Unionized Mixed Oligopoly and Privatization with Excess Burden of Taxation

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Abstract

By introducing the excess burden of taxation into unionized mixed and privatized oligopolies, we show that (i) if the government that maximizes social welfare values with a small weight of excess burden of taxation, privatization matters regardless of the number of firms; however, (ii) when the degree of excess burden of taxation lies within a relatively large range, the results of both desirable privatization and nationalization materialize depending on the critical value of the excess burden of taxation. In contrast to the existing works on mixed oligopoly, we find privatization can enhance social welfare regardless of the number of firms, under mild conditions.
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1 Introduction

The economic implications of mixed oligopoly markets have recently attracted research attention with respect to the change in competition for the efficiency of market structure, as well as with respect to their effects on privatization. Pioneering works by De Fraja and Delbono (1989) and Beato and Mas-Colell (1984) on mixed oligopolies employed game-theoretic analysis of public and private firms. Most other studies on mixed oligopoly have often assumes that the public firm (or government) maximizes social welfare while private firms maximize their own profits.

Although some theoretical studies have succeeded in explaining a mixed duopoly, Willner (2006), Kato and Tomaru (2007), Saha and Sensarma (2008) and Kato (2008) have explicitly investigated different objective functions between the public firm and the government in the mixed duopoly[1]. However, the government directly intervenes with some inefficiency in many mixed markets. As Meade (1944) first pointed out, in the absence of lump-sum transfers, the government must resort to distortionary taxes on income, capital, or consumption. In other words, if the government raises \$1, society pays $\$(1+\lambda)$. The parameter $\lambda > 0$ is usually called the shadow cost of public funds[2]. In the literature on mixed oligopolies, Capuano and De Feo (2010) and Matsumura and Tomaru (2013) have tackled this issue by examining the welfare effect of a change in a public firm's objective function when the government takes into account the shadow cost of public funds (or, henceforth, excess burden of taxation).

As we observed, introducing the excess burden of taxation into an endogenous timing game in a mixed duopoly, and assuming the public firm to be less efficient than private firms, Capuano and De Feo (2010) discovered that without a subsidy, private leadership emerges as more robust. Moreover, Matsumura and Tomaru (2013) investigated optimal subsidy policy with an

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[1]The result of different objective function between the government and the public firm in a mixed oligopoly is a new one since so far the literature same objective function between the government and the public firm in a mixed oligopoly has found various robust results. See also Choi (2011).

[2]This approach is frequently adopted in contract theory. See Laffont and Tirole (1993).

endogenous timing game in mixed and private duopolies, considering both the excess burden of taxation and firms' increasing marginal cost technologies. In contrast to earlier studies on the privatization neutrality theorem, they demonstrated that privatization affects both optimal subsidy rate and resulting welfare. However, while they formulated that the costs of firms are exogenous, our study considers that costs are determined via wage bargaining with the excess burden of taxation[3]. Thus, we extend the analysis to a *unionized* mixed oligopoly and to the effects of privatization by allowing *multiple private firms*. To investigate an optimal privatization policy, we incorporate union behavior into the objectives of the government, with the excess burden of taxation to explain the government's incentive for privatization as a commitment device.

On the issue of unions' wage setting, the results correspond to the empirical findings that in Europe, Japan, and the United States, the government is heavily involved in the setting of public sector wages (Du Caju *et al.*, 2008). Moreover, Bordogna (2003, pp. 62-63) pointed out that "even where bargaining has been decentralized, governments have often maintained strong, centralized, financial controls in order to contain public expenditures and avoid inflationary consequences of the decentralization process." Thus, the issue remaining to be analyzed is whether the above results are robust to changes in the type of competition under unionized mixed oligopoly with excess burden of taxation[4]. As Bordogna (2003) argued, the empirical facts reveal that unions' wage setting under unionized mixed oligopoly is a better approach to tackle this problem, as opposed to competition in the product market. This is why unions' strategic behavior under unionized mixed oligopoly is considered in this paper.

In this paper, following Capuano and De Feo (2010) and Matsumura and Tomaru (2013), we examine whether privatization is desirable with unions' strategic behavior when the excess burden of taxation is introduced in the government's objective function. To study the welfare effect of a change in the government's objective function in a mixed oligopoly, we allow for multiple private firms in the mixed oligopoly setting, rather than the mixed duopoly framework used Capuano and De Feo (2010) and Matsumura and Tomaru (2013). Here, according to our results under general formulation, we remark that there may be limitations in assuming a mixed duopoly when there exists the excess burden of taxation. Consequently, we show that regardless of the number of existing firms, the government's incentive to privatize the public firm always exists when the degree of excess burden of taxation is relatively small, and vice versa when the degree of excess burden of taxation is relatively large.

More specifically, regardless of the number of private firms, the privatization of a public firm is always desirable from the welfare point of view when the degree of excess burden of taxation falls within a small range, that is, $\lambda \in [0, 1]$. Because of the existence of unions, social welfare

[3]In the presence of the excess burden of taxation, Wang and Chen (2011) found for an imposition of the optimal subsidy, the level of welfare with privatization depends on the level of the cost efficiency gap and the excess burden of taxation. This is because they assumed that the public firm is less inefficient than the private firms.

[4]In addition, according to Lewin (1977, p. 140), "an additional shortcoming of the 'union power' thesis with respect to the governmental sector is its de-emphasis, even ignorance, of conditions that make for potentially diverse patterns of labor relations." In fact, this growing importance of the public sector in terms of union membership suggests that a more complete understanding of the role of unions in wage determination requires a better understanding of their role in the public sector (Bahrami *et al.*, 2009, p. 35). See, among others, De Fraja (1993a) and Haskel and Sanchis (1995) for theoretical view.

under either mixed oligopoly or privatization consists of two factors in a reduced form: the representative consumer's utility and the profit of the public or privatized firm. Thus, on the one hand, when the excess burden of taxation is sufficiently small, it forces the public firm's profit to be negative, while the privatized firm's profit is independent of the excess burden of taxation, which emerges as a positive welfare effect of privatization. On the other hand, (the representative) consumer's utility may be higher under mixed oligopoly than under privatization when the excess burden of taxation is sufficiently small. The excess burden of taxation with public funding costs reduces the public firm's profit and, increases the representative consumer's utility as this burden increases under privatization, given the number of firms, and as privatization induces production substitution from the public firm to private firms. The latter two effects under privatization increase welfare, whereas the first effect reduces welfare under mixed oligopoly. For this reason, the government uses total wage as a commitment device to control the unions' wage demands to maintain lower total wage levels under privatization. A decrease in the wages works to improve welfare by increasing the total output under privatization.

When the degree of the excess burden of taxation falls within a large range, that is, $\lambda > 1$, the result of both desirable privatization and nationalization materialize depending on the critical value of the excess burden of taxation. Given that $\lambda > 1$, privatization stimulates total output and consumer's utility under privatization starts to increase. However, after reaching the critical value of the excess burden of taxation, it is dominated by the excess burden effect; and vice versa when the degree of the excess burden of taxation becomes small, given the number of firms. Contrary to the first result, the government may or may not use total wage as a commitment device to control the unions' wage demands to maintain lower total wage levels since higher wages are still maintained under mixed oligopoly when the degree of the excess burden of taxation is sufficiently great.

The main result of our paper is in contrast to the findings of De Fraja and Delbono (1989) regarding mixed oligopoly that privatization can enhance social welfare when the number of existing private firms is relatively large; this finding holds when the effect of excess burden of taxation does not exist. Moreover, our result is crucial to the evaluation of the impact of welfare subsequent to privatization of the public firm to maximize profits. In fact, comparing the privatization with mixed duopoly, Matsumura and Tomaru (2013) argue that privatization reduces welfare when the effect of excess burden of taxation exists under the government's tax-subsidy policies. However, their study does not consider the situation in which the public firm competes with multiple private firms without tax-subsidy policies, and hence, the result that mixed oligopoly reduces welfare does not hold, even with the excess burden of taxation. With the excess burden of taxation, Capuano and De Feo (2010) found under mixed duopoly, privatization is not desirable when assuming that the public firm is less efficient than private firms, which differs from our main result.

2 The Model

Consider a mixed oligopoly situation for a homogeneous good that is supplied by a public firm (indexed by 0) and n private firms. Firm i ($i = 1, 2, ..., n$) is a profit-maximizing private firm, and firm 0 is a public firm that maximizes social welfare. On the demand side of the market, we assume that the representative consumer's utility is a quadratic function given by

$$U = x_0 + \sum_{i=1}^{n} x_i - \frac{(x_0 + \sum_{i=1}^{n} x_i)^2}{2}; \quad i = 1, 2, ..., n,$$

where x_0 is the level of output of the public firm, and x_i is the level of output of the i^{th} private firm ($i = 1, 2, ..., n$). Thus, the inverse demand is characterized by

$$p = 1 - x_0 - \sum_{i=1}^{n} x_i; \quad i = 1, 2, ..., n,$$

where p is the market price.

To analyze the union's wage bargaining, we also assume that the public and private firms are unionized and that the wages, $w_j : j = 0, 1, ..., n$, are determined as a consequence of bargaining between firms and their respective unions. Let $\overline{w}$ and L_j denote the reservation wage and the number of workers who are employed by firm j, respectively. The firms are homogeneous with respect to productivity. Each firm adopts a constant returns-to-scale technology where one unit of labor is turned into one unit of the final good; thus, $x_j = L_j$. Taking $\overline{w}$ as a given, the union's optimal wage-setting strategy regarding firm j, w_j, is defined as

$$\max_{w_j} u_j = (w_j - \overline{w})^{\theta} L_j; j = 0, 1, ..., n,$$

where θ is the weight that the union attaches to the wage level. Following Ishida and Matsushima (2009) in the literature on the unionized mixed duopoly, we assume that $\theta = 1$ and $\overline{w} = 0$ to demonstrate our results simply. That is, the utility function of the union at the firm is its wage bill: $u_j(w_j; L_j) = w_j L_j = w_j x_j$. Thus, we consider the monopoly union model, which assumes that the unions set the wage while the firms choose the employment level once the wage is set by unions (see also Booth, 1995)[5].

Each firm's profit is as following function

$$\pi_j = (p - w_j)x_j, \quad j = 0, 1, ..., n, \tag{1}$$

where the price of labor (i.e., wage) that firm j has to pay is denoted by $w_j, j = 0, 1, ..., n$.

We assume that the public firm maximizes a utilitarian measure of welfare taking into account the excess burden of taxation. That is, let $\lambda > 0$ denote the excess burden of taxation, which implies that distortionary taxation inflicts disutility \$$(1 + \lambda)$ on taxpayers in order to levy \$1 for the state. To compute the real cost incurred by a firm, the firm's cost and revenue are multiplied by $1 + \lambda$. This parameter is a measure of the dead-weight loss due to distortionary taxation. Then, in the presence of the excess burden of taxation, the maximization problem of

[5]As Ishida and Matsushima (2009) and Barcena-Ruiz and Garzon (2009) have suggested, this is because wage claims are decided by the elasticity of labor demand rather than the firm's profit. As a special case of the Nash bargaining solution, the monopoly union model is frequently adopted; see also Oswald and Turnbull (1985). On the other hand, adopting the asymmetric Nash bargaining, De Fraja (1993b) assumed that the public firm maximizes the weight of sum consumer surplus, profit and the union's utility. The weight attached to the union's utility assumed to be exogenously given. However, it is hard to obtain explicit solution due to the asymmetric Nash bargaining under the unionized mixed oligopoly.

the government is given by

$$SW = U - p(x_0 + \sum_{i=1}^{n} x_i) + \sum_{i=1}^{n}(\pi_i + u_i) + u_0 + (1+\lambda)\pi_0, \tag{2}$$

where λ represents the unit of excess burden[6], $U - p(x_0 + \sum_{i=1}^{n} x_i)$ is consumer surplus, each firm π_i and π_0 is the profit of the private and public firm, and u_j is the union's utility of both the private and public firm.

Timing of the three-stage game is as follows. In the first stage, the government chooses whether or not to privatize the public firm. In the second stage, each firm's union negotiates over wages. In the third stage, each firm simultaneously chooses its quantity to maximize its respective objective knowing each union's choice of the wage level.

3 The Market Equilibrium

Before comparing social welfare under the unionized mixed oligopoly with social welfare under the privatization, we first consider all firms' maximization problems. In this paper, since we focus on symmetric Nash equilibrium, we assume that all firms choose the same type of bargaining. Thus, the game is solved by backward induction, i.e., the solution concept used is the subgame perfect Nash equilibrium.

3.1 The Unionized Mixed Oligopoly

In the third stage, given w_j for each firm, the public firm's maximization problem is as follows:

$$\max_{x_0} \quad SW = U + \lambda\pi_0.$$

Given wage level w_j in the third stage, the best-reply functions of the public and private firms are derived, as usual, from the first order condition:

$$\frac{\partial SW}{\partial x_0} = 0 \Leftrightarrow x_0 = \frac{(1+\lambda)(1-nx_i) - \lambda w_0}{1+2\lambda}, \quad \frac{\partial \pi_i}{\partial x_i} = 0 \Leftrightarrow x_i = \frac{1 - w_i - x_0}{n+1}. \tag{3}$$

Solving the first-order conditions (3), we obtain,

$$x_0 = \frac{(1+\lambda)(1+nw_i) - \lambda(1+n)w_0}{1+\lambda(n+2)}, \quad x_i = \frac{\lambda(1+w_0) - (1+2\lambda)w_i}{1+\lambda(n+2)}. \tag{4}$$

In the second stage of this case, each wage is set to maximize its firm's union utility: $U_j = x_j w_j$. To do this, the two independent maximization problems should be considered simultaneously. Using (4), the problem for union $j = 0, 1, ..., n$ is defined as

$$\max_{w_0} \ u_0 = \frac{w_0[(1+\lambda)(1+nw_i) - \lambda(1+n)w_0]}{1+\lambda(n+2)}, \quad \max_{w_i} \ u_i = \frac{w_i[\lambda(1+w_0) - (1+2\lambda)w_i]}{1+\lambda(n+2)}.$$

[6] Usually, it is assumed that $\lambda > 0$. Since λ is a measure of the distortion by taxation, we may be comfortable assuming that $\lambda \in [0, 1]$ which reflects more reality. In Baron and Myerson (1982) regulation model, the principal is a regulator who maximizes social welfare with a weight $\alpha < 1$ for the firm. Since $\alpha < 1$, it is socially costly to give a rent to the firm. However, Capuano and De Feo (2010) assume that λ has some higher-bound restriction.

Solving these problems and noting that superscript "m" denotes the optimal solution in a unionized mixed oligopoly, we have the following result.

Lemma 1: *Suppose $\lambda > 0$. Then, the equilibrium wage, output, union's utility, the profit of private firms and social welfare are given by*

$$w_0^m = \frac{(1+\lambda)[2+\lambda(4+n)]}{\lambda[4+3n+\lambda(8+7n)]}, \quad x_0^m = \frac{(1+n)[2+\lambda(10+n)+\lambda^2(16+3n)+\lambda^3(8+2n)]}{(1+2\lambda)[1+\lambda(n+2)][4+3n+\lambda(8+7n)]};$$

$$w_i^m = \frac{1+\lambda(5+2n)+\lambda^2(6+4n)}{(1+2\lambda)[4+3n+\lambda(8+7n)]}, \quad x_i^m = \frac{1+\lambda(7+2n)+\lambda^2(16+8n)+\lambda^3(12+8n)}{(1+2\lambda)[1+\lambda(n+2)][4+3n+\lambda(8+7n)]};$$

$$u_0^m = \frac{(1+n)(1+\lambda)[2+\lambda(4+n)][2+\lambda(10+n)+\lambda^2(16+3n)+\lambda^3(8+2n)]}{\lambda(1+2\lambda)[1+\lambda(n+2)][4+3n+\lambda(8+7n)]^2};$$

$$u_i^m = \frac{[1+\lambda(5+2n)+\lambda^2(6+4n)][1+\lambda(7+2n)+\lambda^2(16+8n)+\lambda^3(12+8n)]}{(1+2\lambda)^2[1+\lambda(n+2)][4+3n+\lambda(8+7n)]^2};$$

$$\pi_i^m = \frac{[1+\lambda(7+2n)+\lambda^2(16+8n)+\lambda^3(12+8n)]^2}{(1+2\lambda)^2[1+\lambda(n+2)]^2[4+3n+\lambda(8+7n)]^2};$$

$$SW^m = \frac{4+16n+9n^2+\lambda A+\lambda^2 B+\lambda^3 C+\lambda^4 D+\lambda^5 E+\lambda^6 F+\lambda^7 G}{2(1+2\lambda)^2[1+\lambda(2+n)]^2[4+3n+\lambda(8+7n)]^2},$$

where $A = 48+204n+146n^2+18n^3,\ B = 244+1088n+934n^2+206n^3+9n^4;$

$$C = 680+3124n+3088n^2+934n^3+76n^4,\ D = 1120+5152n+5665n^2+2110n^3+241n^4;$$
$$E = 1088+4768n+5606n^2+2404n^3+342n^4,\ F = 576+2176n+2540n^2+1160n^3+188n^4;$$
$$G = 128+320n+264n^2+80n^3+8n^4.$$

3.2 The Unionized Privatized Oligopoly

The previous subsection examined the impact of a unionized mixed oligopoly. This subsection compares the equilibrium of a unionized mixed oligopoly, which would be established in the case of a unionized privatized oligopoly. As discussed in the basic model, consider the situation of a privatized oligopoly for a homogeneous good that is supplied by firm ($k = 1, 2, ..., n+1$), which is a profit-maximizing private firm given that a new inverse demand is characterized by $p = 1 - \sum_{k=1}^{n+1} : n \geq 1$. After privatization, as in Matsumura and Tomaru (2013) and Wang and Chen (2011), social welfare is given by

$$SW = U - px_0 - \sum_{k=1}^{n} px_k + (1+\lambda)R + u_0 + (\pi_0 - R) + \sum_{k=1}^{n}(\pi_k + u_k), \tag{5}$$

where R is the revenue from selling the stocks of public firm 0, u_0 is the union under selling the stocks of public firm 0 (i.e., privatized firm), and $U - px_0 - \sum_{k=1}^{n} px_k$ is consumer surplus. As Matsumura and Tomaru (2013) and Wang and Chen (2011) assumed, we consider the case in where $R = \pi_0 = \pi_{n+1}$ since the financial market is complete[7]. Thus, $SW = U + \lambda\pi_{n+1} = U + \lambda\pi_k$.

[7] As Capuano and De Feo (2010) pointed out, we give full bargaining power to the government, i.e., it is able to extract the whole profit from the privatized firm.

In the third stage, the firm k's profit-maximization problem is to maximize $\pi_k = (p - w_k)x_k$. Hence, solving across the $n+1$ first-order conditions, the $n+1$ best reply functions can be rewritten as follows:

$$x_k = \frac{1-(n+1)w_k + nw_l}{2+n}, \quad k \neq l; k, l = 1, 2, ..., n+1. \tag{6}$$

In the second stage, given the output as a function of wage, each union at each firm sets the wage, w_k, that maximizes union rent, u_k. Thus, similar to the unionized mixed oligopoly in previous section, we have the following result as given, superscript " $*$ " denotes the optimal solution under the unionized privatized oligopoly:

Lemma 2: *Suppose $\lambda > 0$. Then, the equilibrium wage, output, union's utility, social welfare and the profit of private firms are given by*

$$w_k^* = \frac{1}{2+n}, \quad x_k^* = \frac{1+n}{(2+n)^2}, \quad u_k^* = \frac{1+n}{(2+n)^3},$$
$$SW^* = \frac{(1+n)^2(7+6n+n^2+2\lambda)}{2(2+n)^4}, \quad \pi_k^* = \frac{(1+n)^2}{(2+n)^4}.$$

4 Comparisons of Equilibrium Outcomes

Having derived the market equilibrium for the fixed situation in the previous section, we will find Nash equilibrium in the first stage for any given set of utilities of the unions and the profits of firms in both unionized mixed and privatized oligopolies.

First, we observe a result with $\lambda = 0$ as a benchmark.

Result 1: *Suppose that $\lambda = 0$. Then, $SW^m < SW^*$.*

Proof: Comparing the unionized mixed oligopoly with privatization when $\lambda = 0$, straightforward computations show that

$$SW^m - SW^* = -48 - 104n - 111n^2 - 84n^3 - 40n^4 - 8n^5 - 9n^6 < 0. \qquad \text{Q.E.D.}$$

Result 1 indicates that regardless of the number of existing firms, social welfare under privatization is greater than under the unionized mixed oligopoly. This implies that if the public firm's union aims at maximizing wage level and it does not face some budget constraint with a simple union function $u_j = w_j x_j$, the public firm's union can unlimitedly raise its wage because the optimal output level of the public firm is independent of the wage (see Ishida and Matsushima, 2009).

We now present our main proposition as follows.

Proposition 1: *Suppose that $\lambda \in (0, 1]$. Then, $SW^m < SW^*$.*

Proof: See the appendix with Table A-1 and Figure A-1 for more detailed examples. Comparing the mixed oligopoly with privatization when $\lambda \in (0,1]$, straightforward computations show that

$$\begin{aligned} SW^m - SW^* = & -48 - 104n - 111n^2 - 84n^3 - 40n^4 - 8n^5 - 9n^6 \\ & - \lambda(608 + 1448n + 1664n^2 + 1306n^3 + 676n^4 + 180n^5 + 16n^6) \\ & - \lambda^2(4010 + 8208n + 6195n^2 + 8344n^3 + 4619n^4 + 1488n^5 + 223n^6 + 2602n^7) \\ & - \lambda^3(9040 + 25056n + 32458n^2 + 28074n^3 + 18486n^4 + 6064n^5 + 1202n^6 + 100n^7 + 2n^8) \\ & - \lambda^4(14080 + 39368n + 59328n^2 + 52632n^3 + 32295n^4 + 12994n^5 + 3071n^6 + 350n^7 + 12n^8) \\ & - \lambda^5(11776 + 38272n + 54009n^2 + 52544n^3 + 32734n^4 + 13676n^5 + 3540n^6 + 3396n^7 + 22n^8) \\ & - \lambda^6(4096 + 14848n + 24040n^2 + 22560n^3 + 12916n^4 + 4560n^5 + 1004n^6 + 216n^7 + 4n^8) \\ & - \lambda^7(512n + 1440n^2 + 992n^3 - 728n^4 - 1360n^5 - 704n^6 - 72n^7 - 8n^8) > 0. \qquad \text{Q.E.D.} \end{aligned}$$

Proposition 1 suggests that regardless of the number of private firms, the privatization of a public firm is always desirable from a welfare point of view when $\lambda \in (0,1]$. This proposition is different from the corresponding result of De Fraja and Delbono (1989), who found that the privatization of the public firm is desirable in terms of social welfare when the number of private firms is large, and is not desirable when the number of private firms is small.

The following is the intuition behind proposition 1. When comparing welfare before and after the change in the public firm's objective function, the profits of the public firm and privatized firm do matter, that is, while comparing social welfare of reduced form, $SW^m = U^m + \lambda\pi_0^m$ under the mixed oligopoly with its reduced form, $SW^* = U^* + \lambda\pi_k^*$ under privatization. The government confronts two effects when it privatizes its public firm. One, the excess burden of taxation forces the public firm's profit to be negative[8] when $\lambda \in (0,1]$, while the privatized firm's profit is independent of the excess burden of taxation, which is a positive welfare effect of privatization. We call this effect the *"excess burden effect."* The other effect is that the representative consumer's utility may be higher under mixed oligopoly than under privatization when $\lambda \in (0,1]$. This is a positive welfare effect of nationalization[9]. To compare the excess burden effect with this effect, we call as it the *"consumer effect."* The excess burden effect reduces the public firm's profit when $\lambda \in (0,1]$, and increases the representative consumer's utility as λ increases under privatization given the number of firms. Privatization, in turn, induces production substitution from the public firm to the private firms. The latter two effects increase welfare whereas the first effect reduces welfare. This explanation implies that the consumer effect plus the excess burden effect under privatization dominates the effects obtained under mixed oligopoly. That is, the excess burden effect is weaker because of the negative profit of the public firm, while the consumer effect is stronger when λ is larger, given the number of firms. For this reason, the government uses total wage as a commitment device to control the unions' wage demands so as to maintain lower total wage levels under privatization[10]. Thus,

[8] The calculation of the profit of the public firm is relegated to the appendix. However, when λ is sufficiently large (i.e., $\lambda > 1$), its profit may be either positive or negative depending on the critical value of λ. See appendix A-3 and A-4 for more detailed calculations.

[9] See the appendix A-5 and A-6 for each comparison of representative consumer's utility and total output. Precisely speaking, there can exist a critical value of λ^U such that for all $\lambda >$ (resp. $<$)λ^U given $\lambda \in (0,1]$, we obtain the difference as $U^m <$ (resp. $>$)U^* except for $n = 1$.

[10] When comparing total wage, we obtain $(nw_i^m + w_0^m) - (n+1)w_k^* = 4 + 2n + \lambda(16 + 7n - n^2) + \lambda^2(16 + 3n -$

lower total wages under privatization work to improve welfare by increasing total output. This may lead to more output under privatization than under a unionized mixed oligopoly when $\lambda \in (0, 1]$, given that the number of existing firms increases.

However, we should not overemphasize the result that privatization increases welfare. Privatization can be harmful in alternative model formulations. For example, if the public firm faces some budget constraints (an issue ignored in this paper) with a simple union function when the union aims at maximizing wage level, the obtained welfare gain can dominate the welfare gain discussed above with the excess burden of taxation.

On the other hand, the welfare ranking may be reversed when λ is sufficiently large (i.e., when $\lambda > 1$). Thus, we have the following proposition.

Proposition 2: *Suppose that $\lambda > 1$. Then,*

(i) there can exist a critical value of $\lambda^\dagger$ such that for all $\lambda \geq \lambda^\dagger$ and $n \geq 2$, we obtain the difference as $SW^m \geq SW^*$ and for all $\lambda < \lambda^\dagger$, as $SW^m < SW^*$.
(ii) when $n = 1$, we obtain the difference as $SW^m < SW^*$ regardless of λ.

Proof: See the appendix. Q.E.D.

Proposition 2 suggests that depending on the range of the excess burden of taxation, with$\lambda > 1$ and $n > 1$, social welfare is determined under either mixed oligopoly or privatization[11].

Upon recalling welfare of the reduced form, $SW^m = U^m + \lambda\pi_0^m$ under the mixed oligopoly with $SW^* = U^* + \lambda\pi_k^*$ under privatization, the welfare rankings in proposition 2 become intuitive. When $\lambda > 1$, the improvement of social welfare under mixed oligopoly is possible since the excess burden effect dominates the consumer effect under privatization when $\lambda \geq \lambda^\dagger$ and $n \geq 2$. This implies that if the degree of the excess burden of taxation is smaller than that of its critical value, the total output level under mixed oligopoly may be smaller than that under privatization, and vice versa (see the appendix on the public firm's profit and comparison of total output). In other words, given the number of firms, the result of both desirable privatization and nationalization emerge depending on the excess burden of taxation. If $\lambda < \lambda^\dagger$, privatization stimulates total output, hence the consumer effect under the privatization starts to become stronger; however, with $\lambda > \lambda^\dagger$, it is dominated by an excess burden effect, and vice versa if the degree of the excess burden of taxation becomes small with a small number of firms[12]. Consequently, we obtain proposition 2 depending on $\lambda^\dagger$. Whether or not privatization improves welfare depends on which effect is stronger. As pointed out in proposition 1, higher wages are still maintained under mixed oligopoly when $\lambda > \lambda^\dagger$, whereas the excess burden effect starts to dominate the consumer effect. Maintaining higher wage reduces welfare since wages are strategic complements between the unions, whereas the excess burden effect improves welfare even with a higher wages

$n^2 + 2n^3) - \lambda^3(6n - 2n^2 - 4n^3)$. On the other hand, $w_i^m - w_k^* = -(1+n) - \lambda(3 + 2n - n^2) - \lambda^2(2 - 2n^2)$.

[11] For simplicity of explanation, we exclude the case of $n = 1$.

[12] If we adopt the increasing marginal costs without union, it would similar results to proposition 2, except for the case, $n = 1, 2$. The detailed computations are available from author upon request. To provide correct proofs, we present on Supplemental Material of separate page, which is only available for the reviewers and editors.

level under mixed oligopoly, and vice versa if $\lambda < \lambda^{\dagger}$. This result turns out that when $\lambda > \lambda^{\dagger}$, the government may not use total wage as a commitment device to control the unions' wage demands to maintain lower total wage levels under mixed oligopoly. This is because the excess burden effect under mixed oligopoly dominates consumer effect under privatization. However, when $\lambda < \lambda^{\dagger}$, it can use total wage as a commitment device to control lower wage level under privatization, which also works to improve welfare under privatization by increasing the total output.

In sum, propositions 1 and 2 are in sharp contrast to the existing literature that shows that the privatization of the public firm is desirable from the welfare point of view with a relatively large number of firms (De Fraja and Delbono, 1989). With both the excess burden of taxation and firms' endogenous cost via wage bargaining, the intuition of propositions 1 and 2 relies on the different effects exerted by the representative consumer's utility and the profit of firms.

Comparing privatization with mixed duopoly, Matsumura and Tomaru (2013) investigated optimal tax-subsidy policies with the excess burden of taxation. They focused on both the optimal tax-subsidy policies with endogenous timing of production, as well as the privatization neutrality theorem. However, their study does not extend to the situation in which the public firm competes with multiple private firms; hence, the result that mixed oligopoly reduces welfare does not hold even with the excess burden of taxation. Moreover, Capuano and De Feo (2010) demonstrated that with nil or large efficiency gains, an inefficient public firm that maximizes welfare may still be preferred when there exists the excess burden of taxation in the government's objective function, which is different from our results.

5 Concluding Remarks

By introducing the excess burden of taxation into a theoretical framework of unionized mixed oligopoly, this study provides new insight into social welfare within the context of the government's optimal policy in respect to privatization. When $\lambda \in [0, 1]$, privatization matters regardless of the number of firms. However, we show that depending on the range of the excess burden of taxation, social welfare is determined under either the mixed oligopoly or privatization. In this paper, we suggest that under mild conditions, privatization is considered as a powerful instrument to reduce distortionary taxation. However, we show that an inefficient public firm may be preferred even when large inefficiency exists.

We conclude by discussing the limitations of our paper. We have used the simplifying assumption that private and public firms are symmetric due to a decentralized unionization structure with the monopoly union model. By making these assumptions, we do not take into account any cost difference that may arise from the mixed bargaining that occurs between private and public firms. Moreover, in this paper, it is assumed that the public firm is as efficient as the private firm with endogenous input costs (i.e., wages). If the cost between the public and private firms is characterized by increasing and decreasing return to scale, privatization may reduce or improve welfare with the different degree of the excess burden of taxation. Finally, we have not extended the model to consider a situation in which the public firm competes with both domestic and foreign private firms. An extension of our model in these directions is for future research.

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Appendix

A-1. Proof of Proposition 1

Since the comparison of each social welfare becomes complicated when $\lambda \in (0, 1]$, we need to use the plot of the expressions to illustrate the impact of both degree of excess burden of taxation and the number of firms. When $\lambda \in (0, 1]$, the comparison of $SW^m - SW^*$ over the parameter space $\{\lambda, n\}$ is drawn in Figure A-1.

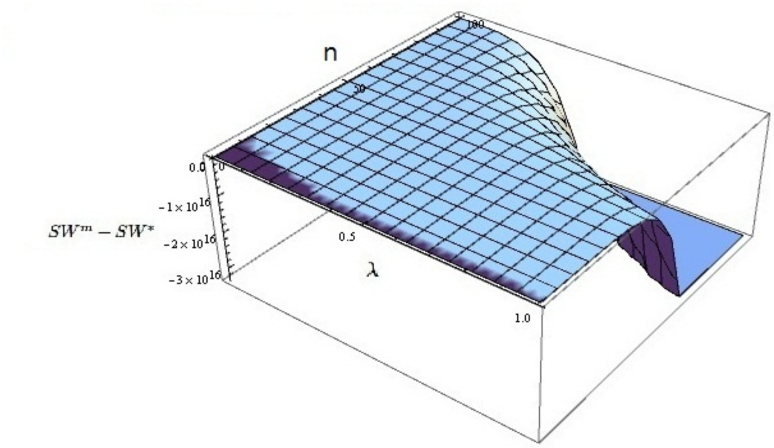

Figure A-1. $\lambda \in (0, 1]$ and $n \in [1, 100]$ when comparing $SW^m - SW^*$

At the same time, using computations, the numerical analysis of Table A-1 shows each comparison of each social welfare.

Table A-1: The number of firms, n with $\lambda \in (0, 1]$ when comparing $SW^m - SW^*$

	$SW^m - SW^*$						
λ	$n = 1$	$n = 2$	$\cdots$	$n = 20$	$\cdots$	$n = 100$	$\cdots$
0.01	-466	-3285	$\cdots$	-961241000	$\cdots$	-35339512327988	$\cdots$
0.1	-1494	-13683	$\cdots$	-34715577801	$\cdots$	-2668026079752860	$\cdots$
0.3	-10944	-115272	$\cdots$	-331139498144	$\cdots$	-26974707984645500	$\cdots$
0.5	-48597	-496448	$\cdots$	-1102889086549	$\cdots$	-96818001098310800	$\cdots$
0.7	-159964	-1594375	$\cdots$	-2876166087553	$\cdots$	-271145850436360000	$\cdots$
0.9	-431732	-4241945	$\cdots$	-6715644736526	$\cdots$	-661425626287401000	$\cdots$
1	-670888	-6555758	$\cdots$	-9929988665978	$\cdots$	-987947372141275000	$\cdots$

$\star$ The negative number in each cell depicts that given the degree of $\lambda \in (0, 1]$, the comparisons of social welfare imply $SW^m < SW^*$. Noting that for simplicity, we omit the decimal place of numbers in each cell, for given the number of firms, $n \geq 1$ and the range, $\lambda \in (0, 1]$.

Q.E.D.

A-2. Proof of Proposition 2

Comparing the mixed oligopoly with privatization when $\lambda > 1$, straightforward computations show that

Table A-2: The number of firms, n with $\lambda > 1$ when comparing $SW^m - SW^*$

	$SW^m - SW^*$						
λ	$n = 1$	$n = 2$	$\cdots$	$n = 50$	$\cdots$	$n = 100$	$\cdots$
1.1	-1011485	-9833240	$\cdots$	-9482371797544900	$\cdots$	-1148552807110180000	$\ddots$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$
3.05	-145710984	-1192542221	$\ddots$	-37760645429656600	$\ddots$	-243295558949306$\times10^3$	$\cdots$
3.06	-148254833	-1211866018	$\ddots$	-37807348191996300	$\ddots$	110123177065542$\times10^4$	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$
3.66	-385863468	-2913691186	$\ddots$	-2513007070883840	$\ddots$	219525485483227$\times10^6$	$\ddots$
3.67	-391585186	-2952733915	$\ddots$	14394729944189400	$\cdots$	226530012745852$\times10^6$	$\ddots$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$
10.35	-132005813464	-827463827	$\ddots$	398248980231828$\times10^7$	$\ddots$	972766339437254$\times10^9$	$\ddots$
10.36	-132744421843	681870899	$\ddots$	401076409416575$\times10^9$	$\ddots$	979574899358374$\times10^9$	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$

$\star$ The positive (negative) number in each cell depicts that given the degree of $\lambda > 1$, the comparisons of social welfare imply $SW^m > (<) SW^*$. Noting that for simplicity, we omit the decimal place of numbers in each cell, for given the number of firms, $n \geq 1$ and the range, $\lambda > 1$.

Since the comparison of each social welfare becomes complicated when $\lambda > 1$, for example, the comparison of $SW^m - SW^*$ over the parameter space $\{\lambda, n\}$ is drawn in Figure A-2.

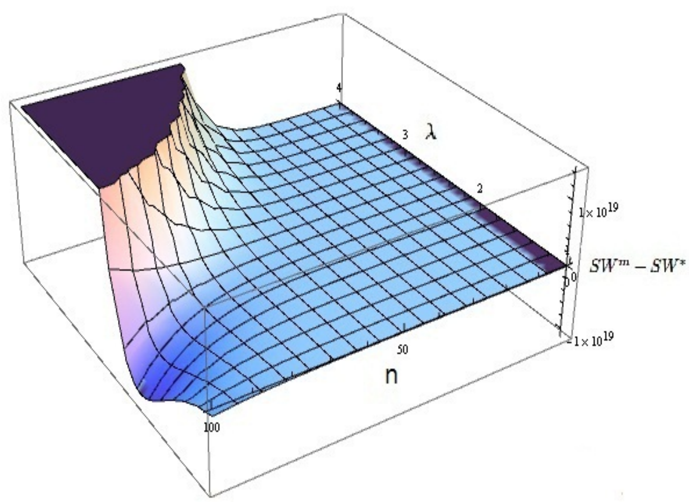

Figure A-2. $\lambda \in (1, 4]$ and $n \in [1, 100]$ when comparing $SW^m - SW^*$

Q.E.D.

A-3. The Profit of Public Firm

$$\lambda\pi_0^m = \frac{-8-8n-\lambda A-\lambda^2 B-\lambda^3 C-\lambda^4 D-\lambda^5 E+\lambda^6 F+\lambda^7 G}{2(1+2\lambda)^2[1+\lambda(2+n)]^2[4+3n+\lambda(8+7n)]^2};$$

where $A = 88+104n+16n^3$, $B = 392+528n+146n^2+10n^3$;

$C = 888+1320n+494n^2+64n^3+2n^4$, $D = 1024+1608n+706n^2+130n^3+8n^4$;

$E = 448+624n+222n^2+52n^3+6n^4$, $F = 128+416n+384n^2+104n^3+8n^4$;

$G = 128+320n+264n^2+80n^3+8n^4$.

A-4. Comparison of Firm's Profit

$$\begin{aligned}\lambda\pi_0^m - \lambda\pi_k^* = {} & -64-192n-224n^2-128n^3-36n^4-4n^5\\ & -\lambda(720+2296n+2921n^2+1892n^3+661n^4+116n^5+8n^6)\\ & -\lambda^2(3328+11208n+15300n^2+10990n^3+4418n^4+986n^5+113n^6+5n^7)\\ & -\lambda^3(8064+28528n+41481n^2+32090n^3+14246n^4+2978n^5+563n^6+40n^7+n^8)\\ & -\lambda^4(10752+39808n+60968n^2+49914n^3+23488n^4+6490n^5+1047n^6+97n^7+4n^8)\\ & -\lambda^5(7424+28800n+46168n^2+39116n^3+18517n^4+4790n^5+664n^6+50n^7+3n^8)\\ & -\lambda^6(2048+8576n+24528n^2+12248n^3+4828n^4+320n^5+340n^6-84n^7-4n^8)\\ & -\lambda^7(256n+720n^2+496n^3-364n^4-680n^5-352n^6-72n^7-4n^8).\end{aligned}$$

As in Tables A-1 and A-2, equation, $\lambda\pi_0^m - \lambda\pi_k^*$ has similar results in each cell when $\lambda \in (0,1]$ and $\lambda > 1$.

A-5. Comparison of Total Output

$$\begin{aligned}X^m(\equiv nx_i^m + x_0^m)) - X^*(\equiv (1+n)x_k^*) = {} & 4+9n+4n^2+\lambda(16+41n+21n^2+n^3)\\ & +\lambda^2(16+52n+27n^2-3n^3-2n^4)+\lambda^3(12n+2n^2-10n^3-4n^4).\end{aligned}$$

A-6. Comparison of Representative Consumer's Utility

$$\begin{aligned}U^m - U^* = {} & 80+280n+337n^2+172n^3+32n^4\\ & +\lambda(1372+3144n+4718n^2+2490n^3+648n^4+52n^5)\\ & +\lambda^2(3456+14208n+20465n^2+13636n^3+4217n^4+304n^5+3n^6)\\ & +\lambda^3(7168+32000n+50104n^2+35964n^3+11686n^4+1036n^5-210n^6-30n^7)\\ & +\lambda^4(7424+37248n+62808n^2+47196n^3+14781n^4-20n^5-977n^6-156n^7-4n^8)\\ & +\lambda^5(3072+19328n+38327n^2+14628n^3+4320n^4-17096n^5-2252n^6-3296n^7-16n^8)\\ & +\lambda^6(2304n+5016n^2+1936n^3-3260n^4-3920n^5-1684n^6-384n^7-12n^8).\end{aligned}$$

Supplemental Material (Not for Publication)

For the reviewers and editors, this Supplemental Material will not be included in the main paper. However, this is only available for the reviewers and editor. In this case where we have been abbreviated, we present on separate page.

S1. The Model: Increasing Marginal Costs without Union

Consider a mixed oligopoly situation for a homogeneous good that is supplied by a public firm (indexed by 0) and n private firms. Firm i ($i = 1, 2, ..., n$) is a profit-maximizing private firm, and firm 0 is a public firm that maximizes social welfare. Assume that the inverse demand is characterized by $p = 1 - x_0 - \sum_{i=1}^{n} x_i$, where p is the market price, x_0 is the level of output of the public firm, and x_i is the level of output of the i^{th} private firm ($i = 1, 2, ..., n$).

Given that each firm shares the same production technology that is represented by a quadratic cost function $\frac{x_j^2}{2}, j = 0, 1, ..., n$, each firm's profit is as shown in

$$\pi_j = px_j - \frac{x_j^2}{2}, \quad j = 0, 1, ..., n. \tag{S-1}$$

where marginal cost is increasing marginal cost.

As in main text, we assume that the public firm maximizes a utilitarian measure of welfare taking into account the shadow cost of public funds. Thus, the maximization problem of the government is given by

$$SW = \frac{(x_0 + \sum_{i=1}^{n} x_i)^2}{2} + \sum_{i=1}^{n} \pi_i + (1 + \lambda)\pi_0, \tag{S-2}$$

where $\lambda \in [0, 1]$ represents the unit of excess burden[13], $\frac{(x_0+\sum_{i=1}^{n} x_i)^2}{2}$ is consumer surplus, each firm π_i and π_0 is the profit of the private and public firm.

Timing of the two-stage game is as follows. In the first stage, the government chooses whether or not to privatize the public firm. In the second stage, each firm simultaneously chooses its quantity to maximize its respective objective knowing type of competition.

S2. The Market Equilibrium

S2-1. The Mixed Oligopoly

For this purpose, we consider two competition regimes: mixed oligopoly, and privatization. To derive the reaction functions of both the public and the private firms, differentiating (S-1) and (S-2) with x_0 and x_i, respectively, we have

$$\frac{\partial SW}{\partial x_0} = 0 \Leftrightarrow x_0 = \frac{(1 + \lambda)(1 - nx_i)}{2 + 3\lambda}, \quad \frac{\partial \pi_i}{\partial x_i} = 0 \Leftrightarrow x_i = \frac{1 - x_0}{2 + n}. \tag{S-3}$$

[13] We do not assume that $\lambda > 1$, since its assumption makes always the mixed oligopoly to be preferred rather than the privatization. We will mention later this reason in the proof of proposition S1.

Solving the first-order conditions (S-3), we obtain,

$$x_0 = \frac{2(1+\lambda)}{4+n+2\lambda(3+n)}, \quad x_i = \frac{1+2\lambda}{4+n+2\lambda(3+n)}.$$

Using each optimal output, we have the following result.

Lemma S1: *Suppose that $\lambda \in (0,1]$. Then, the equilibrium profits of private and public firms and social welfare are given by*

$$\pi_i^m = \frac{3(1+2\lambda)^2}{2[4+n+2\lambda(3+n)]^2}, \quad \pi_0^m = \frac{2(1+\lambda)(1+3\lambda)}{[4+n+2\lambda(3+n)]^2};$$
$$SW^m = \frac{8+7n+n^2+\lambda(28+24n+4n^2)+\lambda^2(32+20n+4n^2)+12\lambda^3}{2[4+n+2\lambda(3+n)]^2}.$$

S2-2. The Privatization

The previous subsection examined the impact of a mixed oligopoly. This subsection compares the equilibrium of a mixed oligopoly, which would be established in the case of a privatized oligopoly. As discussed in the basic model, consider the situation of a privatized oligopoly for a homogeneous good that is supplied by firm ($k = 1, 2, ..., n+1$), which is a profit-maximizing private firm given that a new inverse demand is characterized by $p = 1 - \sum_{k=1}^{n+1} : n \geq 1$.

In the second stage, the firm k's profit-maximization problem is to maximize $\pi_k = px_k - \frac{x_k^2}{2}$. Hence, solving across the $n+1$ first-order conditions, the $n+1$ best reply functions can be rewritten as follows:

$$x_k = \frac{1}{3+n}, \quad k = 1, 2, ..., n+1.$$

After privatization, as in main text, social welfare is given by

$$SW = \frac{(\sum_{k=1}^{n+1} x_k)^2}{2} + \sum_{k=1}^{n} \pi_k + (\pi_0^p - R) + (1+\lambda)R.$$

Using each optimal output, we have the following result.

Lemma S2: *Suppose that $\lambda \in (0,1]$. Then, the equilibrium profit of private firm and social welfare are given by*

$$\pi_k^* = \frac{3}{2(3+n)^2}, \quad SW^* = \frac{4+5n+n^2+3\lambda}{2(3+n)^2}.$$

S3. Comparisons of Equilibrium Outcomes

Having derived the equilibrium for fixed situation in the previous section, we will find the Nash equilibrium in the first stage for any given the profits of firms in both mixed and privatized oligopolies. Note that we do not assume that $\lambda > 1$, since its assumption makes always the mixed oligopoly to be preferred rather than the privatization.

We obtain proposition as follows.

Proposition S1: *Suppose that $\lambda \in (0, 1]$. Then,*

(i) there can exist a critical value of λ^* such that for all $\lambda \geq \lambda^*$ and $n \geq 3$, we obtain the difference as $SW^m \geq SW^*$ and for all $\lambda < \lambda^*$, as $SW^m > SW^*$.
(ii) when $n = 1$ and $n = 2$, we obtain the difference as $SW^m < SW^*$ regardless of λ.
(iii) when $\lambda = 1$, we obtain the difference as $SW^m > SW^*$ regardless of n.

Proof: Comparing both the case of mixed oligopoly with privatization when $\lambda \in (0, 1]$, straightforward computations show that

$$SW^m - SW^* = 8 - n - n^2 + \lambda(12 + 8n + n^2) + \lambda^2(12n + 4n^2). \tag{S-4}$$

By directly applying the above equation to a discriminant when comparing $SW^m - SW^*$, the minimum value is attained because $(12n + 4n^2) > 0$ with $n \geq 1$, then the graph of $SW^m - SW^*$ is a U shaped. Table S-1 gives two roots when comparing each social welfare.

Table S-1: Roots for λ with the number of firms, n

	$SW^m - SW^*$	
n	λ_1	λ^*
1	-0.892182326	-0.420317674
2	-0.731662479	-0.068337521
3	-0.703922796	0.078922796
⋮	⋮	⋮
30	-0.649424096	0.358515005
⋮	⋮	⋮
150	-0.642302046	0.384001393
⋮	⋮	⋮
5000	-0.640446479	0.390196509
⋮	⋮	⋮

Therefore, we know the points λ^* and λ_1 at which the graph of $SW^m - SW^*$ intersects the horizontal axis with $n \geq 1$. However, since we can ignore the negative points, λ_1 by the assumption $\lambda \in (0, 1]$, there can exist a critical value such that for all $\lambda \geq \lambda^*$ when $n \geq 3$, we obtain $SW^m \geq SW^*$, and for all $\lambda < \lambda^*$ as $SW^m < SW^*$. Since we assume that $\lambda \in (0, 1]$, we obtain $SW^m > SW^*$ when $n = 1, 2$. Finally, substituting $\lambda \geq 1$ into Eq. (S-4) yields $SW^m > SW^*$ regardless of n.

Q.E.D.