The Buchanan-Tullock Model: Some Extensions

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22 January 1980

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Introduction

In their seminal work, *The Calculus of Consent* (1962), Buchanan and Tullock develop a decision model which embodies fundamental relationships relevant to institutional choices. However, the Buchanan-Tullock model remains 'general,' thus inviting others to specify details and to develop extensions. This paper seeks to explicate this important model and to extend the model by introducing the group size and group preference heterogeneity as explicit variables.

The original model

The Buchanan-Tullock model considers choices from the point-of-view of the individual. Any formal or informal social or collective arrangement, regulation or 'rule' creates gains and imposes costs which enter the general utility calculus of individuals, and we may expect individuals to select, vie for, or migrate to that arrangement which maximizes private net gain.[1]

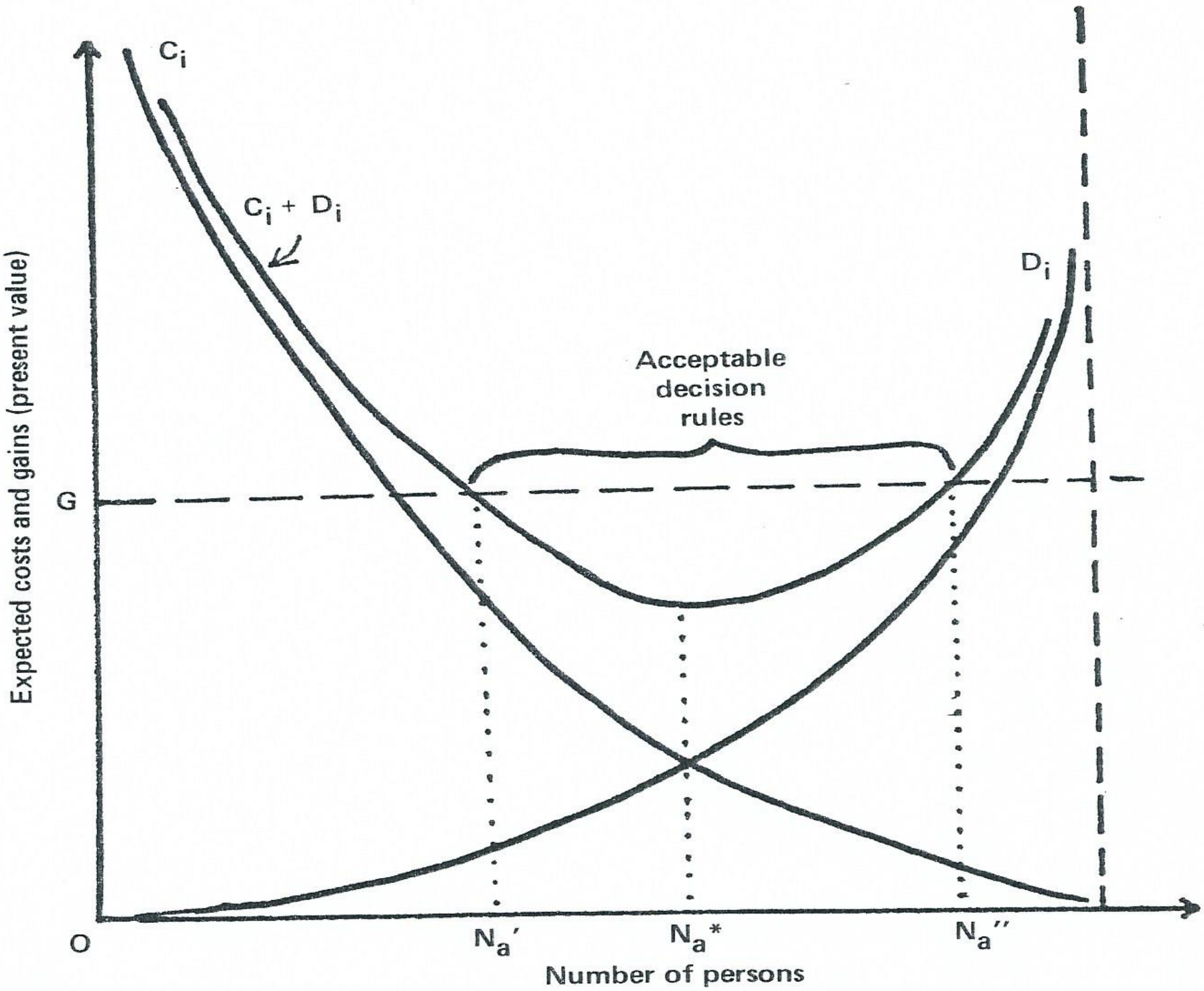
The range of potential organizational choices extends over a broad spectrum, with purely atomistic, 'unorganized' activity at one extreme and completely collective governmental or very highly institutionalized activity at the other. Of course, associated with each organizational choice ordinarily there will be a different bundle of goods. In a purely unorganized regime, the individual is encumbered by few restraints on his own activity but must bear various 'interdependence' costs, including traditional types of external diseconomies which arise as a result of the private activities (or inactivities) of others. Although we can expect some reduction in these interdependence costs through non-market bilateral or small-number transactions, free-riders and high private transactions costs will prevent significant reductions, especially in cases where economies of joint consumption can be attained only with large numbers.

In selecting a group, a location, or an institutional form, the individual may be viewed as seeking to maximize the net gain from social interdependence. However, the process of collectivization, which assimilates externalities and attains economies of joint consumption, generates additional costs which are classified by Buchanan and Tullock (1962) as 'external costs' and 'decision-making costs.'

External costs are those which the individual expects to endure as a result of collective decisions going against him, for example, a coercive tax-price that exceeds his marginal rate of substitution between private and collective goods. The value of external costs is a function of the number of persons required to reach a decision in a group of fixed size. Thus, $C_i = C_i (N_a)$, $N_a \leqslant N$, where C_i is the present value of the expected external cost imposed on the i^{th} individual as a result of an adverse (to him) collective decision, and N_a is the number of persons from a fixed group size of N persons required to agree (the decision rule) before collective action can be taken. In the case of a decision rule which requires unanimity ($N_a = N$), external costs are zero because the individual has veto power. If $N_a = 1$, the external costs will be very high because each individual is potentially at the mercy of every other individual. External costs, then, will be expected to decline as N_a increases. The Buchanan-Tullock external-cost function is represented by curve (C_i) in Figure 1.

The second class of costs defined by Buchanan and Tullock (1962) are *decision costs*, the costs in time and effort required to reach a decision. Decision costs vary with the number of participants required to reach agreement: $D_i = D_i (N_a)$; $N_a \leqslant N$, where D_i is the expected cost in time and effort required to reach a decision imposed on individual i. As shown in Figure 1 by curve (D_i,) decision costs increase as the number required to agree (N_a) increases. The sum of (C_i) and (D_i) (in Figure 1) defines the total cost which the individual expects to bear as a function of the decision rule, N_a. Thus, for individual i, the optimal decision rule is N_a^{c*} persons to reach agreement. These costs are then compared with the expected gains of collective action which, for individual i, are assumed here to have the value OG. Thus, the individual will choose between private and collective activity, depending upon a comparison of OG with the vertical sum of D_i and C_i. If the range of 'acceptable' rules ($N_a' - N_a''$) widens, the probability of successful collective action increases. In this case, individual i will accept any decision rule requiring between N_a' and N_a'' persons to agree. Whether or not collective activity will in fact be selected depends upon the gains-costs calculations of other individuals and the dispersion of acceptable decision rules. Finally, a group can obviously be formed with a decision rule requiring much less agreement than unanimity. Group size and the decision rule will be determined largely by the reciprocal relations between the dispersion of acceptable 'rules' for individuals and for the entire group.

Figure 1. The Buchanan-Tullock model



An extension: Differences in group size

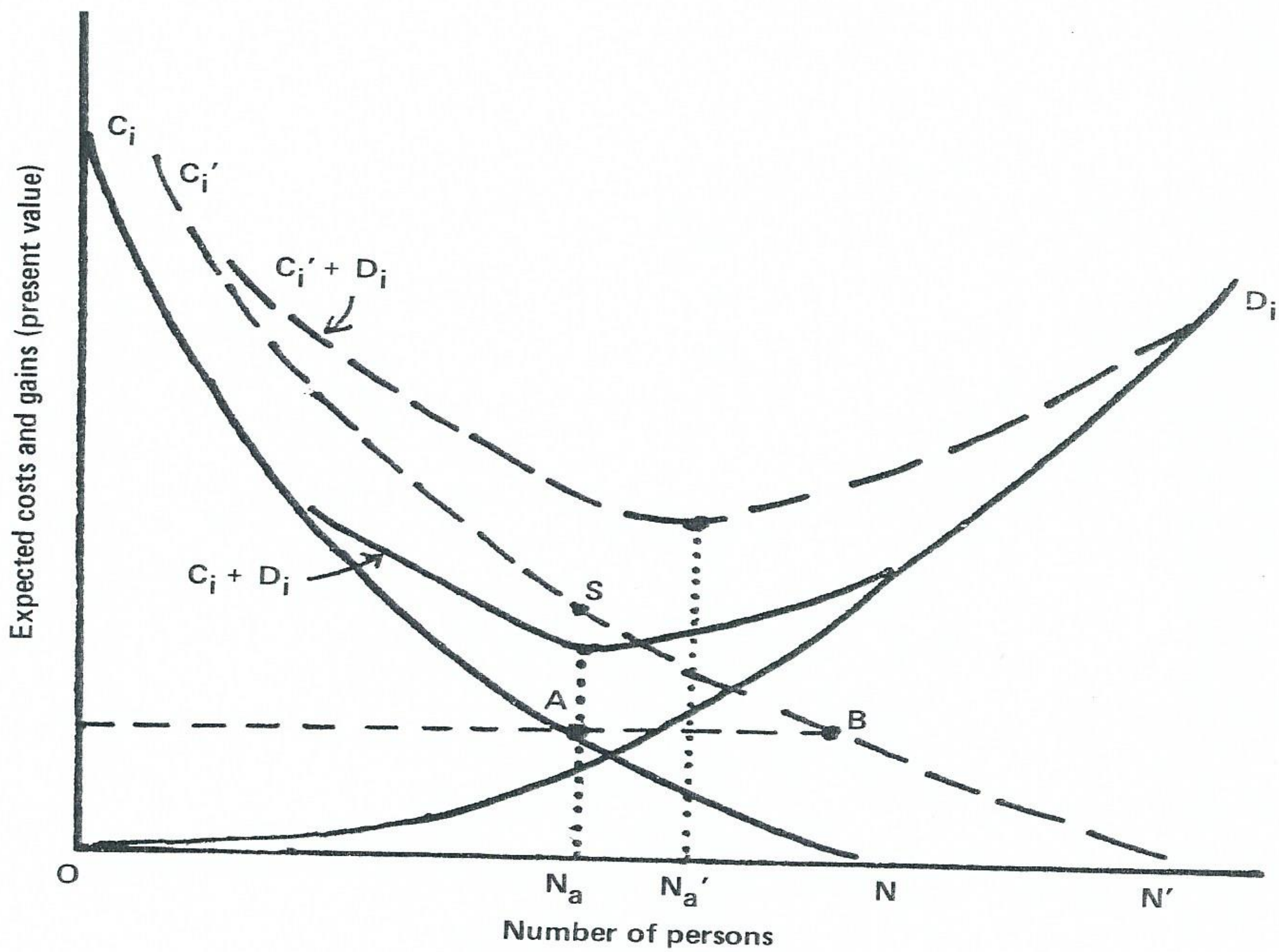
The Buchanan-Tullock model, as originally developed, assumes a group of fixed size and a given degree of preference homogeneity. It indicates nothing about the effects of differences in group size or group heterogeneity, and it remains unrelated to more recent literature concerning the optimal size of jurisdiction for the provision of public goods. We wish to extend the Buchanan-Tullock model so as to account for differences in group size and the degree of preference homogeneity.

Figure 2 modifies the Buchanan-Tullock model for a change in group size, N. As the group size is expanded from N to N', we assume *for simplici-*

ty that the decision cost curve (D_i), which relates to the number of persons required to agree, will not shift. That is, the decision cost curve will not shift with changes in group size.[2]

External costs increase as group size (N) increases, *ceteris paribus*. Assuming N is increased from 50 to 100, while the number of persons required to agree (N_a) remains unchanged, the probability of an adverse decision clearly increases. Thus, if group size is increased from N to N', the external cost curve shifts upwards, say from C_i to C_i'. If this shift is not accompanied by an increase in N_a, then the *proportion* of the group required to agree (N_a/N) will decline. If, on the other hand, N_a is increased so that $N_a'/N' = N_a/N$, external costs and the degree of democracy will remain unchanged (see points A and B in Figure 2). The movement from point A to point B can be envisioned as embodying (1) a shift from point A on curve C_i to point S on curve C_i' because the increase in group size has increased the probability of becoming a loser under the *existing* decision rule and (2) a movement downward along curve C_i' from point S to point B because the

Figure 2. Difference in group size



probability of being a loser has been restored to its original level by changing the decision rule.[3]

Maintenance of the N_a/N ratio (degree of democracy) as the group expands will require N_a to increase, thus leading to a movement up the D_i curve (which is assumed invariant with respect to N). The net effect of an increase in group size is an upward shift in curve $[C_i + D_i]$ to $[C_i' + D_i]$. As N increases, the low point along curve $[C_i' + D_i]$ will be associated with a larger N_a (more persons are required to agree for a decision to be reached), but, since the slope of curve D_i is positive, N_a will not increase in proportion to N. Thus, the expansion of group size will lead to a decline in N_a/N (i.e., to less democracy). The effect of an increase in group size, then, is both to increase N_a and to reduce N_a/N. Thus, committees and small groups frequently use rules of relative unanimity while larger groups move in the direction of majority rule, eliticism, or dictatorship. The obvious implication of this is simply that the total costs of social interdependence are minimized by small groups.

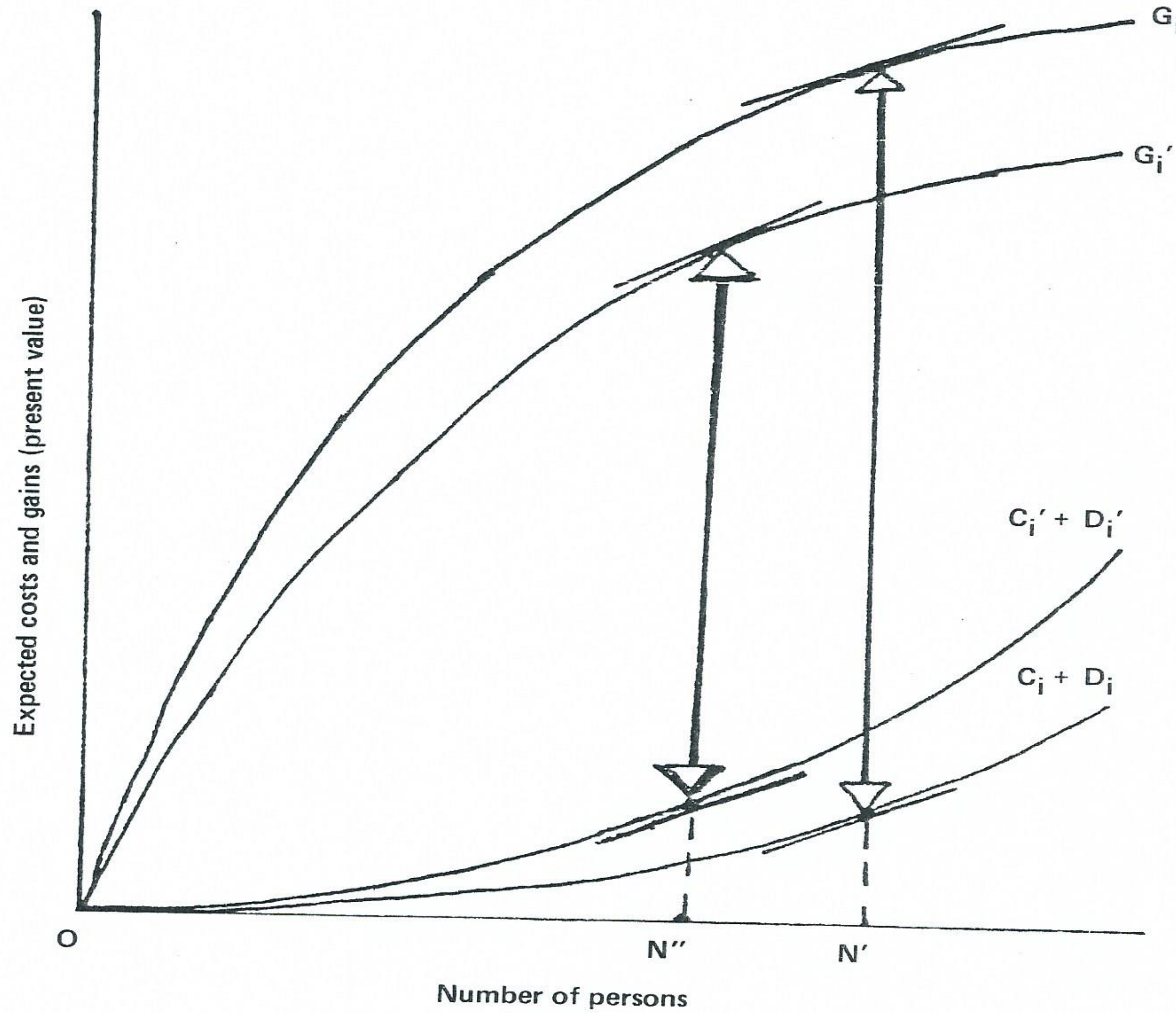
The smaller group, however, cannot enjoy the full assimilation of externality or the full exploitation of economies of scale in joint consumption. The total effect of an increase in group size (keeping preference homogeneity of the group unchanged) is summarized in Figure 3. The rising curve $[C_i + D_i]$ *in Figure 3* is defined here as the locus of *minimum points taken from Figure 2 for various group sizes*. The minimum points of the upward shifting of $[C_i + D_i]$ curves shown in Figure 2 are translated into a continuously rising $[C_i + D_i]$ curve in Figure 3, much as the long-run competitive supply curve is typically taken as a locus of minimum *LAC* points in the presence of rent or rising factor prices. Rightward movements along the $[C_i + D_i]$ curve in Figure 3 reflect the pressure to shift to representative government at some level of N, to dictatorship at another level of N, or at still another level of N to some form of decentralization under federalism, where we will have different $[C_i + D_i]$ curves and different N's and N_a's for each jurisdiction. We would, in general, expect N_a/N to decline at higher levels of government autonomy.

The curve G_i in Figure 3 measures the increase in welfare that is hypothetically available to the individual as a result of his (a) being in a position to internalize externalities through government and (b) being able to enjoy public goods at a lower tax-price. The formal relation G_i is developed nicely by Oates (1972) using compensating variation and needs no elaboration here. The *optimal size of jurisdiction,* then, is determined at the value of N where $G_i - [C_i + D_i]$ is at a maximum (N' in Figure 3).

A second extension: The effect of heterogeneity

We should now develop the effect of increased preference heterogeneity

Figure 3. Optimal group size



on the costs of collective decision-making. We let the symbol H represent the degree of heterogeneity which characterizes the group, i.e., the dispersion of the preferences of the individuals who comprise the group. To the extent that groups are more heterogeneous (H increases), we argue that curves C_i and D_i both shift upwards. In other words, the additional debate or lack of communication that accompanies increased preference heterogeneity within the group will be reflected in increased decision costs, whereas increased diversity of individual demands will result in increased external costs. The effect of the shift in the external cost curve will ordinarily be to increase the required N_a/N; an N_a that is *both* larger and more heterogeneous involves the combined effects of an upward shift in the D_i curve (because of increased heterogeneity) and a movement upward along the new D_i curve (because of the increased N_a required to offset somewhat the higher external costs). Collective action will be selected in such cases only if the gains are sufficiently large.

Let us now adjust Figure 3 to reflect these relationships. With increased heterogeneity, the $[C_i + D_i]$ curve in Figure 3 will shift upward to $[C_i' + D_i']$. If we assume (in order to maintain two-dimensional simplicity) that the increase in N is accompanied by a 'proportionate' increase in heterogeneity, the distance between these two curves will increase as N increase. On the other hand, to the extent that 'in-migrants' make the group more homogeneous, the curve $[C_i' + D_i']$ converges towards curve $[C_i + D_i]$.

The appearance of preference heterogeneity within the group also introduces an important, although heretofore neglected, element into the analysis. Recall that curve G_i represents the gross gain due to joint or collective action. However, as heterogeneity is introduced into the group, collective action will impose a new type of welfare loss. Specifically, all persons have to consume the same amount of the collective good, but as the dispersion of preferences increases, the utility (welfare) of the 'typical' individual in the group decreases. In other words, increased heterogeneity diminishes the potential gains from collectivization. Hence, if curve G_i is drawn initially on the assumption of preference homogeneity within the group, it must now be adjusted for preference heterogeneity. In Figure 3, the curve G_i corresponds to a given preference homogeneity within the group. If preference homogeneity is reduced, the resulting (gross) welfare loss shifts the G_i schedule downwards, say, to G_i'. Under these conditions, the *optimal size of jurisdiction* would be found by maximizing the difference between G_i' and $[C_i' + D_i']$. This would tend to decrease the optimal size jurisdiction, as Figure 3 illustrates, at N''.

Finally, we note that the effect of changes in the homogeneity of preferences of the group on the decision rule (Na and Na/N) cannot be predicted because the effects on external versus decision costs cannot be predicted on an a priori basis. However, since increases in heterogeneity increase costs while reducing potential gains, the probability of gainful collective action is diminished.

Summary

The original Buchanan-Tullock formulation of collective decision-making costs may be expanded to:

$$C_i = C_i(N_a, N, H) \quad (1)$$

$$D_i = D_i(N_a, H) \quad (2)$$

$$G_i = G_i(N, H) \quad (3)$$

Analysis of the effects of group size (N), decision rules (N_a), and homogeneity (H) on external costs (C_i), decision-making costs (D_i), and welfare gains (G_i) can be organized geometrically. Although this analysis extends the Buchanan-Tullock model and suggests a means of geometric manipulation that may be useful in conceptualizing problems of institutional organization much remains to be done. The conditions for optimal group size, degree of heterogeneity, decision rules, and the like require a more extensive effort. Although the general relationships embodied in the external cost and decision-cost functions seem logical and conform to casual observation, their properties – limits, slopes, etc. – have not been developed rigorously or tested empirically; nor have all the insights of other social scientists been brought to bear.

NOTES

1. Related to this notion, see also Buchanan (1968) and Cebula and Schaffer (1975).
2. If D_i does in fact shift with changes in N, our general conclusions do not change.
3. It is possible that the individual will evaluate a given N_a/N differently depending upon the size of the group. Does majority rule, for example, mean the same cost to the individual when practiced in a group of 50 as it does when practiced in a group of 100? The loss of 'indirect' influence will most probably lead the individual to perceive lower costs in the smaller group. This would lead to an adjustment in the C_i curve, lowering it as we move leftward from some given point on the horizontal axis of Figure 2 and raising it as we move rightward from the same point. This possibility does not affect our general results.

REFERENCES

Buchanan, James M. (1968). *The Demand and Supply of Public Goods*. Chicago, Ill.: Rand McNally.

Buchanan, James M., and Tullock, Gordon. (1962). *The Calculus of Consent*. Ann Arbor, Mich.: University of Michigan Press.

Cebula, Richard J., and Schaffer, Beverly K. (1975). 'Revealed Preferences for Pure Public Goods: A Pedagogical Note.' *Rivista Internazionale di Science Economiche e Commerciali*, 22, (Sept.).

Oates, Wallace. (1972). *Fiscal Federalism*. New York: Harcourt, Brace, and Jovanovich.