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Christos Papahristodoulou*

Abstract

The ranking of football players has always engaged media and supporters worldwide. The different views on ranking are due to two reasons. First, leagues are heterogeneous with various qualities. Second, fans often rely on different performance measures and statistics. Despite the fact that team and player bias will never disappear, this paper aims to objectively evaluate the efficiency of 42 top scorers who have played in the UEFA Champions League (UCL) over a period of six years, based on official match-play "multi-input and output" statistics, using input- and output oriented DEA models.

KEYWORDS: efficiency, scorers, forwards, midfielders, Champions League, DEA

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1. Introduction

All over the world, the media and football supporters have always tried to rank teams and players, based on their own subjective views and/or various key parameters. The Union of European Football association (UEFA) asks a number of team managers to nominate the best players in UEFA Champions League (UCL). The Fédération Internationale de Football Association (FIFA) also asks national team managers, team captains and representatives from FIFPro (the worldwide representative organization for professional players) to vote for the world player of the year. The French football magazine, France Football, has awarded the "Ballon D' Or" (the European Footballer of the Year) since 1956, a prize which is considered as the most prestigious individual award in football. The nominee player must have been playing for a European team within UEFA's jurisdiction. France Football asks only a group of in European football journalists participate this voting to (http://en.wikipedia.org/wiki/European_Footballer_of_the_Year).

Obviously, ranking the best player among goalkeepers, defenders, midfielders and forwards is a very difficult task. How should one compare and evaluate amazing savings by goalkeepers, excellent tackling by defenders, wonderful assists by midfielders, and outstanding goals by forwards? Moreover, even if one could observe a defender's tackling, his cooperation with the other defenders and even midfielders, his smart play in terms of offside won or fouls committed etc., and compare him with a top forward, the degree of subjectivity would be very high. Sport journalists do evaluate players with point systems, a system that often differs among countries and media. In addition, low points do not necessarily imply bad performance, if for instance the player followed the instructions given by his manager and might have sacrificed his own performance for the best of his team.

On the other hand, scorers are easier to evaluate because goals scored and other relevant statistics related to goals, are available. The use of "goals scored" though, causes a strong bias mainly against defenders and midfielders. A few defenders score, usually from penalties, foul kicks or other occasions. For instance, in the 96 group matches of the 2005/06 UCL tournament, there were scored 228 goals. Out of 48 players who scored at least two goals, 27 were forward, 19 midfielders and only 2 were defenders.

If some midfielders (usually the offensive ones) who score many goals are to be included in the data set together with the top forward scorers, their goal performance is obviously inferior. Thus, in order to give them a chance to be compared on fair grounds with the forwards, additional performance statistics, such as assists, shots on goal, and fouls suffered can be included. On the other hand, one might doubt whether the additional performance statistics are true output measures, given the fact that only "goals scored" count in matches. In that case, one can follow for instance Despotis et al (2012), and treat assists, shots on goal and fouls suffered as "intermediate" inputs instead, in two-stage decomposition.

The purpose of this simple paper is to evaluate each one of the 42 top scorers and measure his total performance, relative to an envelopment surface, which is composed of other scorers, using a multiple input-multiple output Data Envelopment Approach (DEA) approach. In section two I present two standard Linear Programming (LP) models I used in the estimates; in section three I discuss the input and output variables and the procedure I applied in the estimates; in section four I present and comment on the estimates; finally, section five concludes the paper.

2. Envelopment models

As is well known, the DEA approach envelops a data set of inputs and outputs, as tightly as possible (see, Charnes, et al. (1978), Ali and Seiford (1993), or Ali Emrouznejad's DEA homepage, <u>http://www.deazone.com/</u>).

There are many LP formulations to identify the Data Measurement Units (DMU), i.e. the scorers. When there are multiple criteria, it is harder to find scorers who beat all others in *"more-is-better-case"* (such as goals scored, assists e t c) and in *"less-is-better-case"* (such as played less time). Some of the top scorers will remain at the top in various aspects, while others would probably disregard the selected variables that ranked them as inefficient. The relative efficiency of scorers cannot be decided unless we use as many relevant inputs and outputs as possible, and apply various envelopment models, such as a proportional decrease in inputs or a proportional increase in outputs.

The author's estimates are based on the following two well-known envelopment models, the CCR^1 , Input and Output oriented models and the BCC^2 , Input and Output oriented models.

2.1 CCR: Input oriented model

If scorers are free to adjust their inputs (for instance if their managers let them playing more or less time) in order to achieve some given output(s), an input oriented model is appropriate. Input oriented models are relevant when at least two inputs are used. Since inputs excess is non-negative, the proportional decrease ends when at least one of the excess inputs variables is reduced to zero. The CCR formulation of the input oriented problem is the following:

$$\min \theta - \varepsilon \left(\sum_{i=1}^{m} s_i + \sum_{j=1}^{n} e_j \right)$$
(1)

$$\sum_{u=1}^{q} y_{i,u} \lambda_u - s_i = y_{i,u} \qquad (1a)$$

$$\theta x_{j,u} - \sum_{u=1}^{q} x_{j,u} \lambda_u - e_j = 0 \qquad (1b)$$

$$0 \prec \varepsilon \prec 0.00001 \tag{1c}$$

$$\lambda_{u} \geq 0, \, s_{i} \geq 0, \, e_{j} \geq 0 \qquad (1d)$$

<u>where</u>: s_i , output slack for multi-output i, i = 1, ..., m; e_j , input excess for multi-input j, j = 1, ..., n; $y_{i,u}$, output i of scorer u, u = 1, ..., q; $x_{j,u}$, input j of scorer u; λ_u , weight(s) of u scorer(s); θ_i input efficiency parameter of every u scorer; ε_i is a non-Archimedean positive constant

Constraint (1a) states that the evaluated scorer cannot produce more "output" than the efficient frontier. If he produced as much as the efficient frontier, he would be a part of the efficient frontier too, so that his specific output slack would be zero. If he produced less, he would be inefficient and his inefficiency degree would be equal to his output slack. Constraint (1b) states that the evaluated scorer cannot use less input than the efficient input requirements. If he used as much input as some other efficient input scorers, he would be efficient too, and

¹ CCR stands for Charnes, Cooper, Rhodes (1978), the three authors who formulated that model.

² BCC stands for Banker, Charnes, Cooper (1984), the three authors who formulated that model.

his excess input would be zero. If he used more input, he would be inefficient and his inefficiency degree would be equal to his excess input.

The evaluated scorer *u* is efficient in the strict sense of Koopmans³ if $\lambda_u = 1$, $e_{j}^{u} = 0$, $s_{i}^{u} = 0$ and consequently, $\theta_u = 1$. Moreover, while $\theta_u < 1$ implies inefficiency in the sense of Koopmans, the scorer can be efficient in the weak sense of Debreu and Farrell⁴, if the proportionate inputs reduction (θ_u) left him on the optimum outputs level, i.e. if and only if his output slack $s_{i}^{u} = 0$. Any positive output slack and/or excess input indicates $\theta_u < 1$, i.e. inefficiency. Notice that, the fact that there are neither output slack nor excess input does not necessarily imply that $\lambda_u = 1$. That might happen in the extreme case, if another efficient scorer *k*, envelops the evaluated scorer *u* by 100%, i.e. if $\lambda_k = 1$. Notice finally that the objective function employs a non-Archimedean positive constant ε (determined by the optimal solution) to allow both *e* and *s* to be positive. Given the positive, but unknown constant ε , the problem is in fact a NLP⁵.

The Constant Returns to Scale (CRS), CCR input oriented model is easily modified to the Variable Returns to Scale (VRS), BCC model, by adding the convexity constraint $\sum_{u=1}^{q} \lambda_u = 1$. Normally, the efficiency increases in the VRS frontier.

2.2 CCR: Output oriented model

We turn now to the output orientation model. Output oriented models can be relevant if scorers are not allowed to adjust their inputs to achieve their outputs, for instance if the player is going to play the entire match. The key question in these models is how efficiently the fixed inputs are used to reach the production frontier. In output oriented models one seeks to maximise the proportional increase in outputs.

³ Koopmans (1951) defined technical efficiency as: "a possible point in the commodity space is efficient whenever an increase in one of its coordinates (the net output of one good) can be achieved only at the cost of a decrease in some other coordinate (the net output of another good)" (p. 60).

⁴ Debreu (1951) and Farrell (1957) defined input oriented technical efficiency as $1 - \theta$, so that the production of

a given output will be reached. If $\theta = 0$ the scorer is efficient while if $\theta > 0$ he is inefficient.

⁵ Ali and Seiford (1989) mentioned some computational difficulties when this model is formulated as a one-step non-Archimedean approach. Modern packages, like LINGO that I used to obtain the estimates, can handle that problem very easily. The LINGOs NLP algorithm provided indeed global optimal solutions.

The standard formulation of the output-oriented problem is the following:

$$max \ \phi + \varepsilon \left(\sum_{i=1}^{m} s_i + \sum_{j=1}^{n} e_j \right)$$
 (2)

$$\phi y_{i,u} - \sum_{u=1}^{q} y_{i,u} \lambda_u + s_i = 0$$
 (2a)

$$\sum_{u=1}^{q} x_{j,u} \lambda_{u} + e_{j} = x_{j,u}$$
 (2b)

$$0 \prec \varepsilon \prec 0.00001 \qquad (2c)$$

$$\lambda_{u} \ge 0, s_{i} \ge 0, e_{i} \ge 0 \qquad (2d)$$

where, ϕ_i is the output efficiency parameter of every *u* scorer and all other variables as before.

The interpretation of constraints is similar to the previous model. For instance, all outputs are now multiplied with the efficiency parameter ϕ . If $\phi = 1$, $e^{u}_{j} = 0$ and $s^{u}_{i} = 0$, the evaluated scorer is efficient in the Koopmans sense. If $\phi > 1$, i.e. when the output vector lies below the efficiency frontier, the scorer is inefficient in the sense of Koopmans but efficient in the weak sense of Debreu-Farrell, if and only if $e^{u}_{i} = 0$.

Similarly, this output oriented CCR model turns to BCC model by adding the convexity constraint $\sum_{u=1}^{q} \lambda_{u} = 1$.

3. Variables and Data

The data for the selected input and output variables are collected from the UEFA's official site, <u>http://www.uefa.com/competitions/ucl/history/index.html</u>. They cover six seasons (2006/07 - 2011/12) and are based on 462 match statistics, i.e. 48 matches at group stage, 16 matches at the round of 16 teams, 8 matches at the quarterfinals, 4 matches at semi-finals and the final. Matches at previous qualifying rounds are excluded.

The investigated period is not particularly long, but rather sufficient for most of the included scorers who are in their "best" years. No matter when the period starts (or how long the

investigated period is), there will always be senior good scorers who just play their last season(s) and younger talents who just started their career. Both groups will be disfavoured compared to those who are in the peak of their career and have played for some years. For instance, while I am writing this paper, the first semi-finals of the 2012/13 have just finished, where a young rising star, Robert Lewandofski, scored four goals against Real Madrid, reaching ten goals this year. Since this current period is not included, he is excluded from the observations. Similarly, Andriy Shevchenko, the third-highest goal-scorer in UCL history with 59 goals, is also excluded, because, mainly due to injuries, he made just two goals in the first season 2006/07. In addition to that, the observed statistics do not show why a particular player did not play certain matches. There is no certain information if he was 100% fit, in bad shape or simply the manager decided not to use him for tactical reasons.

Since the study investigates the efficiency of scorers, a number of excellent scorers (42) were selected. The selected scorers should fulfil the following requirements: (i) they must have scored at least 5 goals in one season, or at least 4 goals per season, over two seasons; five scorers, (Del Piero (Juventus), Callejon (Real), Doumbia (CSKA), Shirokov (Zenit) and Cavani (Napoli) scored exactly 5 goals in one season. (ii) all goals count, i.e. even penalty kicks, during the game or after extra time.

A goal scored is obviously the most important "output" variable. Moreover, goals reveal only a part of a scorer's ability. Missing goals and the reason why, would be another important measure to correctly evaluate the scorers' efficiency. Since such statistics do not exist (and it would be questionable to rely on such subjective statistics if it existed), only scored goals count in this study.

Three more "output" variables are used.

Assists

Many "experts" regard assists as "half goals". By definition, an assist is an observation and attributed to the player who passed the ball to a teammate, directly and sometimes indirectly, to score a goal. While a direct pass that leads to goal counts as an assist, the assist is not recorded if the teammate misses the goal. Usually, as indirect passes, which count as assists, are: (for details, see the following site: <u>http://en.wikipedia.org/wiki/Assist_(football</u>)).

(i) A shot by a player X that causes a rebound and then a goal scored by player Z;

- (ii) A run by a player X in the penalty area that results in a penalty kick that player Z scores. On the other hand, if the same player X takes the penalty, is not credited with an assist;
- (iii) A cross, a free kick or a corner kick from player X that leads to goal by player Z, either through volleyed or headed goal. On the other hand, if player Z who receives the pass, cross or rebound must beat at least one opponent before scoring, player X's assist does not count.

Obviously more assists imply better performance. Despite the fact that generally, midfielders or playmakers are better in assists than the scorers are, some top scorers are excellent in assists as well. The problem with "missing goals" mentioned above, appears with assists as well. For instance, the observed statistics improve the efficiency of the players whose assists led to goals and decrease the efficiency of the players whose "assists" were not recorded, simply because the expected scorer missed the goal!

Shots on Goal⁶

A shot on goal is another important measure to evaluate the scorers' performance. Goals are obviously the result of shots on goal. Papahristodoulou (2008) found that shots on goal are strongly significant correlated to goals scored. Moreover, the average return on goals is 0.25, since three out of four shots on goal are saved or deflected. The probability that a shot on goal is converted into goal varies significantly with both the location of the shot and with other factors. For instance, Pollard and Reep in an old study (1997) estimated that the scoring probability is 24% higher for every yard nearer goal and the scoring probability doubles when a player manages to be over 1 yard from an opponent when shooting the ball.

Do shots on goal belong to "more-is-better" or to "less-is-better"? For instance, if one argues that shots on goal should reflect the inability of scorers to convert them into goals, that measure fits better as an input. I believe that this argument is wrong for two reasons. First, unless one obtains information (which is missing) why these shots on goal were not converted into goals, one cannot treat them as identical to "missed goals" and consequently as an indicator of poor performance. Second, if fewer shots on goal should be preferred, and treat that as an input measure, it is difficult to believe that extremely high goal returns per shot on

⁶ "Shots on goal" is the official name, but it includes also the heads on goal.

goal would reflect higher performance and not just true "fortune". It is simply ridiculous to ask for instance Messi to score a goal for, say, every second shot on goal, in order to be equally efficient, as other modest scorers who might have scored a goal out of just two shots on goal. Messi, in this six-year period, scored 50 goals and had 130 shots on goal. He is obviously an outstanding performer in both goals scored and shots on goal. Therefore, the position of the author is just the opposite, that is, scorers who shot more shots on goal must have been more active and therefore performed better in "shots on goal", even if many of their shots did not turn into goals. Moreover, efficiency estimates were obtained by treating shots on goal as an additional input instead of output.

Fouls suffered

All players commit fouls. The main purpose with fouls is to prohibit the opponent players from playing their game, from gaining ground and shooting from favourable positions in order to score goals. (For details regarding the violations of the rules that lead to fouls, see http://www.fifa.com/mm/document/affederation/federation/laws_of_the_game_0708_10565.p df). Offensive players, who suffer many fouls from the opponent players, are obviously regarded as dangerous. The number of fouls they suffer for their team is a credit to them and consequently an indicator of a good performance. Despite the fact that all gained fouls are not equally important, the fouls suffered by forwards and sometimes by midfielders are often nearer the opponent team's area where the scoring probability is higher. Papahristodoulou (2008) found that offensive teams who keep ball possession gain (statistically) more fouls.

When we turn to "input" variables, two measures were used: (i) playing time (in minutes) and (ii) "team power" where the scorer plays.⁷

Playing time in minutes

This is the most frequent match-play input variable. In fact, the simplest performance of scorers that always is used, relates goals scored per minutes played. The longer the playing time a player plays, the higher his output(s) performance is expected to be.

⁷ Fouls committed and offside are also two other "input" proxies that can be used (see Papahristodoulou, 2008). For instance, offensive players who commit many fouls are somehow forced by their opponents to play unsporting and found it pays to teams to commit "soft" fouls, i.e. as long as yellow or red cards do not follow these fouls. Similarly, that study found a weak positive correlation between offside and goals scored for the away teams, but not for the home teams.

This measure treats all matches equally and every minute played is expected to yield the same return, an assumption that might not be very likely. For tactical reasons, or because of injury, scorers play less than 90'per game (or less than 120'in case of extra time). In addition, some scorers play more matches than others, some scorers play "easier" or "home" matches, while others might be kept on the bench for a particular match, especially when their team is already qualified for the next round and some forwards are told to help their midfielders and even their defenders! In this study I treat all played minutes equally, provided that the scorer played at least 90'in a season. Scorers who played less than 90'are normally not expected to score goals and are excluded from that particular season, unless they managed to score a goal and fulfill the goal conditions mentioned earlier.

The "team and player power"

By definition, excellent teams consist of many excellent players, including top scorers. Very often, it is easier to be an excellent scorer for a top team than for an average team. Average teams are often satisfied with draws or keeping clean at their defense and play a more defensive style. Consequently, scorers who play in better teams have better teammates and given the fact that their team plays a more offensive style, have more opportunities to score goals. In order to correct for the "player power," I constructed first an index of the "team power" in which the scorer has played and adjusted that index for the scorer as well, according to the time the scorer has played.

As is known, the group stages at the UCL consist of eight groups with four teams per group. The seeding of teams for the UCL (and the Europa League as well) is based on Bert Kassies estimates. who uses а number of various match results coefficients (http://www.xs4all.nl/~kassiesa/bert/uefa/index.html). Every group contains one top team among the first eight ranked, (1-8), one team with second ranking (9-16), one with third ranking (17-24), and finally one with fourth ranking (25-32). The lottery will then decide the four teams per group. Moreover, since the "team power" is measured as "high as possible", we need to reverse the ranking list, so that the top team is valued with 32 points and the bottom team is valued with 1 point. Consequently, the four power groups are classified as: (A) = 32-25, (B) = 24-16, (C) = 15-9 and (D) = 8-1.

The relative power of every team, in each one of the eight groups, is then measured as:

$$A power = 3A - (B + C + D)$$
$$B power = 2B - (C + D)$$
$$C power = C$$
$$D power = D$$

Depending on the lottery, it is possible that two or more teams in two or more different groups to have the same relative power, even if the teams have different ranking. However, the above condition is sufficient to ensure that no weaker teams can have higher power than the stronger ones. The "team power" for some selected teams per season appears in Appendix (Table A).

Based on the "team power" the scorers were assigned the respective numeric value of their team, weighted by their own playing time, i.e.:

Scorer i Power =
$$\frac{time_i}{540}$$
 Team i Power, if time i ≤ 540 .

The index is divided by 540' (i.e. compared to if the scorer has played all the six groupmatches full time). If his team qualified and played at least two more matches, the "player power" is identical to his "team power." Obviously, scorers who changed teams over seasons are adjusted for the new team's value. Thus, the higher the scorer's power (by playing against relatively weaker teams), the higher his performance should be. Table B in Appendix depicts the power of all selected scorers and their teams.

4. Efficiency estimates

Using four outputs and two inputs, I run simultaneous estimates for all 42 scorers, using Global Solver from the LINGO package. There are 2059 variables, of which 253 non-linear, 295 constraints and one non-linear in the VRS-frontier, and 42 constraints less in the CRS-frontier. The number of iterations in the input oriented exceeded 2 million (in about 20 minutes of computing time), while the global solution to output oriented was very fast (in less than 1 minute). As it was mentioned earlier, I also used three inputs (by treating shots on goal as the third input) and three outputs. The efficiency estimates in the CRS- and VRS-frontiers for both input and output oriented models are given in Table 1. Tables 2 and 3 show the convex combination of efficient scorers who are used to project the inefficient scorers in

input- and output oriented models respectively, for the VRS-frontier with two inputs and four outputs. In addition, Table C in Appendix shows the estimates with two inputs and one, two and three outputs.

	2 inputs, 4 outputs				3 inputs, 3 outputs				
	CRS	VRS	CRS	VRS	CRS	VRS	CRS	VRS	
Player	θ	θ	¢	¢	θ	θ	¢	¢	
КАКА (1)	1	1	1	1	1	1	.7142	infeasible	
DROGBA (2)	.8182	.8183	1.2222	1.2221	.8465	.9316	.9331		
ROONEY (3)	.6317	.6420	1.5830	1.5806	.6034	.7078	1.521		
VILLA (4)	.6531	.6839	1.5311	1.4987	.6985	.6992	1.285		
INZAGHI (5)	.8920	.8936	1.1211	1.1193	.7273	.7694	.9803		
CROUCH (6)	.9003	.9313	1.1107	1.0833	1	1	.8533		
MORIENTES (7)	.7199	.7355	1.3890	1.3782	.8099	.8100	.7733		
VAN NISTELROY (8)	.9901	1	1.0100	1	1	1	.9001		
RAUL (9)	.7049	.7102	1.4186	1.3864	.8122	1	1.102		
RONALDO (10)	1	1	1	1	1	1	.8171		
MESSI (11)	1	1	1	1	1	1	.7679		
TORRES (12)	.7711	.8208	1.2968	1.2421	.8437	.8604	.8656		
GERRARD (13)	.7567	.7597	1.3215	1.3214	.8013	.8570	1.087		
BABEL (14)	.6039	.6527	1.6560	1.6480	.6989	.7320	1.086		
IBRAHIMOVIC (15)	.7639	.7734	1.3091	1.3042	.6901	.6925	1.156		
KANOUTE (16)	.9289	.9971	1.0765	1.0039	.9833	.9949	.8427		
DEIVID (17)	.8328	.8328	1.2007	1.1758	.8522	.8918	1.039		
KUYT (18)	.5033	.5461	1.9870	1.9526	.6059	.6093	1.104		
BENZEMA (19)	1	1	1	1	1	1	.9007		
FABREGAS (20)	.9634	1	1.0379	1	1	1	.6963		
KLOSE (21)	.6842	.7461	1.4615	1.4074	.7946	.8011	1.028		
LISANDRO (22)	.8521	.8521	1.1735	1.1732	.8982	1	.7865		
ADEBAYOR (23)	.5777	.6237	1.7311	1.6676	.5829	.5855	1.484		
DEL PIERO (24)	1	1	1	1	1	1	.8177		
VAN PERSIE (25)	.9591	.9857	1.0426	1.0153	.7752	.7807	1.164		
HENRY (26)	.8759	.9007	1.1416	1.1175	.7484	.7980	.9723		
ETO'O (27)	.8072	.9254	1.2389	1.0684	.8834	1	.9171		
OLIC (28)	.7228	.7340	1.3834	1.3318	.7326	.7359	.9856		
MILITO (29)	.7801	.8596	1.2819	1.1906	.8822	.9134	.8040		
BENDTNER (30)	.6988	.6999	1.4309	1.3757	.8236	.8279	.9323		
СНАМАКН (31)	.9995	1	1.0005	1	1	1	.4018		
PEDRO RODRIGUEZ (32)	.6058	.6128	1.6508	1.6484	.7053	.7054	1.111		
ROBEN (33)	.8099	.8356	1.2347	1.2073	.8156	.8415	.9600		
GOMEZ (34)	.9542	1	1.0479	1	.7737	1	.9165		
ANELKA (35)	.5454	.5562	1.8336	1.8316	.6630	.6640	1.266		
SOLDADO (36)	1	1	1	1	1	1	.5195		
CALLEJON (37)	1	1	1	1	1	1	.5523		
GOMIS (38)	.7101	.7225	1.4083	1.4061	.7379	.7380	1.085		
FREI (39)	.9346	1	1.0699	1	.9438	1	.9280		
DOUMBIA (40)	.9451	.9532	1.0581	1.0507	.9654	1	.8702		
SHIROKOV (41)	.9012	.9812	1.1097	1.0263	.8953	.9359	1.083		
CAVANI (42)	1	1	1	1	1	1	1		

Table 1: Input- and Output oriented estimates

With two inputs and four outputs, both input- and output oriented frontiers show almost similar efficiency and inefficiency estimates. The three "Ballon D'Or" players in the list (Kaká, (2007), Ronaldo, (2008) and Messi (2009-12)) are, as expected, efficient in all frontiers. Apart from them, there are five more efficient scorers in the CRS-frontier (Benzema, Del Piero, Soldado, Callejon and Cavani) and five more efficient scorers in the VRS-frontier (Van Nistelroy (almost), Fabregas, Chamakh, Gomez and Alexander Frei). If we compare the efficiency with respect to goals only (Table C, Appendix), Kaká, Van Nistelroy, Benzema, Fabregas and Del Piero are not efficient.

	VRS													
Player	θ		λ of efficient scorers											
DROGBA	.81829	.3869	10	.2274	11	.3857	24							
ROONEY	.64196	.4270	11	.2247	19	.3483	37							
VILLA	.68394	.1454	1	.0674	11	.2407	19	.5465	37					
INZAGHI	.89360	.1040	10	.0793	11	.8167	37							
CROUCH	.93125	.0905	1	.0214	10	.1399	11	.4553	37	.2929	42			
MORIENTES	.73549	.0222	11	.7534	24	.2233	37							
RAUL	.71016	.0481	1	.0322	10	.1536	11	.3249	19	.4412	42			
TORRES	.82080	.1675	1	.2171	10	.6154	24							
GERRARD	.75968	.3789	1	.3416	19	.2795	37							
BABEL	.65267	.0245	1	.0524	10	.0771	24	.8460	37					
IBRAHIMOVIC	.77337	.0050	1	.2759	11	.5635	19	.1555	37					
KANOUTE	.99707	.2000	1	.1000	37	.7000	42							
DEIVID	.83284	.1111	24	.0912	37	.7977	42							
KUYT	.54613	.3675	1	.0171	10	.5869	24	.0285	37					
KLOSE	.74612	.1121	1	.0838	10	.1677	24	.4099	37	.2264	42			
LISANDRO	.85212	.4373	10	.2479	24	.3148	31							
ADEBAYOR	.62372	.0674	11	.2285	19	.7041	37							
VAN PERSIE	.98572	.3708	11	.0899	19	.5393	37							
HENRY	.90065	.3182	1	.3182	19	.3636	37							
ETO'O	.92544	.7225	1	.0284	10	.1058	19	.1433	42					
OLIC	.73400	.0326	10	.0593	11	.0660	34	.8420	37					
MILITO	.85962	.1510	1	.0798	10	.7692	24							
BENDTNER	.69986	.1120	11	.0480	19	.8400	37							
PEDRO RODRIGUEZ	.61279	.1778	11	.2180	24	.6042	37							
ROBEN	.83557	.3230	10	.3053	24	.3717	37							
ANELKA	.55621	.0836	1	.0077	10	.1295	11	.7792	37					
GOMIS	.72252	.1261	10	.0804	24	.7935	37							
DOUMBIA	.95318	.1997	24	.3326	37	.4677	42							
SHIROKOV	.98124	.0085	1	.0429	19	.1969	37	.7516	42					

Table 2: The projection of inefficient scorers (Input oriented)

1 = Kaká, 10 = Ronaldo, 11 = Messi, 19 = Benzema, 24 = Del Piero, 31 = Chamakh, 34 = Gomez, 37 = Callejon, 42 = Cavani

With three inputs and three outputs, in input oriented VRS-frontier, there are five more scores, Crouch, Raúl, Lissando, Eto'o and Doumbia, who turn efficient. Notice also that, despite the

fact that most inefficient scorers improve their efficiency, compared to two inputs and four outputs, the efficiency of Inzaghi, Ibrahimovic, Adebayor, Van Persie and Henry deteriorates. In output oriented CRS-frontier, only Cavani is efficient, while in VRS- frontier, there is no feasible solution.

	VRS													
Player	¢		λ of efficient scorers											
DROGBA	1.2221	.5163	10	.2708	11	.2129	24							
ROONEY	1.5806	.0229	10	.7731	11	.1694	19	.0346	37					
VILLA	1.4987	.1438	1	.1271	11	.4272	19	.3019	37					
INZAGHI	1.1193	.0994	10	.1145	11	.7861	37							
CROUCH	1.0833	.0838	1	.0120	10	.1726	11	.3956	37					
MORIENTES	1.3782	.0726	11	.8243	24	.1031	37							
RAUL	1.3864	.0142	1	.1394	10	.0733	11	.7336	19					
TORRES	1.2421	.1515	1	.3263	10	.5225	24							
GERRARD	1.3214	.4082	1	.0153	8	.5765	19							
BABEL	1.6480	.1452	11	.0761	24	.7787	37							
IBRAHIMOVIC	1.3042	.0609	1	.4988	11	.4403	19							
KANOUTE	1.0039	.2013	1	.0995	37	.6992	42							
DEIVID	1.1758	.0116	10	.0339	36	.0971	39	.8573	42					
KUYT	1.9526	.5060	1	.2450	10	.2490	24							
KLOSE	1.4074	.0813	1	.1517	10	.0392	11	.4061	37	.3217	42			
LISANDRO	1.1732	.5107	10	.4893	31									
ADEBAYOR	1.6676	.2699	11	.1359	19	.5941	37							
VAN PERSIE	1.0153	.3800	11	.0857	19	.5343	37							
HENRY	1.1175	.3331	1	.3880	19	.2789	37							
ETO'O	1.0684	.7801	1	.1064	10	.1135	42							
OLIC	1.3318	.0116	10	.1182	11	.1252	34	.0397	36	.7053	37			
MILITO	1.1906	.1352	1	.1395	10	.7252	24							
BENDTNER	1.3757	.1635	11	.4627	36	.3738	37							
PEDRO RODRIGUEZ	1.6484	.3631	11	.0147	36	.6221	37							
ROBEN	1.2073	.4069	10	.2715	24	.3215	37							
ANELKA	1.8316	.3435	11	.0764	19	.5802	37							
GOMIS	1.4061	.1545	10	.0568	11	.7887	.37							
DOUMBIA	1.0507	.2130	24	.0423	36	.2870	37	.4577	42					
SHIROKOV	1.0263	.0079	1	.0478	19	.1845	37	.7597	42					

Table 3: The projection of inefficient scorers (Output oriented)

1 = Kaká, 8 = Van Nistelroy, 10 = Ronaldo, 11 = Messi, 19 = Benzema, 24 = Del Piero, 34 = Gomez, 36 = Soldado, 37 = Callejon, 42 = Cavani

From Tables 2 and 3 we observe that Callejon (37) of Real Madrid is the most commonly used efficient scorer. Twenty out of twenty-nine inefficient scorers are tested against him in the input oriented and nineteen out of twenty-nine in the output-oriented models. Callejon in 2011/12 season had an exceptional efficiency. He played about five hours (306⁻) and scored five goals! His score efficiency is almost twice compared to the top scorer of the whole

period, Messi. The three "Ballon D' Or" players, Kaká, Ronaldo and Messi, follow in the next places. These three scorers project every second inefficient scorer.

The first figure below shows the surface based on two inputs and only one output (goals) and all 42 scorers. When there is only one output (goals), there are seven efficient scorers, in both input and output oriented models (VRS-frontier), (see Table C in Appendix). The efficient scorers are depicted as large points. Notice that, when all four outputs are used, the number of efficient scorers increases to thirteen (in the VRS-frontier). Some of them appear to be in the bounds of the conical hull and seem to be (mistakenly) efficient. Seiford and Thrall (1990) have explained that boundary points are not necessarily efficient.



Similarly, in the second figure we aggregate all four outputs and show the thirteen efficient scorers and the twenty-nine inefficient ones. Moreover, due to aggregation, the conical hull is not entirely convex over all thirteen scorers. The convexity is valid over twelve scorers, if we exclude the second highest point, Messi. Ronaldo, Messi and Kaká (the three highest points) seem to have their own convex surface as well.



Table 4 shows the slacks and excesses of the twenty-nine inefficient scorers. All inefficient scorers are Debreu-Farrell inefficient at some slack, and/or at some excess. Notice that, apart from Eto'o, who had a positive excess of playing time, none of the remaining twenty-eight scorers is inefficient in terms of playing more time, i.e. all $e_1 = 0$.

Player	Input Oriented						Output Oriented					
	S ₁	S ₂	S ₃	S4	e ₁	e ₂	S ₁	S ₂	S ₃	S 4	e ₁	e ₂
DROGBA	-	1.2	8.4	-	-	58.7	-	1.9	11.5	-	-	69.6
ROONEY	5.7	-	-	29.7	-	.93	7.6	-	-	54.6	-	-
VILLA	1.4	-	-	-	-	15.4	1.4	-		-	-	27.9
INZAGHI	-	1.5	-	10.9	-	41.7	-	1.8	-	12.5	-	45.3
CROUCH	-	-	5.7	-	-	-	-	-	5.9	-	-	-
MORIENTES	-	.3	3.5	-	-	19.6	-	1.0	5.4	-	-	22.1
RAUL	-	-	11.2	-	-	-	-	-	23.0	-	-	-
TORRES	3.0	-	8.7	-	-	72.5	3.8	-	11.8	-	-	89.5
GERRARD	-	-	1.4	6.2	-	3.4	-	-	2.9	6.9	-	10.9
BABEL	-	-	1.1	-	-	16.2	-	.4	.8	-	-	16.9
IBRAHIMOVIC	9.7	-	-	-	-	35.1	12.1	-	-	9.4	-	35.7
KANOUTE	.2	-	-	4.6	-	17.3	.2	-	-	4.6	-	17.3
DEIVID	-	1.6	-	.1	-	-	-	2.0	-	.2	-	-
KUYT	.6	-	-	-	-	13.1	1.1	-	3.7	-	-	26.9
KLOSE	-	-	2.6	-	-	-	-	-	4.3	-	-	-
LISANDRO	2.4	-	11.6	-	-	35.1	2.2	.09	10.8	-	-	40.3
ADEBAYOR	2.6	-	-	3.5	-	21.9	3.2	-	-	15.1	-	29.3
VAN PERSIE	8.5	-	-	26.9	-	36.6	8.6	-	-	27.5	-	37.0
HENRY	5.9	-	-	10.4	-	40.5	6.4	-	-	11.2	-	47.0
ETO'O	-	-	10.4	-	553	-	-	-	14.6	8.0	546	-
OLIC	-	.4	-	.1	-	-	-	.8	-	-	-	-
MILITO	.3	-	6.4	-	-	5.4	.4	-	8.4	-	-	7.2
BENDTNER	-	-	6.6	1.1	-	15.4	-	-	4.2	2.2	-	32.3
PEDRO RODRIGUEZ	-	.5	1.8	-	-	36.5	-	1.8	3.9	-	-	51.7
ROBEN	6.6	.2	-	-	-	53.3	7.6	.5	-	-	-	64.1
ANELKA	-	-	.2	-	-	34.5	-	-	2.5	3.3	-	67.8
GOMIS	.2	1.6	-	-	-	11.8	-	2.8	-	.3	-	12.4
DOUMBIA	-	.9	1.8	-	-	-	-	.9	1.6	-	-	-
SHIROKOV	9	-	-	19	-	-	9	_	-	19	_	-

Table 4: Slacks and excesses of twenty-nine inefficient scorers (VRS)

There are fifteen and sixteen out of twenty-nine inefficient scores who had zero slack in goals scored, for the input- and output oriented models respectively. The most goals scored inefficient player was Zlatan Ibrahimovic followed by Van Persie, two scorers who perform

<u>Note</u>: s_1 = slack in goals scored; s_2 = slack in assists; s_3 = slack in shots on goal; s_4 = slack in fouls suffered; e_2 = excess in player power.

much better in their national leagues. Similarly, about 2/3 of the inefficient scorers had zero slack in assists. Finally, only eight, respectively nine out of twenty-nine inefficient scorers had zero excess in their power index. The highest excess power had Fernando Torres, followed by Didier Drogba. Among the inefficient scorers, the Debreu-Farrell efficiency differs. For instance, despite the fact that Van Persie's $\theta = 0.986$ and Klose's $\theta = 0.746$ (see Table 1), Klose is more efficient in goals scored, in fouls suffered and in power index, while Van Persie is more efficient only in shots on goal. Also, while Raúl who had a higher inefficiency (higher ϕ -value) than Drogba (see Table 2), he was more efficient in assists and in power index, but less in shots on goal. These differences are because various inefficient scorers.

In combination with Tables 1, 2 and 3, let us check two of the inefficient scorers, one in input oriented and the other in output oriented.

Fernando Torres in the VRS-frontier (input oriented) has an efficiency of 0.82. Torres is compared against the convex combination of Kaká (16.75%), Ronaldo (21.71%) and Del Piero (61.54%). The convex combination of these three efficient scorers has played 2,142' (instead of 2610' that Torres played). Thus, for $\theta = 0.82$, the theoretical playing time of

Torres is: $\hat{T}_{Torres} = \theta_{Torres} T_{Torres} - e_1$, leading to $e_1 = 0$. Similarly, Torres should have had the same power as the weighted power of the three scorers (which is about 91.5 units, instead of 200 units that Torres has). Consequently, Torres has an extra excess of power of about $e_2 =$

72.5, i.e. $\vec{I}_{Torres} = \theta_{Torres} I_{Torres} - e_2$. Torres' power efficiency is worse than his playing time efficiency. The convex combination of these three scorers scores 3 more goals than Torres ($s_1 = 3$) and shots about 9 more shots ($s_3 = 8.7$) than Torres. In summary, Torres is inefficient.

Zlatan Ibrahimovic in the VRS-frontier (output oriented) is about 30% inefficient. The convex combination of Kaká, Messi and Benzema scored about 37 goals. If we subtract Ibrahimovic slack ($s_1 = 12.1$), he should have scored almost 25 goals. Since he scored only 19 goals, (i.e. 5.8 goals less) he is goal-inefficient by 30% (= 5.8/19). Similarly, given his $s_4 = 9.4$ and given that the same convex combination gained 97.8 fouls, he should have gained 88.4 fouls (= 97.8 – 9.4). However, he gained 30% less (68 instead of 97.8). Finally, while the same convex

combination of scorers played exactly the same time as Ibrahimovic, $(e_1 = 0)$, his power is higher $(e_2 = 35.7)$, compared to these three scorers. In summary, Ibrahimovic is inefficient too.

5. Conclusions

Ranking football players is a very difficult task. Everyone who has an opinion weights arbitrarily a number of various "performance" parameters. Some of the parameters are neither directly observed and measured, nor compared. Even if we observe a player who plays creatively, or runs without the ball in order to open spaces, we cannot measure these performances in an objective manner. Nevertheless, the objectively measured parameters, such as goals scored or assists, do not reveal everything, simply because there are "easier" and "tougher" matches and opponents. Everyone should agree that if player X scores the third goal in a 3-0 victory in a group and non-decisive match, while player Y scores an excellent and the decisive goal in a quarterfinal or a semi-final, these goals are not "equal". What people do not agree though is how much higher the performance of scorer Y is. The ranking of scorers should therefore reflect the different weights one sets in these goals.

In this simple paper, I decided not use any weights. None of the goals scored, of assists, of shots on goal and of fouls suffered is worse or better; all are "equally good". Of course, one can repeat the estimates by assigning different weights to goals, assists, shots on goals and fouls suffered, depending upon the importance of the game, or at which round the performance measure was. Similarly, one can assign lower weights in "easy" and "indecisive" goals, assists, shots on goal and fouls suffered. Moreover, such weights might be rather subjective and the data should be re-collected.

If the UEFA official match play statistics are to be taken seriously and measure what they intend to measure, our DEA models rank the following eight players on top: Messi (Barcelona), Ronaldo (Real), Kaká (Real), Benzema (Real), Del Piero (Juventus, retired), Soldado (Valencia), Callejon (Real) and Cavani (Napoli). I believe that very few people would reject the top performance of these players. It would be interesting to find out if these players remained efficient if we weighted the data.

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	2006/07	2007/08	2008/9	2009/10	2010/11	2011/12
Milan	62	54	-	41	34	34
Chelsea	21	38	61	60	61	52
Man Utd	44	40	48	49	56	63
Valencia	40	32	-	-	20	25
Roma	22	22	28	-	26	-
Liverpool	44	45	47	51	-	-
Real	46	49	39	23	25	42
Barcelona	55	49	54	54	65	55
Inter	48	52	45	27	33	37
Lyon	28	25	38	33	38	30
Arsenal	45	48	52	39	30	47
Bayern	24	-	29	45	28	50
Tottenham	-	-	-	-	16	-
CSKA	10	12	-	27	-	16
Porto	33	29	25	27	-	38
Sevilla	-	27	-	45	-	-
Sporting	16	11	18	-	-	-
Napoli	-	-	-	-	-	7
Bordeaux	14	-	12	10	-	-
Werder Br	12	19	24	-	24	-
Schalke	-	15	-	-	13	-
Juventus	-	-	15	24	-	-
Marseille	-	9	16	15	25	14
Fenerbache	-	-	9	-	-	-
Stuttgart	-	14	-	13	-	-
FC Basel	-	-	10	-	12	10
Zenit	-	-	15	-	-	16

Appendix A: Seasonal "team power" of selective UCL teams

Player	2006/07	2007/08	2008/09	2009/10	2010/11	2011/12	Power
КАКА	Milan	Milan	Real	Real	Real	Real	191
DROGBA	Chelsea	Chelsea	Chelsea	Chelsea	Chelsea	Chelsea	272
ROONEY	Man Utd	Man Utd	Man Utd	Man Utd	Man Utd	Man Utd	274
VILLA	Valencia	Valencia	-	Barcelona	Barcelona	Barcelona	163
INZAGHI	Milan	Milan	Milan	Milan	Milan	Milan	122
CROUCH	Liverpool	Liverpool	-	-	Tottenham	-	85
MORIENTES	Valencia	Valencia	Marseille	Marseille	-	-	64
VAN NISTELROY	Real	Real	Real	Real	-	-	120
RAUL	Real	Real	Real	Real	Schalke	-	160
RONALDO	Man Utd	Man Utd	Real	Real	Real	Real	218
MESSI	-	Barcelona	Barcelona	Barcelona	Barcelona	Barcelona	316
TORRES	-	Liverpool	Liverpool	Liverpool	Chelsea	Chelsea	200
GERRARD	Liverpool	Liverpool	Liverpool	Liverpool	-	-	172
BABEL	Liverpool	Liverpool	Liverpool	Liverpool	-	-	83
IBRAHIMOVIC	Inter	Inter	Barcelona	Barcelona	Milan	Milan	267
KANOUTE	-	Sevilla	-	Sevilla			63
DEIVID	-	Fenerbahce	-	Fenerbahce	-	-	12
KUYT	Liverpool	Liverpool	Liverpool	Liverpool	-	-	182
BENZEMA	Lyon	Lyon	Real	Real	Real	Real	141
FABREGAS	Arsenal	Arsenal	Arsenal	Arsenal	Barcelona	Barcelona	286
KLOSE	Werder	-	Bayern	Bayern	Bayern	-	73
LISANDRO	Porto	Porto	Lyon	Lyon	Lyon	Lyon	181
ADEBAYOR	Arsenal	Arsenal	Arsenal	-	Tottenham	-	148
DEL PIERO	-	-	Juventus	Juventus	-	-	20
VAN PERSIE	Arsenal	Arsenal	Arsenal	Arsenal	Arsenal	Arsenal	182
HENRY	Arsenal	Barcelona	Barcelona	Barcelona	-	-	172
ETO'O	Barcelona	Barcelona	Inter	Inter	Inter	-	173
OLIC	CSKA Moscow	-	Hamburg	Bayern	Bayern	Bayern	74
MILITO	-	-	Inter	Inter	Inter	Inter	78
BENDTNER	Arsenal	Arsenal	-	-	-	-	111
СНАМАКН							60
PEDRO	-	Barcelona	Barcelona	Barcelona	Barcelona	Barcelona	
RODRIGUEZ							182
ROBEN	Cheisea	Real	Real	Bayern	Bayern	Bayern	166
GOIVIEZ	-	Stuttgart	Bayern	Bayern	Bayern	Bayern	125
	-	Chelsea	Chelsea	Chelsea	Chelsea	-	201
SOLDADO	кеаг	-	-	-	valencia	valencia	39
CALLEJON	-	-	-	-	-	Real	24
	-	-	Lyon	Lyon	Lyon		83
ALEX. FREI	-	-	FC Basel		FC Basel		22
DOOMBIA	-	-	-	CSKA IVIOSCOW	-	CSKA MOSCOW	16
SHIKUKUV	-	Zenit	Zenit	Zenit	Zenit	Zenit	18
CAVANI	-	-	-	-	-	Napoli	7

Appendix B: The scorer's power and his team(s) over six seasons

<u>Note</u>: Empty spaces denote that his team did not participate in the UCL that season, or the player did not play for at least 90[°], or retired, or moved to another, non-UCL team.

	2 inputs,		2 in	2 inputs,		outs,	2 inputs,		
	go	bal	goal &	assist	goal, assi	st & shot	goal, assi	st & foul	
Player	θ	φ	θ	¢	θ	¢	θ	ф	
КАКА (1)	.4858	1.912	1	1	1	1	1	1	
DROGBA (2)	.6338	1.532	.6347	1.532	.7382	1.355	.8183	1.222	
ROONEY (3)	.4861	1.927	.5691	1.709	.6420	1.581	.5693	1.709	
VILLA (4)	.4792	1.902	.6496	1.552	.6834	1.500	.6511	1.547	
INZAGHI (5)	.7653	1.250	.7653	1.250	.8936	1.119	.7694	1.249	
CROUCH (6)	.7388	1.293	.8969	1.118	.8969	1.118	.9313	1.083	
MORIENTES (7)	.4076	1.989	.4076	1.989	.5128	1.931	.7355	1.378	
VAN NISTELROY (8)	.7379	1.295	1	1	1	1	1	1	
RAUL (9)	.6159	1.453	.6891	1.396	.6891	1.396	.7102	1.386	
RONALDO (10)	1	1	1	1	1	1	1	1	
MESSI (11)	1	1	1	1	1	1	1	1	
TORRES (12)	.3567	2.440	.5521	1.831	.6756	1.514	.8208	1.242	
GERRARD (13)	.5050	1.845	.7597	1.321	.7597	1.321	.7597	1.321	
BABEL (14)	.4984	1.708	.5585	1.702	.6052	1.695	.6527	1.648	
IBRAHIMOVIC (15)	.4489	2.085	.6365	1.523	.7733	1.304	.6717	1.512	
KANOUTE (16)	.4588	1.834	.9949	1.007	.9971	1.004	.9949	1.006	
DEIVID (17)	.7954	1.195	.7954	1.195	.8328	1.176	.8306	1.187	
KUYT (18)	.2385	3.423	.4438	2.307	.5030	2.072	.5461	1.953	
BENZEMA (19)	.9336	1.053	1	1	1	1	1	1	
FABREGAS (20)	.2677	3.231	1	1	1	1	1	1	
KLOSE (21)	.4577	1.793	.6433	1.592	.6854	1.548	.7461	1.407	
LISANDRO (22)	.4713	1.779	.5381	1.734	.6440	1.581	.8522	1.173	
ADEBAYOR (23)	.4271	2.048	.5490	1.845	.6237	1.667	.5496	1.845	
DEL PIERO (24)	.7223	1.451	.7023	1.451	1	1	1	1	
VAN PERSIE (25)	.5810	1.615	.7804	1.287	.9857	1.015	.7807	1.286	
HENRY (26)	.3297	2.544	.7980	1.302	.9007	1.117	.7980	1.302	
ETO'O (27)	.4212	1.931	.9257	1.068	.9257	1.068	.9254	1.068	
OLIC (28)	.6788	1.371	.6803	1.371	.7340	1.338	.7030	1.367	
MILITO (29)	.4292	1.877	.6044	1.694	.6864	1.539	.8596	1.190	
BENDTNER (30)	.6781	1.379	.6999	1.376	.6999	1.376	.6999	1.376	
CHAMAKH (31)	.4019	1.772	.4019	1.771	.5011	1.750	1	1	
PEDRO RODRIGUEZ (32)	.5594	1.653	.5612	1.653	.5660	1.650	.6128	1.648	
ROBEN (33)	.2896	2.862	.4691	2.166	.8202	1.224	.8264	1.234	
GOMEZ (34)	1	1	1	1	1	1	1	1	
ANELKA (35)	.4859	1.863	.5495	1.832	.5531	1.832	.5562	1.832	
SOLDADO (36)	1	1	1	1	1	1	1	1	
CALLEJON (37)	1	1	1	1	1	1	1	1	
GOMIS (38)	.5314	1.672	.5314	1.672	.7151	1.406	.7013	1.443	
FREI (39)	1	1	1	1	1	1	1	1	
DOUMBIA (40)	.8788	1.147	.8788	1.147	.8788	1.119	.9532	1.051	
SHIROKOV (41)	.7399	1.374	.9359	1.104	.9812	1.026	.9359	1.104	
CAVANI (42)	1	1	1	1	1	1	1	1	

Appendix C: Input- and Output oriented estimates with two inputs & various outputs (VRS)