Child Allowances, Educational Subsidies and Economic Growth

Hung-Ju Chen

Department of Economics, National Taiwan University

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ABSTRACT

This paper examines the effects on economic growth attributable to government policies of child allowances and educational subsidies. We show that multiple steady states may arise under these two policies, with club convergence occurring, and the initial condition being of relevance, if the tax rate is fairly high. Under a policy of child allowances, an increase in the tax rate is found to raise the quantity of children, but lower the quality of adults; however, under a policy of educational subsidies, with an increase in the tax rate, corresponding increases are found in both the quantity of children and the quality of adults. We also find that considering the 'threshold' effects of technological externalities, an economy can escape the poverty trap if the threshold is sufficiently low. For developed countries, introducing child allowances may improve or hurt the welfare while introducing educational subsidies is welfare improving.

Keywords: Child allowances; Fertility; OLG, Skill.

JEL Classification: J13, J24, O11.

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* Department of Economics, National Taiwan University, 21 Hsu-Chow Road, Taipei 100, Taiwan; Tel: 886-2-23519641 ext. 535; Fax: 886-2-23582284; e-mail: hjc@ntu.edu.tw.
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1. INTRODUCTION

Child allowances and educational subsidies have become popular governmental policies over recent decades, with such need for government intervention occurring when the competitive economy fails to attain certain important social goals. Over the course of development, there has been a steady decline in fertility in many countries, leading to concerns over the sustainability of social welfare systems. On the one hand, the external effects of children attributable to intra-generational transfers, such as ‘pay-as-you-go’ social security systems, have led to the fertility choices made by individuals falling below the social optimum, thereby giving rise to incentives for governments to provide child allowances in order to raise the fertility rate (van Groezen, Leers and Meijdam, 2003). On the other hand, as a result of the recognition of human capital accumulation as an important engine for growth, in most of the developed nations, not only are there public schooling provisions in place for basic education, but also significant subsidies for higher education.

In this paper, we focus on these trends in our examination of the effects on economic growth attributable to the adoption of policies of child allowances and educational subsidies based on an overlapping-generations (OLG) model. We are particularly interested in the ways in which these two policies affect economic growth through the channel of population structure, such as the quality and quantity of the labor force. Within the considerable number of prior studies on endogenous fertility, there has been a general tendency to adopt the idea originally proposed by Becker, Murphy and Tamura (1990), that the decisions of parents are taken on the basis of both the quantity and quality of their children. Under such a setting, there is no social inter-group mobility, since the children of skilled parents remain skilled and those of unskilled parents remain unskilled (de la Croix and Doepke, 2003, 2004); as such, the fertility choices of skilled and unskilled parents in the current period will clearly determine the future composition of the labor market.

Different from the prior studies, in the present study we assume that adults make the decision on the pursuit of higher education. There are two types of workers in the present study, with those agents receiving (not receiving) higher education being referred to as skilled (unskilled) workers. Besides decision on education, adults also need to make decisions on consumption and fertility; that is, adults need to allocate their time between education, raising children and working, with changes in fiscal policies consequently having immediate impacts on the composition of the labor market. Policies such as the provision of child allowances or educational subsidies which increase the amount of time spent on raising children or on the accumulation of skills may reduce the amount of time available for working, with a corresponding reduction in savings. Within the prior studies based on OLG models, there has been a tendency to separate schooling and working time by assuming that agents accumulate human capital (skills) and work during different periods, thereby ignoring the fact that in order to pursue higher education, agents must sacrifice their working time. We therefore complement the extant literature in the present study by allowing adults to make decisions on the allocation of their time between working and skills accumulation.

Over the postwar period, there are large increases in the relative quantity of skilled labor and the skill premium in U.S. and one explanation of this phenomenon is the increase in the demand for skilled workers due to skill-biased technological changes.[1] The empirical study of Krusell, Rios-Rull and Violante (2000) demonstrates that in addition to low-wage foreign labor, unskilled labor would also compete with cheaper and better capital equipment; we therefore assume that when producing

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[1] See Davis and Haltwanger (1991), Krueger (1993) and Autor, Katz and Krueger (1998).

output, physical capital and skilled labor are more complementary than physical capital and unskilled labor.[2] The complementarity that exists between capital and skilled workers is a novel introduction to the current literature on the analysis of child allowance and educational subsidy policies. Such complementarity between capital and skilled labor in the production function plays an important role in our model, essentially because it changes the concave property of the law of motion of capital per worker, with the potential for the existence of multiple stable steady states.

For a poor economy, where skilled workers and unskilled workers coexist, with an increase in capital, there will be a corresponding increase in the proportion of skilled workers within the working population through the complementarity between capital and skills. This leads to low fertility and high savings, both of which are beneficial to capital accumulation. Therefore, the decreasing returns to capital can be overcome as a result of the complementarity between capital and skills, and thus, the law of motion of capital per worker is convex. Conversely, for a rich economy, where only skilled workers exist, there will be decreasing returns to capital, and the law of motion of capital per worker will be concave. Multiple stable steady states may therefore occur as a result of the convex-concave combination of the law of motion of capital per worker.

In order to provide child allowances and educational subsidies, government levies progressive income tax. These governmental policies would affect agents' decisions with regard to fertility, savings and the accumulation of skills, and would therefore also affect long-run economic growth. In conjunction with the complementarity between capital and skilled workers within the production function,

[2] Studies of Katz and Murphy (1992) and Goldin and Katz (1998) also demonstrate that physical capital and skilled labor are more complementary than physical capital and unskilled labor. Griliches (1969) provides an early analysis of the complementarity of physical capital and skills.

the choices on fertility and skills generate a convex-concave combination of the law of motion of capital per worker. [3]

We find that in an economy with child allowances, a higher tax rate will bring about a quantity-quality trade-off within the population, essentially because it will increase the quantity of children, whereas it will reduce the average educational level within society as a whole. The aggregate savings rate will be reduced, which in turn, will lead to a reduction in the accumulation of capital per worker; such a scenario is obviously detrimental to economic growth.

When a government provides educational subsidies, there will be a general increase in both the quantity of children and the quality of adults; nevertheless, with increases in the amount of time spent on education and raising children, there will be a corresponding reduction in working time. Despite the fact that this would raise the overall quality of adults, the aggregate savings rate would be reduced with the tax rate as a result of the lower savings of skilled workers; such lower aggregate savings would, in turn, further reduce capital accumulation and economic growth.

Our results indicate that under both scenarios, multiple stable non-trivial steady states may occur, depending on the magnitude of the tax rate. A fairly high tax rate may generate club convergence, and thus, the initial condition would be of relevance; those countries starting from high initial levels of capital per worker would converge to a 'good' steady state, whereas others starting from low initial values of capital per worker would remain trapped in poverty. If the tax rate was sufficiently low (high), there would be a unique, stable steady state, with the economy converging to a 'good' ('bad') steady state, irrespective of the state at which it started. Technological progress can help those countries locked in the poverty trap to liberate their

[3] The approach adopted in this study is closely related to that of Galor and Weil (1996) who analyze the relationship between the gender gap, fertility and growth based upon a similar shape of the law of motion of capital per worker.

economies. With the ‘threshold’ effects on the technology externalities, a sufficiently low threshold can help the economy get out of the poverty trap. However, club convergence would still occur if the threshold was sufficiently high.

This study therefore contributes to the extant literature on the poverty trap by demonstrating that multiple stable steady states may occur under a scenario of an economy where provisions are in place for child allowances or educational subsidies. To the best of our knowledge, this is a novel finding within the related literature. Becker, Murphy and Tamura (1990) have shown that endogenizing fertility choice and the degree of altruism can generate non-ergodic growth while Galor and Tsiddon (1997) have shown that multiple stable steady states can easily emerge in a model with human capital accumulation which depends on the home and global externalities. But none of these studies raises policy issues.

It is already well known that increases in child allowances will raise fertility and impede economic growth due to the ‘capital-dilution’ effect (van Groezen, Leers and Meijdam, 2003; van Groezen and Meijdam, 2008). The possibility of child allowances causing multiple equilibria has been examined by the more recent studies. For example, Fanti and Gori (2012) find that if the amount of child allowances provided is higher than the costs attributable to raising children, then the provision of child allowances will cause two positive steady states; however, in their study, the concavity of the law of motion of capital per worker remains invariable, and as a result, only one of the positive steady states is stable, while the other is unstable. Their result is therefore unable to explain the phenomenon of the non-convergence of economic growth found in the prior empirical studies.

A more surprising finding in the present study is that increases in educational subsidies will hamper economic growth. Ever since the seminal work of Lucas (1988), it has been generally recognized within the related studies that human capital accumulation is an important determinant of economic growth; therefore, any policies which are beneficial to human capital accumulation, such as educational subsidies or public schooling, should encourage economic growth.[4] Our finding that an increase in educational subsidies will hinder economic growth is due to three reasons. First, a higher income tax rate will lower the motivation of being skilled workers; second, the increased time spent on education and raising children will result in less time being available for working; and third, an increase in the average fertility rate will give rise to a capital-dilution effect, ultimately reducing the total capital per worker.

Even though introducing child allowances or educational subsidies may reduce consumption and savings, it does not necessarily imply that the introduction of the policy hurts welfare since fertility may increase. Regarding welfare, we find that for developed countries, introducing educational subsidies is welfare improving while introducing child allowances may improve or hurt the welfare, depending on how fertility responses to the policy.

The remainder of this paper is organized as follows. A description of the economic environment is provided in Section 2. This is followed in Section 3 by analysis of the economy with the provision of child allowances, and subsequently in Section 4, by a similar analysis of the economy with the provision of educational subsidies. A comparison of the two policies is then carried out in Section 5. Finally, the conclusions drawn from this study are presented in Section 6.

2. THE MODEL

We consider an infinite-horizon, discrete time overlapping generations model where agents live for three periods - childhood, adulthood (parenthood), and old age. All

[4] See Glomm (1997), Chen (2005) and Chen and Azariadis (2013).

decisions are made in adulthood, with adults also deciding whether or not to receive higher education to become skilled workers (s) or unskilled workers (u).

2.1 Households

Adults in period t care about the number of children (n_t^i) and their old-age consumption (c_{t+1}^i), where i represents the type of agents. The utility function, which is identical for all agents, is defined as:

$$u_t^i = \ln n_t^i + \beta \ln c_{t+1}^i, \qquad i = s, u, \tag{1}$$

where $\beta \in (0,1)$ is the discount factor.

In each period, agents are endowed with one unit of time. Adults need to work to earn the wages (w_t^i). In addition to working, adults also need to choose to spend how much time on raising children (z_t^i) and to acquire higher education or not.[5] We assume that it takes a fixed proportion ($\sigma \in (0,1)$) of time for each adult to acquire higher education to become a skilled worker.

Raising children requires time cost and resource cost. Fertility is assumed to be an increasing function of child allowances (G_t) and time spent on raising children for type-i adults;[6] that is,

$$n_t^i = q\left(z_t^i\right)^{\gamma}(1 + G_t)^{1-\gamma}, \qquad q > 0, \tag{2}$$

where $\gamma, 1-\gamma \in (0,1)$ respectively represent the elasticities of fertility with respect to time devoted to raising children and child allowances.[7]

The government levies progressive income tax in order to provide public subsidies for raising children (G_t) or education (E_t); for simplicity, it is assumed that the tax rate for unskilled workers is zero, while the tax rate imposed on skilled worker

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[5] The estimates by Cigno and Rosati, (1996) provide support the hypothesis that fertility and saving are jointly determined by self-interested agents.

[6] Empirical studies showing a positive effect of child allowances on fertility have been provided by Manski and Mayshar (2003), Cohen, Dehejia and Romanov (2007) and Schellekens (2009).

[7] Similar setting of the fertility function can be also found in Momota (2000), Balestrino, Cigno and Pettini (2003) and Apps and Rees (2004).

is $\tau \in (0,1)$. Following Bovenberg and Jacobs (2005), we assume that educational subsidies compensate foregone labor time of skilled workers due to schooling. Therefore, educational subsidies are assumed to be proportional to the wage loss; that is, $E_t = g\sigma w_t^s$, where $g \in (0,1)$. Savings (a_t^i) for skilled workers are therefore:

$$a_t^s = (1-\tau)(1-\sigma-z_t^s)w_t^s + E_t. \quad (3)$$

If adults decide not to acquire higher education, they become unskilled workers and face with the following budget constraint:

$$a_t^u = (1-z_t^u)w_t^u. \quad (4)$$

The budget constraint for an old agent is therefore:

$$c_{t+1}^i = R_{t+1}a_{t+1}^i, \quad i = s, u, \quad (5)$$

where R_{t+1} is the gross rate of return of savings.

The total population of workers (adults) in period t is denoted by N_t. Let L_t^s denote skilled labor, and L_t^u denote unskilled labor, with the respective ratios of skilled and unskilled workers to the adult population being represented by ϕ_t and $(1-\phi_t)$; thus, skilled labor is $L_t^s = (1-\sigma-z_t^s)\phi_t N_t$ and unskilled labor is $L_t^u = (1-z_t^u)(1-\phi_t)N_t$. We assume that the government runs a balanced budget, and given that the tax revenue is used to provide subsidies for raising children (child allowances) or for providing education, this implies that:

$$G_t N_t = \delta\tau w_t^s L_t^s, \quad (6a)$$

$$E_t \phi_t N_t = (1-\delta)\tau w_t^s L_t^s, \quad (6b)$$

where $\delta = 0, 1$.

2.2 Production

Output is produced, using physical capital (K_t), skilled labor and unskilled labor, based upon the following production function:

$$Y_t = A[K_t^\alpha (L_t^s)^{1-\alpha} + bL_t^u], \quad A, b > 0. \quad (7)$$

Eq. (7) indicates that there is greater complementarity between skilled labor and capital than between unskilled labor and capital.[8] Let $k_t = K_t/N_t$ represent capital per worker. The gross rate of the return on capital and the real wage rates of skilled labor (w_t^s) and unskilled labor (w_t^u) are:

$$R_t = A\alpha\left[\frac{k_t}{(1-\sigma-z_t^s)\phi_t}\right]^{\alpha-1}, \tag{8}$$

$$w_t^s = A(1-\alpha)\left[\frac{k_t}{(1-\sigma-z_t^s)\phi_t}\right]^{\alpha}, \tag{9}$$

$$w_t^u = Ab. \tag{10}$$

Eqs. (8) and (9) imply that the ratio of the wage rate of unskilled workers to that of skilled workers (υ_t) is:

$$\upsilon_t = \frac{w_t^u}{w_t^s} = \frac{b}{(1-\alpha)\left[\frac{k_t}{(1-\sigma-z_t^s)\phi_t}\right]^{\alpha}}. \tag{11}$$

2.3 Equilibrium

The equilibrium of the economy is defined as follows. Given the initial level of capital per worker (k_0), the fiscal policy and the factor prices $\{R_t, w_t^s, w_t^u\}$, an equilibrium comprises of sequences of aggregate physical capital $\{K_t\}$, working population $\{N_t\}$ and households' decision rules $\{a_t^i, z_t^i, c_{t+1}^i\}$, $i = s, u$, such that:

1. Given the factor prices and the fiscal policy, skilled and unskilled workers will make their decisions by maximizing their utility subject to budget constraints;
2. The factor price equations, Eqs. (8)-(10), hold;
3. The markets clear;
4. The government runs a balanced budget;
5. Fertility is governed by Eq. (2).

3. CHILD ALLOWANCES

[8] This type of production is also adopted by Galor and Weil (1996), Kimura and Yasui (2007), Chen (2010) and Azariadis and Chen (2013).

We first consider an economy where the government provides child allowances; that is, $\delta = 1$. Skilled workers will maximize Eq. (1), subject to Eqs. (2), (3) and (5). The optimal choices of z_t^s and s_t^s are:

$$z_t^s = \frac{\gamma(1-\sigma)}{\gamma+\beta}, \tag{12}$$

$$a_t^s = \frac{\beta(1-\tau)(1-\sigma)w_t^s}{\gamma+\beta}. \tag{13}$$

Similarly, unskilled workers will maximize Eq. (1) subject to Eqs. (2), (4) and (5). The optimal decisions of z_t^u and s_t^u are:

$$z_t^u = \frac{\gamma}{\gamma+\beta}, \tag{14}$$

$$a_t^u = \frac{\beta w_t^u}{\gamma+\beta}. \tag{15}$$

Note that $z_t^s < z_t^u$. This is because the opportunity cost of raising children is higher for skilled workers than for unskilled workers and skilled workers will spend less time raising children.

Based on the government budget constraint, Eq. (6a), the amount of children allowances received by each adult is:

$$G_t = \tau\xi k_t^{\alpha}\phi_t^{1-\alpha}, \tag{16}$$

where $\xi = A(1-\alpha)\left[\frac{(1-\sigma)\beta}{\gamma+\beta}\right]^{1-\alpha}$. The fertility of skilled and unskilled workers can be derived by substituting Eq. (16) and the optimal choice of z_t^s and z_t^u into Eq. (2). Based on Eq. (2), skilled workers will have fewer children than unskilled workers.

All workers are freely mobile between the skilled and unskilled labor markets, and thus, at equilibrium, they will be indifferent between becoming skilled or unskilled workers; that is, $u_t^s = u_t^u$. Plugging the optimal decisions on consumption and fertility into the utility functions for skilled and unskilled workers, we can obtain the ratio of the wage rate of unskilled workers to that of skilled workers, which is expressed as:

$$v_t = \frac{w_t^u}{w_t^s} = (1-\tau)(1-\sigma)^{1+\frac{\gamma}{\beta}}. \tag{17}$$

Combining Eqs. (11) and (17), we obtain the ratio of skilled workers to the adult population, which depends on τ and k_t, as follows:

$$\phi(\tau, k_t) = \eta(\tau)k_t, \tag{18}$$

where $\eta(\tau) = \frac{\alpha+\beta}{\beta}\left[\frac{(1-\alpha)(1-\tau)}{b}(1-\sigma)^{1-\alpha+\frac{\gamma}{\beta}}\right]^{\frac{1}{\alpha}}$ and $\eta'(\tau) < 0$.

Since income tax is only levied on skilled workers, Eq. (18) indicates that an increase in the tax rate will lower the ratio of the population of skilled workers to the adult population. We use $\bar{k}$ denote the value that ϕ_t reaches its upper bound, 1; that is, $\phi(\tau, \bar{k}) = 1$. From Eq. (18), we have:

$$\bar{k}(\tau) = \frac{1}{\eta(\tau)} \ .$$

This indicates that an increase in the tax rate will raise $\bar{k}$. Then we have:

$$\phi(\tau, k_t) = \begin{cases} \eta(\tau)k_t & \text{if} \quad k_t < \bar{k} \\ 1 & \phantom{\text{if}} \quad k_t \geq \bar{k} \end{cases}. \tag{19}$$

Using Eq. (19) and fertility choices of skilled and unskilled workers, the average fertility rate of the economy (m_t) can be calculated as:

$$\begin{aligned} m(\tau, k_t) &= \phi(\tau, k_t)n_t^s + (1-\phi(\tau, k_t))n_t^u \\ &= q(1+\tau\xi k_t^{\alpha}\phi_t^{1-\alpha})^{1-\gamma}\left(\frac{\gamma}{\gamma+\beta}\right)^{\gamma}[1-\tilde{\sigma}\phi(\tau, k_t)], \end{aligned} \tag{20}$$

where $\tilde{\sigma} = 1-(1-\sigma)^{\gamma}$, with $0 < \tilde{\sigma} < 1$.

Combining Eqs. (16), (19) and (20), the average fertility is expressed as:

$$m(\tau, k_t) = \begin{cases} q[1+\tau\xi\eta(\tau)^{1-\alpha}k_t]^{1-\gamma}\left(\frac{\gamma}{\gamma+\beta}\right)^{\gamma}[1-\tilde{\sigma}\eta(\tau)k_t] & \text{if} \quad k_t < \bar{k} \\ q(1+\tau\xi k_t^{\alpha})^{1-\gamma}\left[\frac{(1-\sigma)\gamma}{\gamma+\beta}\right]^{\gamma} & \phantom{\text{if}} \quad k_t \geq \bar{k} \end{cases}. \tag{21}$$

We then obtain the following results.

Proposition 1. In an economy with child allowances, an increase in the tax rate will raise the average fertility rate of the economy if $\tau < \alpha$.

Proof: See Appendix A.

If $k_t < \bar{k}$, then tax rate will affect the average fertility through two channels, the ratio of the population of skilled workers to the adult population and the amount of child allowances. Since only skilled workers need to pay the income tax, an increase in the tax rate will lower the ratio of the population of skilled workers to the adult population. Although an increase in the tax rate raises the contribution of tax revenue, the lower ratio of the population of skilled workers to the adult population reduces the tax base. If the tax rate is low enough such that $\tau < \alpha$, the ratio of the population of skilled workers to the adult population will not decrease too much and the first effect will dominate the second effect, leading to an increase in the amount of child allowances. This resultant increase in child allowances will in turn raise the average fertility. Besides, with more adults choosing to become unskilled workers, the average fertility rate will increase even further, since unskilled workers will have higher fertility rate than skilled workers. Based upon annual US time-series data, Krusell, Rios-Rull and Violante (2000) show that between 1963 and 1990, the share of aggregate income attributable to labor was about 70%. Since in our paper, the tax revenue is only used to provide child allowances or educational subsidies, it is reasonable to assume that $\tau < \alpha$. If $k_t \geq \bar{k}$, an increase in the tax rate will raise the amount of child allowances, leaving the ratio of the population of skilled workers to the adult population equal to one. Therefore, the average fertility will increase.

3.1 Dynamics

We are now ready to study the dynamic properties of the economy. The clearing condition of the capital market implies that:

$$K_{t+1} = [\phi_t s_t^s + (1 - \phi_t) s_t^u] N_t,$$

with this equation then being rewritten in terms of capital per worker as:

$$k_{t+1} = \frac{K_{t+1}}{N_{t+1}} = \frac{K_{t+1}}{m_t N_t} = \frac{[\phi_t s_t^s + (1-\phi_t) s_t^u]}{m_t}. \tag{22}$$

Using Eqs. (9), (10), (13) and (15) to substitute wage rates and savings in Eq. (22), we obtain the law of motion of capital per worker within the economy as:

$$k_{t+1} = \frac{(1-\tau)\xi k_t^{\alpha} \phi_t^{1-\alpha} + \frac{A\beta b}{\gamma+\beta}(1-\phi_t)}{m_t}. \tag{23}$$

Substitute ϕ_t and m_t in Eq. (23) by using Eqs. (19) and (21) to, the law of motion of capital per worker becomes:

$$k_{t+1} = f(\tau, k_t) = \begin{cases} f_1(\tau, k_t) & \text{if} \quad k_t < \bar{k} \\ f_2(\tau, k_t) & \phantom{\text{if}} \quad k_t \geq \bar{k} \end{cases}, \tag{24}$$

where

$$f_1(\tau, k_t) = \frac{\frac{A\beta b}{(\gamma+\beta)^{1-\gamma}\gamma^{\gamma}} \left\{1 + \eta(\tau) \left[(1-\sigma)^{\frac{-\gamma}{\beta}} - 1\right] k_t \right\}}{q[1 + \tau\xi\eta(\tau)^{1-\alpha} k_t]^{1-\gamma} [1 - \tilde{\sigma}\eta(\tau) k_t]}. \tag{25}$$

and

$$f_2(\tau, k_t) = \frac{(1-\tau)\xi k_t^{\alpha}}{q\left[\frac{(1-\sigma)\gamma}{\gamma+\beta}\right]^{\gamma} (1 + \tau\xi k_t^{\alpha})^{1-\gamma}}. \tag{26}$$

In order to analyze dynamic behavior of the economy with the provision of child allowances, we make the following restriction with regard to τ.

Assumption 1. In an economy with child allowances, the tax rate is small enough such that $\tau < \min\{\alpha, \bar{\tau}\}$, where $\bar{\tau} = \frac{\tilde{\sigma}}{\tilde{\sigma} + \frac{A\beta b}{\gamma+\beta}(1-\sigma)^{-\frac{\gamma}{\beta}}}$.[9]

Appendix B shows that under assumption 1, $f_1(\tau, k_t)$ is convex in k_t and $f_2(\tau, k_t)$ is a concave function in k_t. Finally, we have:

$$f_1(\tau, 0) = \frac{A\beta b}{q(\gamma+\beta)^{1-\gamma}\gamma^{\gamma}} > 0, \quad \lim_{k_t \to \infty} \frac{\partial f_2(\tau, k_t)}{\partial k_t} = 0. \tag{27}$$

[9] The assumption that $\tau < \bar{\tau}$ is made in order to determine the shape of $f_1(\tau, k_t)$. See Appendix B.

Therefore, we can determine that, if $k_t < \bar{k}$, then the law of motion of capital per worker is convex, and if $k_t \geq \bar{k}$, then it becomes concave; together with Eq. (27), this implies the existence of a steady state. However, these conditions are not sufficient to guarantee the uniqueness of the steady state. There are, at most, three steady states, which are referred to as k_L^*, k_M^* and k_H^*, where k_L^* and k_M^* are the solutions for $k^* = f_1(\tau, k^*)$ and k_H^* is the non-trivial solution for $k^* = f_2(\tau, k^*)$, with $k_L^* < k_M^* < k_H^*$. Note that both skilled and unskilled workers exist in the economy at k_L^*, whereas the economy comprises of only skilled workers at k_H^*.

As regards the dynamic behavior of the economy, there are three possible situations. If $f(\tau, \bar{k}) > \bar{k}$ and $f(\tau, k_t) > k_t$ for all $k_t < \bar{k}$, then there will be a unique, stable steady state, k_H^*, and the economy will converge to a high level of steady state, k_H^*, regardless of its initial condition (see $f(\tau_A, k_t)$ in Figure 1).

<Figure 1 is inserted about here>

In Figure 1, $f(\tau_B, k_t)$ describes the dynamic system of the economy for the case where $f(\tau, \bar{k}) > \bar{k}$ and there exists $k_t < \bar{k}$ such that $f(\tau, k_t) < k_t$. In this case, there are multiple steady states (k_L^*, k_M^* and k_H^*) (see $f(\tau_C, k_t)$ in Figure 1). Of these three steady states, k_L^* and k_H^* are stable, whilst k_M^* is unstable; as a result, club convergence occurs and the initial value of capital per worker matters. If $k_0 < k_M^*$, the economy will converge to an underdevelopment trap (k_L^*) with the coexistence of skilled and unskilled workers. On the other hand, if $k_0 > k_M^*$, the economy will converge to a high level of capital per worker (k_H^*) where only skilled workers will exist. If $f(\tau, \bar{k}) < \bar{k}$, there will be a unique steady state (k_L^*). The economy will converge to this steady state and be trapped in poverty no matter where it starts (see $f(\tau_D, k_t)$ in Figure 1).

3.2 Effects of the tax rate

We go on to examine the effects of the tax rate on the economic behavior in this sub-section. First, we note that with an increase in the tax rate, there is a corresponding rise in $\bar{k}$, whilst $f_1(\tau,0)$ remains unchanged. We can obtain the impact of τ on k_H^* from Equation (26), as follows:

$$k_H^{*\prime}(\tau) = -\frac{(1-\gamma+\gamma\tau)\xi(k_H^*)^{1+\alpha}}{(1-\tau)\{1-\alpha+(1-\alpha\gamma)\tau\xi(k_H^*)^{\alpha}\}} < 0;$$

that is, with an increase in τ, there is a corresponding reduction in k_H^*. The following proposition illustrates how the tax rate affects the law of motion of capital per worker.

Proposition 2. In an economy with child allowances, an increase in the tax rate will result in a downward rotation of the locus of the law of motion of capital per worker if Assumption 1 holds.

Proof: See Appendix C.

We first consider the case when $k_t \geq \bar{k}$. In this case, only skilled workers exist in the economy. As demonstrated in the Proposition 1, an increase in the tax rate will raise the average fertility, while also reducing the after-tax income and savings for skilled workers. Both effects will lead to a reduction in capital per worker, and as such, an increase in the tax rate will lead to a downward shift of the locus of $f_2(\tau, k_t)$.

In the case where $k_t < \bar{k}$, a higher tax rate will directly reduce the after-tax income for skilled workers. However, the lower ratio of the population of skilled adults to the adult population implies an increase in the wage rate for skilled workers. Savings for skilled workers will remain unchanged since the positive and negative effects caused by the increase in the tax rate will cancel each other out, and the aggregate savings for the economy as a whole will be reduced as a results of the lower

ϕ_t.[10] Thus, in conjunction with the increase in the average fertility, there will be a downward rotation of $f_1(\tau, k_t)$.

Figure 1 presents the locus of the law of motion of capital per worker $f(\tau_A, k_t)$, $f(\tau_B, k_t)$, $f(\tau_C, k_t)$ and $f(\tau_D, k_t)$ with $\tau_A < \tau_B < \tau_C < \tau_D$. The parameterization in Figure 1 is assumed to satisfy the conditions that $f(\tau, \bar{k}) > \bar{k}$ and $f(\tau, k_t) > k_t$ for all $k_t < \bar{k}$ when τ equals 0 and $f(\tau, \bar{k}) < \bar{k}$ when τ is close to 1. We begin our analysis by considering the dynamic transition of an economy with a low tax rate (τ_A). Under this scenario, the economy will converge to $k_H^*(\tau_A)$ regardless of where it starts from. Once when the tax rate exceeds τ_B, club convergence will begin to occur. When the tax rate reaches τ_C, multiple stable non-trivial steady states will exist. If the initial level of capital per worker in the economy is sufficiently large, then the economy will be able to converge to $k_H^*(\tau_C)$; however, those economies that are particularly poor, and those which start from low initial values of capital per worker, will be stuck in poverty ($k_L^*(\tau_C)$). Under a scenario where the tax rate is sufficiently large (such as τ_D), the economy will be trapped in the underdevelopment state $k_L^*(\tau_D)$.

4. EDUCATIONAL SUSIDIES

We now go on to consider an economy where the government provides educational subsidies; that is, $\delta = 0$. Recall that such subsidies are proportional to the wage loss due to schooling; that is, $E_t = g\sigma w_t^s$, where $g \in (0,1)$. Since government runs a balanced budget and only skilled workers pay for tax, this implies that $g = \tau$.

The optimal choices for skilled workers are affected by the educational subsidies and become:

$$z_t^s = \frac{\gamma(1-\sigma)}{\gamma + \beta(1-\tau)}, \tag{28}$$

[10] Based on Eqs. (9), (12) and (13), we can derive the savings for skilled workers as $s_t^s = (1-\tau)\xi k_t^\alpha \phi_t^{-\alpha}$. Substituting Eq. (19) into the saving function, we have $s_t^s = A\beta b(1-\sigma)^{\frac{-\gamma}{\beta}}/(\gamma+\beta)$, which is independent of the tax rate.

$$a_t^s = \frac{\beta(1-\tau)(1-\sigma)w_t^s}{\gamma+\beta(1-\tau)}. \quad (29)$$

The optimal decisions for unskilled workers are the same as those in Eqs. (15) and (16). In order to guarantee that $z_t^s < z_t^u$, we assume $\tau < \frac{\sigma(\gamma+\beta)}{\beta}$ in an economy with educational subsidies. The free mobility between being skilled unskilled workers implies that the ratio of the wage rate of unskilled workers to that of skilled workers is:

$$v_t = \frac{w_t^u}{w_t^s} = (1-\tau)\left[\frac{(1-\sigma)(\gamma+\beta)}{\gamma+\beta(1-\tau)}\right]^{1+\frac{\gamma}{\beta}}. \quad (30)$$

Combining Eqs. (12) and (30), we can express the ratio of skilled workers to the adult population, in terms of τ and k_t:

$$\phi(\tau, k_t) = \mu(\tau)k_t, \quad (31)$$

where $\mu(\tau) = \frac{1}{\beta}\left[\frac{1-\alpha}{b}(1-\tau)^{1-\alpha}(\gamma+\beta)^{1+\frac{\gamma}{\beta}}\left[\frac{1-\sigma}{\gamma+\beta(1-\tau)}\right]^{1-\alpha+\frac{\gamma}{\beta}}\right]^{\frac{1}{\alpha}}$. Notice that $\mu'(\tau) = \frac{\gamma(\alpha-\tau)\mu(\tau)}{\alpha(1-\tau)[\gamma+\beta(1-\tau)]}>0$.

We use $\tilde{k}$ to represent the value that $\phi(\tau, \tilde{k}) = 1$. Based on Eq. (31), we can then calculate that:

$$\tilde{k}(\tau) = \frac{1}{\mu(\tau)}.$$

Since $\mu'(\tau) > 0$, we have that $\tilde{k}(\tau)$ is a decreasing function in τ.

Therefore, we have:

$$\phi(\tau, k_t) = \begin{cases} \mu(\tau)k_t & \text{if} \quad k_t < \tilde{k} \\ 1 & \phantom{\text{if}} \quad k_t \geq \tilde{k} \end{cases}. \quad (32)$$

Using Eq. (32), the average fertility rate of the economy is derived as:

$$m(\tau, k_t) = \begin{cases} q\left[\left(\frac{\gamma}{\gamma+\beta}\right)^{\gamma} - \zeta(\tau)\mu(\tau)k_t\right] & \text{if} \quad k_t < \tilde{k} \\ q\left[\frac{\gamma(1-\sigma)}{\gamma+\beta(1-\tau)}\right]^{\gamma} & \phantom{\text{if}} \quad k_t \geq \tilde{k} \end{cases}, \quad (33)$$

where $\zeta(\tau) = \left(\frac{\gamma}{\gamma+\beta}\right)^{\gamma} - \left[\frac{\gamma(1-\sigma)}{\gamma+\beta(1-\tau)}\right]^{\gamma} > 0$ and $\zeta'(\tau) < 0$. Note that $q\zeta(\tau)$ measures the fertility difference between unskilled and skilled workers. Using Eq. (33), we can state the following proposition with regard to the impact of the tax rate on the average fertility rate.

Proposition 3. In an economy with educational subsidies and $\tau < \min\left\{\alpha, \frac{\sigma(\gamma+\beta)}{\beta}\right\}$, an increase in the tax rate will raise the average fertility rate of the economy, provided that the fertility difference between unskilled and skilled workers is not too large; that is, $\zeta(\tau) < \bar{\zeta}(\tau) = \frac{\alpha\beta\gamma(1-\sigma)(1-\tau)}{[\gamma+\beta(1-\tau)](\alpha-\tau)}$.

Proof: See Appendix D.

If $k_t < \tilde{k}$, an increase in the tax rate will raise the amount of educational subsidies and the motivation of being skilled workers. However, a higher tax rate on skilled workers' income lowers the motivation of being skilled workers. If the tax rate is low enough ($\tau < \alpha$), the former effect will dominate the latter, and there will be an increase in the ratio of the population of skilled workers to the adult population. Eq. (28) indicates that an increase in the tax rate will also raise the fertility rate for skilled workers since it reduces the opportunity cost of raising children, with a resultant increase in the average fertility. Conversely, an increase in the ratio of skilled workers to the total population will lower the average fertility because the fertility rate of skilled workers is lower than that of unskilled workers. The average fertility is therefore will increase if the fertility difference between unskilled and skilled workers is not too large. If $k_t \geq \tilde{k}$, with an increase in the tax rate, there will an increase in the fertility rate of skilled workers, while the ratio of the population of skilled workers to the adult population will remain equal to 1.

Note that $\zeta(\tau)$ is a decreasing function in τ while $\bar{\zeta}(\tau)$ is an increasing function in τ. The condition that $\zeta(\tau) < \bar{\zeta}(\tau)$ will be satisfied if $\tau > \tilde{\tau}$, where $\tilde{\tau}$ satisfies $\zeta(\tilde{\tau}) < \bar{\zeta}(\tilde{\tau})$. In order to facilitate our analysis of the effects of educational subsidies, instead of Assumption 1, we make the following assumption.

Assumption 2. In an economy with educational subsidies, we assume that $\tau \in \left(\tilde{\tau}, \min\left\{\alpha, \frac{\sigma(\gamma+\beta)}{\beta}\right\}\right)$, where $\tilde{\tau}$ satisfies $\zeta(\tilde{\tau}) = \frac{\alpha\beta(1-\tilde{\tau})}{(\alpha-\tau)}\left[\frac{\gamma(1-\sigma)}{\gamma+\beta(1-\tilde{\tau})}\right]^{\gamma}$.

4.1 Dynamics

The law of motion of capital per worker within the economy can be expressed as:

$$k_{t+1} = A\frac{(1-\alpha)\left[\frac{(1-\tau)(1-\sigma)\beta}{\gamma+\beta(1-\tau)}\right]^{1-\alpha} k_t^{\alpha}\phi_t^{1-\alpha} + \frac{\beta b}{\gamma+\beta}(1-\phi_t)}{m_t}. \tag{34}$$

Using Eqs. (32) and (33) to substitute ϕ_t and m_t in Eq. (34), we can derive the law of motion of capital per worker as:

$$k_{t+1} = g(\tau, k_t) = \begin{cases} g_1(\tau, k_t) & \text{if} \quad k_t < \tilde{k} \\ g_2(\tau, k_t) & \phantom{\text{if}} \quad k_t \geq \tilde{k} \end{cases}, \tag{35}$$

where

$$g_1(\tau, k_t) = A\frac{\left\{(1-\alpha)\left[\frac{\beta(1-\tau)(1-\sigma)}{\gamma+\beta(1-\tau)}\mu(\tau)\right]^{1-\alpha} - \frac{\beta b\mu(\tau)}{\gamma+\beta}\right\}k_t + \frac{\beta b}{\gamma+\beta}}{q\left[\left(\frac{\gamma}{\gamma+\beta}\right)^{\gamma} - \zeta(\tau)\mu(\tau)k_t\right]}. \tag{36}$$

and

$$g_2(\tau, k_t) = \frac{A(1-\alpha)\left[\frac{(1-\tau)(1-\sigma)\beta}{\gamma+(1-\tau)\beta}\right]^{1-\alpha} k_t^{\alpha}}{q\left[\frac{\gamma(1-\sigma)}{\gamma+\beta(1-\tau)}\right]^{\gamma}}. \tag{37}$$

Besides, we also have:

$$g_1(\tau, 0) = \frac{A\beta b}{q(\gamma+\beta)^{1-\gamma}\gamma^{\gamma}} > 0, \quad \lim_{k_t\to\infty}\frac{\partial g_2(\tau, k_t)}{\partial k_t} = 0. \tag{38}$$

In Appendix E, we show that $g_1(\tau, k_t)$ is convex in k_t and $g_2(\tau, k_t)$ is a concave function in k_t. Together with Eq. (38), this implies the existence of a steady state.

There are, at most, three steady states which are referred to as $k_L^\#$, $k_M^\#$ and $k_H^\#$, where $k_L^\#$ and $k_M^\#$ are the solutions for $k^\# = g_1(\tau, k^\#)$ and $k_H^\#$ is the non-trivial solution for $k^\# = g_2(\tau, k^\#)$ and with $k_L^\# < k_M^\# < k_H^\#$. Since the locus of $g(\tau, k_t)$ is similar to the locus of $f(\tau, k_t)$, our analysis on the dynamic behavior of the economy under taken in the previous section can be also applied here.

4.2 Effects of educational subsidies

In order to examine the effects of the tax rate, we first derive the impact of τ on $k_H^\#$ using Eq. (37):

$$k_H^{\#\prime}(\tau) = \frac{k_H^\#}{1-\alpha}\left\{\frac{\zeta'(\tau)}{\left(\frac{\gamma}{\gamma+\beta}\right)^{\gamma} - \zeta(\tau)} - \frac{(1-\alpha)\gamma}{(1-\tau)[\gamma+\beta(1-\tau)]}\right\} < 0.$$

Thus, an increase in τ there will reduce k_H^*. The following proposition explains the way in which the tax rate affects the law of motion of capital per worker.

Proposition 4. In an economy with educational subsidies, an increase in the tax rate will cause a downward rotation of the law of motion of capital per worker if Assumption 2 holds.

Proof: See Appendix E.

As indicated earlier in Proposition 3, with an increase in the tax rate, there will be a corresponding increase in the average fertility rate, and together with the lower after-tax income and savings for skilled workers, this will reduce the overall capital per worker. Therefore, if $k_t \geq \tilde{k}$, an increase in the tax rate will cause a downward shift of the locus of the capital per worker.

If $k_t < \tilde{k}$, an increase in the tax rate will induce a higher ratio of skilled adults to the total adults population, which in turn, will lower the wage rate for skilled workers. Therefore, savings for skilled workers will decrease. Although an increase in the ratio of skilled adults to the total adults population is beneficial to the capital accumulation since skilled workers will save more than unskilled workers, this effect is not sufficiently large to offset the negative effect on the capital accumulation caused by the reduced savings of skilled workers. Hence, the aggregate savings for the economy will be reduced. Together with an increase in the average fertility, a higher tax rate will induce a downward rotation of the law of motion of capital per worker.

Figure 2 presents the economic dynamics under four tax rates with $\tau_A < \tau_B < \tau_C < \tau_D$. We assume that the parameterization in Figure 2 satisfies the conditions that $g(\tau, \tilde{k}) > \tilde{k}$ and $g(\tau, k_t) > k_t$ for all $k_t < \tilde{k}$ when τ equals 0 and $g(\tau, \tilde{k}) < \tilde{k}$ when τ is close to 1. Since the impact of the tax rate on the dynamic performance in an economy with educational subsidies is similar to that of an economy with child allowances, the results in Proposition 3 can also be applied here. Note that under educational subsidies, $\tilde{k}$ is reduced with the tax rate, while under child allowances, $\overline{k}$ is increased with the tax rate.

<Figure 2 is inserted about here>

5. DISCUSSION

In this section, we consider two possible extensions of our model: technological progress and social welfare. We first summarize our findings and a summary of the effects of the tax rate on the macroeconomic variables, based on the two alternative policies, is provided in Table 1. A comparison of the two policies reveals that the tax rate has quite diverse effects on the ratio of the population of skilled workers to the adult population, although it does have similar effects on both the average fertility rate and capital accumulation. With an increase in the tax rate, the ratio of the population of skilled workers to the adult population increases under child allowances decreases while it increases under educational subsidies.

<Table 1 is inserted about here>

Before ϕ_t reaches 1, with an increase in the tax rate, the average fertility will increase due to the increases in fertility rates for both skilled and unskilled workers as well as the increase in the population of unskilled workers under the policy of child allowances. However, under educational subsidies, the increase in the average fertility rate is mainly attributable to the increase in the fertility rate of skilled workers. Under both policies, higher average fertility rates will reduce the capital per worker, thereby giving rise to a 'capital-dilution' effect.

As regards savings, savings for both skilled and unskilled workers unchanged under a policy of child allowances. The national savings will be reduced, essentially as a result of the reduction in the ratio of skilled workers to the total population. However, under educational subsidies, with an increase in the tax rate, the national savings would be reduced as a result of the reduced savings of skilled workers.

When ϕ_t reaches 1, an increase in the tax rate will cause the same effects on fertility and savings under both policies. The fertility rate will increase while the savings will decrease with the tax rate.

5.1 Technological progress

In the absence of technological progress, a country trapped in a low output, low educational attainment equilibrium would remain there forever. As demonstrated by Azariadis and Drazen (1990), one explanation for the phenomenon of non-convergent long-run growth paths within the extant economic growth literature is the 'threshold' property of technological externalities. We therefore go on to introduce this feature into our analysis by assuming that total factor productivity (*A*) is dependent upon capital per worker:

$$A(k_t) = \begin{cases} \underline{A} & \text{if} \quad k_t < \hat{k} \\ \overline{A} & \phantom{\text{if}} \quad k_t \geq \hat{k} \end{cases}.$$

We consider an economy where child allowances are provided and the tax rate is set at τ_C, as illustrated in Figure 3, with Figure 3(i) illustrating a low-threshold scenario, such that $\hat{k} < k_L^*(\underline{A}, \tau_C) < \overline{k}(\overline{A}, \tau_C)$. All economies begin with a low initial level of capital per worker ($k_0 < \hat{k}$), and initially converge towards $\hat{k}$ along $f_1(\underline{A}, \tau, k_t)$; such economies would then jump to $f_1(\overline{A}, \tau, k_t)$ and on reaching the threshold $k^{\#}$, would converge to a high steady state level, $k_H^*(\overline{A}, \tau_C)$.

However, club convergence may still occur if the threshold is not sufficiently low, such as the case where $k_L^*(\underline{A}, \tau_C) < \hat{k} < \overline{k}(\overline{A}, \tau_C)$, as illustrated in Figure 3(ii). Under this scenario, initial value is of relevance. If $k_0 < \hat{k}$, then the economy would converge to the underdevelopment trap ($k_L^*(\underline{A}, \tau_C)$); however, if $k_0 \geq \hat{k}$, then the economy would converge to a high steady state level, $k_H^*(\overline{A}, \tau_C)$.

<Figure 3 is inserted about here>

5.2 Welfare

Finally, we examine the effects of the introduction of these two policies on welfare. The social welfare is defined by the representative adult's steady-state lifetime utility. Because these two policies are more common in developed countries than in developing countries, we focus our analysis on the effects for developed countries. Therefore, the welfare is defined at the high steady state (k_H^*); that is:

$$u_H^* = \ln n_H^* + \beta \ln c_H^*,$$

where n_H^* and c_H^* are the steady-state fertility and consumption of skilled workers at k_H^*, respectively.

With an introduction of child allowances, government levies income tax and the steady-state fertility and consumption of skilled workers are $n_H^* = q\left[\frac{\gamma(1-\sigma)}{\gamma+\beta}\right]^{\gamma} [1 +$

$\tau\xi(k_H^*)^\alpha]^{1-\gamma}$ and $c_H^* = A^2\alpha(1-\alpha)(1-\tau)\left[\frac{(1-\sigma)\beta}{\gamma+\beta}\right]^{2(1-\alpha)}(k_H^*)^{2\alpha-1}$. Taking the derivative of u_H^* with respect to τ, we can obtain:

$$u_H^{*\prime}(\tau) = \frac{(1-\gamma)\xi[(k_H^*)^\alpha + \tau\alpha(k_H^*)^{\alpha-1}k_H^{*\prime}(\tau)]}{1+\tau\xi(k_H^*)^\alpha} + \beta\left[\frac{-1}{1-\tau} + \frac{(2\alpha-1)k_H^{*\prime}(\tau)}{k_H^*}\right]. \quad (39)$$

Evaluating Eq. (39) at $\tau = 0$, we have:

$$u_H^{*\prime}(\tau)\big|_{\tau=0} = (1-\gamma)\left[1+\frac{\beta(1-2\alpha)}{1-\alpha}\right]\left\{\frac{\xi^{\frac{1}{\alpha}}}{q\left[\frac{(1-\sigma)\gamma}{\gamma+\beta}\right]^\gamma}\right\}^{\frac{\alpha}{1-\alpha}} - \beta. \quad (40)$$

Eq. (40) indicates that $u_H^{*\prime}(\tau)$ will be positive if q is sufficiently small and vice versa. Therefore, we have the following result.

Proposition 5. For developed countries, an introduction of child allowances will improve (hurt) welfare if q is sufficiently small (large).

An introduction of child allowances will raise fertility rate which will be beneficial to welfare. However, an increase in the population growth will cause a 'capital-dilution' effect which will lower k_H^* and c_H^*. This effect will in turn reduce welfare. If q is sufficiently small, the former will dominate the latter and introducing of child allowances will improve welfare. However, the situation will be reversed if q is sufficiently large.

On the other hand, if the government introduces educational subsidies, the steady-state fertility and consumption of skilled workers are $n_H^* = q\left[\frac{\gamma(1-\sigma)}{\gamma+\beta(1-\tau)}\right]^\gamma$ and $c_H^* = A^2\alpha(1-\alpha)\left[\frac{(1-\tau)(1-\sigma)\beta}{\gamma+\beta(1-\tau)}\right]^{2(1-\alpha)}(k_H^*)^{2\alpha-1}$. Taking the derivative of u_H^* with respect to τ, we can obtain:

$$u_H^{*\prime}(\tau) = \frac{\beta[\gamma+2\beta(1-\alpha)]}{\gamma+\beta(1-\tau)} - \frac{2\beta(1-\alpha)}{1-\tau} - \frac{\beta(1-2\alpha)k_H^{*\prime}(\tau)}{k_H^*}. \quad (41)$$

Evaluating Eq. (41) at $\tau = 0$, we have:

$$u_H^{*\prime}(\tau)\Big|_{\tau=0} = \frac{\beta^2\gamma(1-2\alpha)}{(\gamma+\beta)(1-\alpha)} > 0.$$

We then obtain the following result.

Proposition 6. For developed countries, an introduction of educational subsidies will improve welfare.

Similar to the introduction of child allowances, introducing educational subsidies will improve welfare by raising fertility; however, it will also reduce welfare by generating the 'capital-dilution' effect. The former always dominates the latter so that this policy is welfare improving.

6. CONCLUSIONS

Our primary aim in this study is to examine the impacts of child allowances and educational subsidies on economic growth based on an overlapping-generations model, analyzing the effects of these two policies on fertility and the proportion of skilled workers to the adult population. The provision of child allowances and educational subsidies is financed by levying income tax on skilled workers; the production function is assumed to exhibit the complementarity of capital and skills; and agents are allowed to make decisions on consumption, savings and personal investment in higher education.

We demonstrate that these two policy provisions may result in countries being trapped in underdevelopment and that the tax rate is an important determinant of long-run economic growth. If the tax rate is sufficiently high, the economy will converge to a steady state with a low proportion of high skilled workers to the adult population, as well as low output per worker. With a reduction in the tax rate, club convergence may occur; thus, the initial condition is of relevance. Only when the tax rate is sufficiently low will the economy converge to a 'good' steady state, regardless of its initial condition. The threshold effect of technological externalities can prevent the economy from becoming trapped in poverty, provided that the threshold was sufficiently low; under such a scenario, there would be a unique, stable and ‘good’ steady state. However, if the threshold was not sufficiently low, then club convergence would still occur. These two policies will affect welfare differently. While an introduction of educational subsidies is always welfare improving, an introduction of child allowances may improve or hurt welfare.

As regards the effects on the future tax burden, our results demonstrate that both child allowances and educational subsidies can be used as a means of raising fertility, and hence, the population growth rate; the increase in the population growth rate will increase the tax base, thereby mitigating the future tax burden. However, our analysis also shows that in order to mitigate the future tax burden, the government needs to raise the current tax burden, which may result in the economy becoming stuck in the poverty trap.

The focus of this paper has been on the effects of these two policies on economic growth, not on social welfare, which is invariably the focus in the prior studies; thus, as an extension of this research, it may prove interesting to include intra-general transfer mechanisms, such as social security systems, within our model, in order to study the optimal allocation of tax revenue.

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Table 1 The effects of an increase in the tax rate

	Child Allowances	Educational Subsidies
Panel A: $k_t < \bar{k}$		
ϕ_t	↓	↑
z_t^s	→	↑
z_t^u	→	→
n_t^s	↑	↑
n_t^u	↑	→
m_t	↑	↑
c_t^s	↓	↓
c_t^u	↓	↓
a_t^s	→	↓
a_t^u	→	→
$\bar{k}/\tilde{k}$	↑	↓
$f_1(\tau, k_t)/ g_1(\tau, k_t)$	↓	↓
Panel B: $k_t \geq \bar{k}$ ($\phi_t = 1$)		
z_t^s	→	↑
n_t^s	↑	↑
c_t^s	↓	↓
a_t^s	↓	↓
$f_2(\tau, k_t)/ g_2(\tau, k_t)$	↓	↓

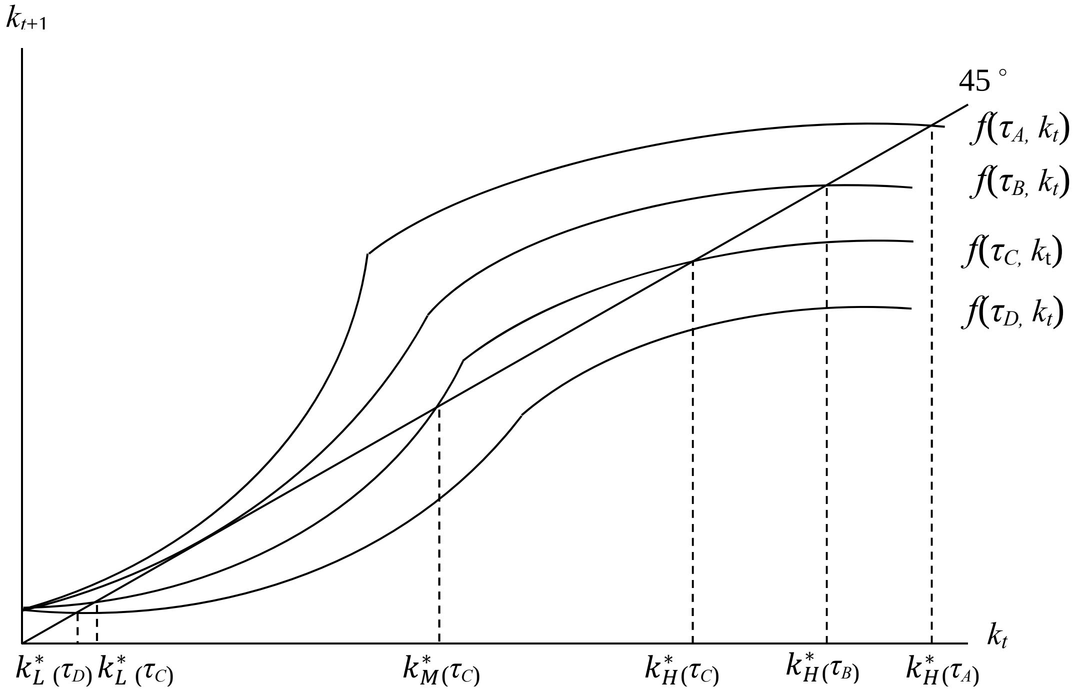

Figure 1 The effects of tax rate on the dynamic systems of capital per worker under a policy of child allowances

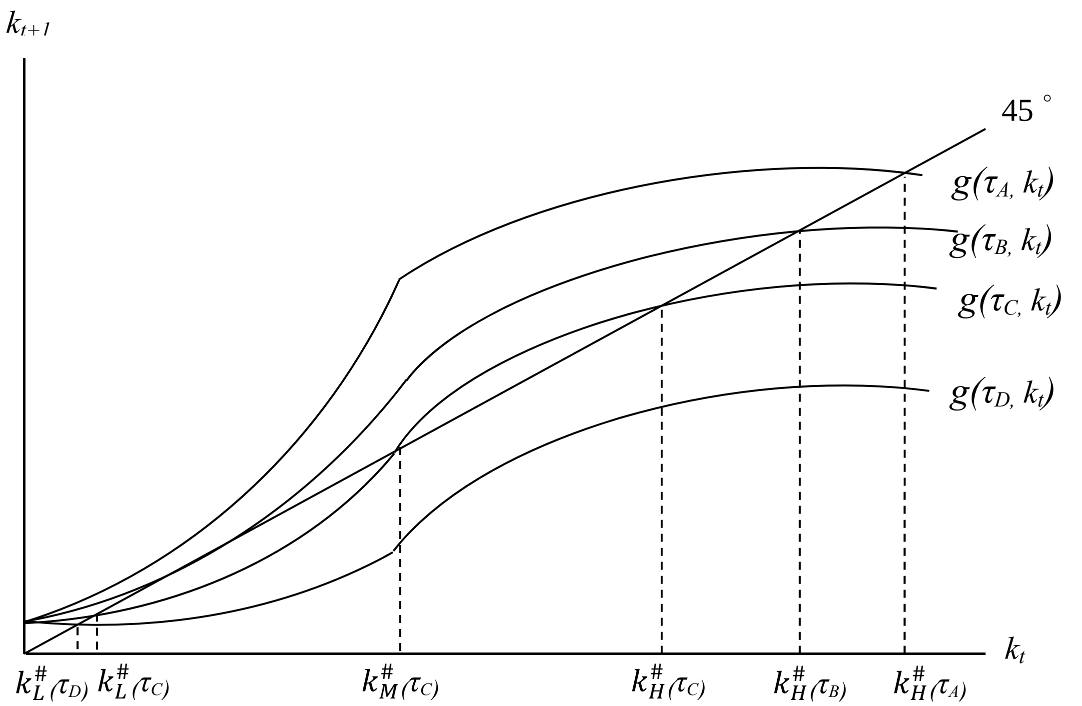

Figure 2 The effects of tax rate on the dynamic systems of capital per worker under a policy of educational subsidies

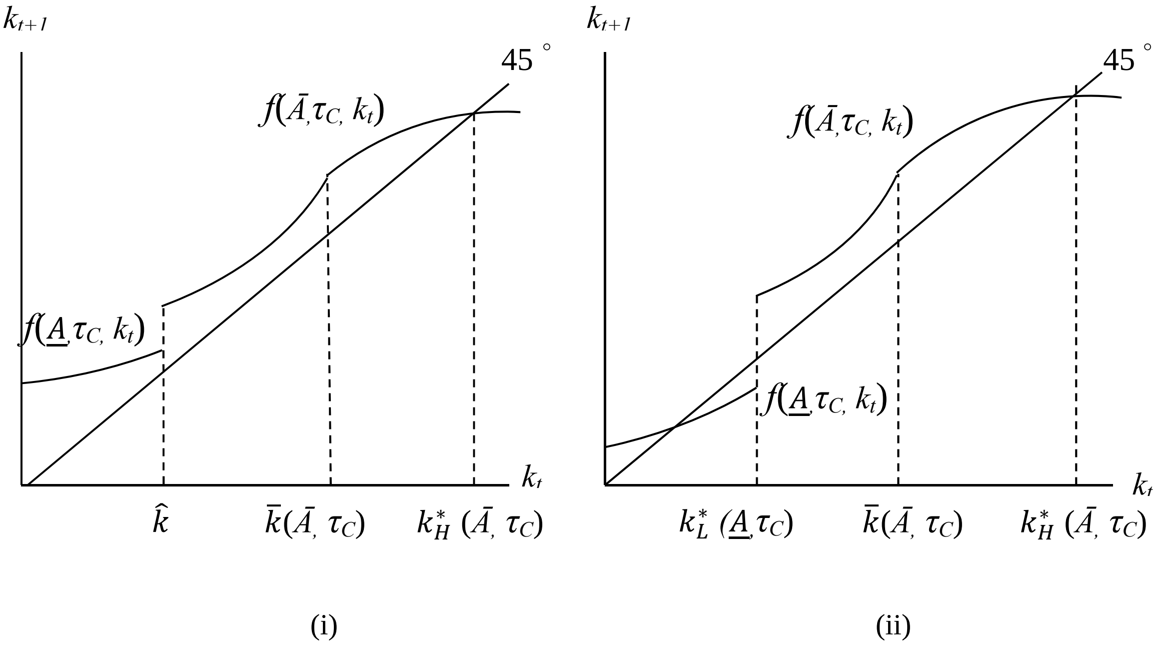

Figure 3 Dynamic system of capital per worker when
(i) $\hat{k} < k_L^*(\underline{A}, \tau_C)$*, and (ii)* $k_L^*(\underline{A}, \tau_C) < \hat{k} < \bar{k}(\bar{A}, \tau_C)$

APPENDIX A

Proof of Proposition 1

Using Eq. (21) to differentiate $m(\tau, k_t)$ with respect to τ, we obtain that if $k_t \geq \bar{k}$, then:

$$\frac{\partial m(\tau, k_t)}{\partial \tau} = \frac{q\left(\frac{\gamma}{\gamma+\beta}\right)^{\gamma}[1+\tau\xi\eta(\tau)^{1-\alpha}k_t]^{-\gamma}k_t}{\alpha(1-\tau)}$$
$$\times\left\{(1-\gamma)\tau\xi\eta(\tau)^{1-\alpha}\left(\frac{\alpha-\tau}{\tau}\right)[1-\tilde{\sigma}\eta(\tau)k_t]+\tilde{\sigma}\eta(\tau)[1+\tau\xi\eta(\tau)^{1-\alpha}k_t]\right\}.$$

Therefore, $\frac{\partial m(\tau,k_t)}{\partial \tau} > 0$ if $\alpha > \tau$.

If $k_t \geq \bar{k}$, then:

$$\frac{\partial m(\tau, k_t)}{\partial \tau} = q\theta\xi k_t^{\alpha}(1+\tau\xi k_t^{\alpha})^{-\gamma}\left(\frac{\gamma}{\gamma+\beta}\right)^{\gamma}(1-\sigma)^{\gamma} > 0.$$

QED.

APPENDIX B

Locus of $f(\tau, k_t)$

From Eq. (25), the first partial derivative of $f_1(\tau, k_t)$, with respect to k_t, is:

$$\frac{\partial f_1(\tau, k_t)}{\partial k_t} = \frac{\frac{A\beta b}{q(\gamma+\beta)^{1-\gamma}\gamma^{\gamma}}(\iota_1+\iota_2 k_t+\iota_3 k_t^2)}{[1+\tau\xi\eta(\tau)^{1-\alpha}k_t]^{2-\gamma}[1-\tilde{\sigma}\eta(\tau)k_t]^2}, \qquad \text{(B1)}$$

where

$$\iota_1 = \eta(\tau)\left[(1-\sigma)^{\frac{-\gamma}{\beta}} - (1-\sigma)^{\gamma} - (1-\gamma)\tau\xi\eta(\tau)^{-\alpha}\right];$$

$$\iota_2 = \tau\xi\eta(\tau)^{1-\alpha}\left\{\gamma\eta(\tau)\left[(1-\sigma)^{\frac{-\gamma}{\beta}} - 1\right] + (2-\gamma)\tilde{\sigma}\eta(\tau)\right\} > 0;$$

$$\iota_3 = (1-\gamma)\left[(1-\sigma)^{\frac{-\gamma}{\beta}} - 1\right]\tau\xi\eta(\tau)^{3-\alpha}\tilde{\sigma} > 0.$$

Note that $(1-\sigma)^{\frac{-\gamma}{\beta}} > 1$ and $(1-\sigma)^{\gamma} < 1$. Moreover, we can rewrite ι_1 as $\iota_1 = \eta(\tau)\left[(1-\sigma)^{\frac{-\gamma}{\beta}} - 1 + \tilde{\sigma} - (1-\gamma)\tau\xi\eta(\tau)^{-\alpha}\right]$. Under Assumption 1, $\tilde{\sigma} > \tau\xi\eta(\tau)^{-\alpha} >$

$(1-\gamma)\tau\xi\eta(\tau)^{-\alpha}$; we therefore have $\iota_1 > 0$ and $\frac{\partial f_1(\tau,k_t)}{\partial k_t} > 0$.

The second partial derivative of $f_1(\tau, k_t)$, with respect to k_t, can be calculated as:

$$\frac{\partial^2 f_1(\tau,k_t)}{\partial k_t^2} = \frac{\frac{A\beta b}{q(\gamma+\beta)^{1-\gamma}\gamma^\gamma}(\iota_4 + \iota_5 k_t + \iota_6 k_t^2 + \iota_7 k_t^3)}{[1+\tau\xi\eta(\tau)^{1-\alpha}k_t]^{3-\gamma}[1-\tilde{\sigma}\eta(\tau)k_t]^3}\,, \tag{B2}$$

where

$\iota_4 = \iota_2 + \eta(\tau)[2\tilde{\sigma} - (2-\gamma)\tau\xi\eta(\tau)^{-\alpha}]\iota_1$;

$\iota_5 = 2\iota_3 + \iota_2\eta(\tau)[\tilde{\sigma} - (1-\gamma)\tau\xi\eta(\tau)^{-\alpha}] + \iota_1(4-\gamma)\tilde{\sigma}\tau\xi\eta(\tau)^{2-\alpha}$;

$\iota_6 = \tau\xi\eta(\tau)^{1-\alpha}[\iota_3\gamma + \iota_2(3-\gamma)\tilde{\sigma}\eta(\tau)] > 0$;

$\iota_7 = \iota_3(2-\gamma)\tau\xi\eta(\tau)^{2-\alpha}\tilde{\sigma} > 0$.

Under our Assumption 1, $\iota_4 > 0$ and $\iota_5 > 0$, which implies that $\frac{\partial^2 f_1(\tau,k_t)}{\partial k_t^2} > 0$. Therefore, $f_1(\tau, k_t)$ is convex in k_t.

Using Eq. (26), we can obtain that the first and second partial derivatives of $f_2(\tau, k_t)$, with respect to k_t, are as follows:

$$\frac{\partial f_2(\tau,k_t)}{\partial k_t} = \frac{\alpha(1-\tau)\xi k_t^{\alpha-1}[1+\gamma\tau\xi k_t^\alpha]}{q\left[\frac{(1-\sigma)\gamma}{\gamma+\beta}\right]^\gamma (1+\tau\xi k_t^\alpha)^{2-\gamma}} > 0. \tag{B3}$$

$$\frac{\partial^2 f_2(\tau,k_t)}{\partial k_t^2} = \frac{-(1-\tau)\xi\iota_8 k_t^{\alpha-2}}{q\left[\frac{(1-\sigma)\gamma}{\gamma+\beta}\right]^\gamma (1+\tau\xi k_t^\alpha)^{3-\gamma}} < 0, \tag{B4}$$

where

$\iota_8 = \alpha(1-\alpha) + \tau\xi\alpha k_t^\alpha[1+\alpha+\gamma-3\alpha\gamma+\tau\xi\gamma(1-\alpha\gamma))k_t^\alpha] > 0$.

Therefore, $f_2(\tau, k_t)$ is a concave function in k_t.

APPENDIX C

Proof of Proposition 2

When $k_t < \bar{k}$, the partial derivative of $f_1(\tau, k_t)$, with respect to τ, is:

$$\frac{\partial f_1(\tau,k_t)}{\partial\tau}=\frac{-\dfrac{A\beta bk_t}{\alpha(1-\tau)q(\gamma+\beta)^{1-\gamma}\gamma^{\gamma}}(\iota_9+\iota_{10})}{[1+\tau\xi\eta(\tau)^{1-\alpha}k_t]^{2-\gamma}[1-\tilde{\sigma}\eta(\tau)k_t]^2}<0,$$

where

$$\iota_9=\eta(\tau)\left[(1-\sigma)^{\frac{-\gamma}{\beta}}-(1-\sigma)^{\gamma}\right][1+\tau\xi\eta(\tau)^{1-\alpha}k_t]>0;$$

$$\iota_{10}=\frac{\alpha-\tau}{\tau}\left\{1+\eta(\tau)\left[(1-\sigma)^{\frac{-\gamma}{\beta}}-1\right]k_t\right\}[1-\tilde{\sigma}\eta(\tau)k_t](1-\gamma)\tau\xi\eta(\tau)^{1-\alpha}>0.$$

When $k_t\geq\bar{k}$, the partial derivative of $f_2(\tau,k_t)$, with respect to τ, is:

$$\frac{\partial f_2(\tau,k_t)}{\partial\tau}=-\frac{\xi k_t^{\alpha}\{1+\xi k_t^{\alpha}[\tau+(1-\tau)(1-\gamma)]\}}{q\left[\dfrac{(1-\sigma)\gamma}{\gamma+\beta}\right]^{\gamma}(1+\tau\xi k_t^{\alpha})^{2-\gamma}}<0.$$

Therefore, an increase in τ will lower the locus of $f_1(\tau,k_t)$ and $f_2(\tau,k_t)$.

QED.

APPENDIX D

Proof of Proposition 3

First note that:

$$\mu'(\tau)=\frac{\gamma(\alpha-\tau)\mu(\tau)}{\alpha(1-\tau)[\gamma+\beta(1-\tau)]},$$

which implies that $\mu'(\tau)>0$ if $\alpha>\tau$ and vice versa. Besides, we can also derive that

$$\zeta'(\tau)=-\frac{\beta\gamma[\gamma(1-\sigma)]^{\gamma}}{[\gamma+\beta(1-\tau)]^{1+\gamma}}<0$$

Using Eq. (33) to differentiate $m(\tau,k_t)$ with respect to τ, we obtain the results that if $k_t\geq\bar{k}$:

$$\frac{\partial m(\tau,k_t)}{\partial\tau}=-qk_t[\zeta'(\tau)\mu(\tau)+\zeta(\tau)\mu'(\tau)]$$
$$=\frac{qk_t\gamma\mu(\tau)}{\gamma+(1-\tau)\beta}\left\{\beta\left[\frac{\gamma(1-\sigma)}{\gamma+\beta(1-\tau)}\right]^{\gamma}-\zeta(\tau)\frac{\alpha-\tau}{\alpha(1-\tau)}\right\}.$$

Therefore, $\frac{\partial m(\tau,k_t)}{\partial\tau}>0$ if $\zeta(\tau)<\frac{\alpha\beta(1-\tau)}{(\alpha-\tau)}\left[\frac{\gamma(1-\sigma)}{\gamma+\beta(1-\tau)}\right]^{\gamma}$.

If $k_t\geq\bar{k}$, then:

$$\frac{\partial m(\tau,k_t)}{\partial \tau} = -qk_t\zeta'(\tau) > 0.$$

QED.

APPENDIX E

Locus of $g(\tau,k_t)$

Using Eq. (36), the first partial derivative of $g_1(\tau,k_t)$, with respect to k_t, is:

$$\frac{\partial g_1(\tau,k_t)}{\partial k_t} = \frac{A\beta b\mu(\tau)}{q(\gamma+\beta)} \frac{\left\{\left[\frac{\gamma+\beta(1-\tau)}{(1-\sigma)(\gamma+\beta)}\right]^{\frac{\gamma}{\beta}} - 1\right\}\left(\frac{\gamma}{\gamma+\beta}\right)^{\gamma} + \zeta(\tau)}{\left[\left(\frac{\gamma}{\gamma+\beta}\right)^{\gamma} - \zeta(\tau)\mu(\tau)k_t\right]^2}, \tag{E1}$$

$$\frac{\partial^2 g_1(\tau,k_t)}{\partial k_t^2} = \frac{2A\beta b\zeta(\tau)\mu(\tau)^2}{q(\gamma+\beta)} \frac{\left\{\left[\frac{\gamma+\beta(1-\tau)}{(1-\sigma)(\gamma+\beta)}\right]^{\frac{\gamma}{\beta}} - 1\right\}\left(\frac{\gamma}{\gamma+\beta}\right)^{\gamma} + \zeta(\tau)}{\left[\left(\frac{\gamma}{\gamma+\beta}\right)^{\gamma} - \zeta(\tau)\mu(\tau)k_t\right]^3}. \tag{E2}$$

Since $z_t^s < z_t^u$, we have that $\frac{1}{\gamma+\beta} > \frac{1-\sigma}{\gamma+\beta(1-\tau)}$, which implies that $\frac{\gamma+\beta(1-\tau)}{(1-\sigma)(\gamma+\beta)} > 1$, and hence, $\left[\frac{\gamma+\beta(1-\tau)}{(1-\sigma)(\gamma+\beta)}\right]^{\frac{\gamma}{\beta}} > 1$. We can then obtain that $\frac{\partial g_1(\tau,k_t)}{\partial k_t} > 0$ and $\frac{\partial^2 g_1(\tau,k_t)}{\partial k_t^2} > 0$. Therefore, $g_1(\tau,k_t)$ is convex in k_t,.

Using Eq. (37), the first and second partial derivatives of $g_2(\tau,k_t)$, with respect to k_t, are as follows:

$$\frac{\partial g_2(\tau,k_t)}{\partial k_t} = \frac{A\alpha(1-\alpha)\left[\frac{\beta(1-\tau)(1-\sigma)}{\gamma+\beta(1-\tau)}\right]^{1-\alpha} k_t^{\alpha-1}}{q\left[\left(\frac{\gamma}{\gamma+\beta}\right)^{\gamma} - \zeta(\tau)\right]} > 0. \tag{E3}$$

$$\frac{\partial^2 g_2(\tau,k_t)}{\partial k_t^2} = \frac{-A\alpha(1-\alpha)^2\left[\frac{\beta(1-\tau)(1-\sigma)}{\gamma+\beta(1-\tau)}\right]^{1-\alpha} k_t^{\alpha-2}}{q\left[\left(\frac{\gamma}{\gamma+\beta}\right)^{\gamma} - \zeta(\tau)\right]} < 0. \tag{E4}$$

Therefore, $g_2(\tau,k_t)$ is a concave function in k_t.

APPENDIX F

Proof of Proposition 4

Using Eq. (36), we can calculate:

$$\frac{\partial g_1(\tau,k_t)}{\partial \tau}=\frac{A\beta b k_t}{q(\gamma+\beta)\left[\left(\frac{\gamma}{\gamma+\beta}\right)^{\gamma}-\zeta(\tau)\mu(\tau)k_t\right]}\left\{\iota_{11}+\frac{\iota_{12}\iota_{13}}{\left(\frac{\gamma}{\gamma+\beta}\right)^{\gamma}-\zeta(\tau)\mu(\tau)k_t}\right\},$$

where

$$\iota_{11}=-\frac{\gamma\mu(\tau)\left\{\tau(1-\alpha)\left[\frac{\gamma+\beta(1-\tau)}{(1-\sigma)(\gamma+\beta)}\right]^{\frac{\gamma}{\beta}}+(\alpha-\tau)\right\}}{[\gamma+(1-\tau)\beta]\alpha(1-\tau)}<0;$$

$$\iota_{12}=1+\mu(\tau)\left\{\left[\frac{\gamma+\beta(1-\tau)}{(1-\sigma)(\gamma+\beta)}\right]^{\frac{\gamma}{\beta}}-1\right\}k_t>0;$$

$$\iota_{13}=\zeta'(\tau)\mu(\tau)+\zeta(\tau)\mu'(\tau)<0.$$

Therefore, we have $\frac{\partial g_1(\tau,k_t)}{\partial \tau}<0$.

Using Eq. (37), we can obtain:

$$\frac{\partial g_2(\tau,k_t)}{\partial \tau}=\frac{A(1-\alpha)\left[\frac{\beta(1-\tau)(1-\sigma)}{\gamma+\beta(1-\tau)}\right]^{1-\alpha}k_t^{\alpha}}{q\left[\left(\frac{\gamma}{\gamma+\beta}\right)^{\gamma}-\zeta(\tau)\right]}\iota_{14},$$

where $\iota_{14}=\frac{\zeta'(\tau)}{\left(\frac{\gamma}{\gamma+\beta}\right)^{\gamma}-\zeta(\tau)}-\frac{(1-\alpha)\gamma}{(1-\tau)[\gamma+\beta(1-\tau)]}<0$. Thus, we have $\frac{\partial g_2(\tau,k_t)}{\partial \tau}<0$.

QED.