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Abstract

The hybrid New Keynesian Phillips curve has been criticized for lacking a micro-foundation. In this paper, an alternative purely forward-looking model of the Phillips curve is constructed on the basis of a micro-foundation of trend inflation. In addition, another source of output gaps other than frictions—a Nash equilibrium of a Pareto inefficient path—is considered. The model indicates that the role of frictions has been overestimated and that frictions are less important than previously have been thought. The conventional monetary policy of utilizing frictions cannot necessarily stabilize inflation. In contrast, the monetary policy of controlling the government’s preference is very effective. A problem is that the effects of both types of monetary policy are not distinguishable.

JEL Classification code: E31, E58, E63
Keywords: Trend inflation; inflation persistence; central bank independence; The New Keynesian Phillips curve; the fiscal theory of the price level

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INTRODUCTION

The pure New Keynesian Phillips curve (NKPC) has been criticized for possessing the serious problem that it is not consistent with the observed highly persistent nature of inflation (e.g., Fuhrer and Moore, 1995; Galí and Gertler, 1999). Mankiw (2001) argues that the NKPC is ultimately a failure and is not consistent with standard stylized facts about the dynamic effects of monetary policy. Since the work of Galí and Gertler (1999), a modified version of the NKPC—that is, a hybrid NKPC that includes lagged inflation—has been intensely studied. The hybrid NKPC well captures the persistent nature of inflation, but it remains puzzling why rational agents would behave in backward-looking manners, even if only partially so. Galí et al. (2005) argue that an important unresolved issue is the provision of a more coherent rationale for the role of lagged inflation in the hybrid NKPC. Furthermore, Fuhrer (2006) concluded that inflation in the hybrid NKPC inherits relatively little persistence from the driving process and that a micro-founded mechanism that generates substantial intrinsic persistence in inflation is required.

Recently, an alternative approach has been presented that argues that high intrinsic inflation persistence is spurious as a result of trend inflation. Cogley and Sbordone (2005, 2006) show that, if trend inflation is incorporated into the pure NKPC, its performance on fitting actual inflation data improves greatly. They conclude that trend inflation has been historically quite volatile and that, if these fluctuations of long-run moving trend inflation are taken into account, a purely forward-looking model approximates the short-run dynamics of inflation quite well. Woodford (2007) considers that Cogley and Sbordone (2005) present an alternative interpretation of the apparent need for lagged inflation terms in the NKPC (see also Hornstein, 2007). Indeed, data on inflation in most industrial economies show high levels of volatility and a transition from high inflation to low inflation in the 1980s, which strongly implies the existence of trends in inflation (e.g., Stock and Watson, 2006; Sbordone, 2007). Ascari (2004) argues that disregarding trend inflation is very far from being an innocuous assumption and that the results obtained by models log-linearized around a zero inflation steady state are misleading (see also Bakhshi et al., 2003). These studies suggest that the puzzle of inflation persistence in the NKPC will be solved by incorporating trend inflation into the NKPC. However, if we proceed further in this research direction, another serious theoretical problem arises, that is, the lack of a micro-foundation of trend inflation. Can trend inflation be explained as a consequence of rational agents’ optimizations? Why do monetary policymakers often allow upward trends in inflation? This paper presents a micro-foundation of trend inflation.

The fiscal theory of the price level (FTPL) argues that a problem with conventional inflation theory is that it largely neglects the importance of the government’s borrowing behavior in inflation dynamics (e.g., Leeper, 1991; Sims, 1994, 1998, 2001; Woodford, 1995, 2001; Cochrane, 1998a, 1998b, 2005). The FTPL implies that, if a government’s borrowing behavior is well modeled, the mechanism of severely deviated inflation paths can be explained without assuming ad hoc frictions or irrationality. In this paper, this possibility is explored and a model of trend inflation that is firmly based on a micro-foundation is constructed (see Harashima, 2008b). The model indicates that trend inflation accelerates or decelerates if the time preference rates of the government and the representative household are heterogeneous.

Another important factor in the Phillips curve that should also be carefully examined is the nature of output gaps. In the NKPC, output gaps are assumed to be generated only by frictions. Without frictions, no output gaps can exist because, if an economy is under full price flexibility, its equilibrium output level is always sustained. However, this New Keynesian explanation has not generally been regarded as sufficiently successful, because price rigidity has been criticized for its fragile theoretical (micro-) foundation and its inability to explain the persistent nature of inflation. As shown above, Mankiw (2001) severely criticized the NKPC. This criticism implies that there will be other sources of output gaps. In this paper, I consider
another source of output gaps that are generated even under full price flexibility (see Harashima, 2012, 2013a, 2013b). Rational agents will usually not allow Pareto inefficiency (e.g., output gaps) to remain for a long period; it will disappear soon after it is generated under full price flexibility. However, an exception is possible because a Nash equilibrium can conceptually coexist with Pareto inefficiency. If a Nash equilibrium that consists of strategies that generate Pareto inefficient payoffs is rationally selected, rigidity-like phenomena may be observed. This paper shows that a Nash equilibrium consisting of strategies of choosing a Pareto inefficient transition path of consumption to the steady state (hereafter called a “Nash equilibrium of a Pareto inefficient path”) is generated even in a frictionless economy if—and probably only if—the rate of time preference shifts. An essential reason for the generation of this path is that households are intrinsically risk averse and not cooperative. In a strategic environment, this generates the possibility that, if consumption needs to be substantially and discontinuously increased to keep Pareto optimality, a non-cooperative household’s strategy to deviate from the Pareto optimal path gives a higher expected utility than the strategy of choosing the Pareto optimal path.

The above-mentioned two factors (a model of trend inflation and a mechanism of output gaps under full price flexibility) are considered in analyses of monetary policies, and an alternative model of the Phillips curve is constructed. In contrast to the NKPC, both factors are fully based on micro-foundations. Comparisons between this new model and the NKPC indicate that the role of frictions has been overestimated and that frictions are less important than has been thought.

The paper is organized as follows. In Section 2, I construct a model of trend inflation that assumes an economically Leviathan government, in which the government and the representative household behave in purely forward-looking manners and achieve simultaneous optimization. Section 3 shows that a Nash equilibrium of a Pareto inefficient path is rationally generated when the time preference rates of risk-averse and non-cooperative households shift. In Section 4, a new model of the Phillips curve is constructed and compared with the NKPC. Finally, I offer concluding remarks in Section 5.

2 TREND INFLATION

2.1 The model of trend inflation

2.1.1 The government

2.1.1.1 The government budget constraint

The government budget constraint is

$$\dot{B}_t = B_t i_t + G_t - X_t - \partial_t,$$

where $B_t$ is the nominal obligation of the government to pay for its accumulated bonds, $i_t$ is the nominal interest rate for government bonds, $G_t$ is the nominal government expenditure, $X_t$ is the nominal tax revenue, and $\partial_t$ is the nominal amount of seigniorage at time $t$. The tax is assumed to be lump sum, the government bonds are long term, and the returns on the bonds are realized only after the bonds are held during a unit period (e.g., a year). The government bonds are redeemed in a unit period, and the government successively refines the bonds by issuing new ones at each time $t$. Let $b_t = \frac{B_t}{P_t}$, $g_t = \frac{G_t}{P_t}$, $x_t = \frac{X_t}{P_t}$, and $\partial_t = \frac{\partial_t}{P_t}$, where $P_t$ is the price level

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1 The model of the optimal trend inflation in this paper is based on the inflation model in Harashima (2007). Harashima (2004b, 2008a, 2013a) are also related to the model and analyses in this paper.
at time \( t \). Let also \( \pi_t = \frac{\dot{P}_t}{P_t} \) be the inflation rate at time \( t \). By dividing by \( P_t \), the budget constraint is transformed to

\[
\dot{b}_t = b_t i_t + g_t - x_t - \varphi_t, \quad \text{which is equivalent to}
\]

\[
\dot{b}_t = b_t i_t + g_t - x_t - \varphi_t = b_t (i_t - \pi_t) + g_t - x_t - \varphi_t.
\] (1)

Because the returns on government bonds are realized only after holding the bonds during a unit period, investors buy the bonds if

\[
\int_{t-1}^{t+1} (\pi_s + r_t) ds \geq i_t \geq \int_{t-1}^{t+1} \pi_s ds
\]

at time \( t \), where \( i_t \) is the nominal interest rate for bonds bought at \( t \) and \( r_t \) is the real interest rate in markets at \( t \). Hence, by arbitrage,

\[
\int_{t-1}^{t+1} (\pi_s + r_t) ds \quad \text{and if } r_t \text{ is constant such that } r_t = r \quad \text{(i.e., if it is at steady state), then}
\]

\[
i_t = \int_{t-1}^{t+1} \pi_s ds + r.
\]

The nominal interest rate \( i_t = \int_{t-1}^{t+1} \pi_s ds + r \) means that, during a sufficiently small period between \( t \) and \( t + dt \), the government’s obligation to pay for the bonds’ return in the future increases not by \( dt (\pi_t + r_t) \) but by \( dt \left( \int_{t-1}^{t+1} \pi_s ds + r_t \right) \). If \( \pi_t \) is constant, then \( \int_{t-1}^{t+1} \pi_s ds = \pi_t \) and \( i_t = \pi_t + r_t \), but if \( \pi_t \) is not constant, these equations do not necessarily hold.

Since bonds are redeemed in a unit period and successively refinanced, the bonds the government is holding at \( t \) have been issued between \( t - 1 \) and \( t \). Hence, under perfect foresight, the average nominal interest rate for all government bonds at time \( t \) is the weighted sum of \( i_t \) such that

\[
i_t = \int_{t-1}^{t} i_t \left( \int_{t-1}^{t} \frac{B_s}{B_v} dv \right) ds = \int_{t-1}^{t} \int_{t-1}^{s+1} \pi_s dv \left( \int_{t-1}^{t} \frac{B_s}{B_v} dv \right) ds + r,
\]

where \( \frac{B_s}{B_v} \) is the nominal value of bonds at time \( t \) that were issued at time \( s \). If the weights \( \int_{t-1}^{t} \frac{B_s}{B_v} dv \) between \( t - 1 \) and \( t \) are not so different from each other, then approximately

\[
i_t = \int_{t-1}^{t} \int_{s}^{s+1} \pi_s dv ds + r.
\]

To be precise, if the absolute values of \( \pi_t \) for \( t - 1 < s \leq t + 1 \) are sufficiently smaller than unity, the differences among the weights are negligible and then approximately

\[
i_t = \int_{t-1}^{t} \int_{s}^{s+1} \pi_s dv ds + r
\] (2)

(see Harashima, 2008). The average nominal interest rate for the total government bonds,
therefore, develops by \( i_t = \int_{t-1}^{t} \int_{s}^{s+1} \pi_v \, dv \, ds + r \). If \( \pi_t \) is constant, then
\[
\int_{t-1}^{t} \int_{s}^{s+1} \pi_v \, dv \, ds = \pi_t; \quad \text{thus,} \quad i_t = \pi_t + r.
\]
If \( \pi_t \) is not constant, however, the equations
\[
\int_{t-1}^{t} \int_{s}^{s+1} \pi_v \, dv \, ds = \pi_t
\]
and \( i_t = \pi_t + r \) do not necessarily hold.

2.1.1.2 An economically Leviathan government
Under a proportional representation system, the government represents the median household whereas the representative household from an economic perspective represents the mean household. Because of this difference, they usually have different preferences. To account for this essential difference, a Leviathan government is assumed in the model. There are two extremely different views regarding government’s behavior in the literature on political economy: the Leviathan view and the benevolent view (e.g., Downs 1957; Brennan and Buchanan 1980; Alesina and Cukierman 1990). From an economic point of view, a benevolent government maximizes the expected economic utility of the representative household, but a Leviathan government does not. Whereas the expenditure of a benevolent government is a tool used to maximize the economic utility of the representative household, the expenditure of a Leviathan government is a tool used to achieve the government’s own policy objectives. For example, if a Leviathan government considers national security to be the most important political issue, defense spending will increase greatly, but if improving social welfare is the top political priority, spending on social welfare will increase dramatically, even though the increased expenditures may not necessarily increase the economic utility of the representative household.

Is it possible, however, for such a Leviathan government to hold office for a long period? Yes, because a government is generally chosen by the median of households under a proportional representation system (e.g., Downs 1957), whereas the representative household usually presumed in the economics literature is the mean household. The economically representative household is not usually identical to the politically representative household, and a majority of people could support a Leviathan government even if they know that the government does not necessarily pursue only the economic objectives of the economically representative household. In other words, the Leviathan government argued here is an economically Leviathan government that maximizes the political utility of people, whereas the conventional economically benevolent government maximizes the economic utility of people. In addition, because the politically and economically representative households are different (the median and mean households, respectively), the preferences of future governments will also be similarly different from those of the mean representative household. In this sense, the current and future governments presented in the model can be seen as a combined government that goes on indefinitely; that is, the economically Leviathan government always represents the median representative household.

The Leviathan view generally requires the explicit inclusion of government expenditure, tax revenue, or related activities in the government’s political utility function (e.g., Edwards and Keen 1996). Because an economically Leviathan government derives political utility from expenditure for its political purposes, the larger the expenditure is, the happier the Leviathan government will be. But raising tax rates will provoke people’s antipathy, which

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2 See the literature on the median voter theorem (e.g., Downs 1957). Also see the literature on the delay in reforms (e.g., Alesina and Drazen 1991).

3 The most prominent reference to Leviathan governments is Brennan and Buchanan (1980).

4 The government behavior assumed in the fiscal theory of the price level reflects an aspect of a Leviathan government. Christiano and Fitzgerald (2000) argue that non-Ricardian policies correspond to the type of policies in which governments are viewed as selecting policies and committing themselves to those policies in advance of prices being determined in markets.
increases the probability of being replaced by the opposing party that also nearly represents the median household. Thus, the economically Leviathan government regards taxes as necessary costs to obtain freedom of expenditure for its own purposes. The government therefore will derive utility from expenditure and disutility from taxes. Expenditure and taxes in the political utility function of the government are analogous to consumption and labor hours in the economic utility function of the representative household. Consumption and labor hours are both control variables, and as such, the government’s expenditure and tax revenue are also control variables. As a whole, the political utility function of economically Leviathan government can be expressed as $u_G(g_t, x_t)$. In addition, it can be assumed on the basis of previously mentioned arguments that $\frac{\partial u_G}{\partial g_t} > 0$ and $\frac{\partial^2 u_G}{\partial g_t^2} < 0$, and therefore that $\frac{\partial u_G}{\partial x_t} < 0$ and $\frac{\partial^2 u_G}{\partial x_t^2} > 0$. An economically Leviathan government therefore maximizes the expected sum of these utilities discounted by its time preference rate under the constraint of deficit financing.

2.1.3 The optimization problem
The optimization problem of an economically Leviathan government is

$$\text{Max } E \int_0^\infty u_G(g_t, x_t) \exp(-\theta_G t) dt$$

subject to the budget constraint

$$\dot{b}_t = b_t (i_t - \pi_t) + g_t - x_t - \varphi_t,$$

where $u_G$ is the constant relative risk aversion utility function of the government, $\theta_G$ is the government’s rate of time preference, and $E$ is the expectation operator. All variables are expressed in per capita terms, and population is assumed to be constant. The government maximizes its expected political utility considering the behavior of the economically representative household that is reflected in $i_t$ in its budget constraint.

2.1.2 Households
The economically representative household maximizes its expected economic utility. Sidrauski (1967)’s well-known money in the utility function model is used for the optimization problem. The representative household maximizes its expected utility

$$E \int_0^\infty u_P(c_t, m_t) \exp(-\theta_P t) dt$$

5 It is possible to assume that governments are partially benevolent. In this case, the utility function of a government can be assumed to be $u_G(g_t, c_t, i_t)$, where $c_t$ is real consumption and $i_t$ is the leisure hours of the representative household. However, if a lump-sum tax is imposed, the government’s policies do not affect steady-state consumption and leisure hours. In this case, the utility function can be assumed to be $u_G(g_t, x_t)$.

6 Some may argue that it is more likely that $\frac{\partial u_G}{\partial x_t} > 0$ and $\frac{\partial^2 u_G}{\partial x_t^2} < 0$. However, the assumption used is not an important issue here because $-x_t \left[ \frac{\partial u_G}{\partial x_t} (g_t, x_t) \right]^{-1} \frac{\partial^2 u_G}{\partial x_t^2} (g_t, x_t) \left[ \frac{\partial x_t}{\partial x_t} \right] - x_t = 0$ at steady state, as will be shown in the solution to the optimization problem later in the paper. Thus, the results are not affected by which assumption is used.
subject to the budget constraint

$$\dot{a}_t = \left( r_t a_t + w_t + \sigma_t \right) - \left[ c_t + (\sigma_t + r_t) m_t \right] - g_t,$$

where $u_P$ and $\theta_P$ are the utility function and the time preference rate of the representative household, $c_t$ is real consumption, $w_t$ is real wage, $\sigma_t$ is lump-sum real government transfers, $m_t$ is real money, $a_t = k_t + m_t$, and $k_t$ is real capital. It is assumed that $r_t = f(k_t)$, $w_t = f(k_t) - k_t f'(k_t)$, $u_P > 0$, $u_P' < 0$, $\frac{\partial u_P(c_t, m_t)}{\partial m_t} > 0$, and $\frac{\partial^2 u_P(c_t, m_t)}{\partial m_t^2} < 0$, where $f(\cdot)$ is the production function. Government expenditure ($g_t$) is an exogenous variable for the representative household because it is an economically Leviathan government. It is also assumed that, although all households receive transfers from a government in equilibrium, when making decisions, each household takes the amount it receives as given, independent of its money holdings. Thus, the budget constraint means that the real output $f(k_t)$ at any time is demanded for the real consumption $c_t$, the real investment $\dot{k}_t$, and the real government expenditure $g_t$ such that $f(k_t) = c_t + \dot{k}_t + g_t$. The representative household maximizes its expected economic utility considering the behavior of government reflected in $g_t$ in the budget constraint. In this discussion, a central bank is not assumed to be independent of the government; thus, the functions of the government and the central bank are not separated. This assumption can be relaxed, and the roles of the government and the central bank are explicitly separated in Section 2.2.

Note that the time preference rate of government ($\theta_G$) is not necessarily identical to that of the representative household ($\theta_P$) because the government and the representative household represent different households (i.e., the median and mean households, respectively). In addition, the preferences will differ because (1) even though people want to choose a government that has the same time preference rate as the representative household, the rates may differ owing to errors in expectations (e.g., Alesina and Cukierman 1990); and (2) current voters cannot bind the choices of future voters and, if current voters are aware of this possibility, they may vote more myopically as compared with their own rates of impatience in private economic activities (e.g., Tabellini and Alesina 1990). Hence, it is highly likely that the time preference rates of a government and the representative household are heterogeneous. It should be also noted, however, that even though the rates of time preference are heterogeneous, an economically Leviathan government behaves based only on its own time preference rate, without hesitation.

### 2.1.3 The simultaneous optimization

First, I examine the optimization problem of the representative household. Let Hamiltonian $H_P$ be

$$H_P = u_P(c_t, m_t) \exp(-\theta_P t) + \lambda_P t \left[ r_t a_t + w_t + \sigma_t - c_t - (\pi_t + r_t) m_t - g_t \right],$$

where $\lambda_P t$ is a costate variable, $c_t$ and $m_t$ are control variables, and $a_t$ is a state variable. The optimality conditions for the representative household are;

$$\frac{\partial u_P(c_t, m_t)}{\partial c_t} \exp(-\theta_P t) = \lambda_P t, \quad (4)$$

$$\frac{\partial u_P(c_t, m_t)}{\partial m_t} \exp(-\theta_P t) = \lambda_P t (\pi_t + r_t), \quad (5)$$
\[ \dot{\lambda}_{t,t} = -\lambda_{t,t} r_t, \quad (6) \]

\[ \dot{a}_t = (r a_t + w_t + \sigma_t) - \left[ c_t + \left( \pi_t + r_t \right) m_t - g_t \right], \quad (7) \]

\[ \lim_{t \to \infty} \lambda_{t,t} a_t = 0. \quad (8) \]

By conditions (4) and (5),

\[ \left[ \frac{\partial u_p(c_t, m_t)}{\partial c_t} \right]^{-1} \frac{\partial u_p(c_t, m_t)}{\partial m_t} = \pi_t + r_t, \quad \text{and by conditions (4) and (6),} \]

\[ -c_t \left[ \frac{\partial u_p(c_t, m_t)}{\partial c_t} \right]^{-1} \frac{\partial^2 u_p(c_t, m_t)}{\partial c_t^2} \frac{\dot{c}_t}{c_t} + \theta_p = r_t. \quad (9) \]

Hence,

\[ \theta_p = r_t = r \quad (10) \]

at steady state such that \( \dot{c}_t = 0 \) and \( \ddot{k}_t = 0 \).

Next, I examine the optimization problem of the economically Leviathan government. Let Hamiltonian \( H_G \) be \( H_G = u_G(g_t, x_t) \exp(-\theta_G t) + \lambda_{G,t} \left[ \dot{t}_t - \pi_t \right] + g_t - x_t - \varphi_t \), where \( \lambda_{G,t} \) is a costate variable. The optimality conditions for the government are:

\[ \frac{\partial u_G(g_t, x_t)}{\partial g_t} \exp(-\theta_G t) = -\dot{\lambda}_{G,t}, \quad (11) \]

\[ \frac{\partial u_G(g_t, x_t)}{\partial x_t} \exp(-\theta_G t) = \dot{\lambda}_{G,t}, \quad (12) \]

\[ \dot{\lambda}_{G,t} = -\lambda_{G,t} \left( \dot{t}_t - \pi_t \right), \quad (13) \]

\[ \dot{t}_t = b_t \left( \dot{t}_t - \pi_t \right) + g_t - x_t - \varphi_t, \quad (14) \]

\[ \lim_{t \to \infty} \lambda_{G,t} b_t = 0. \quad (15) \]

Combining conditions (11), (12), and (13) and equation (2) yields the following equations:

\[ -g_t \left[ \frac{\partial u_G(g_t, x_t)}{\partial g_t} \right]^{-1} \frac{\partial^2 u_G(g_t, x_t)}{\partial g_t^2} \frac{\dot{g}_t}{g_t} + \theta_G = \dot{t}_t - \pi_t = r_t + \int_{t-1}^t \int_s^{t+1} \varphi_v d\nu d s - \pi_t, \quad (16) \]

and

\[ -x_t \left[ \frac{\partial u_G(g_t, x_t)}{\partial x_t} \right]^{-1} \frac{\partial^2 u_G(g_t, x_t)}{\partial x_t^2} \frac{\dot{x}_t}{x_t} + \theta_G = \dot{t}_t - \pi_t = r_t + \int_{t-1}^t \int_s^{t+1} \varphi_v d\nu d s - \pi_t. \quad (17) \]
Here, \[ g_t \left[ \frac{\partial u_G(g_t, x_t)}{\partial g_t} \right]^{-1} \frac{\partial^2 u_G(g_t, x_t)}{\partial g_t^2} \hat{g}_t = 0 \quad \text{and} \quad x_t \left[ \frac{\partial u_G(g_t, x_t)}{\partial x_t} \right]^{-1} \frac{\partial^2 u_G(g_t, x_t)}{\partial x_t^2} \hat{x}_t = 0 \] at steady state such that \( \hat{g}_t = 0 \) and \( \hat{x}_t = 0 \); thus,

\[ \theta_G = r_t + \int_{t-1}^{t} \int_s^{s+1} \pi_o \, dv \, ds - \pi_t. \]  \hspace{1cm} (18)

Hence, by equation (10),

\[ \int_{t-1}^{t} \int_s^{s+1} \pi_o \, dv \, ds = \pi_t + \theta_G - \theta_p \]  \hspace{1cm} (19)

at steady state such that \( \hat{g}_t = 0, \hat{x}_t = 0, \hat{c}_t = 0, \) and \( \hat{\kappa}_t = 0. \)

Equation (19) is a natural consequence of simultaneous optimization by the economically Leviathan government and the representative household. If the rates of time preference are heterogeneous between them, then

\[ i_t - r = \int_{t-1}^{t} \int_s^{s+1} \pi_o \, dv \, ds \neq \pi_t. \]

This result might seem surprising because it has been naturally conjectured that \( i_t = \pi_t + r \). However, this is a simple misunderstanding because \( \pi_t \) indicates the instantaneous rate of inflation at a point such that \( \pi_t = \frac{P_t}{P_t^0} \), whereas \( \int_{t-1}^{t} \int_s^{s+1} \pi_o \, dv \, ds \) roughly indicates the average inflation rate in a period. Equation (19) indicates that \( \pi_t \) develops according to the integral equation \( \int_{t-1}^{t} \int_s^{s+1} \pi_o \, dv \, ds - \theta_G + \theta_p \). If \( \pi_t \) is constant, the equations \( i_t = \pi_t + r \) and \( \int_{t-1}^{t} \int_s^{s+1} \pi_o \, dv \, ds = \pi_t \) are true. However, if \( \pi_t \) is not constant, the equations do not necessarily hold. Equation (19) indicates that the equations \( i_t = \pi_t + r \) and \( \int_{t-1}^{t} \int_s^{s+1} \pi_o \, dv \, ds = \pi_t \) hold only in the case where \( \theta_G = \theta_p \) (i.e., a homogeneous rate of time preference). It has been previously thought that a homogeneous rate of time preference naturally prevails; thus, the equation \( i_t = \pi_t + r \) has not been questioned. As argued previously, however, a homogeneous rate of time preference is not usually guaranteed.

### 2.1.4 The law of motion for trend inflation

Equation (19) indicates that inflation accelerates or decelerates as a result of the government and the representative household reconciling the contradiction in heterogeneous rates of time preference. If \( \pi_t \) is constant, the equation \( \pi_t = \int_{t-1}^{t} \int_s^{s+1} \pi_o \, dv \, ds \) holds; conversely, if \( \pi_t \neq \int_{t-1}^{t} \int_s^{s+1} \pi_o \, dv \, ds \), then \( \pi_t \) is not constant. Without the acceleration or deceleration of inflation,

\[ \text{If and only if } \theta_G = \frac{g_t - x_t - \phi_t}{b_t} \text{ at steady state, then the transversality condition } (15) \lim_{t \to \infty} \lambda_{\theta_G} b_t = 0 \text{ holds.} \]

The proof is shown in Harashima (2008b).
therefore, equation (19) cannot hold in an economy in which $\theta_G \neq \theta_P$. In other words, it is not until $\theta_G \neq \theta_P$ that inflation can accelerate or decelerate. Heterogeneous time preferences ($\theta_G \neq \theta_P$) bend the path of inflation and enables inflation to accelerate or decelerate. The difference of time preference rates ($\theta_G - \theta_P$) at each time needs to be transformed to the accelerated or decelerated inflation rate $\pi_t$ at each time.

Equation (19) implies that inflation accelerates or decelerates nonlinearly in the case in which $\theta_G \neq \theta_P$. For a sufficiently small period $dt$, $\pi_{t+1} = \pi_t + 6(\theta_G - \theta_P) t^2$ is determined with $\pi_0 \leq \pi_s \leq \pi_{t+1}$ that satisfies

$$
\int_{t-1}^{t} \int_{s}^{s+1} \pi_v \, dv \, ds - \pi_s = \theta_G - \theta_P,
$$

so as to hold the equation

$$
\int_{t-1}^{t} \int_{s}^{s+1} \pi_v \, dv \, ds = \int_{t-1}^{t+1} \int_{s}^{s+1} \pi_v \, dv \, ds + \pi_{t+1} - \pi_t.
$$

A solution of the integral equation (19) for given $\theta_G$ and $\theta_P$ is

$$
\pi_t = \pi_0 + 6(\theta_G - \theta_P) t^2.
$$

(20)

Generally, the path of inflation that satisfies equation (19) for $0 \leq t$ is expressed as

$$
\pi_t = \pi_0 + 6(\theta_G - \theta_P) \exp[z_t \ln(t)]
$$

(21)

where $z_t$ is a time dependent variable. The stream of $z_t$ is various depending on the boundary condition, i.e., the past and present inflation during $-1 < t \leq 0$ and the path of inflation during $0 < t \leq 1$ that is set to make $\pi_0$ satisfy equation (19). However, $z_t$ has the following important property. If $\pi_t$ satisfies equation (19) for $0 \leq t$, and $-\infty < \pi_t < \infty$ for $-1 < t \leq 1$, then

$$
\lim_{t \to +\infty} z_t = 2.
$$

Proof is shown in Harashima (2008b). Any inflation path that satisfies equation (19) for $0 \leq t$ therefore asymptotically approaches the path of equation (20). The mechanism behind the law of motion for inflation (equation [20]) is examined more in detail in Harashima (2008b).

### 2.1.5 The optimal trend inflation

The trend inflation should be consistent with equation (21). The discrete-time version of equation (21) is

$$
\pi_t^T = \pi_0^T + 6(\theta_G - \theta_P) \exp[z_t \ln(t - \sigma)]
$$

(22)

and equivalently

$$
\pi_{t+1}^T = \pi_t^T + 6(\theta_G - \theta_P) \exp[z_{t+1} \ln(t - \sigma + 1)] - \exp[z_t \ln(t - \sigma)]
$$

(23)

where $\pi_t^T$ is the trend component in inflation in period $t$, and $\sigma (\leq t)$ is the period when the latest shock on $\theta_G$ occurred. It will be explained later in Section 2.2 that $\theta_G$ should be time-variable and shocks on $\theta_G$ play an important role in inflation dynamics. When a shock on $\theta_G$ occurs and the value of $\theta_G$ is changed in period $\sigma$, the trend inflation need be adjusted to be consistent with the new value of $\theta_G$ for the new initial period $\sigma$. The value of $z_t$ is
determined by the mechanism explained in Section 2.1.2. Equations (22) and (23) are used in the model as the trend component in inflation.

2.2 The central bank
In Section 2.1, central banks are not explicitly considered because they are not assumed to be independent of governments. However, in actuality, central banks are independent organizations in most countries even though some of them are not sufficiently independent. Furthermore, in the conventional inflation model, it is the central banks that control inflation and governments have no role in controlling inflation. Conventional inflation models show that the rate of inflation basically converges at the target rate of inflation set by a central bank. The target rate of inflation therefore is the key exogenous variable that determines the path of inflation in these models.

Both the government and the central bank can probably affect the development of inflation, but they would do so in different manners, as equation (21) and conventional inflation models indicate. However, the objectives of the government and the central bank may not be the same. For example, if trend inflation is added to conventional models by replacing their aggregate supply equations with equation (21), inflation cannot necessarily converge at the target rate of inflation because another key exogenous variable ($\theta_G$) is included in the models.

A government makes inflation develop consistently with the equation (21), which implies that inflation will not necessarily converge at the target rate of inflation. Conversely, a central bank makes inflation converge at the target rate of inflation, which implies that inflation will not necessarily develop consistently with equation (21). That is, unless either $\theta_G$ is adjusted to be consistent with the target rate of inflation or the target rate of inflation is adjusted to be consistent with $\theta_G$, the path of inflation cannot necessarily be determined. Either $\theta_G$ or the target rate of inflation need be an endogenous variable. If a central bank dominates, the target rate of inflation remains as the key exogenous variable and $\theta_G$ should then be an endogenous variable. The reverse is also true.

A central bank will be regarded as truly independent if $\theta_G$ is forced to be adjusted to the one that is consistent with the target rate of inflation set by the central bank. For example, suppose that $\theta_G > \theta_P$ and a truly independent central bank manipulates the nominal interest rate. Here,

$$i_t = \int_{t-1}^{t} \int_{t}^{s+1} \pi_s \, dv \, ds + r = \theta_G + \pi_t$$

(24)

at steady state such that $\dot{g}_t = 0$, $\dot{x}_t = 0$, $\dot{c}_t = 0$, and $\dot{k}_t = 0$ by equations (1), (7), and (13). If the accelerating inflation rate is higher than the target rate of inflation, the central bank can raise the nominal interest rate from $i_t = \theta_G + \pi_t$ (equation (24)) to

$$i_t = \theta_G + \pi_t + \psi$$

by positive $\psi$ by intervening in financial markets to lower the accelerating rate of inflation. In this case, the central bank keeps the initial target rate of inflation because it is truly independent. The government thus faces a rate of increase of real obligation that is higher than $\theta_G$ by the extra rate $\psi$. If the government lowers $\theta_G$ so that $\theta_G < \theta_P$ and inflation stops accelerating,

---

8 The extra rate $\psi$ affects not only the behavior of government but also that of the representative household, in which
the central bank will accordingly reduce the extra rate $\psi$. If, however, the government does not accommodate $\theta_G$ to the target rate of inflation, the extra rate $\psi$ will increase as time passes because of the gap between the accelerating inflation rate and the target rate of inflation widens. Because of the extra rate $\psi$, the government has no other way to achieve optimization unless it lowers $\theta_G$ to one that is consistent with the target rate of inflation. Once the government recognizes that the central bank is firmly determined to be independent and it is in vain to try to intervene in the central bank’s decision makings, the government would not dare to attempt to raise $\theta_G$ again anymore.

Equation (22) implies that a government allows inflation to accelerate because it acts to maximize its expected utility based only on its own preferences. A government is hardly the only entity that cannot easily control its own preferences even when these preferences may result in unfavorable consequences. It may not even be possible to manipulate one’s own preferences at will. Thus, even though a government is fully rational and is not weak, foolish, or untruthful, it is difficult for it to self-regulate its preferences. Hence, an independent neutral organization is needed to help control $\theta_G$. Delegating the authority to set and keep the target rate of inflation to an independent central bank is a way to control $\theta_G$. The delegated independent central bank will control $\theta_G$ because it is not the central bank’s preference to stabilize the price level—it is simply a duty delegated to it. An independent central bank is not the only possible choice. For example, pegging the local currency with a foreign currency can be seen as a kind of delegation to an independent neutral organization. In addition, the gold standard that prevailed before World War II can be also seen as a type of such delegation.

Note also that the delegation may not be viewed as bad from the Leviathan government’s point of view because only its rate of time preference is changed, and the government can still pursue its political objectives. One criticism of the argument that central banks should be independent (e.g., Blinder 1998) is that, since the time-inconsistency problem argued in Kydland and Prescott (1977) or Barro and Gordon (1983) is more acute with fiscal policy, why is it not also necessary to delegate fiscal policies? An economically Leviathan government, however, will never allow fiscal policies to be delegated to an independent neutral organization because the Leviathan government would then not be able to pursue its political objectives, which in a sense would mean the death of the Leviathan government. The median household that backs the Leviathan government, but at the same time dislikes high inflation, will therefore support the delegation of authority but only if it concerns monetary policy. The independent central bank will then be given the authority to control $\theta_G$ and oblige the government to change $\theta_G$ in order to meet the target rate of inflation.

Without such a delegation of authority, it is likely that generally $\theta_G > \theta_P$ because $\theta_G$ represents the median household whereas $\theta_P$ represents the mean household. Empirical studies indicate that the rate of time preference negatively correlates with permanent income (e.g., Lawrance 1991), and the permanent income of the median household is usually lower than that of the mean household. If generally $\theta_G > \theta_P$, that suggests that inflation will tend to accelerate unless a central bank is independent. The independence of the central bank is therefore very important in keeping the path of inflation stable.

Note also that the forced adjustments of $\theta_G$ by an independent central bank are exogenous shocks to both the government and the representative household because they are

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the conventional inflation theory is particularly interested. In this sense, the central bank’s instrument rule that concerns and simultaneously affects both behaviors of the government and the representative household is particularly important for price stability.
planned solely by the central bank. When a shock on $\theta_G$ is given, the government and the representative household must recalculate their optimal paths including the path of inflation by resetting $\theta_G$, $\pi_t$, and $\phi$ in equation (22).

3 OUTPUT GAPS

3.1 Model with non-cooperative households

This section examines another source of output gaps other than frictions. A Ramsey type growth model with non-cooperative households is constructed to examine economic fluctuations.

3.1.1 The shock

The model describes the utility maximization of households after an upward time preference shock. This shock was chosen because it is one of the few shocks that result in a Nash equilibrium of a Pareto inefficient path. Another important reason for selecting an upward time preference shock is that it shifts the steady state to lower levels of production and consumption than before the shock, which is consistent with the phenomena actually observed in a recession.

Although the rate of time preference is a deep parameter, it has not been regarded as a source of shocks for economic fluctuations, possibly because the rate of time preference is thought to be constant and not to shift suddenly. There is also a practical reason, however. Models with a permanently constant rate of time preference exhibit excellent tractability (see Samuelson, 1937). However, the rate of time preference has been naturally assumed and actually observed to be time-variable. The concept of a time-varying rate of time preference has a long history (e.g., Böhm-Bawerk, 1889; Fisher, 1930). More recently, Lawrance (1991) and Becker and Mulligan (1997) showed that people do not inherit permanently constant rates of time preference by nature and that economic and social factors affect the formation of time preference rates. Their arguments indicate that many incidents can affect and change the rate of time preference throughout a person’s life. For example, Parkin (1988) examined business cycles in the United States, explicitly considering the time-variability of the time preference rate, and showed that the rate of time preference was as volatile as technology and leisure preference.

3.1.2 Households

Households are not intrinsically cooperative. Except in a strict communist economy, households do not coordinate themselves to behave as a single entity when consuming goods and services. The model in this paper assumes non-cooperative, identical, and infinitely long living households and that the number of households is sufficiently large. Each of them equally maximizes the expected utility

$$E_0 \int_0^\infty \exp(\theta t) u(c_t) dt,$$

subject to

$$\frac{dk_t}{dt} = f(A, k_t) - \delta k_t - c_t,$$

where $y_t$, $c_t$, and $k_t$ are production, consumption, and capital per capita in period $t$, respectively;

---

9 The model in Section 3 is based on the model by Harashima (2012). See also Harashima (2004a, 2013b, 2013c).
$A$ is technology and constant; $u$ is the utility function; $y_t = f(A,k_t)$ is the production function; $\theta (> 0)$ is the rate of time preference; $\delta$ is the rate of depreciation; and $E_0$ is the expectations operator conditioned on the agents’ period 0 information set. $y_t$, $c_t$, and $k_t$ are monotonously continuous and differentiable in $t$, and $u$ and $f$ are monotonously continuous functions of $c_t$ and $k_t$, respectively. All households initially have an identical amount of financial assets equal to $k_0$, and all households gain the identical amount of income $y_t = f(A,k_t)$ in each period. It is assumed that $\frac{du(c_t)}{dc_t} > 0$ and $\frac{d^2u(c_t)}{dc_t^2} < 0$; thus, households are risk averse. For simplicity, the utility function is specified to be the constant relative risk aversion utility function

$$u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma} \quad \text{if} \quad \gamma \neq 1$$
$$u(c_t) = \ln(c_t) \quad \text{if} \quad \gamma = 1,$$

where $\gamma$ is a constant and $0 < \gamma < \infty$. In addition, $\frac{\partial f(A,k_t)}{\partial k_t} > 0$ and $\frac{\partial^2 f(k_t)}{\partial k_t^2} < 0$. Both technology ($A$) and labor supply are assumed to be constant.

The effects of an upward shift in time preference are shown in Figure 1. Suppose first that the economy is at steady state before the shock. After the upward time preference shock, the vertical line $dt = 0$ moves to the left (from the solid vertical line to the dashed vertical line in Fig. 1). To keep Pareto efficiency, consumption needs to jump immediately from the steady state before the shock (the prior steady state) to point $Z$. After the jump, consumption proceeds on the Pareto efficient saddle path after the shock (the posterior Pareto efficient saddle path) from point $Z$ to the lower steady state after the shock (the posterior steady state). Nevertheless, this discontinuous jump to $Z$ may be uncomfortable for risk-averse households that wish to smooth consumption and not to experience substantial fluctuations. Households may instead take a shortcut and, for example, proceed on a path on which consumption is reduced continuously from the prior steady state to the posterior steady state (the bold dashed line in Fig. 1), but this shortcut is not Pareto efficient.

Choosing a Pareto inefficient consumption path must be consistent with each household’s maximization of its expected utility. To examine the possibility of the rational choice of a Pareto inefficient path, the expected utilities between the two options need be compared. For this comparison, I assume that there are two options for each non-cooperative household with regard to consumption just after an upward shift in time preference. The first is a jump option, $J$, in which a household’s consumption jumps to $Z$ and then proceeds on the posterior Pareto efficient saddle path to the posterior steady state. The second is a non-jump option, $NJ$, in which a household’s consumption does not jump but instead gradually decreases from the prior steady state to the posterior steady state, as shown by the bold dashed line in Figure 1. The household that chooses the $NJ$ option reaches the posterior steady state in period $s(\geq 0)$. The difference in consumption between the two options in each period $t$ is $b_t(\geq 0)$. Thus, $b_0$ indicates the difference between $Z$ and the prior steady state. $b_t$ diminishes continuously and becomes zero in period $s$. The $NJ$ path of consumption ($c_t$) after the shock is monotonously continuous and differentiable in $t$ and $\frac{dc_t}{dt} < 0$ if $0 \leq t < s$. In addition,
\[ \tau < c_t < \hat{c}_t \quad \text{if} \quad 0 \leq t < s \]
\[ c_t = \bar{c} \quad \text{if} \quad 0 \leq s \leq t , \]

where \( \hat{c}_t \) is consumption when proceeding on the posterior Pareto efficient saddle path and \( \bar{c} \) is consumption in the posterior steady state. Therefore,

\[ b_t = \hat{c}_t - c_t > 0 \quad \text{if} \quad 0 \leq t < s \]
\[ b_t = 0 \quad \text{if} \quad 0 \leq s \leq t . \]

It is also assumed that, when a household chooses a different option from the one the other households choose, the difference in the accumulation of financial assets resulting from the difference in consumption \( (b_t) \) before period \( s \) between that household and the other households is reflected in consumption after period \( s \). That is, the difference in the return on financial assets is added to (or subtracted from) the household’s consumption in each period after period \( s \). The exact functional form of the addition (or subtraction) is shown in Section 3.1.4.

3.1.3 Firms

Unutilized products \( (b_t) \) are eliminated quickly in each period by firms because holding \( b_t \) for a long period is a cost to firms. Elimination of \( b_t \) is accomplished by discarding the goods or preemptively suspending production, thereby leaving some capital and labor inputs idle. However, in the next period, unutilized products are generated again because the economy is not proceeding on the Pareto efficient saddle path. Unutilized products are therefore successively generated and eliminated. Faced with these unutilized products, firms dispose of the excess capital used to generate \( b_t \). Disposing of the excess capital is rational for firms because the excess capital is an unnecessary cost, but this means that parts of the firms are liquidated, which takes time and thus disposing of the excess capital will also take time. If the economy proceeds on the \( NJ \) path (that is, if all households choose the \( NJ \) option), firms dispose of all of the remaining excess capital that generates \( b_t \) and adjust their capital to the posterior steady-state level in period \( s \), which also corresponds to households reaching the posterior steady state. Thus, if the economy proceeds on the \( NJ \) path, capital \( k_t \) is

\[ \bar{k} < k_t \leq \hat{k}_t \quad \text{if} \quad 0 \leq t < s \]
\[ k_t = \bar{k} \quad \text{if} \quad 0 \leq s \leq t , \]

where \( \hat{k}_t \) is capital per capita when proceeding on the posterior Pareto efficient saddle path and \( \bar{k} \) is capital per capita in the posterior steady state.

The real interest rate \( i_t \) is

\[ i_t = \frac{\partial f (A, k_t)}{\partial k_t} , \]

Because the real interest rate equals the rate of time preference at steady state, if the economy proceeds on the \( NJ \) path,

\[ \bar{\theta} \leq i_t < \theta \quad \text{if} \quad 0 \leq t < s \]
\[ i_t = \theta \quad \text{if} \quad 0 \leq s \leq t , \]
where \( \tilde{\theta} \) is the rate of time preference before the shock and \( \theta \) is the rate of time preference after the shock. \( i_t \) is monotonously continuous and differentiable in \( t \) if \( 0 \leq t < s \).

### 3.1.4 Expected utility after the shock

The expected utility of a household after the shock depends on its choice of the \( J \) or \( NJ \) path. Let \( Jalone \) indicate that the household chooses option \( J \), but the other households choose option \( NJ \); \( NJalone \) indicate that the household chooses option \( NJ \), but the other households choose option \( J \); \( Jtogether \) indicate that all households choose option \( J \); and \( NJtogether \) indicate that all households choose option \( NJ \). Let \( p \) be the subjective probability of a household that the other households choose the \( J \) option (e.g., \( p = 0 \) indicates that all the other households choose option \( NJ \)). With \( p \), the expected utility of a household when it chooses option \( J \) is

\[
E_0(J) = p E_o(Jtogether) + (1-p) E_o(Jalone) ,
\]

and when it chooses option \( NJ \) is

\[
E_0(NJ) = p E_o(NJalone) + (1-p) E_o(NJtogether) ,
\]

where \( E_o(Jalone) \), \( E_o(NJalone) \), \( E_o(Jtogether) \), and \( E_o(NJtogether) \) are the expected utilities of the household when choosing \( Jalone \), \( NJalone \), \( Jtogether \), and \( NJtogether \), respectively. Given the properties of \( J \) and \( NJ \) shown in Sections 3.1.2 and 3.1.3,

\[
E_0(J) = p E_o\left[ \int_0^{\infty} \exp(-\theta t)u(c_t+b_t)dt + \int_0^{\infty} \exp(-\tilde{\theta} t)u(c_{t+1})dt \right] \\
+ (1-p) E_o\left[ \int_0^{\infty} \exp(-\theta t)u(c_t+b_t)dt + \int_0^{\infty} \exp(-\tilde{\theta} t)u(\tilde{c}_t-\tilde{a})dt \right] ,
\]

and

\[
E_0(NJ) = p E_o\left[ \int_0^{\infty} \exp(-\theta t)u(c_t)dt + \int_0^{\infty} \exp(-\tilde{\theta} t)u(c_{t+1}+a_t)dt \right] \\
+ (1-p) E_o\left[ \int_0^{\infty} \exp(-\theta t)u(c_t)dt + \int_0^{\infty} \exp(-\tilde{\theta} t)u(\tilde{c}_t)dt \right] ,
\]

where

\[
\tilde{\alpha} = \theta \int_0^{\infty} b_t \exp \left[ \int_r^s i_q dq \right] dr ,
\]

and

\[
a_t = i_t \int_0^{\infty} b_t \exp \left[ \int_r^s i_q dq \right] dr ,
\]

and the shock occurred in period \( t = 0 \). Figure 2 shows the paths of \( Jalone \) and \( NJalone \). Because there is a sufficiently large number of households and the effect of an individual household on the whole economy is negligible, in the case of \( Jalone \), the economy almost proceeds on the \( NJ \) path. Similarly, in the case of \( NJalone \), it almost proceeds on the \( J \) path. If
the other households choose the NJ option (Jalone or NJtogether), consumption after \( s \) is constant as \( \bar{c} \) and capital is adjusted to \( \bar{k} \) by firms in period \( s \). In addition, \( a \) and \( i \) are constant after \( s \) such that \( a \) equals \( \bar{a} \) and \( i \) equals \( \theta \), because the economy is at the posterior steady state. Nevertheless, during the transition period before \( s \), the value of \( i \) changes from the value of the prior time preference rate to that of the posterior rate. If the other households choose option \( J \) (Jalone or Jtogether), however, consumption after \( s \) is \( \tilde{c} \) and capital is not adjusted to \( \bar{k} \) by firms in period \( s \) and remains at \( \tilde{k} \).

As mentioned in Section 3.1.2, the difference in the returns on financial assets for the household from the returns for each of the other households is added to (or subtracted from) its consumption in each period after period \( s \). This is described by \( a \) and \( i \) in equations (27) and (28), and equations (29) and (30) indicate that the accumulated difference in financial assets resulting from \( b \) increases by compound interest between the period \( r \) to \( s \). That is, if the household takes the NJalone path, it accumulates more financial assets than each of the other \( J \) households, and instead of immediately consuming these extra accumulated financial assets after period \( s \), the household consumes the returns on them in every subsequent period. If the household takes the Jalone path, however, its consumption after \( s \) is \( \tilde{c} \), as shown in equation (27). \( \bar{a} \) is subtracted because the income of each household, \( \left( \frac{t}{A_k} \right) \), including the Jalone household, decreases equally by \( b_t \). Each of the other NJ households decreases consumption by \( b_t \) at the same time, which compensates for the decrease in income; thus, its financial assets (i.e., capital per capita; \( k_t \)) are kept equal to \( \bar{k} \). The Jalone household, however, does not decrease its consumption, and its financial assets become smaller than those of each of the other NJ households, which results in the subtraction of \( \bar{a} \) after period \( s \).

### 3.2 Pareto inefficient transition path

#### 3.2.1 Rational Pareto inefficient path

**3.2.1.1 Rational choice of a Pareto inefficient path**

Before examining the economy with non-cooperative households, I first show that, if households are cooperative, only option \( J \) is chosen as the path after the shock because it gives a higher expected utility than option \( NJ \). Because there is no possibility of Jalone and NJalone if households are cooperative, then \( E_0(J) = E_0(Jtogether) \) and \( E_0(NJ) = E_0(NJtogether) \). Therefore,

\[
E_0(J) - E_0(NJ) = E_0 \left[ \int_0^s \exp(-\theta t)u(c + b_t)dt + \int_s^\infty \exp(-\theta t)u(\tilde{c}_t)dt \right] - E_0 \left[ \int_0^s \exp(-\theta t)u(c)dt + \int_s^\infty \exp(-\theta t)u(\bar{c})dt \right]
\]

\[
= E_0 \left[ \int_0^s e^{-\theta t} \left( u(c + b_t) - u(c) \right)dt + \int_s^\infty e^{-\theta t} \left( u(\tilde{c}_t) - u(\bar{c}) \right)dt \right] > 0
\]

because \( c_t < c + b_t \) and \( \bar{c} < \tilde{c} \).

Next, I examine the economy with non-cooperative households. First, the special case with a utility function with a sufficiently small \( \gamma \) is examined.

**Lemma 1:** If \( \gamma (0 < \gamma < \infty) \) is sufficiently small, then \( E_0(Jalone) - E_0(NJtogether) > 0 \).

**Proof:** \( \lim_{\gamma \to 0} \left[ E_0(Jalone) - E_0(NJtogether) \right] \)

---

10 The idea of a rationally chosen Pareto inefficient path was originally presented by Harashima (2004b).
\[ E_0 \int_0^\gamma \exp(-\theta t) \lim_{\gamma \to 0} \left[ u(c_i + b_i) - u(c_i) \right] dt + E_0 \int_\alpha^\gamma \exp(-\theta t) \lim_{\gamma \to 0} \left[ u(\bar{c} - \bar{a}) - u(\bar{c}) \right] dt \\
= E_0 \int_0^\gamma \exp(-\theta t) \lim_{\gamma \to 0} \left[ u(c_i + b_i) - u(c_i) \right] dt - E_0 \int_\alpha^\gamma \exp(-\theta t) \bar{a} dt \\
= E_0 \int_0^\gamma \exp(-\theta t) \lim_{\gamma \to 0} \left[ \int_0^\gamma \left( b_i \exp \left[ \int_i^\gamma dq \right] \right) dq \right] \int_\alpha^\gamma \exp(-\theta t) dt \\
= E_0 \int_0^\gamma \exp(-\theta t) \lim_{\gamma \to 0} \left[ \int_0^\gamma \left( b_i \exp \left[ \int_i^\gamma dq \right] \right) dq \right] dt \\
= E_0 \lim_{\gamma \to 0} \left[ b_i \left[ e \times \theta(s-t) - e \times \int_i^\gamma dq \right] dt > 0 \right],
\]

because, if \( 0 \leq t < s \), then \( i, < \theta \) and \( \exp[\theta(s-t)] > \exp[\int_i^\gamma dq] \). Hence, because \( \exp[\theta(s-t)] \)

\[ > \exp[\int_i^\gamma dq], \quad E_0(Jalone) - E_0(NJtogether) > 0 \quad \text{for sufficiently small} \ \gamma. \]

Second, the opposite special case (i.e., a utility function with a sufficiently large \( \gamma \)) is examined.

**Lemma 2:** If \( \gamma(0 < \gamma < \infty) \) is sufficiently large and if \( 0 < \lim_{\gamma \to \infty} \frac{a}{\gamma} < 1 \), then \( E_0(Jalone) - E_0(NJtogether) < 0 \).

**Proof:** Because \( 0 < b_i \), then

\[ \lim_{\gamma \to \infty} \frac{1 - \gamma}{\gamma} \left[ u(c_i + b_i) - u(c_i) \right] = \lim_{\gamma \to \infty} \left[ \left( \frac{c_i + b_i}{\bar{c}} \right)^{1-\gamma} - \left( \frac{c_i}{\bar{c}} \right)^{1-\gamma} \right] = 0 \]

for any period \( t(<s) \). On the other hand, because \( 0 < \bar{a} \), then for any period \( t(<s) \), if

\[ 0 < \lim_{\gamma \to \infty} \frac{a}{\gamma} < 1,
\]

\[ \lim_{\gamma \to \infty} \frac{1 - \gamma}{\gamma} \left[ u(\bar{c} - \bar{a}) - u(\bar{c}) \right] = \lim_{\gamma \to \infty} \left[ \left( 1 - \frac{\bar{a}}{\bar{c}} \right)^{1-\gamma} - 1 \right] = \infty. \]

Thus,

\[ \lim_{\gamma \to \infty} \frac{1 - \gamma}{\gamma} \left[ E_0(Jalone) - E_0(NJtogether) \right] \]

\[ = \lim_{\gamma \to \infty} \frac{1 - \gamma}{\gamma} \int_0^\gamma \exp(-\theta t) \lim_{\gamma \to \infty} \left[ u(c_i + b_i) - u(c_i) \right] dt \]

\[ + \lim_{\gamma \to \infty} \frac{1 - \gamma}{\gamma} \int_\alpha^\gamma \exp(-\theta t) \lim_{\gamma \to \infty} \left[ u(\bar{c} - \bar{a}) - u(\bar{c}) \right] dt \]

\[ = 0 + \infty > 0. \]

Because \( \frac{1 - \gamma}{\gamma} < 0 \) for any \( \gamma(1 < \gamma < \infty) \), then if \( 0 < \lim_{\gamma \to \infty} \frac{a}{\gamma} < 1, \ E_0(Jalone) - E_0(NJtogether) \)
< 0 for sufficiently large $\gamma(< \infty)$. \hfill ■

The condition $0 < \lim \frac{\bar{a}}{c} < 1$ indicates that path $NJ$ from $c_0$ to $\bar{c}$ deviates sufficiently from the posterior Pareto efficient saddle path and reaches the posterior steady state $\bar{c}$ not taking much time. Because steady states are irrelevant to the degree of risk aversion ($\gamma$), both $c_0$ and $\bar{c}$ are irrelevant to $\gamma$.

By Lemmas 1 and 2, it can be proved that $E_0(Jalone) - E_0(NJtogether) < 0$ is possible.

**Lemma 3:** If $0 < \lim_{\gamma \to \infty} \frac{\bar{a}}{c} < 1$, then there is a $\gamma^*(0 < \gamma^* < \infty)$ such that if $\gamma^* < \gamma < \infty$, $E_0(Jalone) - E_0(NJtogether) < 0$.

**Proof:** If $\gamma(> 0)$ is sufficiently small, then $E_0(Jalone) - E_0(NJtogether) > 0$ by Lemma 1, and if $\gamma(< \infty)$ is sufficiently large and if $0 < \lim_{\gamma \to \infty} \frac{\bar{a}}{c} < 1$, then $E_0(Jalone) - E_0(NJtogether) < 0$ by Lemma 2. Hence, if $0 < \lim_{\gamma \to \infty} \frac{\bar{a}}{c} < 1$, there is a certain $\gamma^*(0 < \gamma^* < \infty)$ such that, if $\gamma^* < \gamma < \infty$, then $E_0(Jalone) - E_0(NJtogether) < 0$. \hfill ■

However, $E_0(Jtogether) - E_0(NJalone) > 0$ because both $Jtogether$ and $NJalone$ indicate that all the other households choose option $J$; thus, the values of $i_t$ and $k_t$ are the same as those when all households proceed on the posterior Pareto efficient saddle path. Faced with these $i_t$ and $k_t$, deviating alone from the Pareto efficient path ($NJalone$) gives a lower expected utility than $Jtogether$ to the $NJ$ household. Both $Jalone$ and $NJtogether$ indicate that all the other households choose option $NJ$ and $i_t$ and $k_t$ are not those of the Pareto efficient path. Hence, the sign of $E_0(Jalone) - E_0(NJtogether)$ varies depending on the conditions, as Lemma 3 indicates.

By Lemma 3 and the property $E_0(Jtogether) - E_0(NJalone) > 0$, the possibility of the choice of a Pareto inefficient transition path, that is, $E_0(J) - E_0(NJ) < 0$, is shown.

**Proposition 1:** If $0 < \lim_{\gamma \to \infty} \frac{\bar{a}}{c} < 1$ and $\gamma^* < \gamma < \infty$, then there is a $p^*\{0 \leq p^* \leq 1\}$ such that if $p = p^*$, $E_0(J) - E_0(NJ) = 0$, and if $p < p^*$, $E_0(J) - E_0(NJ) < 0$.

**Proof:** By Lemma 3, if $\gamma^* < \gamma < \infty$, then $E_0(Jalone) - E_0(NJtogether) < 0$ and $E_0(Jtogether) - E_0(NJalone) > 0$. By equations (25) and (26),

$$E_0(J) - E_0(NJ) = p[E_0(Jtogether) - E_0(NJalone)] + (1 - p)[E_0(Jalone) - E_0(NJtogether)]$$.

Thus, if $0 < \lim_{\gamma \to \infty} \frac{\bar{a}}{c} < 1$ and $\gamma^* < \gamma < \infty$, $\lim_{p \to 0} [E_0(J) - E_0(NJ)] = E_0(Jalone) - E_0(NJtogether) < 0$ and $\lim_{p \to 1} [E_0(J) - E_0(NJ)] = E_0(Jtogether) - E_0(NJalone) > 0$. Hence, by the intermediate value theorem, there is $p^*\{0 \leq p^* \leq 1\}$ such that if $p = p^*$, $E_0(J) - E_0(NJ) = 0$ and if $p < p^*$, $E_0(J) - E_0(NJ) < 0$. \hfill ■
Proposition 1 indicates that, if \( 0 < \lim_{s \to 0} \gamma < 1 \), \( \gamma^* < \gamma < \infty \), and \( p < p^* \), then the choice of option \( NJ \) gives the higher expected utility than that of option \( J \) to a household; that is, a household may make the rational choice of taking a Pareto inefficient transition path. The lemmas and proposition require no friction, so a Pareto inefficient transition path can be chosen even in a frictionless economy. This result is very important because it offers counter-evidence against the conjecture that households never rationally choose a Pareto inefficient transition path in a frictionless economy.

### 3.2.1.2 Conditions for a rational Pareto inefficient path

The proposition requires several conditions. Among them, \( \gamma^* < \gamma < \infty \) may appear rather strict. If \( \gamma^* \) is very large, path \( NJ \) will rarely be chosen. However, if path \( NJ \) is such that consumption is reduced sharply after the shock, the \( NJ \) option yields a higher expected utility than the \( J \) option even though \( \gamma \) is very small. For example, for any \( \gamma (0 < \gamma < \infty) \),

\[
\lim_{s \to 0} E_0(Jaltogether) + E_0(NJtogether)
\]

\[
= \lim_{s \to 0} \int_s^\infty \exp(-\theta t)[u(c_s + b_t) - u(c_s)]dt + \lim_{s \to 0} \int_s^\infty \exp(-\theta t)[u(\bar{c} - \bar{a}) - u(\bar{c})]dt
\]

\[
= u(c_s + b_t) - u(c_s) - \frac{1}{\theta} \lim_{s \to 0} \int_s^\infty \frac{u(\bar{c}) - u(\bar{c} - s\theta b_t)}{s} = u(c_s + b_t) - u(c_s) - \frac{du(\bar{c})}{d\bar{c}}
\]

\[
= \left[ \frac{(c_s + b_t)^{1-\gamma} - c_s^{1-\gamma}}{1-\gamma} \right] < 0
\]

because \( \lim_{\gamma \to 0+} \left[ \frac{(c_s + b_t)^{1-\gamma} - c_s^{1-\gamma}}{1-\gamma} \right] = \bar{c}[\ln(c_s + b_t) - \ln(c_s)] = \bar{c} \ln\left(1 + \frac{b_t}{c_s}\right) < b_t \) and

\[
\lim_{\gamma \to 0+} \left[ \frac{(c_s + b_t)^{1-\gamma} - c_s^{1-\gamma}}{1-\gamma} \right] = \lim_{\gamma \to 0+} \left[ \frac{1 + \frac{b_t}{c_s}}{1-\gamma} \right] = 0
\]

consider an example in which path \( NJ \) is such that \( b_t \) is constant and \( b_t = \bar{b} \) before \( s \) (Figure 3); thus, \( E_0[\int_0^s b_t = s\bar{b}] \). In this \( NJ \) path, consumption is reduced more sharply than it is in the case shown in Figure 2. In this case, because \( \bar{a} > E_0[\int_0^s b_t = \theta s\bar{b}] \), \( 0 < \gamma \), and \( c_s < c_t \), for \( t < s \), then \( E_0[\int_t^s \exp(-\theta t)[u(c_s + b_t) - u(c_s)]dt < E_0[\int_t^s \exp(-\theta t)dt[u(c_s + b_t) - u(c_s)] = E_0[\frac{1 - \exp(-\theta s)}{\theta}][u(c_s + b_t) - u(c_s)], \) and in addition, \( E_0[\int_s^\infty \exp(-\theta t)[u(\bar{c} - \bar{a}) - u(\bar{c})]dt = E_0[\int_s^\infty \exp(-\theta t)dt[u(\bar{c} - \bar{a}) - u(\bar{c})]] = E_0[\frac{\exp(-\theta s)}{\theta}[u(\bar{c} - \bar{a}) - u(\bar{c})] < E_0[\exp(-\theta b)[u(\bar{c} - \bar{a}) - u(\bar{c})]]. \)
Hence,
\[ E_0(\text{Jtogether}) + E_0(\text{NJtogether}) \]
\[ = E_0 \int_0^\infty \exp(-\theta t)[u(c_x + b_t) - u(c_x)] dt + E_0 \int_0^\infty \exp(-\theta t)[u(c_x - \bar{\eta}) - u(c_x)] dt \]
\[ < E_0 \frac{1 - \exp(-\theta \bar{\gamma})}{\theta} [u(c_x + \bar{\eta}) - u(c_x)] + E_0 \frac{\exp(-\theta \bar{\gamma})}{\theta} [u(c_x - \theta \bar{\eta}) - u(c_x)] \]
\[ = E_0 \frac{1 - \exp(-\theta \bar{\gamma})}{\theta} \left[ u(c_x + \bar{\eta}) - u(c_x) \right] - \frac{\exp(-\theta \bar{\gamma})}{1 - \exp(-\theta \bar{\gamma})} \left[ u(\bar{\gamma}) - u(\bar{\gamma} - \theta \bar{\eta}) \right] . \]

As \( \gamma \) increases, the ratio \( \frac{u(c_x + \bar{\eta}) - u(c_x)}{u(\bar{\gamma}) - u(\bar{\gamma} - \theta \bar{\eta})} \) decreases; thus, larger values of \( s \) can satisfy \( E_0(\text{Jalone}) - E_0(\text{NJtogether}) < 0 \). For example, suppose that \( \bar{\gamma} = 10, c_x = 10.2, \bar{\eta} = 0.3, \) and \( \theta = 0.05 \). If \( \gamma = 1 \), then \( s^* = 1.5 \) at the minimum, and if \( \gamma = 5 \), then \( s^* = 6.8 \) at the minimum. This result implies that, if option \( NJ \) is such that consumption is reduced relatively sharply after the shock (e.g., \( b_t = \bar{\eta} \)) and \( \gamma > \gamma^* \), option \( NJ \) will usually be chosen. Choosing option \( NJ \) is not a special case observed only if \( \gamma \) is very large, but option \( NJ \) can normally be chosen when the value of \( \gamma \) is within usually observed values. Conditions for generating a rational Pareto inefficient transition path therefore are not strict. In a recession, consumption usually declines sharply after the shock, which suggests that households have chosen the \( NJ \) option.

### 3.3 Nash equilibrium

#### 3.3.1 A Nash equilibrium consisting of \( NJ \) strategies

A household strategically determines whether to choose the \( J \) or \( NJ \) option, considering other households’ choices. All households know that each of them forms expectations about the future values of its utility and makes a decision in the same manner. Since all households are identical, the best response of each household is identical. Suppose that there are \( H \in N \) identical households in the economy where \( H \) is sufficiently large (as assumed in Section 3.1). Let \( q_\eta (0 \leq q_\eta \leq 1) \) be the probability that a household \( \eta \in H \) chooses option \( J \). The average utility of the other households almost equals that of all households because \( H \) is sufficiently large. Hence, the average expected utilities of the other households that choose the \( J \) and \( NJ \) options are \( E_0(J\text{together}) \) and \( E_0(NJ\text{together}) \), respectively. Hence, the payoff matrix of the \( H \)-dimensional symmetric mixed strategy game can be described as shown in Table 1. Each identical household determines its behavior on the basis of this payoff matrix.

In this mixed strategy game, the strategy profiles
\[ (q_1, q_2, \ldots, q_H) = \{(1,1,\ldots,1), (p^*, p^*, \ldots, p^*), (0,0,\ldots,0)\} \]
are Nash equilibria for the following reason. By Proposition 1, the best response of household \( \eta \) is \( J \) (i.e., \( q_\eta = 1 \)) if \( \gamma > \gamma^* \), indifferent between \( J \) and \( NJ \) (i.e., any \( q_\eta \in [0,1] \)) if \( \gamma = \gamma^* \), and \( NJ \) (i.e., \( q_\eta = 0 \)) if \( \gamma < \gamma^* \). Because all households are identical, the best-response correspondence of each household is identical such that \( q_\eta = 1 \) if \( \gamma > \gamma^* \), [0,1] if \( \gamma = \gamma^* \), and 0 if \( \gamma < \gamma^* \) for any household \( \eta \in H \). Hence, the mixed strategy profiles \( (1,1,\ldots,1), (p^*, p^*, \ldots, p^*), \) and \( (0,0,\ldots,0) \) are the intersections of the graph of the best-response correspondences of all households. The Pareto efficient saddle path solution \((1,1,\ldots,1)\) (i.e., \( J\text{together} \)) is a pure strategy Nash equilibrium, but a Pareto inefficient transition path \((0,0,\ldots,0)\) (i.e., \( NJ\text{together} \)) is
also a pure strategy Nash equilibrium. In addition, there is a mixed strategy Nash equilibrium 
\((p^*, p^*, \ldots, p^*)\).

### 3.3.2 Selection of equilibrium

Determining which Nash equilibrium, either \(NJ_{together} (0,0,\ldots,0)\) or \(J_{together} (1,1,\ldots,1)\), is dominant requires refinements of the Nash equilibrium, which necessitate additional criteria. Here, if households have a risk-averse preference in the sense that they avert the worst scenario when its probability is not known, households suppose a very low \(p\) and select the \(NJ_{together} (0,0,\ldots,0)\) equilibrium. Because

\[
\begin{align*}
E_0(J_{alone}) & - E_0(NJ_{alone}) \\
= E_0 \left[ \int_0^s \exp(-\theta t) [u(c_t, b_t) - u(c_t)] dt + \int_s^\infty \exp(-\theta t) [u(c_t - \bar{a}) - u(c_t + a)] dt \right] \\
< E_0 \left[ \int_0^s \exp(-\theta t) [u(c_t + b_t) - u(c_t)] dt + \int_s^\infty \exp(-\theta t) [u(c_t - \bar{a}) - u(c_t)] dt \right] \\
= E_0(J_{alone}) - E_0(NJ_{together})(\theta) 
\end{align*}
\]

by Lemma 3, \(J_{alone}\) is the worst choice in terms of the amount of payoff, followed by \(NJ_{together}\), and \(NJ_{alone}\), and \(J_{together}\) is the best. The outcomes of choosing option \(J\) are more dispersed than those of option \(NJ\). If households have a risk-averse preference in the above-mentioned sense and avert the worst scenario when they have no information on its probability, a household will prefer the less dispersed option \((NJ)\), fearing the worst situation that the household alone substantially increases consumption while the other households substantially decrease consumption after the shock. This behavior is rational because it is consistent with preferences. Because all households are identical and know inequality (31), all households will equally suppose that they all prefer the less dispersed \(NJ\) option; therefore, all of them will suppose a very low \(p\), particularly \(p = 0\), and select the \(NJ_{together} (0,0,\ldots,0)\) equilibrium, which is the Nash equilibrium of a Pareto inefficient path. Thereby, unlike most multiple equilibria models, the problem of indeterminacy does not arise, and “animal spirits” (e.g., pessimism or optimism) are unnecessary to explain the selection.

### 3.4 Amplified generation of unutilized resources

A Nash equilibrium of a Pareto inefficient path successively generates unutilized products \((b_t)\). They are left unused, discarded, or preemptively not produced during the path. Unused or discarded goods and services indicate a decline in sales and an increase in inventory for firms. Preemptively suspended production results in an increase in unemployment and idle capital. As a result, profits decline and some parts of firms need to be liquidated, which is unnecessary if the economy proceeds on the \(J\) path (i.e., the posterior Pareto efficient path). If the liquidation is implemented immediately after the shock, \(b_t\) will no longer be generated, but such a liquidation would generate a tremendous shock. The process of the liquidation, however, will take time because of various frictions, and excess capital that generates \(b_t\) will remain for a long period. During the period when capital is not reduced to the posterior steady-state level, unutilized products are successively generated. In a period, \(b_t\) is generated and eliminated, but in the next period, another, new, \(b_t\) is generated and eliminated. This cycle is repeated in every period throughout the transition path, and it implies that demand is lower than supply in every period. This phenomenon may be interpreted as a general glut or a persisting disequilibrium by some definitions of equilibrium. That is, \(b_t\) is another source of output gaps than frictions.

### 2.5 Time preference shock as the exceptional shock
Not all shocks result in a Nash equilibrium of a Pareto inefficient path. If anything, this type of shock is limited because such a shock needs to force consumption to fluctuate very jaggedly to maintain Pareto efficiency. A Pareto inefficient path is preferred, because these substantially jagged fluctuations can be averted. An upward time preference shock is one shock that necessitates a substantially jagged fluctuation as shown in Figure 1. Other examples are rare because shocks that do not change the steady state (e.g., monetary shocks) are not relevant. One other example is technology regression, which would move the vertical line \( \frac{dc_t}{dt} = 0 \) to the left in Figure 1 and necessitate a jagged consumption path to keep Pareto efficiency. In this sense, technology and time preference shocks have similar effects on economic fluctuations. However, a technology regression also simultaneously moves the curve \( \frac{dk_t}{dt} = 0 \) downwards, and accordingly, the Pareto efficient saddle path also moves downwards. Therefore, the jagged consumption is smoothed out to some extent. As a result, the substantially jagged consumption that can generate a recession would require a large-scale, sudden, and sharp regression in technology, which does not seem very likely. An upward time preference shock, however, only moves the vertical line \( \frac{dc_t}{dt} = 0 \) to the left.

In some macro-economic models with multiple equilibria, changing equilibria may necessitate substantially jagged consumption to keep Pareto optimality. There are many types of multiple equilibria models that depend on various types of increasing returns, externalities, or complementarities, but they are vulnerable to a number of criticisms (e.g., insufficient explanation of the switching mechanism; see, e.g., Morris and Shin, 2001). Examining the properties, validity, and plausibility of each of these many and diverse models is beyond the scope of this paper.

## 4 PHILLIPS CURVE

### 4.1 Models of the Phillips curve

#### 4.1.1 Trend inflation and inflation

The micro-foundation of trend inflation discussed in Section 2 indicates that inflation \( \pi_t \) is a function of trend inflation \( \pi_t^T \), in particular such that

\[
\pi_t = \pi_t^T + \nu_{1,t}
\]

where \( \nu_{1,t} \) is a variable that represents factors other than trend inflation in period \( t \). Equation (32) indicates that the aggregate supply equation (the Phillips curve) is modeled as a variable moving around a trend and occasionally diverting from the trend because of other factors.

#### 4.1.2 Output gaps and inflation

Section 2 shows that shifts in \( \theta_p \) change the path of trend inflation \( \pi_t^T \) unless \( \theta_G \) is immediately changed in the same direction and by the same magnitude as \( \theta_p \). Usually \( \theta_G \) will not change immediately after a shift in \( \theta_p \), so the path of trend inflation will usually change after a shift in \( \theta_p \). Hence, \( \pi_t \) changes as \( \theta_p \) shifts; thus, \( \pi_t \) is a function of \( \theta_p \) such that

\[
\pi_t = h_0(\theta_{p,t})
\]

In Section 3, I showed that outputs \( y_t \) fluctuate with shifts of \( \theta_p \) and unutilized
resources \((b_t)\) are generated. The unutilized resources indicate the existence of output gaps. The output gaps \(x_t\) can be described as follows:

\[
x_t = \ln(y_t) - \ln(y_t^*) ,
\]

where \(y_t^*\) is \(y_t\) at the steady state or on the saddle path. Let also

\[
x_{b,t} = \ln(y_t) - \ln(y_t + b_t) .
\]

That is, \(x_{b,t}\) is the output gap generated owing to \(b_t\) in period \(t\) and is a part of \(x_t\). Because \(b_t\) is a function of \(\theta_p\), \(x_{b,t}\) is also a function of \(\theta_p\) such that

\[
x_{b,t} = h_b(\theta_{p,t}) ,
\]

where \(\theta_{p,t}\) is \(\theta_p\) in period \(t\). Suppose that \(\theta_{p,t}\) is a Markov process and shifts in \(\theta_p\) occasionally occur. By equations (33) and (34), \(x_{b,t}\) will be observed to correlate with \(\pi_t\) such that

\[
\pi_t = h_b^{-1}(x_{b,t}) .
\]

Equation (35) does not indicate causation; it merely indicates that there is a correlation between \(\pi_t\) and \(x_{b,t}\). The causations are described by equations (33) and (34).

There is, however, a conventional correlation between inflation and output gaps, and it is caused by frictions in price flexibility. The output gaps generated by frictions are traditionally thought to be the only sources of output gaps in the NKPC. Suppose that the sources of output gaps are only \(b_t\) and frictions. Thus, the output gaps that are generated by frictions in period \(t\) are

\[
x_{F,t} = \ln(y_t + b_t) - \ln(y_t^*) .
\]

That is,

\[
x_t = x_{b,t} + x_{F,t} .
\]

According to the micro-foundation of the NKPC, \(x_{F,t}\) is correlated with \(\pi_t\) in a forward-looking manner such that

\[
\pi_t = h_F(x_{F,t+i|t-1}) + \nu_{2,t} ,
\]

for \(i = 0,1,2,\ldots\)

(36)

Where \(\nu_{2,t}\) is a variable that represents factors other than trend inflation and \(x_{F,t+i|t-1}\) is the \(x_{F,t+i}\) expected in period \(t-1\).

### 4.1.3 Three models of inflation in the aggregate supply equation

Combining equations (32) and (36), inflation can be modeled as

\[
\pi_t = \pi_t^F + h_F(x_{F,t+i|t-1}) ,
\]

for \(i = 0,1,2,\ldots\)

With i.i.d. shocks \(\varepsilon_t\), the data generation mechanism of \(\pi_t\) can be modeled as
\[ \pi_t = \pi_t - \pi_t^{FP} + h_t(F_{t+1},t) + \epsilon_{t+1} \quad \text{for } i = 0,1,2, \ldots \] 

or more simply

\[ \pi_t = \alpha_1 \pi_t^{FP} + \phi_1 x_{F_t,t-1} + \epsilon_{t+1} \quad , \]

where \( \alpha_1 \) and \( \phi_1 \) are constants and expected to be positive, and \( \epsilon_{t+1} \) is an i.i.d. shock in period \( t \). It is important to note that equations (37) and (38) are aggregate supply equations that are firmly constructed on a micro-foundation basis. Another important point is that equations (37) and (38) do not include the correlation indicated by equation (35).

Equations (37) and (38) superficially resemble the pure NKPC and the hybrid NKPC, but they are actually completely different. Typical pure and hybrid NKPCs can be described, respectively, as

\[ \pi_t = \phi_2 x_{F_t,t-1} + \epsilon_{2,t} \quad (39) \]

and

\[ \pi_t = \alpha_3 \pi_{t-1} + \phi_3 x_{F_t,t-1} + \epsilon_{3,t} \quad , \]

where \( \alpha_3 \), \( \phi_2 \), and \( \phi_3 \) are constants and expected to be positive, and \( \epsilon_{2,t} \) and \( \epsilon_{3,t} \) are disturbances in period \( t \). That is, a pure NKPC indicates that inflation is a function of \( x_{F_t} \), and a hybrid NKPC indicates that inflation is a function of both lagged inflation and \( x_{F_t} \). An important difference between equation (38) and equations (39) and (40) is that equation (38) includes trend inflation but the others do not.

Conceptually, most models of NKPC assume that \( x_{b,t} \) does not exist and \( x_t \) consists only of \( x_{F,t} \) as shown in equations (39) and (40), and data of \( x_t \) are usually regarded to be identical to those of \( x_{F,t} \). However, if \( x_{b,t} \) does exist, estimations of equations (39) and (40) using data of \( x_t \) as those of \( x_{F,t} \) are in reality estimations of the following aggregate supply equations, respectively:

\[ \pi_t = \phi_2 x_{F_t,t-1} + \epsilon_{2,t} = \phi_2 \left( x_{F_t,t-1} + x_{b,t-1} \right) + \epsilon_{2,t} \]

and

\[ \pi_t = \alpha_3 \pi_{t-1} + \phi_3 x_{F_t,t-1} + \epsilon_{3,t} = \alpha_3 \pi_{t-1} + \phi_3 \left( x_{F_t,t-1} + x_{b,t-1} \right) + \epsilon_{3,t} \quad , \]

where \( x_{F_t,t-1} \) and \( x_{b,t-1} \) are \( x_t \) and \( x_{b,t} \) expected in period \( t-1 \), respectively. Although conceptually \( x_t = x_{F_t} \) in the NKPC, in reality, inflation is a function of \( x_t = x_{F_t} + x_{b,t} \) in estimation models of the pure NKPC and a function of lagged inflation and \( x_t = x_{F_t} + x_{b,t} \) in hybrid NKPC models. Equations (41) and (42) are therefore actual estimation models of the pure NKPC and the hybrid NKPC, respectively. In the following discussion, equation (38) is referred to as Model 1, and equations (41) and (42) are referred to as Models 2 and 3, respectively.

4.1.4 Superiority of Model 1

In the sense that Model 1 is constructed on the basis of purely forward-looking micro-foundations of both trend inflation and friction, it is superior to Model 3 (hybrid NKPC), which lacks a micro-foundation for including lagged inflation. Model 2 (pure NKPC), however,
does have a micro-foundation for the friction component, but it is usually empirically rejected, whereas the results of estimates from Model 3 are usually empirically accepted. Model 1 cannot be easily estimated empirically because it is difficult to distinguish between $x_{F,t}$ and $x_{b,t}$ in the data, but it is highly likely that Model 1 would be empirically supported because the trend inflation in Model 1 and the lagged inflation in Model 3 play almost the same role in the estimation of both models. Model 1 is therefore superior to Model 2 in the sense that it would most likely be empirically supported. As a whole, therefore, it is likely that Model 1 is the closest to the true mechanism of the three models.

4.2 Are frictions important?

4.2.1 Inappropriateness of the pure NKPC (Model 2)
Most empirical research has rejected Model 2, and the reason can be understood by comparing Model 2 with Model 1. If Model 1 is the true mechanism, Model 2 will be naturally rejected empirically because the movement of trend inflation $(\pi_T)$ cannot be captured sufficiently only by $x_t (= x_{F,t} + x_{b,t})$, as shown in Section 2. The estimates of $\varphi_2$ in Model 2 therefore will be always statistically non-significant as shown in many empirical researches. This result seems natural, because it is well known that inflation is persistent, and output gaps caused by frictions $(x_{F,t})$ cannot, by their nature, be persistent. Trend inflation, however, can be persistent. For these reasons, Model 1 is superior to Model 2.

4.2.2 A problem in hybrid NKPC (Model 3)
Unlike Model 2, Model 3 lacks a micro-foundation, but the results generated from the model match with empirical data. If Model 1 is the true mechanism, it is natural that the results from Model 3 would fit the empirical data. Suppose for simplicity that $z_t = 2$ in equation (2) because $\lim_{z_t \to \infty} z_t$ as shown in Section 2. By equation (2),

$$
\pi_t^T = \pi_{t-1}^T + 6(\theta_G - \theta_p)[(t - \sigma)^2 - (t - \sigma - 1)^2]
= \pi_{t-1}^T + 6(\theta_G - \theta_p)[2(t - \sigma) + 1]
$$

for $t \geq s$. Note that equation (1) indicates that the path of $\pi_t^T$ just after a shift of $\theta_p$ is more complex than what is shown in equation (43) because $\pi_t^T$ is influenced by its past path. Nevertheless, for simplicity, I assume that equation (43) holds even just after a shift in $\theta_p$ because $\lim_{z_t \to \infty} z_t = 2$, and the path of $\pi_t^T$ will soon approach the path indicated in equation (43).

By combining equation (43) with Model 1,

$$
\pi_t = \pi_{t-1} + \varphi_1(\pi_{F,t-1} - \pi_{F,t-M-2}) + 6 \alpha_1(\theta_G - \theta_p)[2(t - \sigma) + 1] .
$$

Model 1 is transformed to be a function of lagged inflation ($\pi_{t-1}$); that is, Model 1 indicates that $\pi_t$ is auto-correlated, as Model 3 also indicates. Model 3 includes lagged inflation without showing its micro-foundation, but Model 1 provides this micro-foundation and thus validates the inclusion of lagged inflation in an aggregate supply equation.

At the same time, however, Model 1 (equation (44)) indicates that the coefficient of $\pi_{t-1}$ should be unity. As is well known, estimates of $\alpha_1$ in Model 3 are usually far less than unity (e.g., 0.5). The reason for the difference is that, although Models 1 and 3 are similar in that they include lagged inflation, other explanatory variables are not the same. The explanatory variables of $\pi_t$ in Model 1 are $x_{F,t}$ and $\theta_p$, and those in Model 3 are $x_{F,t}$ and $x_{b,p}$. Combining Model 3 with
equation (44) (i.e., Model 1) yields
\[
(1-a_3)x_{t-1} = \varphi_3(x_{F,t-1} + x_{b,t-1}) + \varepsilon_{x,t} - \varphi_1(x_{F,t-1} - x_{F,t-2}) - (c_{x,t} - c_{x,t-1}) - 6a_1(\theta_G - \theta_P)[2(t-\sigma) + 1].
\]
Suppose for simplicity that \( \varepsilon_{x,t} = \varepsilon_{1,t} = \varepsilon_{t-1,t} = 0 \) and \( x_{F,t-1} - x_{F,t-2} = 0 \); thus,
\[
(1-a_3)x_{t-1} + 6a_1(\theta_G - \theta_P)[2(t-\sigma) + 1] = \varphi_3(x_{F,t-1} + x_{b,t-1})
\]
should always hold. If \( a_3 = 1 \) as Model 1 indicates,
\[
\varphi_3(x_{F,t-1} + x_{b,t-1}) = 6a_1(\theta_G - \theta_P)[2(t-\sigma) + 1]
\]
should be always held. Both \( \varphi_3(x_{F,t-1} + x_{b,t-1}) \) and \( 6a_1(\theta_G - \theta_P)[2(t-\sigma) + 1] \) are negative, and thus estimates of Model 3 for \( a_3 = 1 \) can be statistically significant. However, even if \( a_3 < 1 \),
\[
\varphi_3(x_{F,t-1} + x_{b,t-1}) = (1-a_3)x_{t-1} + 6a_1(\theta_G - \theta_P)[2(t-\sigma) + 1]
\]
can be fallaciously satisfied if \( (1-a_3)x_{t-1} + 6a_1(\theta_G - \theta_P)[2(t-\sigma) + 1] < 0 \) and a larger value of \( \varphi_3 \) is given. In particular, when \( \pi_i \) is low, the probability that \( (1-a_3)x_{t-1} + 6a_1(\theta_G - \theta_P)[2(t-\sigma) + 1] \) is negative will be high, and the probability that \( a_3 \) is estimated to be far less than unity will be also high. In this case, the estimated value of \( \varphi_3 \) is fallaciously larger than the case of \( a_3 = 1 \). On the other hand, when \( \pi_i \) is high, \( a_3 \) will be estimated to be close to unity because \( (1-a_3)x_{t-1} + 6a_1(\theta_G - \theta_P)[2(t-\sigma) + 1] \) is positive unless \( a_3 \) is close to unity. Even if \( a_3 \) is estimated to be far less than unity and statistically significant, therefore, Model 1 indicates that this is a fallacious result.

### 4.2.3 Frictions are less important than previously thought

In addition to erroneously small values of \( a_3 \), the fact that estimated values of \( \varphi_3 \) will be fallaciously larger is also important because it indicates that the influence of frictions (\( \varphi_3 \)) will also be overestimated in Model 3. Furthermore, another factor influences the overestimation of frictions. \( \varphi_3 \) is the coefficient not of \( x_{F,t} \) but of \( x_{t} = x_{F,t} + x_{b,t} \); thus, \( \varphi_3 \) reflects not only frictions but also \( b_t \). Model 1’s micro-foundation indicates that the output gaps caused by time preference shifts (\( x_{b,t} \)) are irrelevant to the data generation mechanism of \( \pi_t \). Equation (35) merely indicates that \( \pi_t \) is superficially correlated with \( x_{b,t} \), but there is no causation between the two. Because \( \varphi_3 \) reflects both correlations between \( \pi_t \) and \( x_{F,t} \) and \( \pi_t \) and \( x_{b,t} \), the estimates of \( \varphi_3 \) will be influenced not only by frictions but also by the movement of \( b_t \). With this effect, therefore, the influence of frictions, if it is measured by \( \varphi_3 \), will be overestimated.

The above two factors combined will greatly bias estimates of \( \varphi_3 \) upwards. It is likely therefore that the influence of frictions is largely overestimated if Model 3 is used for the evaluation. This finding has an important implication. Frictions have been regarded as an important factor in economic activities, but their role may be far smaller than has been previously thought. Even though some degree of frictions may actually exist and have real impacts, the results presented here indicate that the importance of frictions should not be exaggerated. Furthermore, this is most likely true not only for inflation but also for more general economic activities. This conclusion seems very natural, because it is highly likely that humans are sufficiently rational and can quickly and fully exploit the opportunities frictions provide and minimize the obstruction caused to economic activities by frictions.
4.3 Monetary policies

Monetary policies have usually been implemented on the basis of Model 2 or Model 3. If monetary policies were to be implemented on the basis of Model 1, then the effects could be different, so monetary policies based on Model 1 are examined in this section.

4.3.1 Aggregate demand equation

An examination of monetary policies requires not only an aggregate supply equation but also an aggregate demand equation. The following is a typical forward-looking New Keynesian aggregate demand equation (e.g., Clarida et al., 1999; Svensson and Woodford, 2003):

\[ x_t = x_{F,t} + x_{b,t} = x_{F,t} + x_{b,t} \]

where \( i_t \) is the nominal interest rate; \( r \) is the real interest rate at steady state; \( \beta_r \) is a constant coefficient; and \( \eta_t \) is an i.i.d. shock with zero mean. Equation (45) is obtained under the assumption that \( x_t \) is generated only by frictions. In other words, equation (45) assumes that \( x_t = x_{F,t} \). However, in Model 1, \( x_t = x_{F,t} + x_{b,t} \). Hence, to be consistent with Model 1, equation (45) should be changed to

\[ x_{F,t} = x_{F,t} + x_{b,t} = x_{F,t} + x_{b,t} = x_{F,t} + x_{b,t} \]

Equation (47) indicates that \( x_t \) is influenced not only by \( i_{q,t-1} - \pi_{r,t} - r \) but also by \( x_{b,t} \). For example, when \( \theta_{P,t} \) shifts upwards, \( x_{b,t} \) increases and \( T_t \) also increases. The response of conventional monetary policy is to raise nominal interest rates to make \( x_{F,t} \) decrease through equation (46) (the aggregate demand equation) and consequently make \( \pi_t \) decrease through the aggregate supply equation.

This conventional operation focuses only on \( x_{F,t} \) and does not consider the effect of the shift of \( \theta_{P,t} \) on \( \pi^F_t \). Model 1, however, indicates that \( \pi_t \) depends on \( \pi^F_t \), which is not affected by \( x_{F,t} \). Hence, Model 1 indicates that \( \pi_t \) is not necessarily sufficiently controlled through the use of
conventional monetary policy. To stabilize \( \pi_t \) by the conventional monetary policy, nominal interest rates should be raised far more than would be done with the conventional policy, at least up to the point where the effect of \( x_{F,t} \) on \( \pi_t \) overwhelms the effect of \( \pi_t^T \) on \( \pi_t \). Even if nominal interest rates are raised to this far higher rate, \( \pi_t^T \) will accelerate unless \( \theta_G \) is sufficiently reduced, as shown in Section 3. Nominal interest rates therefore should continue to be increased successively and indefinitely to stabilize inflation. Conversely, if \( \theta_P \) has a large shift upward, the nominal interest rate will have to be reduced to zero (the lower bound of the nominal interest rate) unless \( \theta_G \) is sufficiently increased. In this case, deflation will accelerate if \( \theta_G \) is not sufficiently increased.

There is a great deal of evidence, however, that inflation has been stabilized by conventional monetary policy. I explore the possible reasons for this in the following sections.

### 4.3.2.2 Controlling the government’s time preference

Although trend inflation \( \pi_t^T \) cannot be controlled by conventional monetary policy, it can be controlled through other types of monetary policy. The central bank can stabilize \( \pi_t^T \) by controlling the time preference rate of government (\( \theta_G \)). As shown in Section 2, by manipulating nominal interest rates, the central bank can force the government to change \( \theta_G \). If \( \theta_G \) changes according to the central bank’s plan, then \( \pi_t^T \) will eventually stabilize. Model 1 indicates that, if \( \pi_t^T \) is stabilized at the target rate, \( \pi_t \) will also stabilize in the sense that \( \pi_t \) does not accelerate or decelerate and will remain near the target rate. For example, when \( \theta_P \) shifts downwards and \( \pi_t^T \) begins to accelerate, the central bank should raise nominal interest rates and force the government to lower \( \theta_G \) to stabilize \( \pi_t^T \). If \( \theta_G \) is successfully lowered as planned, \( \pi_t^T \) will stabilize.

Section 2 shows that acceleration and deceleration of trend inflation are caused by the difference between \( \theta_G \) and \( \theta_P \). Therefore, only monetary policy aimed at controlling the government’s time preference rate can eventually stabilize inflation in the sense that \( \pi_t \) does not accelerate or decelerate. Conversely, the monetary policy of utilizing frictions plays only a minor role in the process of inflation stabilization.

### 4.3.2.3 Indistinguishable effects of monetary policies

The monetary policy of utilizing frictions (conventional monetary policy) nevertheless has been regarded as the main player in inflation stabilization because the tools used in both types of monetary policy (utilizing frictions and controlling \( \theta_G \)) are the same. Both types of policy manipulate nominal interest rates. In addition, the directions of the effects of both policy types are the same; for example, if nominal interest rates are raised, inflation decreases. Hence, the effects of the two types of monetary policy are not easily distinguishable. Even if a central bank consciously implements a monetary policy of utilizing frictions, it automatically also implements the monetary policy of controlling \( \theta_G \) at the same time. If inflation stabilizes as a result of the operation, the central bank may believe that the monetary policy of utilizing frictions was effective, even though it was the policy of controlling \( \theta_G \) that was effective. This indistinguishable nature of the effects of the policy therefore will lead to the incorrect belief that the monetary policy of utilizing frictions is very effective for inflation stabilization even when \( \theta_P \) shifts.

### 4.3.2.4 Power to control output gaps

The aggregate demand equation that is consistent with Model 1 (equations [46] and [47]) indicates another important nature of monetary policy. Monetary policies, whether utilizing frictions or controlling \( \theta_G \), do not have enough power to stabilize output gaps. Because \( x_{b,t} \) is
exogenously given for the central bank, monetary policies cannot eliminate \( x_{b,t} \). By decreasing \(|x_{F,t}|\) through equation (46), \(|x_t|\) becomes smaller to some extent, but a large \(|x_t|\) will continue to exist because \( x_{b,t} \) continues to exist. The results from Model 1 indicate that we should not expect to stabilize large output gaps through monetary policies, although small output gaps caused by frictions may be stabilized by them. In contrast, monetary policies—particularly the monetary policy of controlling \( \theta_G \)—are very effective for stabilizing inflation.

5 CONCLUDING REMARKS

Pure and hybrid NKPCs have been criticized for empirical failures and the lack of micro-foundation, respectively. An alternative approach to the Phillips curve is to focus on trend inflation. In this paper, a micro-foundation of trend inflation is shown. Another important factor in the Phillips curve is the nature of output gaps. In the NKPC, output gaps are assumed to be generated only by frictions, but in this paper another source of output gaps is considered. These output gaps are generated as a Nash equilibrium consisting of strategies of choosing a Pareto inefficient transition path of consumption to the steady state.

The model presented in this paper is superior to the hybrid NKPC because it is constructed on the basis of purely forward-looking micro-foundations of both trend inflation and friction, and it is superior to the pure NKPC in the sense that it can be empirically supported. Comparisons between the new model and both types of NKPC indicate that the role of frictions has been overestimated and that frictions are less important than previously thought. Even though some amount of frictions may actually exist, their importance should not be exaggerated.

The model also indicates that the conventional monetary policy of utilizing frictions cannot necessarily stabilize inflation. In contrast, the monetary policy of controlling \( \theta_G \) is very effective. A problem is that the effects of both types of monetary policy are not distinguishable. This indistinguishable nature results in the incorrect belief that the monetary policy of utilizing frictions (conventional monetary policy) is very effective for inflation stabilization even when \( \theta_P \) shifts.
References


Slump in Japan,” EconWPA Working Papers, ewp-mac0402015.
Figure 1: A time preference shock
Figure 2: The paths of Jalone and NJalone
Figure 3: A Pareto inefficient transition path

Path of $NJ_{together}$

Posterior Pareto efficient saddle path
Table 1  The payoff matrix

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<th>Any other household</th>
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<td>NJ</td>
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<td></td>
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<td>$E_0(NJ_{alone}), E_0(J_{together})$</td>
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