Leapfrogging Cycles in International Competition

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Abstract

Technological leadership has shifted at various times from one country to another. We propose a mechanism that endogenously explains this perpetual cycle of technological leapfrogging in a two-country model including the dynamic optimization of an infinitely-lived consumer. In the model, the stock of knowledge accumulates in each country over time because of domestic innovation and spillovers from foreign innovation. We show that if the international knowledge spillovers are reasonably efficient, technological leadership may shift first from one country to another, and then alternate between countries along an equilibrium path.

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1 Introduction

Throughout history, technological leadership has shifted at various times from one country to another. For instance, during the early 17th century, Venice and Spanish Lombardy were among the technologically most advanced regions in Europe (Davids 2008, p. 2). Over the centuries, the "technological center of gravity of Europe then moved, residing at various times in Italy, southern Germany, the Netherlands, France, England, and then again in Germany" (Mokyr 1990, p. 207). Some economic historians even claim that the US had begun to lose its technological leadership as early as the early 1990s (Nelson and Wright 1992).

An important question is why such economic and technological leapfrogging takes place. An equally fundamental question is why technological leapfrogging has repeatedly occurred. The first question has been investigated in existing literature, in which technological leapfrogging is triggered by major exogenous changes in technology (Brezis, Krugman, and Tsiddon 1993).[1] In contrast, the cause of perpetual cycles in technological leapfrogging has scarcely been studied. While we may regard the perpetual cycles of leapfrogging as responses to the perpetual exogenous changes in technology, this explanation for leapfrogging cycles is essentially exogenous based on macro shocks in technology. The present paper will show otherwise.

The aim of this analysis is to develop an endogenous theory that explains the perpetual cycle in technological leadership as a market-driven equilibrium phenomenon. For this purpose, we develop a new growth model that can capture how national technological leadership endogenously moves between countries along an equilibrium dynamic path. In doing this, we focus on endogenous innovation and international spillovers in a two-country setting with the dynamic optimization of consumption and saving by an infinitely-lived consumer. As the firms in a country develop innovations by investing resources, the stock of knowledge accumulates in the home country, and this subsequently but only partially contributes to the accumulation of foreign knowledge because of international spillovers through foreign direct investment (FDI).[2]

By regarding technological leadership as the state whereby a given country innovates most among all countries, we demonstrate that technological leadership by that country may shift to another country and then may alternate perpetually between the countries. Specifically, we obtain two main results. (a) The profitability of innovation is low, only the leading country innovates in equilibrium. In this case, leapfrogging never takes place. (b) If the profitability of innovation is sufficiently high, both leading and lagging countries engage in innovation. In this case, technological leadership can shift over time and will

[1] See also Ohyama and Jones (1995), Motta, Thisse, and Cabrales (1997), Brezis and Tsiddon (1998), van de Klundert and Smulders (2001), and Desmet (2002). The present paper essentially differs from those analyses in its focus on endogenous and perpetual cycles of leapfrogging, which will complement them by clarifying the intrinsically cyclical nature of national technological leadership.

[2] As argued by Brezis (1995), foreign capital plays a role in industrialization and development processes. We may also accept that international capital flows, as well as imports, are important channels for international knowledge spillover, as discussed in the literature (Grossman and Helpman 1991; Feenstra 1996). See Branstetter (2006) for recent empirical evidence.

perpetually move back and forth between countries along an equilibrium path if the international knowledge spillovers are reasonably efficient.

The key driving force behind endogenous and perpetual leapfrogging is the ability of a country to learn from foreign innovations. For example, a lagging country may learn much more from foreign innovations developed in a leading country than the leading country learns from those developed in the lagging country. Meanwhile, domestic innovations take place and build the knowledge stock in each country. The analysis formally shows that leapfrogging is possible only when both countries innovate, where the lagging country has a dual engine of knowledge growth consisting of domestic innovation and foreign innovation diffusing by the spillovers. If a country can learn efficiently from diffuse foreign innovations, technological leadership will perpetually alternate between the countries. We can easily elaborate on why both countries innovate in equilibrium; the profitability of innovation can be sufficiently high that innovation pays even for the technologically lagging country. When the profitability is low, the lagging country does not innovate and simply receives the spillovers from foreign innovation, where no leapfrogging occurs. This implies that the spillovers by themselves can only make the lagging country, at most, as innovative as the leading country, but *not more* innovative.

The endogenous occurrence of perpetual leapfrogging is *not* new in the context of price competition between firms. For instance, the important paper by Giovannetti (2001) considers a duopoly in which firms considering infinite technological adoption set prices with Bertrand competition in the product market. Using this model, Giovannetti identifies the conditions whereby firms alternate in adopting the new technology, thereby representing a leapfrogging process. He shows that demand conditions, such as price elasticities, play a role in determining whether leapfrogging can be perpetual in Bertrand competition. Lee, Kim, and Lim (2011) have provided recent empirical support for this contention. In addition, some studies in the field of economic geography address both the theory and the empirical evidence of technological leapfrogging at the regional level (for example, Quah 1996a, b).[3]

The present study relates to the literature on innovation and growth cycles. In order to capture the cyclical growth phenomena in the simplest fashion, we follow Shleifer (1986), Deneckere and Judd (1992), Gale (1996), Francois and Shi (1999), and Matsuyama (1999, 2001) by assuming that patents last only for a single period in a discrete time model. This assumption implies that a single period is sufficiently long, which can be somewhere around 20 years. Given that in reality, many innovated consumption goods become obsolete before their patents expire, for the sake of simplicity, we assume that innovations are made obsolete in a single period (which is fairly long). In line with existing studies, we assume the temporary nature of the monopoly enjoyed by innovators, which plays a role in explaining leapfrogging cycles in the growth process.[4]

[3]See Athreye and Godley (2009), Giovannetti (2013), and Petrakos and Rodríguez-Pose, and Rovolis (2005) for more recent research. In the political economy literature, Krusell and Ríos-Rull (1996) provide an endogenous explanation on a long cycle of stagnation and growth, similar to perpetual leapfrogging in the present paper, by focusing on vested interests in determining policies. See also Aghion et al. (2002) for perpetual leapfrogging at the firm level.

[4]See also Iwaisako and Tanaka (2012) for endogenous cycles in a North–South product-cycle model

In relation to this, in the present study, we view leapfrogging cycles as a discrete phenomenon.[5] This is in line with the literature on nonlinear equilibrium dynamics, in which a discrete-time growth model is commonly used for explaining complicated, real-world cycles (Nishimura and Yano 2008). Given that innovations often come in a cluster (Shleifer 1986), we believe that the discrete-time model can be a reasonable choice for explaining innovation-driven cycles such as leapfrogging in our model, although it is also essential to address this issue in a continuous-time setting as in Francois and Lloyd-Ellis (2003), who explain endogenous growth cycles in a continuous-time model of Schumpeterian growth.[6]

This study provides an important contribution to the theoretical literature by developing a new growth model with the dynamic optimization of an infinitely-lived consumer that can explicitly capture how the (relative) national leadership in cutting-edge technology endogenously moves between countries along an equilibrium path. The beauty of the present model is that the equilibrium dynamical system is derived from the model as an autonomous one-dimensional system, so that we can completely track and illustrate an entire equilibrium path of the national technological leadership between countries for any initial condition, by means of a tractable phase-diagram analysis. The result is novel to existing literature in demonstrating the intrinsically cyclical nature of national technological leadership; in our model, technological leadership endogenously fluctuates between countries on an equilibrium path. No research has addressed the equilibrium trajectory of national technological leadership or its endogenous cycles between countries.

2 Model

Time is discrete and extends from $-\infty$ to $+\infty$. Consider two countries, A and B that have identical preferences and differ only in their initial levels of innovation productivity. The countries are denoted by i or h ($i = h,\ B$; $h = A,\ B$), using a superscript for variables pertaining to the production side and a subscript for those pertaining to the consumption side.

There is a continuum of differentiated consumption goods in each period t. Each good is indexed by j. We follow the R&D-based endogenous growth model with expanding variety (Romer 1990, Grossman and Helpman 1991) by assuming innovation as generating new varieties of goods. Given that we later allow for FDI, the country where a particular firm innovates and manufactures may change. Let $\Gamma^i(t)$ be the set of goods that are innovated in country i in period t, and let $\Lambda^i(t)$ be the set of goods that are manufactured in country i in period t.

with overlapping generations, in which innovation and imitation interact with each other to generate perpetual fluctuations in the world growth rate. In their model, however, there is no leapfrogging.

[5] See the discussion at the end of Section 3.3 on the use of a discrete-time model. See also footnote 12.

[6] See also Francois and Lloyd-Ellis (2008, 2009, 2013) for related studies.

2.1 Consumption

In each country, an infinitely lived representative consumer inelastically supplies L units of labor for production and research and development (R&D) in every period. Note that the two countries are assumed to have equal labor forces, L. Each consumer is endowed with the same intertemporal utility function

$$U_i = \sum_{t=0}^{\infty} \beta^t \ln u_i(t),$$

where $\beta \in (0, 1)$ is the time preference rate. Temporary utility $u_i(t)$ is defined on the set $\{\Lambda^A(t) \cup \Lambda^B(t)\}$ of goods manufactured in both countries (free trade), taking the standard Dixit–Stiglitz form:

$$u_i(t) = \left(\int_{j \in \{\Lambda^A(t) \cup \Lambda^B(t)\}} x_i(j,t)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}, \tag{1}$$

where $x_i(j,t)$ is the consumption of good j in country i. Parameter $\theta \in (0, 1)$ denotes an inverse measure of the elasticity of substitution. Let $E_i(t) \equiv \int_{j \in \{\Lambda^A(t) \cup \Lambda^B(t)\}} p(j,t) x_i(j,t) dj$ be the spending in country i, where $p(j,t)$ denotes the price of good j. Solving the utility maximization problem in (1) leads to the demand function for good j, $x_i(j,t) = p(j,t)^{-(1/\theta)} E_i(t)/P(t)^{1-(1/\theta)}$, where $P(t)$ is the price index.[7] Aggregating these expressions, we obtain the derived aggregate demand, $x_A(j,t) + x_B(j,t) \equiv x(j,t)$, as

$$x(j,t) = \frac{E(t) p(j,t)^{-(1/\theta)}}{P(t)^{1-(1/\theta)}}, \tag{2}$$

where $E(t) = E_A(t) + E_B(t)$ is the aggregate spending in period t. The price elasticity of demand is constant at θ^{-1} for any j.

Solving the dynamic optimization of the consumer's utility for consumption and saving decisions under the intertemporal budget constraint results in the usual Euler equation $E_i(t+1)/E_i(t) = \beta(1 + r(t))$, where $r(t)$ is the interest rate in period t. We obtain

$$\frac{E(t+1)}{E(t)} = \beta(1 + r(t)). \tag{3}$$

2.2 Innovation, FDI, and manufacture

A single firm innovates and monopolistically supplies each differentiated consumption good.[8] Innovating a new good takes one period. In each period, say $t-1$, a firm in

[7] As is well known, the index is defined as $P(t) = \left(\int_{j \in \{\Lambda^A(t) \cup \Lambda^B(t)\}} p(j,t)^{1-(1/\theta)} dj \right)^{\frac{1}{1-(1/\theta)}}$.

[8] We assume this in order to make the analysis as simple as possible, consistent with the standard endogenous growth framework. This implies that there is no adoption of innovation or an explicit market for innovation. However, without any change in the results, it is possible to assume a more general model with an innovation market in which innovation and manufacture are by different firms.

country i can innovate one technology to produce a new differentiated good at the end of the period, $t-1$, by investing $1/K^i(t-1) \equiv k^i(t-1)$ units of domestic labor in R&D activity.[9] Here $K^i(t-1)$ denotes the technology level in innovation for country i in period $t-1$. In the subsequent period t, the firm will set up a production plant. In doing this, the firm can choose the country in which to manufacture the good in order to maximize monopolistic profits. In equilibrium, as foreign profits may be greater, the firm may transfer production to a foreign country through FDI. This is the channel for innovation diffusion in our model.[10]

We assume a simple production technology. There are constant returns to scale in the production of any good j and the productivity of labor is the same in both countries, which is normalized to be one.[11] The marginal cost in country i is thus equal to the wage rate in country i, $w^i(t)$. When the firm chooses to manufacture in country i in period t, captured by $j \in \Lambda^i(t)$, it produces $x(j,t)$ units of good j by using labor in country i. The standard profit maximization problem is written as

$$\max_{(p(j,t),\ x(j,t))} \pi(j,t) = p(j,t)x(j,t) - w^i(t)x(j,t)$$

subject to the market demand function (2). Since, by (2), the price elasticity of each good j is constant at $1/\theta$, the firm sets a monopolistic price of $p(j,t) = w^i(t)/(1-\theta) \equiv p^i(t)$. By substituting this into (2), we obtain the demand and profit functions as

$$x(j,t) = \frac{E(t)p^i(t)^{-(1/\theta)}}{P(t)^{1-(1/\theta)}} \equiv x^i(t) \tag{4}$$

and

$$\pi(j,t) = \theta E(t)\left(\frac{p^i(t)}{P(t)}\right)^{1-(1/\theta)} \equiv \pi^i(t) \tag{5}$$

for $j \in \Lambda^i(t)$ $(i = A,\ B)$. As firms prefer the country where profits are higher, the discounted present value of the firm innovating in country i in period $t-1$ is expressed as

$$V^i(t-1) = \frac{\max\{\pi^A(t), \pi^B(t)\}}{1+r(t-1)} - w^i(t-1)\ k^i(t-1). \tag{6}$$

In order to capture cyclical phenomena in the simplest fashion possible, we follow Shleifer (1986), Deneckere and Judd (1992), Gale (1996), Francois and Shi (1999) and

[9] We can extend the current deterministic innovation process to a stochastic form without any qualitative change of results.

[10] In line with the literature on international trade and growth (Lai 1998), we do not distinguish between the various forms of production transfer, including fully and partly owned subsidiaries and licensing.

[11] Here we simply consider that efficiency in manufacturing normalizes across countries. We can extend this simple setting by allowing for country-specific manufacturing efficiency and endogenous technological progress. In such an extended model, we can easily verify that the comparative advantage between R&D and manufacturing (rather than the absolute advantage in R&D) plays an important role in perpetual leapfrogging, although the results and their implications for perpetual leapfrogging do not change fundamentally.

Matsuyama (1999, 2001) by assuming that patents last only for one period.[12] This assumption implies that the length of a unit period is sufficiently long, which can be somewhere around 20 years given the duration of real-world patents. Given that in reality, many innovated consumption goods become obsolete before their patent expires, we may assume that innovations are made obsolete within a single period (which in our model is fairly long).[13] As shown below, this assumption makes the analysis tractable without any fundamental change in the results.[14] Finally, free entry guarantees that the net value of a firm should not be positive in equilibrium: $V^i(t-1) \leq 0$ for each i.

2.3 Knowledge accumulation and spillovers

Technology in innovation $K^i(t)$ advances with knowledge accumulation. Following Romer (1990), we assume intertemporal knowledge spillovers in innovation: current innovations contribute to the accumulation of the stock of knowledge $K^i(t)$, with which the cost of innovation, $k^i(t) = 1/K^i(t)$, reduces over time. Here, as is standard, the technology level in innovation $K^i(t)$ is interpreted as the knowledge stock in innovation.

The knowledge stock of a country consists of cumulative innovations of two types: home and foreign innovations. Define

$$N^i(t) \equiv \int_{\Gamma^i(t)} dj \text{ and } M^i(t) \equiv \int_{j \in \Gamma^f(t-1) \cap \Lambda^i(t)} dj. \tag{7}$$

Here $N^i(t)$ denotes the number of innovations developed in country i in period t and $M^i(t)$ denotes the number of innovations that occur in period $t-1$ in country f and then flows into country i from country f in period t. Following Romer (1990), we assume that the knowledge stock $K^i(t)$ linearly depends on the sum of domestic innovations that are developed up to the beginning of period t; i.e., $N^i(t-1) + N^i(t-2) + \cdots$, where $N^i(s)$ is a familiar proxy for the flow of knowledge generated as a by-product of the innovations in period s. We also assume that the international knowledge spillovers as an externality accompany FDI, such that each country learns from its foreign innovation inflows. Hence the knowledge stock of country i also depends on the sum, $M^i(t-1) + M^i(t-2) + \cdots$. Accordingly, we simply describe the knowledge stock using

$$K^i(t) = \sum_{s=-\infty}^{t} \left(N^i(s-1) + \mu M^i(s-1)\right) \text{ with } \mu \leq 1, \tag{8}$$

[12] This assumption implies that all patents start and expire at the same time, although in reality patents overlap. We may deal with this undesirable property by interpreting the length of a period as very long (e.g., 40 years) and dividing each period into subperiods (e.g., two 20-year periods), although we need a continuous-time model to fix this problem completely. In the present paper, we view leapfrogging as a discrete-time phenomenon and leave this issue for future work.

[13] This assumption may also be justified if each innovation is interpreted as fairly specific. For example, "innovation" in this model would be represented by the specific innovation associated with iPhone 4S or smartphones instead of cell phones or information technology more generally.

[14] Note that, in the next subsection, we assume that obsolete innovations stay "alive" in the sense that they continue to contribute to the current knowledge stock, although they are not explicitly traded in the marketplace. Whether they are traded or not is not important for our main story explained later.

where the parameter $\mu \in [0, 1]$ captures the efficiency of the contribution of international knowledge spillovers through foreign innovation inflows to knowledge accumulation and thus technological progress occurs. The efficiency of international knowledge spillovers increases with μ. If $\mu = 1$, spillovers are as efficient as domestic spillovers; if $\mu = 0$, there is no learning at all from foreign innovations. For the sake of explanation, we rewrite (8) as a flow as follows

$$K^i(t+1) - K^i(t) = N^i(t) + \mu M^i(t). \tag{9}$$

Considering (9), one may conjecture that spillovers $M^i(t)$ by themselves can cause a reversal of $K^i(t) > K^f(t)$: $K^i(t+1) < K^f(t+1)$ might hold by taking a sufficiently large $M^f(t)$. Leapfrogging may be able to occur simply through spillover $M^f(t)$. However, this conjecture is not the case with the present model because the example (of such a large $M^B(t)$) is not consistent with (8) and (9) for the following reason. Equation (8) says that innovation in country i, $N^i(t-1)$, not only contributes to the foreign knowledge, $K^f(t+1)$, through spillovers of $M^f(t)(= N^i(t-1))$ but also increases the domestic knowledge $K^i(t)$ in the previous period. Therefore, it is not possible to arbitrarily take a large $M^f(t)$ with $K^i(t)$ constant. *As $M^f(t)$ becomes large, by (8), $K^i(t)$ and $K^i(t+1)$ must also become large at a higher rate than, or at least the same rate as, $K^f(t+1)$ does.* We cannot artificially make $K^i(t+1) < K^f(t+1)$ by controlling $M^f(t)$ only. As we will see later, a sufficient number of domestic innovations, $N^f(t)$, is essential for the reversal of $K^i(t) > K^f(t)$ (i.e., leapfrogging). In summary, so long as we choose an identical equilibrium path, spillovers $M^f(t)$ by themselves cannot cause a reversal of $K^i(t) > K^f(t)$.

It is also worth pointing out that in (8) and (9), we assume that knowledge develops horizontally, rather than vertically. That is, we assume that knowledge accumulates as innovations are added to old innovations, not as innovations replace old innovations. What we see by this way of modeling knowledge accumulation is that current cutting-edge technologies/products (e.g., motor engine; smartphone; hybrid car) are often based on obsolete ones (e.g., steam engine; landline phone; gas-fueled car). The old technologies/products are made obsolete by the new ones and disappear from the marketplace, but the underlying concepts with the old technologies are incorporated into the new state-of-the-art technologies/products. Furthermore, our horizontal modeling of knowledge accumulation also implies that a country innovates along different lines to the other country, rather than along the same line. This captures that technologies or products made in different countries are sometimes differentiated, at least slightly. Thus, the knowledge stock of country i can accumulate as foreign innovations ($M^i(t)$) are simply added to, rather than replace or be replaced by, the own innovations ($N^i(t)$). Although clearly the results would become richer if the model also included the knowledge stock as a vertical ladder or the replacement of technologies, in this study, we focus on the above-mentioned horizontal aspect of knowledge with (8) and (9), which can help us highlight our main point.

3 Technological Leadership in Equilibrium Dynamics

In this section, we prove the main result that technological leadership may endogenously fluctuate over time, thereby perpetually moving back and forth between countries along an equilibrium path. Before proceeding, we provide a formal definition of the concept of technological leadership. Taking into account the notion in economic history (Davids 2008),[15] we refer to a country that develops the most innovations among the countries as the technological leader, and a country that develops few innovations as a lagging country. In the present model, and as will be made apparent later, this definition is equivalent to that in existing literature, which defines leadership as the state whereby a given country has the highest productivity among the countries. Thus, in equilibrium, country i innovates more if and only if its innovation productivity is higher; $N^i(t) > N^f(t)$ if and only if $K^i(t) > K^f(t)$. For simplicity, we use $K^i(t) > K^f(t)$ to designate country i as the technological leader, and we refer to any reversal of the leading position as technological leapfrogging.

Without loss of generality, we assume that country A is the leading country in period t, $K^A(t) > K^B(t)$ (and thus $N^A(t) > N^B(t)$ to be shown in equilibrium), and we refer to this situation as regime A. If $K^A(t) < K^B(t)$ (and thus $N^A(t) < N^B(t)$ to be shown in equilibrium), we refer to it as regime B.

In any period of time, this model can be regarded as a variant of a conventional two-good Ricardian model, where the two outputs considered are innovation and production. Given $K^A(t) > K^B(t)$, there are potentially three possible specialization patterns in period t: (1) one in which both countries engage in manufacturing, (2) one in which both countries engage in R&D, and (3) one in which both countries are specialized. It is useful to define a new variable, $N(t) = \int_{j\in\{\Lambda^A(t)\cup\Lambda^B(t)\}} dj$, which is the total number of goods manufactured in t, satisfying $N(t) = N^A(t-1) + N^B(t-1)$.

3.1 No Leapfrogging

Suppose that both countries produce goods. In this case, only the leading country A innovates, and both countries manufacture in equilibrium. That is, $\Gamma^A(t) \neq \emptyset$ and $\Gamma^B(t) = \emptyset$. By (7), we have $N^A(t) > 0$ and $N^B(t) = 0$. By $N(t+1) = N^A(t) + N^B(t)$, we also have $N(t+1) = N^A(t)$. As this situation is similar to the North–South product-cycle model à la Krugman (1979) and Helpman (1993) where only the North innovates and both the North and the South manufacture, we may refer to this pattern as a "North–South regime."[16] As manufacturing takes place in both countries, the wages are internationally equalized, $w^A(t) = w^B(t) = w(t)$, implying $p^i(t) = p(t)$ and thus $x^i(t) = x(t)$ by (4).

Only the leading country A innovates, so the free-entry condition holds as $V^A(t) = 0 > V^B(t)$.[17] Under the equalization of wages, $p^i(t) = p(t)$, using (5), the discounted

[15] Davids (2008) considered that a country that has technological leadership plays an initiating role in the development of new technologies across a wide variety of fields.

[16] Here, we assume that the North is a country that innovates; however, if the North (the South) was defined as a country where the wage rate is higher (lower) as is also usual in the literature, these North/South labels could be misleading. Nevertheless, we use these labels because we can easily control the international wage differential in the present model by incorporating into the model an international differential in manufacturing productivity.

[17] More specifically, free entry in country A requires $V^A(t) = 0$. No innovation in country B implies $V^B(t) < 0$.

present value of an innovation in country i in (6) can be expressed as

$$V^i(t) = \frac{1}{1+r(t)} \frac{\theta E(t+1)}{N(t+1)} - w^i(t)\ k^i(t). \tag{10}$$

Substituting into (10) the Euler equation $1 + r(t) = E(t+1)/\left(\beta E(t)\right)$ from (3), the free-entry condition $V^A(t) = 0 > V^B(t)$ requires

$$\frac{\beta\theta E(t)}{N(t+1)} = w(t)\ k^A(t) < w(t)\ k^B(t). \tag{11}$$

The first equality in (11) ensures that the discounted value of an innovation $(\beta\theta E(t)/N^A(t))$ and the cost $(w(t)\ k^A(t))$ are balanced in the leading country A. The second inequality in (11) ensures that the cost of an innovation is lower in the leading country $(w(t)k^A(t))$ than the lagging country B $(w(t)k^B(t))$. Noting $k^i(t) = 1/K^i(t)$, this implies $K^A(t) > K^B(t)$; the leading country A has accumulated more knowledge up to period t.

The labor market-clearing conditions for country i is given by

$$L = \int_{\Gamma^i(t)} k^i(t)dj + \int_{\Lambda^i(t)} x^i(t)dj. \tag{12}$$

The quantity of a good, $x^i(t)$, can be derived from (4) and (7) as $x^i(t) = (1-\theta)E(t)/\left(w(t)N(t)\right)$ for each i. Together with this, by eliminating the country index i from (12),[18] we can obtain the world labor constraint as

$$2L = N^A(t)\ k^A(t) + (1-\theta)\frac{E(t)}{w(t)}, \tag{13}$$

in which $\Gamma^B(t) = \emptyset$ is used. The left-hand side in (13) is the world supply of labor, and the right-hand side is the world demand for labor from both the innovation sector in leading country A and the manufacturing sectors in both countries.

In order to determine the equilibrium flow of innovation made in period t, we will eliminate the term $E(t)/w(t)$ from the world labor market-clearing condition (13), using the free-entry condition (11). Then, noting $k^A(t) = 1/K^A(t)$ and $N(t+1) = N^A(t)$, the flow of innovation in period t is derived as

$$N^A(t) = K^A(t)\frac{2L\Theta}{1+\Theta} \quad \text{and } N^B(t) = 0, \tag{14}$$

where $\Theta \equiv \beta\theta/(1-\theta)$. Here Θ captures the discounted present value of a markup over the firm's marginal cost, which increases with the time preference β and decreases with the elasticity of substitution θ^{-1}.[19] We refer to Θ as the profitability of an innovation.

[18] We do this by summing up both sides of (12) over i.

[19] Li (2001) argues that the evidence regarding whether there is any conventional value or a range of values for the elasticity of substitution is inconclusive. For example, Broda and Weinstein (2006) show that the elasticity of substitution is, on average, greater than two, but tends to decline over time and is actually less than two in some sectors (e.g., motor vehicles).

Equation (14) shows that the innovation flow $N^A(t)$ increases with the knowledge stock $K^A(t)$ and the profitability of an innovation.

Now we can derive the number of goods that are manufactured in each country. Noting (12) with $\Gamma^B(t) = 0$, we have $L = \int_{\Lambda^B(t)} x^B(t)dj$. Then, by (11) and (14), we obtain

$$\left(\int_{\Lambda^A(t)} dj\right) = \frac{1-\Theta}{2}N(t) \text{ and } \left(\int_{\Lambda^B(t)} dj\right) = \frac{1+\Theta}{2}N(t). \tag{15}$$

To ensure that the leading country A manufactures, i.e., $\int_{\Lambda^A(t)} dj > 0$, we need to impose

$$\Theta < 1.^{20} \tag{16}$$

This requires that the time preference β is small and the price elasticity of substitution θ^{-1} is high.

So far, we have four important conditions. Inequality (16) guarantees that only the leading country innovates in equilibrium. Inequality $K^A(t) > K^B(t)$ requires that country A becomes the leading country. Equations (14) and (15) determine the innovation flow and the fractions of manufactured goods, respectively.

In what follows, we demonstrate that in the North–South regime, leapfrogging never takes place even if the spillovers are completely efficient ($\mu = 1$). By (14) and (9), the growth of knowledge can be expressed as follows:

$$K^A(t+1) = \left(\frac{2L\Theta}{1+\Theta} + 1\right) K^A(t) \tag{17}$$

and

$$K^B(t+1) = \mu M^B(t) + K^B(t), \tag{18}$$

where $M^B(t) = \int_{\Lambda^B(t)} dj = (1+\Theta)\, N(t)/2$ by (15).

As $K^A(t)$ is given by history, (17) fully determines the growth of knowledge for the leading country A. Apparently, (18) does not determine $K^B(t+1)$ without any additional historical assumption because the level of $M^B(t) = (1+\Theta)\, N(t)/2$ depends on $N(t) = N^A(t-1) + N^B(t-1)$, which is determined by innovation activities undertaken in the previous period, $t-1$. So far, we do not have any assumption on innovation activities in period $t-1$ or before. Nevertheless, as shown in our first theorem, regardless of innovation activities in the past, leapfrogging never takes place in the North–South regime.

Theorem 1 (No leapfrogging with lower profitability) *Suppose that the profitability of an innovation* Θ *falls below* 1. *Then, under the infinitely lived agent's dynamic optimization, only the leading country innovates in equilibrium (the North-South regime). In this case, leapfrogging never takes place.*

[20] This condition also guarantees that the resource condition for the leading country A, $N^A(t)/A^A(t) < L$, is satisfied.

Proof. By (16), $\Theta < 1$ ensures that only the leading country innovates. By (15), $M^B(t) = (1+\Theta)\,N(t)/2$. (a) Assume $K^A(t-1) > K^B(t-1)$. By the expression of $N^A(t-1)$ in (14), with $K^A(t-1) = N^A(t-1) + K^A(t)$ from (9), substituting $N(t) = N^A(t-1)$ into (18) derives

$$K^B(t+1) = K^A(t)\frac{\mu L\Theta\,(1+\Theta)}{(2L+1)\,\Theta+1} + K^B(t). \tag{19}$$

From (17) and (19), we can show that $K^A(t+1) > K^B(t+1)$ holds so long as $K^A(t) > K^B(t)$, noting $\Theta < 1$ and $\mu < 1$. (b) Assume $K^A(t-1) < K^B(t-1)$. By symmetry, noting $N(t) = N^B(t-1)$, the analogous procedures derive

$$K^B(t+1) = \left(\frac{\mu L\Theta\,(1+\Theta)}{(2L+1)\,\Theta+1} + 1\right)K^B(t). \tag{20}$$

From (17) and (20), $K^A(t+1) > K^B(t+1)$ holds so long as $K^A(t) > K^B(t)$, given $\Theta < 1$ and $\mu < 1$. This proves that $K^A(t) > K^B(t)$ cannot be reversed for the subsequent period when $\Theta < 1$, whether either country is a leading country in the previous period $t-1$. ■

We now elaborate upon the intuition why economies with the lower profitability of innovation cannot experience leapfrogging. There are potentially two sources of leapfrogging: knowledge growth from domestic innovation and spillovers from foreign innovation. In this regime, however, the lagging country (country B) only has spillovers from $M^B(t)$, the foreign innovations developed in the leading country (country A). No domestic innovations take place in the lagging country. Given that the leading country also gains from the innovations $M^B(t)$ (which are included in $K^A(t)$) even more efficiently than, or at least equally to, the lagging country, *the spillovers alone can only make the lagging country as innovative as, but not more innovative than, the leading country.* Thus, leapfrogging never takes place. As shown later, if the lagging country not only has spillovers from the leading country, but also innovates by itself, leapfrogging will be possible.

Why does only the leading country innovate when $\Theta < 1$? Since $\Theta = \beta\theta/(1-\theta)$, lower Θ is associated with lower β and higher θ^{-1}. A lower time preference β results in a higher interest rate $r(t)$, which decreases the discounted value of the profit $\pi(t)$. A higher elasticity of substitution θ^{-1} implies a lower markup ratio $(1/1-\theta)$ and a lower profit $\pi(t)$. The inequality condition $\Theta < 1$ intuitively requires that the discounted value for an innovation is fairly low. That is, the value is too low for the lagging country B to innovate by itself. In other words, where the profitability of an innovation Θ (depending on the time preference rate β and the elasticity of substitution θ^{-1}) is higher, the discounted benefit from an innovation would be higher and thus innovation would be profitable, even for firms in the lagging country. Finally, we may summarize this by stating that: when the profitability of an innovation is low, leapfrogging does not take place because the lagging country does not innovate.

3.2 An illustration

To illustrate further the international dynamics of knowledge in the North–South regime, we assume that the leading country A has retained leadership in the past; i.e., $N^A(s) >$

$0 = N^B(s)$ and thus $K^A(s) > K^B(s)$ for $s = t,\ t-1,\ \cdots$. This consideration is reasonable given that Theorem 1 shows that leapfrogging never takes place. The growth of knowledge follows (17) and (19) for any $s \geq t$. Define $\psi(t) = K^A(t)/(K^A(t) + K^B(t))$, which stands for the knowledge ratio for country A. We can derive the dynamic system for $\psi(t)$ as follows. Noting $K^A(t) > K^B(t)$,

$$\psi(t+1) = \frac{(a_1 + 1)\,\psi(t)}{1 + (a_1 + \mu a_2)\,\psi(t)} \text{ for } \psi(t) \in (0.5, 1), \tag{21}$$

where a_1 and a_2 are positive numbers determined by β, θ, and L.[21] By applying the above procedures to the case of $\psi(t) \in (0, 0.5)$ where country B is the leading country, we can easily derive the following dynamic system:

$$\psi(t+1) = \frac{(1 - \mu a_2)\,\psi(t) + \mu a_2}{1 + (1 - \psi(t))\,(a_1 + \mu a_2)} \text{ for } \psi(t) \in (0, 0.5). \tag{22}$$

Note that $a_1 < 1$ and $a_2 < 1$ if $\Theta < 1$. We thus can verify that so long as $\Theta < 1$, the steady state is unique and higher than 0.5 for (21) and lower than 0.5 for (22).

Figure 1 illustrates the phase diagram for systems (21) and (22) with their steady states, ψ_A^* and ψ_B^*. As shown, any path starting in the situation where country A (B) is the leading country stably converges to a steady state; $\psi(s) > (<) 0.5$ for all $s > t$ if $\psi(t) > (<) 0.5$. Thus, this phase diagram shows that no leapfrogging occurs in the case where the profitability of an innovation is lower.

3.3 Leapfrogging cycles

Let us now consider the situation where both countries innovate (pattern (2)), which is realized when $\Theta > 1$, as explained below. In this case, only the lagging country, B, manufactures. Then we have $N^A(t) > 0$ and $N^B(t) > 0$; $N(t+1) = N^A(t) + N^B(t)$. In contrast to the previous case, both countries innovate in equilibrium. For convenience sake, we refer to this specialization pattern as a North–North regime. Because innovation takes place in both countries, by the free-entry condition $V^A(t) = V^B(t) = 0$, the cost of an innovation is internationally equated; $w^A(t)k^A(t) = w^B(t)k^B(t)$. Manufacturing takes place only in country B, so that $\pi^A(t+1) < \pi^B(t+1)$ must hold, which implies $w^A(t) > w^B(t)$. Thus, the following inequality must hold in this case: $k^A(t) < k^B(t)$.

Substituting the Euler equation (3) into the value of innovation (10), the free-entry condition of $V^A(t) = V^B(t) = 0$ implies

$$\frac{\beta\theta E(t)}{N(t+1)} = w^A(t)k^A(t) = w^B(t)k^B(t). \tag{23}$$

The interpretation of (23) is similar to that of (11).

[21] The formal definitions are:

$$a_1 \equiv \frac{2L\Theta}{1+\Theta} \text{ and } a_2 \equiv \frac{L\Theta(1+\Theta)}{1+\Theta(2L+1)}.$$

By applying $\Lambda^A(t) = \emptyset$ to the labor condition (12), we can easily determine the flow of innovation for the leading country A as

$$N^A(t) = LK^A(t). \tag{24}$$

By using (4) and the monopolistic pricing $p^B(t) = w^B/(1-\theta)$, $x^B(t) = (1-\theta)E(t)/(N(t)w^B(t))$ is obtained. Substituting this into the labor market condition (12), with (23), we can also determine the flow of innovation for the lagging country B as

$$N^B(t) = \frac{1}{1+\Theta}\left(\Theta LK^B(t) - LK^A(t)\right). \tag{25}$$

Equation (25) shows that the innovation flow in the lagging country $N^B(t)$ increases with the domestic knowledge stock $K^B(t)$ but decreases with the foreign knowledge stock $K^A(t)$. In order to ensure that the lagging country also innovates, $N^B(t) > 0$, by (25), we need to assume

$$\Theta > \frac{K^A(t)}{K^B(t)} > 1, \tag{26}$$

which requires that the international technological gap, $K^A(t)/K^B(t)$, is not very large. To allow (26) to be feasible, we need to impose

$$\Theta > 1. \tag{27}$$

Condition (27) requires that the profitability of an innovation is high; the time preference rate β is large and/or and the elasticity of substitution θ^{-1} is low.

What if the knowledge gap, $K^A(t)/K^B(t)$, is larger than Θ? The specialization pattern then goes to case (3), where both countries are specialized such that the leading country innovates and the lagging country manufactures. We may refer to this as a full North–South regime. In this case, the wage rate in the lagging country is determined by its labor market-clearing condition $L = N(t)x^B(t)$ as $w^B(t) = (1-\theta)E(t)/L$. The innovation flow in the leading country A $N^A(t)$ does not change from (24) while that in the lagging country B is zero ($N^B(t) = 0$). The condition for the full North–South regime is

$$\frac{K^A(t)}{K^B(t)} > \Theta > 1. \tag{28}$$

To prove that leapfrogging may take place here, we assume (26). In proving this, we suppose that country A retains leadership for two consecutive periods. That is, (26) holds for two periods, t and $t-1$. This implies that spillovers $M^B(t)$ are equal to $N^A(t-1) = LK^A(t-1)$ because innovations developed by country A in period $t-1$ all flow to the lagging country B.

By substituting (24) and (25) into (9), with $M^A(t) = 0$ and $M^B(t) = LK^A(t-1)$, the growth of knowledge follows

$$K^A(t+1) = \underbrace{LK^A(t)}_{N^A(t):\ \text{domestic innovation}} + K^A(t), \tag{29}$$

$$K^B(t+1) = \underbrace{\frac{\Theta LK^B(t) - LK^A(t)}{1+\Theta}}_{N^B(t):\ \text{domestic innovation}} + \underbrace{\frac{\mu L}{L+1}K^A(t)}_{M^B(t):\ \text{spillovers}} + K^B(t). \tag{30}$$

In (30), the lagging country B has two sources of knowledge growth, namely, domestic innovation $N^B(t)$ and spillovers from foreign innovation $M^B(t)$, which sharply contrast with the North–South regime where the lagging country does not innovate. By combining (29) and (30), we derive the international dynamics of knowledge as

$$\psi(t+1) = \frac{(L+1)\,\psi(t)}{\frac{\mu L}{L+1}\psi(t) + \left(\frac{\Theta L}{1+\Theta} + 1\right)}, \tag{31}$$

given $0.5 < \psi(s) < \frac{\Theta}{1+\Theta}$ for $s = t,\, t-1$ coming from (26).

Using (31), the following theorem formally proves the perpetual cycle of leapfrogging as an equilibrium phenomenon.

Theorem 2 (Leapfrogging cycles with higher profitability) *Suppose that the profitability of an innovation Θ exceeds $1 + 2L$ and that the international technological gap $K^A(s)/K^B(s)$ is lower than Θ for $s = t,\, t-1$. Then, under dynamic optimization by the infinitely lived agent, both the leading country and the lagging country innovate in equilibrium (the North–North regime). In this case, neither country may be able to retain its technological leadership for infinite sequential periods; i.e., leapfrogging may take place repeatedly and perpetually along an equilibrium path. Specifically, this occurs if*

$$\mu \in \left(\frac{2\,(L+1)}{1+\Theta}, 1\right]. \tag{32}$$

Proof. First, by (26), $\Theta > 1$ and $K^A(s)/K^B(s) < \Theta$ ensure that both countries innovate. The steady state of system (31) is uniquely given by

$$\psi^* = \frac{1}{\mu}\frac{L+1}{1+\Theta},$$

which is less than 0.5 so long as (32) holds. If $\psi^* < 0.5$, given (31), $\psi(t)$ will stably decrease and eventually fall below 0.5. This shows that when country A has leadership for two periods (t and $t-1$), it can never retain its leadership for an infinite number of sequential periods. Put differently, technological leadership is always temporary. By symmetry, it is straightforward to show the opposite case where country B initially has leadership for two periods. This proves the perpetual occurrence of leapfrogging, taking into account the fact that for the case where a country initially has leadership for just one period, either the country retains leadership for two periods or is immediately leapfrogged. Finally, by the assumption of $\Theta > 1 + 2L$, the condition (32) is feasible. ■

Theorem 2 implies that when the profitability of an innovation Φ is high enough, the discounted benefit from an innovation is high and thus innovation can be profitable even for firms in the lagging country. Both countries innovate in equilibrium, in which case leapfrogging can take place if the efficiency of international spillovers μ is reasonably high. It is worth pointing out that leapfrogging can take place even if $\mu < 1$, i.e., the international spillovers are *not* completely efficient. We summarize this as a proposition.

Proposition 1 *So long as the profitability of innovation is sufficiently high, leapfrogging can take place endogenously and perpetually if the international knowledge spillovers are reasonably efficient. Even if the spillovers are not completely efficient, leapfrogging is still possible to occur in equilibrium.*

The key driving force behind leapfrogging cycles is the dual growth engine of a lagging country. In the North–North regime, which results from higher profitability Θ, the lagging country both innovates and manufactures. Thus, the lagging country's knowledge accumulates not only through its own innovations but also through the flow of spillovers from the leading country's innovations. In this sense, the growth engine of knowledge in the lagging country is dual: innovating by itself and learning from abroad.[22] Although the leading country innovates faster than the lagging country, knowledge growth in the leading country is driven only by domestic innovations. This creates the possibility of leapfrogging.

Needless to say, our analysis has some limitations that arise from the model specification. Let us briefly discuss them. First, specialization takes place in the present model, which is a dynamic version of the Ricardian model. In reality, the leading country also manufactures foreign innovations (those in the lagging country) and may also learn from them. In addition, in history, no country seems to have ever been specialized in R&D. Therefore, this model would capture some particular aspect of real-world behavior. That is, lagging countries may have an advantage in international technology competition with the leading country because they can learn from the leader's active innovation as well as their own experience in innovation. Otherwise, we could easily remove this unrealistic aspect concerning specialization from the model by assuming, for instance, a strictly concave production function in manufacturing. Since this would make the analysis intractable without adding new insights, we adopt the present setting for simplicity.

Second, given the historical fact that technology leadership has often shifted between countries, it is important to provide an extended case comprising more than two countries. We can demonstrate that three or more countries on an equilibrium path can perpetually experience various forms of leapfrogging including, for example, growth miracles (Matsuyama 2007), in which the least productive country leapfrogs all rival countries with higher productivity levels in a single burst. Such growth miracles may take place sporadically or consecutively or in some complex combination; see Furukawa (2012) for a formal analysis.

Third, we assume that countries are basically homogeneous, in order to explain the mechanism through which leapfrogging occurs endogenously in a clear-cut way. Allowing for country heterogeneity, we can demonstrate that leapfrogging may take place finite times in the model where the countries have different labor endowments and/or efficiency levels of the spillovers.[23]

Finally, and most importantly, because the model assumes discrete time, it is implicitly assumed that the leading country takes a long time to exploit its technological advantage

[22] Recall that in the North–South regime, knowledge growth in the lagging country results only from foreign innovations, in which leapfrogging is not possible (Theorem 1).

[23] The formal analysis is available upon request from the author.

and in the meantime allows the lagging country to take over. One may wonder *why the leading country waits so long*. The simple answer to this question is that, in each period, firms in the leading country only pursue a one-period monopoly and thus they do not care if the leadership of their home country persists or not. For example, if we introduce into the model a welfare-maximizing government that gives subsidies to innovating firms, a policy game between international governments may lead to a different situation where the leading country does not wait so long. However, because innovations often take a very long time from startup to implementation and come in bunches,[24] the government in the leading country cannot see *in real time* what happens in the lagging country, because of asymmetric information. Given this, the only thing the leading country can do may be simply to wait and see what happens in the lagging country.

Of course, it is potentially necessary to extend our discrete-time analysis to continuous time. In a continuous-time setting, it is essential to consider what happens as the technology levels of two countries become equal in the process of leapfrogging. One possible way forward would be to focus on technological complementarity between countries. Spillovers from the leading country then combine with the backward technology of the lagging country, so our leapfrogging mechanism should work in a continuous-time setting. We leave this fundamental issue associated with discrete time for future work.

3.4 An illustration

To illustrate, we again use a phase diagram. However, the configuration of the phase diagram depends on the history, i.e., which country was a leading country in the previous period. As this is simply a problem of visual complication, to clarify the illustration, we assume that innovation activities are completed within one period. Thus, the innovation value in (6) should be replaced by

$$V^i(t) = \max\{\pi^A(t), \pi^B(t)\} - w^i(t)k^i(t). \tag{33}$$

Noting that $\theta > 0.5$ holds if (26) or (28) holds, we can describe the international dynamics of knowledge as follows.[25]

$$\psi(t+1) = \Phi(\psi(t)) \equiv \begin{cases} \frac{\mu L + (1-\mu L)\psi(t)}{1+(1+\mu)L(1-\psi(t))} & \text{for } \psi(t) \in (0, 1-\theta) \\ \frac{(\theta-(1-\mu))L+(1+(1-\mu)L)\psi(t)}{\theta L + 1 + \mu L(1-\psi(t))} & \text{for } \psi(t) \in (1-\theta, 0.5) \\ (L+1)\frac{\psi(t)}{1+\theta L + \mu L \psi(t)} & \text{for } \psi(t) \in (0.5, \theta) \\ (L+1)\frac{\psi(t)}{1+(1+\mu)L\psi(t)} & \text{for } \psi(t) \in (\theta, 1) \end{cases}. \tag{34}$$

The equilibrium dynamic system Φ is autonomous and nonlinear. Figure 2 depicts the phase diagram of system Φ for $\mu < 2(1-\theta)$. There are two steady states, both of which are stable. For all initial points, technological leadership can never alternate internationally. In this case of $\mu < 2(1-\theta)$ where the spillovers are less efficient (small μ), the result

[24] This issue is intensively investigated in the literature on innovation cycles (see Shleifer 1986).
[25] See Appendix A for the derivations.

is essentially identical to that in the North–South regime; that is, no leapfrogging takes place.

There are two subcases with (a) $\mu < (1-\theta)/\theta$ and (b) $(1-\theta)/\theta < \mu$. In case (a), even if the advantage of the leading country is initially very small ($\psi(t)$ is around 0.5), the knowledge gap stably widens and the world economy finally converges to the steady state (ψ_i^{**}) in the full North–South regime. The two countries, even though both innovate initially, will eventually split into innovative and non-innovative countries, in which the outcome is ultimately determined by the initial (small) knowledge difference. In case (b), $\psi(t)$ converges to the steady state (ψ_i^{*}), in which case both countries innovate in the long run.

Most importantly, Figure 3 depicts a typical path for the case in which $\mu > 2(1-\theta)$. Given that no steady state exists, the international knowledge fraction $\psi(t)$ will move perpetually back and forth between the two regimes $(0, 0.5)$ and $(0.5, 1)$. Finally, note that the condition of leapfrogging cycles in the simplified model, $\mu > 2(1-\theta)$, is analogous to (32).

4 Concluding remarks

In this paper, we developed a two-region endogenous innovation model with dynamic optimization of the infinitely lived consumer, in which knowledge diffuses internationally through FDI. The major finding is that technological leadership may shift internationally, perpetually moving back and forth between countries if the profitability of an innovation is higher and the spillovers are relatively efficient. Specifically, if the profitability of an innovation is lower, in equilibrium, only the leading country innovates. In this case, leapfrogging never arises. If the profitability of an innovation is higher, in equilibrium, both countries innovate. In this case, leapfrogging perpetually takes place along an equilibrium path if international spillovers are reasonably efficient. In a big picture, we may say that the growth process of an international economy can be intrinsically cyclical depending on the factors such as the profitability of innovation and the efficiency of international spillovers.

Our result shows the possibility that lagging countries leapfrog *in spite of innovating less*, by focusing on learning from foreign innovation as a missing link. In this sense, the present paper is close to Glass (1999) in spirit. With some examples from Asian countries including South Korea or China,[26] Glass (1999) considers whether learning from foreign innovation through imitation can serve as a stepping stone enabling firms from lagging countries to undertake innovation. The present paper extends Glass's (1999) view by demonstrating that the effectiveness of learning may be a key factor in enabling the lagging country to leapfrog the leading country by shifting to an innovative economy.

To grasp the essence of leapfrogging cycles, we have left some important issues for future work. First, we have conceptualized essentially homogeneous countries. Departing

[26]See Carolan, Singh, and Talati (1998).

from this, we would be able to investigate various patterns of leapfrogging, including one-time or terminal leapfrogging. Second, given heterogeneous countries, to consider which country finally prevails may attract policy-related researchers to leapfrogging issues. For example, the government of a country may affect the process of leapfrogging by means of policy, including subsidies, tariffs, competition policies, and institutional reforms. In pursuing this line of research, it could be interesting to investigate the Nash equilibrium in a policy game where each government maximizes domestic welfare. Third, as an alternative, it may be fruitful to relate the degree of international spillovers to the legal protection of intellectual property, a prominent issue in international relations. Strengthening the domestic level of intellectual property protection may or may not delay the timing of a country to leapfrog (or be leapfrogged). It may even deprive it of the opportunity to leapfrog.

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Appendix A

In the North–North regime:

(A) Assume $\frac{\theta}{1-\theta} > \frac{K^A(t)}{K^B(t)} > 1$ (i.e., $\psi(t) \in (0.5, \theta)$). The free-entry condition (23) becomes

$$\frac{\theta E(t)}{N^A(t) + N^B(t)} = \frac{w^i(t)}{K^i(t)}. \tag{A1}$$

Given the labor condition for the leading country A, we have

$$N^A(t) = LK^A(t). \tag{A2}$$

By the labor condition for the lagging country, $L = \left(N^B(t)/K^B(t)\right) + \left(N^A(t) + N^B(t)\right) x^B(t)$, with $x^B(t) = \frac{(1-\theta)E(t)}{(N^A(t)+N^B(t))w^B(t)}$, we thus have

$$N^B(t) = \theta LK^B(t) - (1-\theta)\, LK^A(t), \tag{A3}$$

in which $w^B(t)$ is eliminated using (A1). Noting $M^B(t) = N^A(t)$ here, by (9), we can have the dynamic system as follows:

$$\psi(t+1) = \frac{(L+1)\,\psi(t)}{\mu L\psi(t) + \theta L + 1} \text{ for } \psi(t) \in (0.5, \theta)\,. \tag{A4}$$

Note that $\theta > 0.5$ holds in the North–North regime.

(B) Assume $\frac{1-\theta}{\theta} < \frac{K^A(t)}{K^B(t)} < 1$ (i.e., $\psi(t) \in (1-\theta, 0.5)$). Due to the symmetry,

$$N^A(t) = \theta LK^A(t) - (1-\theta)\, LK^B(t) \text{ and } N^B(t) = LK^B(t) \tag{A5}$$

hold. We can also derive

$$\psi(t+1) = \frac{(\theta + (\mu-1))L + (1 + (1-\mu)\, L)\,\psi(t)}{\theta L + 1 + \mu L\,(1-\psi(t))} \text{ for } \psi(t) \in (1-\theta, 0.5)\,. \tag{A6}$$

In the full North–South regime:

(A) Assume $\frac{K^A(t)}{K^B(t)} > \frac{\theta}{1-\theta} > 1$ (i.e., $\psi(t) \in (\theta, 1)$). The leading country innovates following

$$N^A(t) = LK^A(t) \text{ and } N^B(t) = 0. \tag{A7}$$

The lagging country receives spillovers $\mu M^B(t) = \mu N^A(t)$, with $\mu \leq 1$.[27] Then, the knowledge dynamics is as follows:

$$\psi(t+1) = \frac{(L+1)\,\psi(t)}{1 + (1+\mu)\, L\psi(t)} \text{ for } \psi(t) \in (\theta, 1)\,. \tag{A8}$$

[27] By using the labor market condition for the lagging country B and the free-entry condition, we can easily verify that

$$\frac{A^A(t)}{A^B(t)} > \frac{w^A(t)}{w^B(t)} = \frac{\theta}{1-\theta} > 1$$

holds.

(B) Assume $\frac{K^A(t)}{K^B(t)} < \frac{1-\theta}{\theta} < 1$ (i.e., $\psi(t) \in (0, 1-\theta)$). Due to the symmetry, $N^A(t) = 0$ and $N^B(t) = LK^B(t)$, with $\mu M^A(t) = \mu N^B(t)$. We can easily have

$$\psi(t+1) = \frac{\mu L + (1-\mu L)\,\psi(t)}{1 + (1+\mu)\,L\,(1-\psi(t))} \text{ for } \psi(t) \in (0, 1-\theta)\,. \tag{A9}$$

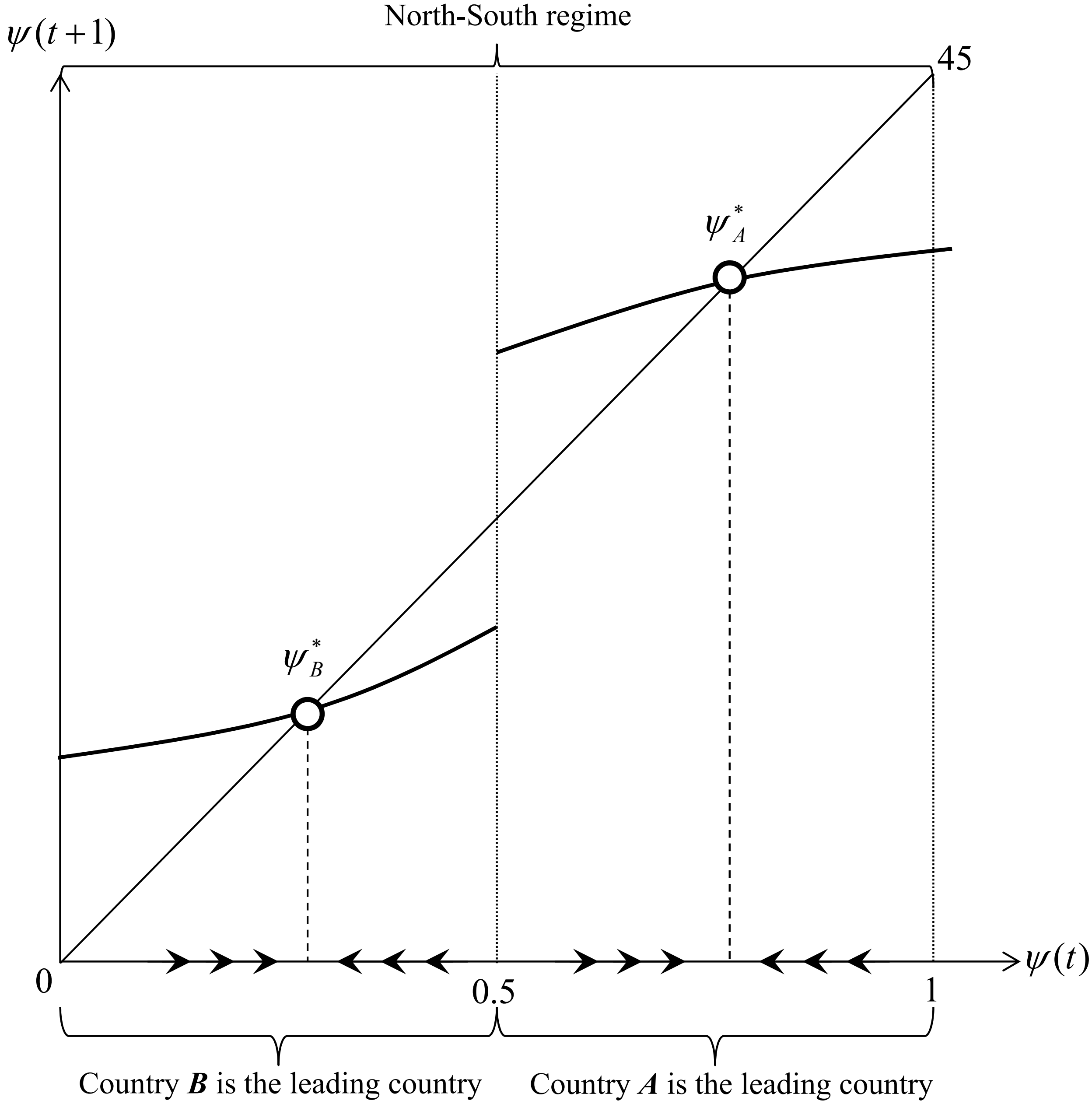

Figure 1: No leapfrogging in the North-South regime

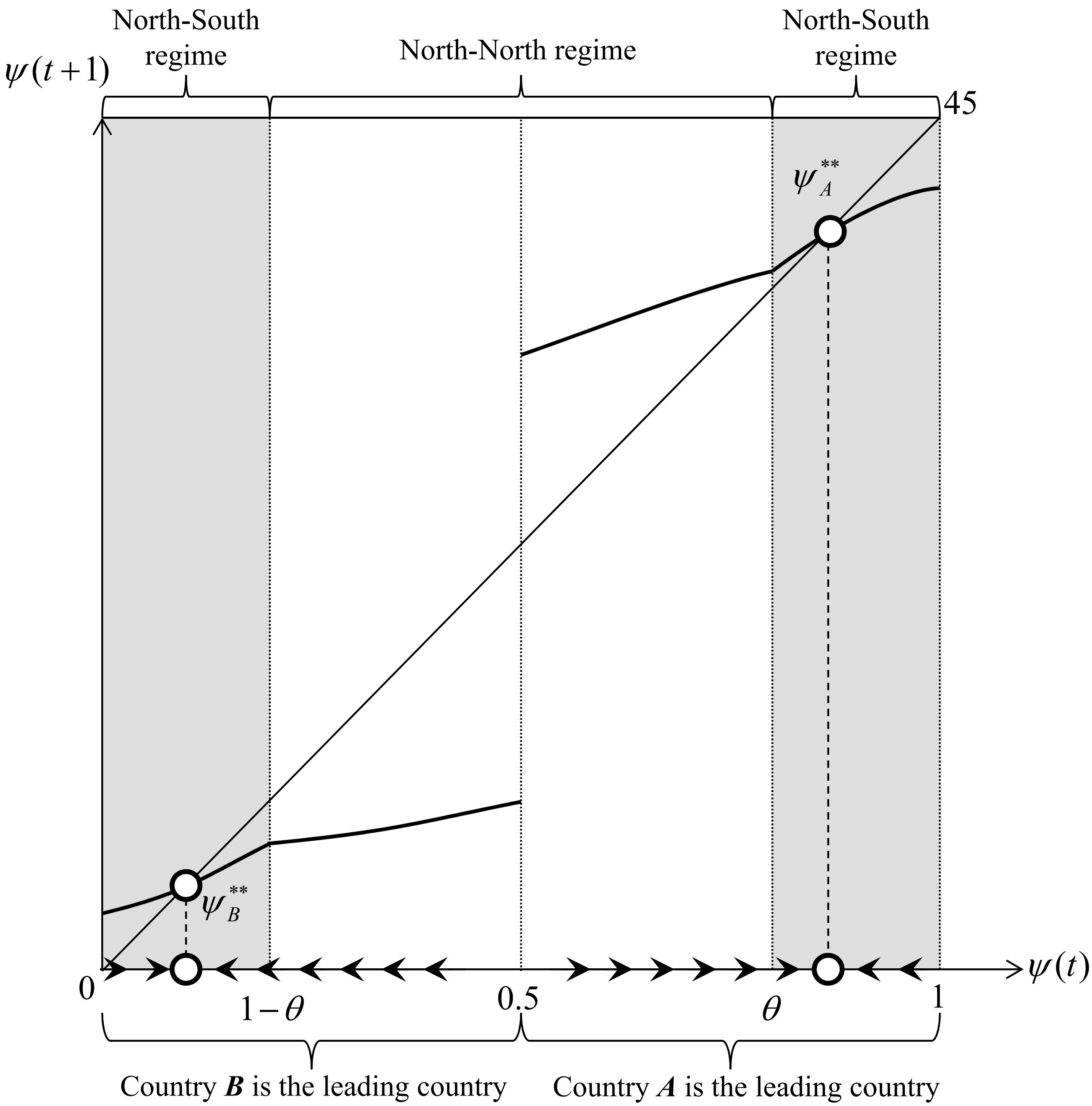

Figure 2: No leapfrogging in the North-South regime
(a) Converging to ψ_i^{**} as $\mu < (1-\theta)/\theta$

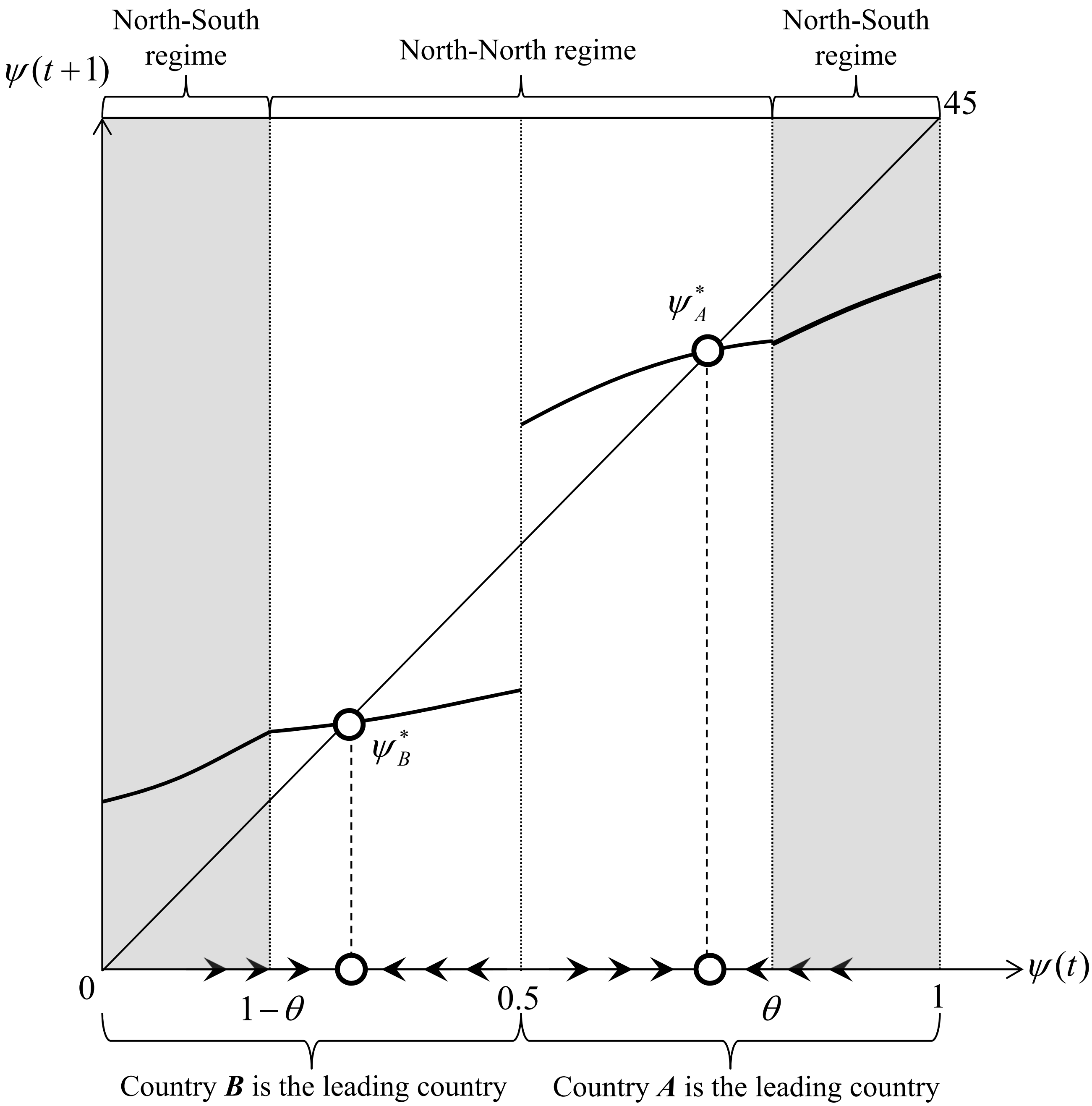

Figure 2: No leapfrogging in the North-North regime
(b) Converging to ψ_i^* as $\mu > (1-\theta)/\theta$

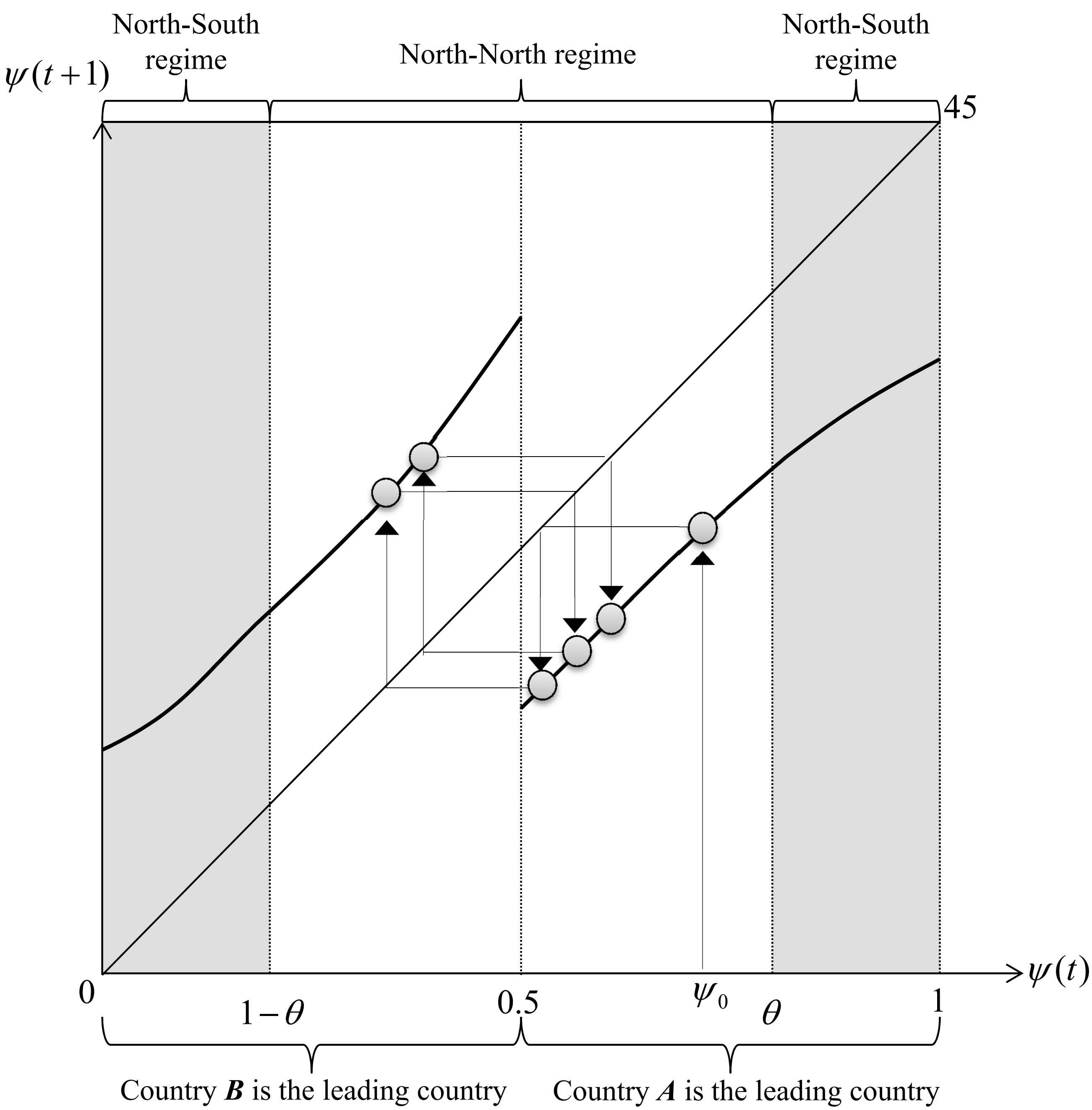

Figure 3: Perpetual leapfrogging in the North-North regime