/(b). ■

Proof of Proposition 2. When $(1-s(0))[A_{s2}h_s - A_{u2}h_u] \geq (1+r)c_h$, the same condition holds for group 1 from Assumption 5, which is the case covered in Section 5.1. When $A_{u2} < 1$ and $(1-s(0))[A_{s2}h_s - A_{u2}h_u] < (1+r)c_h$, given $p_{s1}H_1 > 0$, stable H_2 does not exist for some F_2 from Proposition 1 (ii)(b). Given $p_{s1}H_1 = H_1 = 0$, $H_2 = F_2$ from the proposition, but $(H_1, H_2) = (0, F_2)$ is an equilibrium for any F_1 and F_2 only if $A_{u1} \geq 1$, $(1-s(0))[A_{s1}h_s - A_{u1}h_u] < (1+r)c$ (see Figure 2), *and* $H_1 = 0$ *happens to* hold ($H_1 = F_1$ too could hold depending on F_1 and F_2) from Assumption 5 and Proposition 1 (ii)(a). Equilibria that are stable for any F_1 and F_2 may not exist and thus this case is not considered in the proposition (briefly discussed in footnote 30).

$p_{si} = 1$ when $H_i > 0$ ($i = 1, 2$) from Proposition 1 (ii). Then, from Proposition 1 (ii)(a), given $p_{s1}H_1 = H_1 = 0$, $H_2 = F_2$, and given $H_1 > 0$, both $H_2 = F_2$ and $H_2 = 0$ ($H_2 = 0$) when $s(\frac{F_2N_2}{F_2N_2+H_1N_1}) < (\geq)\ 1 - \frac{(1+r)c_h}{A_{s2}h_s - A_{u2}h_u}$. (i) In this case, from Proposition 1 (ii)(a), $H_1 = F_1$ always. Hence, $H_1 = F_1$ and both $H_2 = F_2$ and $H_2 = 0$ are equilibria ($H_2 = 0$ is the equilibrium) when $s(\frac{F_2N_2}{F_2N_2+F_1N_1}) < (\geq)\ 1 - \frac{(1+r)c_h}{A_{s2}h_s - A_{u2}h_u}$. (ii) Given H_2, the value of H_1 is determined in the same way as H_2. Hence, $(H_1, H_2) = (0, F_2), (F_1, 0)$, and, when $s(\frac{F_iN_i}{F_iN_i+F_jN_j}) < 1 - \frac{(1+r)c_h}{A_{si}h_s - A_{ui}h_u}$ for any $i, j = 1, 2$ ($j \neq i$), $(H_1, H_2) = (F_1, F_2)$ as well. ■

Proof of Lemma 3. An equilibrium with $p_{si} \in (0,1)$ and $p_{ui} = 1$, if exists, is not stable, because the return to the primary sector for the skilled becomes positive whenever p_{si} increases. An equilibrium with $p_{si} \in (0,1)$ and $p_{ui} = 0$ does not exist from $A_{si} > 1$. An equilibrium with $p_{si}, p_{ui} \in (0,1)$, which satisfies $(1-s_i)A_{si}h_s + s_iE[A_{ki}h_k] - h_s = (1-s_i)A_{ui}h_u + s_iE[A_{ki}h_k] - h_u = 0$, is not stable because, whenever p_{si} increases and p_{ui} non-increases, the return for the skilled becomes positive and p_{si} does not have a tendency to return to the original value: the effect of p_{si} on the return for the skilled is greater than for the unskilled from $A_{si}h_s > A_{ui}h_u$ and the effect of p_{ui} on the return is same for both types of workers.

Thus, if a stable equilibrium exists, $p_{si} = 0$ or 1. As shown in the proof of Lemma 1 (i), $p_{si} = p_{ui} = 0$ is not an equilibrium. $p_{si} = 0$ and $p_{ui} \in (0,1)$ is not an equilibrium when $A_{ui} \neq 1$, while it is not stable when $A_{ui} = 1$ (since the return for the unskilled becomes positive whenever p_{si} increases). $p_{si} = 0$ and $p_{ui} = 1$ is not an equilibrium when $A_{ui} < 1$. When $A_{ui} \geq 1$, it is a stable equilibrium if the return for the skilled is negative, i.e. $(1-s(0))A_{si}h_s + s(0)A_{ui}h_u - h_s < 0 \Leftrightarrow s(0) > \frac{(A_{si}-1)h_s}{A_{si}h_s - A_{ui}h_u}$, because the return for the unskilled is positive (when $A_{ui} > 1$) or it increases whenever p_{si} increases (when $A_{ui} = 1$). (When $s(0) = \frac{(A_{si}-1)h_s}{A_{si}h_s - A_{ui}h_u}$, it is not stable since the returns for the skilled increases with p_{si}.)

As for possible equilibria with $p_{si} = 1$, if $(1-s_i)A_{si}h_s - h_s > (1-s_i)A_{ui}h_u - h_u$ with $p_{si} = 1 \Leftrightarrow s(\frac{H_iN_i}{H_iN_i + p_{sj}H_jN_j}) < \frac{(A_{si}-1)h_s + (1-A_{ui})h_u}{A_{si}h_s - A_{ui}h_u}$, the proof of Lemma 1 (i) and (ii) can be applied with a slight modification, thus the result of the lemma holds. If $(1–s_i)A_{si}h_s–h_s \le (1–s_i)A_{ui}h_u–h_u$ with $p_{si}=1$, $p_{si}\le p_{ui}$ and thus $p_{ui}=1$ must hold. $p_{si}=p_{ui}=1$ is a stable equilibrium when $(1–s_i)A_{si}h_s+s_iE[A_{ki}h_k]–h_s=(1–s_i)A_{si}h_s+s_i[H_iA_{si}h_s+(1–H_i)A_{ui}h_u]–h_s>0$ with $p_{si}=1 \Leftrightarrow s\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big)<\frac{(A_{si}-1)h_s}{(1–H_i)(A_{si}h_s-A_{ui}h_u)}$, since the returns for both types are positive. When $s\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big)=\frac{(A_{si}-1)h_s}{(1–H_i)(A_{si}h_s-A_{ui}h_u)}$, it is not stable because the return for the skilled falls with a decrease in p_{si}. (When $(1-s_i)A_{si}h_s-h_s=(1-s_i)A_{ui}h_u-h_u$ with $p_{si}=1$, the additional reason is that the effect of p_{si} on the return for the skilled is greater than for the unskilled and the effect of p_{ui} on the returns are same.)

To summarize, when $A_{ui}\ge 1$, $p_{ui}=1$, and since $\frac{(A_{si}-1)h_s}{(1–H_i)(A_{si}h_s-A_{ui}h_u)}\ge\frac{(A_{si}-1)h_s+(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}$, $p_{si}=1$ if $s\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big)<\frac{(A_{si}-1)h_s}{(1–H_i)(A_{si}h_s-A_{ui}h_u)}$ and $p_{si}=0$ if $s(0)>\frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}$. Hence, because $s(0)>\frac{(A_{si}-1)h_s+(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}$ (from Assumption 6), $\frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}\le\frac{(A_{si}-1)h_s}{(1–H_i)(A_{si}h_s-A_{ui}h_u)}$, and $s(0)>s\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big)$ hold, the stable equilibrium(a) is $p_{si}=p_{ui}=1$ when $s(0)\le\frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}$; both $p_{si}=p_{ui}=1$ and $p_{si}=0$, $p_{ui}=1$ when $s(0)>\frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}$ and $s\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big)<\frac{(A_{si}-1)h_s}{(1–H_i)(A_{si}h_s-A_{ui}h_u)}$; and $p_{si}=0$ and $p_{ui}=1$ otherwise.

When $A_{ui}<1$, if $s\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big)<\frac{(A_{si}-1)h_s+(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}$, the result of Lemma 1 (ii) applies, otherwise, the stable equilibrium is $p_{si}=p_{ui}=1$ (no stable equilibrium exists) if $s\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big)<(\ge)\frac{(A_{si}-1)h_s}{(1–H_i)(A_{si}h_s-A_{ui}h_u)}$. ■

Proof of Lemma 4. (Proof when $p_{sj}H_j=0$ for $j\ne i$) The proof of Lemma 2 applies.

(Existence/nonexistence of $H_i>0$ when $p_{sj}H_j>0$)

(i) If $(1-s_i)A_{si}h_s-h_s>(1-s_i)A_{ui}h_u-h_u$ with $p_{si}=1$ and $H_i=F_i$, and $p_{si}=1$ for given $H_i=F_i$ hold, i.e. $s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)<1-\frac{h_s-h_u}{A_{si}h_s-A_{ui}h_u}$ (from Lemma 3 (i) and (ii)), the corresponding part of Lemma 2 (i) applies and thus $H_i=F_i$ is the only stable equilibrium with $H_i>0$.[38] Instead, if $(1-s_i)A_{si}h_s-h_s\le(1-s_i)A_{ui}h_u-h_u$ with $p_{si}=1$ and $H_i=F_i$ and $p_{si}=1$ for given $H_i=F_i$ hold, i.e. $s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)\in\big[\frac{(A_{si}-1)h_s+(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u},\frac{(A_{si}-1)h_s}{(1–F_i)(A_{si}h_s-A_{ui}h_u)}\big)$ (from Lemma 3 (i) and (ii)), $p_{si}=p_{ui}=1$ from the lemma and the net return to investment with $H_i=F_i$ equals $\big[1-s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)\big](A_{si}h_s-A_{ui}h_u)-(1+r)c_h$. Hence, if the return is positive, i.e. if $s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)<1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$, $H_i=F_i$ is the only stable equilibrium with $H_i>0$, otherwise, no stable equilibrium with $H_i>0$ exists. (If the return is positive, an equilibrium with $H_i\in(0,F_i)$ too may exist, and if it is zero, $H_i=F_i$ is the only equilibrium with $H_i>0$, both of which are not stable.) Finally, when $A_{ui}\ge1$, if $p_{si}=0$ for $H_i=F_i$ holds, i.e. $s(0)>\frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}$ (from Lemma 3 (i)), the net return is $h_s-(1+r)c_h-A_{ui}h_u$, thus, if it is non-negative, $H_i=F_i$ is the only stable equilibrium with $H_i>0$ (note Assumption 2), otherwise, no stable equilibrium with $H_i>0$ exists.

To summarize, when $A_{ui}\ge1$, $H_i=F_i$ if $h_s–(1+r)c_h\ge A_{ui}h_u$ and $s(0)>\frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}$ or if $s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)<\min\big\{\frac{(A_{si}-1)h_s}{(1–F_i)(A_{si}h_s-A_{ui}h_u)},\frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}\big\}$. Note that, when $h_s-(1+r)c_h\ge A_{ui}h_u$, $\frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}\le$

[38] To be exact, if $s(0)>\frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}$, for given F_i, there exists $\widetilde{H}_i\in(0,F_i)$ such that $s\big(\frac{\widetilde{H}_iN_i}{\widetilde{H}_iN_i+p_{sj}H_jN_j}\big)=\frac{(A_{si}-1)h_s+(1-A_{ui})h_u}{(1-\widetilde{H}_i)(A_{si}h_s-A_{ui}h_u)}$, and when $A_{ui}\ge1$, $p_{si}=0$ and $p_{ui}=1$ is the only equilibrium for $H_i\in(0,\widetilde{H}_i]$. However, such H_i ($p_{hi}\in(0,1)$) is not an equilibrium since the net return is $h_s-(1+r)c_h-A_{ui}h_u$ (note Assumption 2). The same reasoning applies to the next case and the corresponding cases of (ii) too.

$\min\{\frac{(A_{si}-1)h_s}{(1-F_i)(A_{si}h_s-A_{ui}h_u)}, \frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}\}$ and thus $H_i = F_i$ always. Hence, when $A_{ui} \geq 1$, $H_i = F_i$ if $h_s-(1+r)c_h \geq A_{ui}h_u$ or if $h_s-(1+r)c_h < A_{ui}h_u$ and $s(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}) < 1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$. When $A_{ui} < 1$, if $1-\frac{h_s-h_u}{A_{si}h_s-A_{ui}h_u} \geq \frac{(A_{si}-1)h_s}{(1-F_i)(A_{si}h_s-A_{ui}h_u)}$, $H_i = F_i$ when $s(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}) < 1-\frac{h_s-h_u}{A_{si}h_s-A_{ui}h_u}$; otherwise, $H_i = F_i$ when $s(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}) < \min\{\frac{(A_{si}-1)h_s}{(1-F_i)(A_{si}h_s-A_{ui}h_u)}, 1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}\}$.

(ii) If $(1-s_i)A_{si}h_s-h_s > (1-s_i)A_{ui}h_u-h_u$ with $p_{si}=1$ and $H_i=F_i$ and $p_{si}=1$ for $H_i=F_i$ hold, i.e. $s(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}) < 1-\frac{h_s-h_u}{A_{si}h_s-A_{ui}h_u}$ holds (see the proof of (i)), the corresponding part of the proof of Lemma 2 (ii) applies. In particular, since $1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u} < 1-\frac{h_s-h_u}{A_{si}h_s-A_{ui}h_u}$, $H_i = F_i$ is a stable equilibrium in the same cases as Lemma 2 (ii), except that now $s(0) \leq 1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$ is not possible from Assumption 6. No equilibrium with $H_i > 0$ exists in the remaining cases: if $(1-s_i)A_{si}h_s-h_s \leq (1-s_i)A_{ui}h_u-h_u$ with $p_{si}=1$ and $H_i=F_i$ and $p_{si}=1$ hold, $p_{ui}=1$ and the net return is $(1-s_i)[A_{si}h_s-A_{ui}h_u]-(1+r)c_h \leq h_s-(1+r)c_h-h_u < 0$; and when $A_{ui} \geq 1$, if $p_{si}=0$, $p_{ui}=1$ and the net return is $h_s-(1+r)c_h-A_{ui}h_u \leq h_s-(1+r)c_h-h_u < 0$.

(Existence/nonexistence of $H_i = 0$ when $p_{sj}H_j > 0$) Given $H_i = 0$, if $A_{ui} < 1$ (thus $p_{ui} = 0$), the corresponding part of the proof of Lemma 2 applies and thus $H_i=0$ is not an equilibrium in (i)(b) and when $A_{ui}<1$ in (ii). Given $H_i=0$, if $A_{ui} \geq 1$, the net return is $max\{(1-s_i)A_{si}h_s+s_iA_{ui}h_u, h_s\}-(1+r)c_h-A_{ui}h_u = max\{(1-s(0))[A_{si}h_s-A_{ui}h_u], h_s-A_{ui}h_u\}-(1+r)c_h < h_s-(1+r)c_h-h_u$. Thus, $H_i=0$ is always an equilibrium when $A_{ui} \geq 1$ in (ii), while it is (is not) an equilibrium if $max\{(1-s(0))[A_{si}h_s-A_{ui}h_u], h_s-A_{ui}h_u\}-(1+r)c_h < (\geq) 0$ in (i)(a). That is, $H_i=0$ if $(1-s(0))[A_{si}h_s-A_{ui}h_u] < h_s-A_{ui}h_u \Leftrightarrow s(0) > \frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}$ and $h_s-(1+r)c_h < A_{ui}h_u$, or if $s(0) \leq \frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}$ and $s(0) > \frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$. Since $\frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u} > \frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u} \Leftrightarrow h_s-(1+r)c_h < A_{ui}h_u$, $H_i=0$ if $h_s-(1+r)c_h < A_{ui}h_u$ and $s(0) > \frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$ in (i)(a). ■

Proof of Proposition 3. (When $p_{sj}H_j=0$ for $j \neq i$) Since H_i, p_{si}, and p_{ui} are determined independent of $s(\cdot)$, the corresponding result of Proposition 1 applies.

(When $p_{sj}H_j > 0$) (i)(a)1 From Lemmas 4 (i)(a)1 and 3 (i). (a)2 From Lemmas 4 (i)(a)2 and 3 (i). Note that $p_{si} = 1$ when $H_i = F_i$, since, if $p_{si} = 0$, $H_i = 0$ from $h_s-(1+r)c_h-A_{ui}h_u < 0$. (b)1 From Lemmas 4 (i)(b) and 3 (ii)(a) and Proposition 1 (i)(a). (b)2 From Lemmas 4 (i)(b) and 3 (ii)(b). (Since $s(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}) \geq 1-\frac{h_s-h_u}{A_{si}h_s-A_{ui}h_u}$, $\max[\frac{(A_{si}-1)h_s}{(1-F_i)(A_{si}h_s-A_{ui}h_u)}, 1-\frac{h_s-h_u}{A_{si}h_s-A_{ui}h_u}] = \frac{(A_{si}-1)h_s}{(1-F_i)(A_{si}h_s-A_{ui}h_u)}$.)

(ii) H_i is determined as in Lemma 4 (ii). When $H_i = 0$, p_{si} and p_{ui} are determined independent of $s(\cdot)$ and Proposition 1 applies. When $H_i = F_i$, from Lemma 4 (ii) and Lemma 2 (ii), $s(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}) < \frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$. Hence, when $A_{ui} \geq 1$, $p_{si} = p_{ui} = 1$ from Lemma 3 (i), since $s(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}) < \frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u} < \frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}$ and $H_i=0$ holds if $p_{si}=0$. When $A_{ui}<1$, from Lemma 3 (ii)(a), p_{si} and p_{ui} are determined as in Lemma 1 (ii) from $s(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}) < 1-\frac{(1+r)c_h}{A_{si}h_s-A_{ui}h_u} < 1-\frac{h_s-h_u}{A_{si}h_s-A_{ui}h_u}$. ■