Relative Performance Concerns, Attention Allocation and Complementarities in Information Acquisition

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Abstract

This article studies how relative performance concerns affect institutional investors' information choices in the context of a multi-security market. I show that due to relative performance concerns and learning capacity constraint, institutional investors tend to acquire the same piece of information and the same asset as their peers. I also show that the change of distribution of capacity constraint can affect the price efficiency and cost of capital, but only through its effect on investors' average capacity constraint.

Keywords: relative performance concerns, attention allocation, complementarities in information acquisition

1. Introduction

"Institutions are herding animals. We watch the same indicators and listen to the same prognostications. Like lemmings, we tend to move in the same direction and at the same time."

Wall Street Journal,17 Oct 1989

Empirical evidence suggests that institutional investors tend to buy and sell the same stocks at the same time (see, among others, Bikhchandani and Sharma (2000), Hirshleifer and Hong Teoh (2003), Choi and Sias (2009), Sias (2004) for the evidence of institutional herding). There are several potential reasons for institutional herding.[1]In this article, I identify an additional

[1]For an overview of theoretical research on rational herd behavior in financial markets, see Bikhchandani and Sharma (2000) and Hirshleifer and Hong Teoh (2003).

channel though which institutional investors herd. By incorporating relative performance concerns into a rational expectations equilibrium model with rational inattention, I show how such payoff externalities can give merit to the quotation above, that is, complementarities in information acquisition and hence, herding on a particular asset or asset class.

I study the effect of relative performance concerns by extending the model developed by Van Nieuwerburgh and Veldkamp (2010). In particular, I take the relative performance contract as given, in which an institutional investor's marginal utility of realized trading profits increases in the average realized trading profits of the other investors in the economy. I focus on a three-period, noisy rational expectations equilibrium with independent assets and independent signals and assume that investors have entropy learning technology so they have to choose to learn about specific assets or to allocate his capacity to all assets in the first stage. After allocating his capacity, every agent receives the signal and chooses optimal portfolio in the second stage. In the third stage, institutional investors

There are several reasons why institutional investors may have to take relative performance concerns into consideration. First, individual investors usually base their investment decision on the relative past performance of funds.[2] Second, as is well known from contract theory (Holmstrom (1982)), relative performance-based contracts may mitigate agency problems.[3]

I show that there exists a linear noisy rational expectations equilibrium in the second stage. In the first stage, institutional investors are faced with information processing capacity which bounds the distance between priors and posteriors variance-covariance matrix so they use their capacity to learn from prices and signals. Then I show that there is strategic complementarities in information choice. This is in contrast with the indifference result in Van Nieuwerburgh and Veldkamp (2010), that investors who have CARA utility functions are indifferent about how to allocating the capacity. So this paper provides another explanation to the herd on particular information and hence, herd on particular assets.

The intuition for the result is the following. If more people choose to learn more about the payoff of asset j, they will also hold more shares of

[2]See, among others, Chevalier and Ellison (1997), Palomino (2005) and Sirri and Tufano (1998).

[3]See Gümbel (2005).

asset j because the variance is reduced by learning. Agent i, due to relative performance concerns, also want to hold more asset j, which result in raising the value of learning the asset. As a result, agent i also want to learn more about the asset.

I also investigate how the distribution of capacity constraint will change the price efficiency and, as a result, the cost of capital. I conclude that if the average N^{th} root of capacity is the same, where N is the number of assets, the distribution will not affect the price efficiency and the cost of capital. In particular, given the average of N^{th} root of capacity, the inequality of capacity constraints among investors does not matter for price efficiency. If some agents have low capacity and other agents have high capacity, it is obvious that some agents will have little information about assets' payoff and other agents have much more information about assets' payoff. My conclusion is that the information asymmetry will not affect the cost of capital and price efficiency, given the mean of N^{th} root of capacity constraint.

This article is related to several strands of literature. First, it is related to relative performance concerns and herding. Gümbel (2005) explores the optimal design of compensation contract between an investor and his agent, showing that investors herd in their asset allocation decision when the cost of information is high and managers are sufficiently risk averse. Maug and Naik (1996) examines the circumstances under which the fund managers decide to acquire the same information as their peer, who only learn and trade in one asset. . Whereas they derive the optimal contract which gives rise to herding behavior in the framework of Kyle (1985). This paper's focus is not on optimal contract and I incorporate relative performance concerns into the framework of Admati (1985), taking the optimal contract as given. I conclude that herding may rise because fund managers choose to learn similar information simultaneously due to relative performance objectives.

There are many papers that produce complementarities on information acquisition. In a static environment, Froot et al. (1992), Barlevy and Veronesi (2007), Garcia and Strobl (2011) and Veldkamp (2006) among others generate information complementarities.[4] Avdis (2012) and Hellwig and Veldkamp (2009) obtain complementarities in a dynamic setting. This article is different from these papers in that I incorporate relative performance concerns and information learning capacity in a noisy rational expectations equilibrium.

[4]See Avdis (2012) for a review of mechanisms in a static environment.

As a result, agents have to decide which assets to learn under the learning constraint. Our model and the one in Garcia and Strobl (2011) both assume that the information acquisition cost is a nonpecuniary cost, but the cost in Garcia and Strobl (2011) is the same for all agents, while our model allows for heterogeneous capacity constraint. Garcia and Strobl (2011) supposes that investors choose to acquire information or not according to how many other investors pay to acquire information. Our paper can be viewed as a multi-asset extension of Garcia and Strobl (2011) where investors choose to allocate learning capacity according to other investors' choices.

The existing model closest to ours is that of Van Nieuwerburgh and Veldkamp (2009). They investigate how precisely a signal an investor wants to observe about each asset's payoff when there is a constraint on the total amount of signal precision he can observe. Mondria (2010) models the interplay between the attention allocation of portfolio investors and price comovement. This article differs from this literature in that I examine the consequences that relative performance concerns have on information choice. In contrast to the indifference results in the Van Nieuwerburgh and Veldkamp (2009), our model predicts that with CARA preferences and entropy learning technology, investors' information choice is complementary with the average precision. What's more, while Van Nieuwerburgh and Veldkamp (2009) focuses on the partial equilibrium so agents ignore the information included in the price, I consider the noisy rational expectations equilibrium with CARA utility function so agents will learn from price.

This article is also related to the literature that studies relative wealth concerns. Abel (1990) and Gali (1994) consider the effects of consumption externalities on asset pricing and portfolio choice. Gali (1994) concludes that positive externalities ($0 < \gamma < 1$) tend to reduce the size of the equity premium investors require to absorb the entire equity supply because the optimal individual risky share will increase. I get similar results in noisy rational expectations model. As is stated by Garcia and Strobl (2011), investors hedge the relative wealth risk by imitating the average portfolio. As a result, the demand shift will decrease the price and hence, the risk premium. Like ours, Andrei (2010) uses the standard Admati (1985) model to investigate the interaction between relative wealth concerns and information advantage and propose a new mechanism to explain home bias.

The remainder of this article is organized as follows. Section 2 describes the model with emphasis on information leaning. Section 3 describes the equilibrium at the second stage and agent's optimal information choice according to average precision. Section 4 presents an extension of our model. I introduce the heterogeneous capacity constraints and demonstrate the effect of inequality of capacity constraints on the market efficiency. Section 5 summarizes our contribution and concludes.

2. The Model

This paper studies a three-period economy and there is a continuum of investors. In period 1, institutional investors make decisions as to the precision of signals about asset payoffs, subject to limited information-processing capacity. In period 2, the agents observe signals and then choose what assets to purchase. In period 3, agents receive the asset payoffs and realize their utility. I extend the multiple asset rational expectations model developed by Admati (1985) to allow for externalities. In particular, I assume that agents care not only about their own wealth, but also about their peers'.

2.1. Information Sets

I assume that all random variables belong to a linear space of jointly normally distributed random variables. In particular, I assume that there are N risky assets and they pay off f. All investors are endowed with a prior belief that $f \sim N(\mu, \Sigma)$. At time 1, agent i chooses how to allocate his information capacity by choosing the signal variance Σ_i where $\eta_i = f + e_i$ is the signal and $e_i \sim N(0, \Sigma_i)$. So the signal is noisy but unbiased. At time 2, agent i combines his signal , the price and his prior belief, using Bayes'law. Let $\hat{\mu_i}$ and $\widehat{\Sigma}_i$ be the posterior mean and variance of payoffs , conditional on all information known to the investor in period 2:

$$\begin{aligned} \hat{\mu_i} &\equiv E[f|\mu, \eta_i, p] && (1)\\ &= (\Sigma^{-1} + \Sigma_i^{-1} + \Sigma_p^{-1})^{-1}(\Sigma^{-1}\mu + \Sigma_i^{-1}\eta_i + \Sigma_p^{-1}E[f|p]), && (2)\\ \widehat{\Sigma}_i &\equiv Var[f|\mu, \eta, p] = (\Sigma^{-1} + \Sigma_i^{-1} + \Sigma_p^{-1})^{-1}, && (3) \end{aligned}$$

where $\Sigma_p = Var[f|p]$.

Following Van Nieuwerburgh and Veldkamp (2010), investor i is faced with information processing constraint: $|\Sigma|/|\widehat{\Sigma}_i| \leq K_i$.

2.2. Preferences and Assets

I assume that investor i has preferences of the form

$$U_i = -exp(-\tau(W_i - \gamma\bar{W})). \tag{4}$$

I study a competitive market which is populated by a continuum of agents, indexed by $i \in [0, 1]$. There are N+1 assets in the market: a riskless asset in perfectly elastic supply with a price normalized to 1 and the payoff is R. The aggregate supply of the risky asset is random and equals $x + \bar{x}$ where $x \sim N(0, \sigma_x^2 I)$. To simplify our analysis, I assume that the assets are independent, so the payoff variance-covariance matrix is diagonal. I also assume that Σ and Σ_i for all i are diagonal.

2.3. Definition of Equilibrium

2.3.1. The Second Stage

Given the signals, a rational expectation equilibrium is characterized by a set of trading strategies q_i where $i \in [0, 1]$, and a price function P such that

(1).Each agent i chooses her trading strategy q_i so as to maximize her expected utility conditional on her information set $\mathcal{F}_i$, i.e., q_i solves

$$\max E[-exp(-\tau(W_i - \gamma\bar{W}))|\mathcal{F}_i], \quad i \in [0, 1]. \tag{5}$$

(2).Markets clear, i.e.,

$$\int_0^1 q_i = x + \bar{x}. \tag{6}$$

2.3.2. The First Stage

Knowing the utility in the second stage for different signals and other investors' signal precision choice, investor i makes the optimal information choice $\widehat{\Sigma}_i$ such that

$$\max_{\widehat{\Sigma}_i} E[E[U_i|\mathcal{F}_i]], \tag{7}$$

$$s.t. |\Sigma|/|\widehat{\Sigma}_i| \leq K_i. \tag{8}$$

3. Characterization of Equilibrium

In this section I solve for the equilibrium defined above by backward induction, starting with the optimal portfolio decision. In period 2, investors have three pieces of information which aggregate to form the conditional expectation of the assets' payoffs: the prior beliefs, the signals and the equilibrium price.

Proposition 1. *There exists a rational expectations equilibrium price*

$$p = \frac{1}{R}(A + Bf + Cx), \tag{9}$$

where

$$A = [\Sigma^{-1} + \frac{\psi'\psi}{\rho^2\sigma_x^2} + (1-\gamma)\psi]^{-1}[\Sigma^{-1}\mu - (1-\gamma)\rho\bar{x}], \tag{10}$$

$$B = [\Sigma^{-1} + \frac{\psi'\psi}{\rho^2\sigma_x^2} + (1-\gamma)\psi]^{-1}[(1-\gamma)\psi + \frac{\psi'\psi}{\rho^2\sigma_x^2}], \tag{11}$$

$$C = -[\Sigma^{-1} + \frac{\psi'\psi}{\rho^2\sigma_x^2} + (1-\gamma)\psi]^{-1}[(1-\gamma)\rho I + \frac{\psi'}{\rho\sigma_x^2}], \tag{12}$$

and $\psi = \int_0^1 \Sigma_i^{-1} di$.

Proof is in appendix.

Remark: The equilibrium price is reduced to the one in Garcia and Strobl (2011) where there is only one risky asset.

The price is also a function of the posterior mean and variance of the "average" investor. Let

$$\widehat{\Sigma}_a^{-1} \equiv \Sigma^{-1} + \frac{\psi'\psi}{\rho^2\sigma_x^2} + (1-\gamma)\psi, \tag{13}$$

$$\hat{\mu}_a \equiv [\Sigma^{-1} + \frac{\psi'\psi}{\rho^2\sigma_x^2} + (1-\gamma)\psi]^{-1}[\Sigma^{-1}\mu + (1-\gamma)\psi f + \frac{\psi'\psi}{\rho^2\sigma_x^2}(f - \psi^{-1}\rho x)]. \tag{14}$$

Then (9) can be written as $pR = \hat{\mu}_a - \rho\widehat{\Sigma}_a(1-\gamma)(x+\bar{x})$, where $\hat{\mu}_a$ and $\widehat{\Sigma}_a$ can be interpreted as the "average" posterior mean and variance, respectively. The price of the asset is equal to the average posterior mean minus a discount that result from the risk the market associates with holding all the supply of risky assets. $\rho\widehat{\Sigma}_a(1-\gamma)\bar{x}$ is the cost of capital which comports with the definition in Easley and O'hara (2004) and Lambert et al. (2012).

In Gali (1994), positive consumption externalities tend to reduce the size of the equity premium investors require to absorb the entire supply. Here I get similar result that the relative wealth concerns makes the effective supply decrease to $(1-\gamma)(x+\bar{x})$.

An interesting result is that with relative performance concerns, the "average" posterior variance $\widehat{\Sigma}_a$ is different from the"real" average posterior variance $\int_0^1 \widehat{\Sigma}_i^{-1} di$,

$$\Sigma^{-1} + \frac{\psi'\psi}{\rho^2\sigma_x^2} + (1-\gamma)\psi < \int_0^1 \widehat{\Sigma}_i^{-1} di, \tag{15}$$

as long as $\gamma \neq 0$, where $\widehat{\Sigma}_i^{-1} = \Sigma^{-1} + \Sigma_i^{-1} + \frac{\psi'\psi}{\rho^2\sigma_x^2}$.

Also, the information content of prices is independent of γ, i.e., $Var[f|p] = (\frac{1}{\sigma_x^2\rho^2}\psi'\psi)^{-1}$. The results are summarized in Corollary 1.

Corollary 1. *The relative performance concern, denoted as γ, alters the cost of capital in two different ways. It reduces the effective supply of assets and increase the average posterior variance. Given ψ, γ has no effect on price efficiency.*

The optimal portfolio choice of agent i is given in the appendix. It can be rearranged as

$$q_i = \frac{\widehat{\Sigma}_i^{-1}(\hat{\mu}_i - pR)}{\rho} + \gamma(x + C^{-1}Bf) + \gamma\bar{x} - \gamma C^{-1}BpR.$$

$x + C^{-1}Bf$ is the information about the supply of assets agent i get from the equilibrium prices. The higher the $x + C^{-1}Bf$, the more shares agent i want to buy. It can be viewed as the herding effect. Agent i would deviate from the standard portfolio choice and buy what other people are buying because he will suffer more from lagging behind other institutional investors than beating them.

The period-2 expected utility is

$$\begin{aligned} E[U_i|\mathcal{F}_i] &= -E[\exp(-\tau(W_i - \gamma\bar{W}))|\mathcal{F}_i]] \\ &= -|I - 2\widehat{\Sigma}_i(-\rho\gamma C^{-1}B)|^{-\frac{1}{2}} \exp(-\frac{1}{2}(\hat{\mu}_i - pR)'\widehat{\Sigma}_i^{-1}(\hat{\mu}_i - pR)). \end{aligned}$$

By standard results from statistics, if X and Y are arbitrary random variables for which the necessary expectations and variance exist, then $Var(Y) = E[Var(Y|X)] + Var(E[Y|X])$. As a result,

$$\hat{\mu}_i - pR \sim N((I-B)\mu - A, V_{ER}), \tag{16}$$

where

$$\begin{aligned} V_{ER} &= Var(f-pR) - Var(f-pR|\mathcal{F}_i) \\ &= Var(f - Bf + Cx) - Var(f|\mathcal{F}_i) \\ &= (I-B)\Sigma(I-B)' + \sigma_x^2 CC' - \widehat{\Sigma}_i \\ &= \Sigma + B\Sigma B' - 2\Sigma B' - \widehat{\Sigma}_i + \sigma_x^2 CC'. \end{aligned} \tag{17}$$

Let $Q = \Sigma + B\Sigma B' - 2\Sigma B' + \sigma_x^2 CC'$, then $V_{ER} = Q - \widehat{\Sigma}_i$. By standard results from statistics, if $X \in R^n$ is a normally distributed random vector with mean ϑ and covariance matrix Ω and $I - 2\Omega A^o$ is positive definite, so $E[\exp(X^{\mathrm{T}} A^o X + b^{\mathrm{T}})]$ is well-defined and is given by

$$E[\exp(X'A^oX + b'X)] = |I - 2\Omega A^o|^{-\frac{1}{2}} \exp\Bigg(b'\vartheta + \vartheta' A^o \vartheta \\ + \frac{1}{2}(b + 2A^o\vartheta)' \times (I - 2\Omega A^o)^{-1}\Omega(b + 2A^o\vartheta)\Bigg).$$

Using the previous result about the expectation of the exponential function and the definition of V_{ER}, the period-1 expected utility is

$$E[E[U_i|\mathcal{F}_i]] = -|I - 2\widehat{\Sigma}_i(-\rho\gamma C^{-1}B)|^{-\frac{1}{2}} |I - 2V_{ER}(-\frac{1}{2}\widehat{\Sigma}_i^{-1})|^{-\frac{1}{2}} \\ \times \exp\Bigg(-\frac{1}{2}((I-B)\mu - A)'\widehat{\Sigma}_i^{-1} \times ((I-B)\mu - A) \\ + \frac{1}{2}((I-B)\mu - A)' \times \widehat{\Sigma}_i^{-1}(I + V_{ER}\widehat{\Sigma}_i^{-1})^{-1} V_{ER}\widehat{\Sigma}_i^{-1}((I-B)\mu - A)\Bigg).$$

Noting that

$$\begin{aligned} \widehat{\Sigma}_i^{-1}(I + V_{ER}\widehat{\Sigma}_i^{-1})^{-1}V_{ER}\widehat{\Sigma}_i^{-1} - \widehat{\Sigma}_i^{-1} &= \widehat{\Sigma}_i^{-1}(I + (Q - \widehat{\Sigma}_i)\widehat{\Sigma}_i^{-1})^{-1}(Q - \widehat{\Sigma}_i)\widehat{\Sigma}_i^{-1} - \widehat{\Sigma}_i^{-1} \\ &= \widehat{\Sigma}_i^{-1}(Q - \widehat{\Sigma}_i)\widehat{\Sigma}_i^{-1} - \widehat{\Sigma}_i^{-1} \\ &= Q^{-1}(Q - \widehat{\Sigma}_i)\widehat{\Sigma}_i^{-1} - \widehat{\Sigma}_i^{-1} \\ &= -Q^{-1}. \end{aligned}$$

As a result,

$$\begin{aligned} E[E[U_i|\mathcal{F}_i]] &= -|I - 2\widehat{\Sigma}_i(-\rho\gamma C^{-1}B)|^{-\frac{1}{2}}|I - 2V_{ER}(-\tfrac{1}{2}\widehat{\Sigma}_i^{-1})|^{-\frac{1}{2}} \\ &\quad \times \exp\left(-\tfrac{1}{2}((I-B)\mu - A)'Q^{-1}((I-B)\mu - A)\right). \end{aligned}$$

Then agent i's period-1 information choice problem can be written as

$$\max -|I - 2\widehat{\Sigma}_i(-\rho\gamma C^{-1}B)|^{-\frac{1}{2}}|I - 2V_{ER}(-\frac{1}{2}\widehat{\Sigma}_i^{-1})|^{-\frac{1}{2}}. \tag{18}$$

Substitute $-\rho C^{-1}B = \psi$ and $V_{ER} = (Q - \widehat{\Sigma}_i)$ into (18) yields,

$$\max |\widehat{\Sigma}_i^{-1} - 2\gamma\psi|. \tag{19}$$

Given the entropy capacity constraint in (8), the solution to the capacity allocation problem is indeterminate.

Proposition 2. *When the amount of capacity K an investor is endowed with limits how much his signal η can reduce payoff uncertainty as $|\Sigma|/|\widehat{\Sigma}_i| \leq K$, the investor who has no relative performance concerns is indifferent between any allocation of his capacity.*

Proof. When $\gamma = 0$, the object reduces to $|\widehat{\Sigma}_i^{-1}|$. When Σ is exogenous, the object depends only on the capacity K, not on how that capacity is allocated across assets. □

Note that the proposition is consistent with the "indifferent results" in Van Nieuwerburgh and Veldkamp (2009). However, Van Nieuwerburgh and Veldkamp (2009) get the result in a partial equilibrium while I get the result in the noisy rational expectation equilibrium.

According to Veldkamp (2011), indifference arises because the desire for specialization and diversification just offset each other. On one hand, when many investors learn about asset j, the equilibrium price of j contains more information of asset j. If agent i choose also to learn about asset j, the portfolio choice in period 2 is much harder to predict on the basis of time-1 information. However, the CARA investors are averse to the time-1 portfolio uncertainty. On the other hand, as more investors choose to learn about asset j, more information is revealed by the equilibrium price, the uncertainty is reduced and agent i would choose to hold more asset j. As a result, devoting capacity to asset j becomes more valuable. When CARA investors are faced with entropy constraint, they are indifferent between any allocation of their capacity.

Proposition 3. *When the agent has relative performance concerns, indifferent result is violated and investors' information choices are complementary. That is, when there are more investors learning about asset j, other investors also want to learn about asset j.*

Proof. When there are only two assets, the problem is greatly simplified. In this situation, $\Sigma = \begin{pmatrix} \Sigma^{11} & 0 \\ 0 & \Sigma^{22} \end{pmatrix}$, $\Sigma_i = \begin{pmatrix} \Sigma_i^{11} & 0 \\ 0 & \Sigma_i^{22} \end{pmatrix}$. Define Σ_ψ^{ii} where i=1,2 such that $\psi = \begin{pmatrix} \frac{1}{\Sigma_\psi^{11}} & 0 \\ 0 & \frac{1}{\Sigma_\psi^{22}} \end{pmatrix}$. The capacity constraint can be written as $\Sigma^{11}\Sigma^{22}/\widehat{\Sigma}_i^{11}\widehat{\Sigma}_i^{22} = K$, which equals to

$$\left(1 + \frac{\Sigma^{11}}{\Sigma_i^{11}} + \frac{\Sigma^{11}}{\rho^2\sigma_x^2(\Sigma_\psi^{11})^2}\right)\left(1 + \frac{\Sigma^{22}}{\Sigma_i^{22}} + \frac{\Sigma^{22}}{\rho^2\sigma_x^2(\Sigma_\psi^{22})^2}\right) = K, \tag{20}$$

because $\widehat{\Sigma}_i = [\Sigma^{-1} + \Sigma_i^{-1} + (\frac{1}{\rho^2\sigma_x^2}\psi'\psi)]^{-1}$

Then

$$\widehat{\Sigma}_i = \begin{pmatrix} \frac{1}{\Sigma^{11}} + \frac{1}{\Sigma_i^{11}} + \frac{1}{\rho^2\sigma_x^2(\Sigma_\psi^{11})^2} & 0 \\ 0 & \frac{1}{\Sigma^{22}} + \frac{1}{\Sigma_i^{22}} + \frac{1}{\rho^2\sigma_x^2(\Sigma_\psi^{22})^2} \end{pmatrix}^{-1} \tag{21}$$

$$= \begin{pmatrix} \frac{\Sigma^{11}\Sigma_i^{11}\rho^2\sigma_x^2(\Sigma_\psi^{11})^2}{\Sigma^{11}+\Sigma_i^{11}+\rho^2\sigma_x^2(\Sigma_\psi^{11})^2} & 0 \\ 0 & \frac{\Sigma^{22}\Sigma_i^{22}\rho^2\sigma_x^2(\Sigma_\psi^{22})^2}{\Sigma^{22}+\Sigma_i^{22}+\rho^2\sigma_x^2(\Sigma_\psi^{22})^2} \end{pmatrix}. \tag{22}$$

The objective is rewritten as the following equation

$$\begin{aligned}
&|\widehat{\Sigma}_i^{-1} - 2\gamma\psi| \\
= &\begin{vmatrix} \frac{1}{\Sigma^{11}} + \frac{1}{\Sigma_i^{11}} + \frac{1}{\rho^2\sigma_x^2(\Sigma_\psi^{11})^2} - \frac{2\gamma}{\Sigma_\psi^{11}} & 0 \\ 0 & \frac{1}{\Sigma^{22}} + \frac{1}{\Sigma_i^{22}} + \frac{1}{\rho^2\sigma_x^2(\Sigma_\psi^{22})^2} - \frac{2\gamma}{\Sigma_\psi^{22}} \end{vmatrix} \\
= &\left(\frac{1}{\Sigma^{11}} + \frac{1}{\Sigma_i^{11}} + \frac{1}{\rho^2\sigma_x^2(\Sigma_\psi^{11})^2} - \frac{2\gamma}{\Sigma_\psi^{11}}\right)\left(\frac{1}{\Sigma^{22}} + \frac{1}{\Sigma_i^{22}} + \frac{1}{\rho^2\sigma_x^2(\Sigma_\psi^{22})^2} - \frac{2\gamma}{\Sigma_\psi^{22}}\right) \\
= &\left(1 + \frac{\Sigma^{11}}{\Sigma_i^{11}} + \frac{\Sigma^{11}}{\rho^2\sigma_x^2(\Sigma_\psi^{11})^2} - \frac{2\gamma}{\Sigma_\psi^{11}}\Sigma^{11}\right)\left(1 + \frac{\Sigma^{22}}{\Sigma_i^{22}} + \frac{\Sigma^{22}}{\rho^2\sigma_x^2(\Sigma_\psi^{22})^2} - \frac{2\gamma}{\Sigma_\psi^{22}}\Sigma^{22}\right)\frac{1}{\Sigma^{11}\Sigma^{22}}.
\end{aligned}$$

So agent i's problem is equivalent to

$$\min_{\Sigma_i^{11},\Sigma_i^{22}} \left(1+\frac{\Sigma^{11}}{\Sigma_i^{11}}+\frac{\Sigma^{11}}{\rho^2\sigma_x^2(\Sigma_\psi^{11})^2}\right)\frac{2\gamma}{\Sigma_\psi^{22}}\Sigma^{22}+\left(1+\frac{\Sigma^{22}}{\Sigma_i^{22}}+\frac{\Sigma^{22}}{\rho^2\sigma_x^2(\Sigma_\psi^{22})^2}\right)\frac{2\gamma}{\Sigma_\psi^{11}}\Sigma^{11}$$

under the constraint (21). Simple calculations show that

$$\left(1+\frac{\Sigma^{11}}{\Sigma_i^{11}}+\frac{\Sigma^{11}}{\rho^2\sigma_x^2(\Sigma_\psi^{11})^2}\right)=\sqrt{K\frac{\Sigma_\psi^{22}\Sigma^{11}}{\Sigma_\psi^{11}\Sigma^{22}}}. \tag{23}$$

Similarly,

$$\left(1+\frac{\Sigma^{22}}{\Sigma_i^{22}}+\frac{\Sigma^{22}}{\rho^2\sigma_x^2(\Sigma_\psi^{22})^2}\right)=\sqrt{K\frac{\Sigma_\psi^{11}\Sigma^{22}}{\Sigma_\psi^{22}\Sigma^{11}}}. \tag{24}$$

Substitute (23) and (24) into (22),

$$\widehat{\Sigma}_i=\begin{pmatrix}\sqrt{K\frac{\Sigma_\psi^{22}}{\Sigma_\psi^{11}\Sigma^{22}\Sigma^{11}}} & 0\\ 0 & \sqrt{K\frac{\Sigma_\psi^{11}}{\Sigma_\psi^{22}\Sigma^{11}\Sigma^{22}}}\end{pmatrix}^{-1}. \tag{25}$$

When there are N risky assets, define Σ_ψ^{jj} in a similar way and agent i wants to

$$\max \ \prod_{j=1}^N\left(1+\frac{\Sigma^{jj}}{\Sigma_i^{jj}}+\frac{\Sigma^{jj}}{\rho^2\sigma_x^2(\Sigma_\psi^{jj})^2}-\frac{2\gamma\Sigma^{jj}}{\Sigma_\psi^{jj}}\right), \tag{26}$$

$$s.t. \prod_{j=1}^N\left(1+\frac{\Sigma^{jj}}{\Sigma_i^{jj}}+\frac{\Sigma^{jj}}{\rho^2\sigma_x^2(\Sigma_\psi^{jj})^2}\right)=K. \tag{27}$$

Use the Lagrange method and some simple calculations show that

$$1+\frac{\Sigma^{jj}}{\Sigma_i^{jj}}+\frac{\Sigma^{jj}}{\rho^2\sigma_x^2(\Sigma_\psi^{jj})^2}=\frac{2\gamma\Sigma^{jj}}{\Sigma_\psi^{jj}}\left(\frac{K}{\prod_{j=1}^N a_j}\right)^{\frac{1}{N}}, \tag{28}$$

where $a_j=\frac{2\gamma\Sigma^{jj}}{\Sigma_\psi^{jj}}$. Then

$$\widehat{\Sigma}_i^{jj-1}=\frac{K^{\frac{1}{N}}}{\Sigma_\psi^{jj}}\left(\prod_{j=1}^N\frac{\Sigma_\psi^{jj}}{\Sigma^{jj}}\right)^{\frac{1}{N}}. \tag{29}$$

So it is clear that when there are two assets, the precision of agent i about the first asset , denoted as $(\widehat{\Sigma}_i^{11})^{-1}$, equals to $\sqrt{K\frac{\Sigma_\psi^{22}}{\Sigma_\psi^{11}\Sigma^{22}\Sigma^{11}}}$ and is increasing in $\frac{1}{\Sigma_\psi^{11}}$ and decreasing in $\frac{1}{\Sigma_\psi^{22}}$, as is stated in (25). When there are N risky assets, similar results can be derived from (29). □

The optimal response of agent i's information choice to the average information choice is complementary. It can be viewed as a multi-asset generalization of Garcia and Strobl (2011), where the marginal value of information is increasing in the number of agents who acquire information so there are complementarities in information acquisition. When there are multiple assets, agents have to choose to optimally allocate the capacity, which depends on other agents choice. If more fund managers or other institutional investors choose to allocate more capacity in the first asset, due to relative performance concerns, agent i should also learn more about the payoff of the first asset.

The intuition is that if most people choose to learn more about the payoff of asset j, they will also hold more shares of asset j because the variance is reduced by learning. Agent i, due to relative performance concerns, also want to hold more asset j, which result in raising the value of learning the asset. As a result, agent i also want to learn more about the asset. The effect of herding, combined with the effect of information revelation, dominates the effect of aversion to time-1 uncertainty.

Note that the precision of the signal is increasing in the capacity, as in (23), (24) and (29). The result is consistent with the one derived by Peress (2004), that studies the relationship between wealth and information acquisition. Peress (2004) shows that demand for information is increasing with wealth. However, Peress (2004) assumes that absolute risk aversion is decreasing with wealth, while I assume CARA utility and impose capacity constraint instead of wealth and the cost of information.

4. Distribution of Capacity and Price Efficiency

A natural question is how the distribution of capacity of information acquisition will affect the information efficiency of the price. Hellwig (1980) states that price aggregates the disperse information possessed by agents, while the stochastic supply of assets prevents prices from fully revealing the payoff f. The information content of prices is given by $var[f|p]^{-1} = \Sigma^{-1} + \frac{1}{\rho^2\sigma_x^2}\psi'\psi$. When there are only two assets, $\psi = \begin{pmatrix} \frac{1}{\Sigma_\psi^{11}} & 0 \\ 0 & \frac{1}{\Sigma_\psi^{22}} \end{pmatrix}$, where

$$\frac{1}{\Sigma_\psi^{11}} = \int_0^1 \Sigma_i^{11} di, \tag{30}$$

$$\frac{1}{\Sigma_\psi^{22}} = \int_0^1 \Sigma_i^{22} di. \tag{31}$$

Integrating (23) and (24) over all agents yields

$$\left(1 + \frac{\Sigma^{11}}{\Sigma_\psi^{11}} + \frac{\Sigma^{11}}{\rho^2\sigma_x^2(\Sigma_\psi^{11})^2}\right) = \sqrt{\frac{\Sigma_\psi^{22}\Sigma^{11}}{\Sigma_\psi^{11}\Sigma^{22}}} \int_0^1 \sqrt{K_i} di, \tag{32}$$

$$\left(1 + \frac{\Sigma^{22}}{\Sigma_\psi^{22}} + \frac{\Sigma^{22}}{\rho^2\sigma_x^2(\Sigma_\psi^{22})^2}\right) = \sqrt{\frac{\Sigma_\psi^{11}\Sigma^{22}}{\Sigma_\psi^{22}\Sigma^{11}}} \int_0^1 \sqrt{K_i} di. \tag{33}$$

When there are N risky assets, integrating (28) over all agents yields

$$1 + \frac{\Sigma^{jj}}{\Sigma_\psi^{jj}} + \frac{\Sigma^{jj}}{\rho^2\sigma_x^2(\Sigma_\psi^{jj})^2} = \frac{\Sigma^{jj}}{\Sigma_\psi^{jj}} \left(\prod_{j=1}^{N} \frac{\Sigma_\psi^{jj}}{\Sigma^{jj}}\right)^{\frac{1}{N}} \int_0^1 \sqrt[N]{K_i} di. \tag{34}$$

The results above are summarized in the following proposition.

Proposition 4. *The distribution of capacity K will influence the information content of the price and cost of capital, but only through the mean of the N^{th} root of each agents' capacity, i.e., $\sqrt[N]{K_i}$. If the distribution of the capacity is changed but the mean of the N^{th} of capacity remain unchanged, the information content of price and cost of capital would stay the same.*

Note that the information content of the price is independent of the coefficient γ, which is consistent with the information content of prices given by Garcia and Strobl (2011).

Proposition 4 states that given $\int_0^1 \sqrt[N]{K_i} di$, the change of distribution of K would not affect Σ_ψ, and hence, can not affect the stock price. The result is in contrast with the conclusion in Peress (2004) which says the more unequal the distribution of wealth, the higher the stock price.

5. Conclusion

This paper investigates the effects of relative performance concerns on information acquisition when there are multiple assets and a constraint on the total amount of signal precision each investors can observe. I extend the multiple asset model developed by Admati (1985) to allow for externalities and derive a one-period, noisy rational expectations equilibrium with independent assets and independent signals. With entropy learning technology, I show that investors' choices of learning are complementary. As a result, investors choose similar asset portfolios. So I propose a theoretical explanation of the phenomenon that institution investors follow each other into and out of the same securities.

Appendix A. Proof of Proposition 1

I speculate that the equilibrium price takes the form $pR = D + Bf + C(x+\overline{x})$. Then for agent i, $W_i - \gamma\bar{W} = q_i'(f-pR) - \gamma(x+\overline{x})'(f-pR)$ where q_i is agent i's portfolio choice. Given the form of equilibrium price,

$$\begin{aligned} W_i - \gamma\bar{W} &= q_i'(f-pR) - \gamma(x+\bar{x})'(f-pR) \\ &= (f-pR)'[q_i - \gamma C^{-1}(Rp - D - Bf)] \\ &= (f-Rp)'[q_i + \gamma C^{-1}D + \gamma C^{-1}(B-I)Rp] + (f-Rp)'\gamma C^{-1}B(f-pR). \end{aligned}$$

Then the expected utility in period 3 is

$$E[-\exp(-\rho(W_i - \gamma\overline{W}))|p,\eta_i] = -E[\exp((f-pR)'G(f-pR) + (f-pR)'b)|p,\eta_i].$$

where $G = -\rho\gamma C^{-1}B$, $b = -\rho[q_i + \gamma C^{-1}D + \gamma C^{-1}(B-I)Rp]$.

By equation (17) and the definition of $\hat{\mu}_i$ and $\widehat{\Sigma}_i$, the problem of agent i is to chooses q_i to maximize

$$\begin{aligned} b'(\hat{\mu}_i - Rp) + (\hat{\mu}_i - Rp)'G(\hat{\mu}_i - Rp) &+ \frac{1}{2}[b + 2G(\mu_i - Rp)]' \\ &\times (I - 2\widehat{\Sigma}_i G)^{-1}\widehat{\Sigma}_i[b + 2G(\hat{\mu}_i - Rp)]. \end{aligned}$$

Differentiating the expression above w.r.t b yields the first order condition

$$\hat{\mu}_i - Rp + (I - 2\widehat{\Sigma}_i G)^{-1}\widehat{\Sigma}_i[b + 2G(\hat{\mu}_i - Rp)] = 0.$$

Then $b = -\widehat{\Sigma}_i^{-1}(\hat{\mu}_i - Rp)$. By the definition of b,

$$q_i = \frac{\widehat{\Sigma}_i^{-1}(\hat{\mu}_i - pR)}{\rho} - \gamma[C^{-1}D + C^{-1}(B - I)Rp]. \tag{A.1}$$

The market clearing condition requires that $\int_0^1 q_i di = x + \overline{x}$.

Substituting

$$E[f|p] = E[B^{-1}(Rp - D - Cx)|p] = B^{-1}(Rp - D)$$

and

$$\Sigma_p \equiv Var[f|p] = \sigma_x^2 B^{-1}C(B^{-1}C)'$$

into the market clearing condition and matching coefficients yields the solution of D, B and C. Let $A = D + C\bar{x}$, then we have the solution of A, B and C.

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