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8 November 2013

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November 2013

Abstract

The hybrid New Keynesian Phillips curve has been criticized for lacking a micro-foundation. In this paper, an alternative purely forward-looking model of the Phillips curve is constructed on the basis of a micro-foundation of trend inflation. In addition, another source of output gaps other than frictions—a Nash equilibrium of a Pareto inefficient path—is considered. The model indicates that the role of frictions has been overestimated and that frictions are less important than previously have been thought. The conventional monetary policy of utilizing frictions cannot necessarily stabilize inflation. In contrast, the monetary policy of controlling the government's preference is very effective. A problem is that the effects of both types of monetary policy are not distinguishable.

JEL Classification code: E31, E58, E63
Keywords: Trend inflation; inflation persistence; central bank independence; The New Keynesian Phillips curve; the fiscal theory of the price level

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1 INTRODUCTION

The pure New Keynesian Phillips curve (NKPC) has been criticized for possessing the serious problem that it is not consistent with the observed highly persistent nature of inflation (e.g., Fuhrer and Moore, 1995; Galí and Gertler, 1999). Mankiw (2001) argues that the NKPC is ultimately a failure and is not consistent with standard stylized facts about the dynamic effects of monetary policy. Since the work of Galí and Gertler (1999), a modified version of the NKPC—that is, a hybrid NKPC that includes lagged inflation—has been intensely studied. The hybrid NKPC well captures the persistent nature of inflation, but it remains puzzling why rational agents would behave in backward-looking manners, even if only partially so. Galí et al. (2005) argue that an important unresolved issue is the provision of a more coherent rationale for the role of lagged inflation in the hybrid NKPC. Furthermore, Fuhrer (2006) concluded that inflation in the hybrid NKPC inherits relatively little persistence from the driving process and that a micro-founded mechanism that generates substantial intrinsic persistence in inflation is required.

Recently, an alternative approach has been presented that argues that high intrinsic inflation persistence is spurious as a result of trend inflation. Cogley and Sbordone (2005, 2006) show that, if trend inflation is incorporated into the pure NKPC, its performance on fitting actual inflation data improves greatly. They conclude that trend inflation has been historically quite volatile and that, if these fluctuations of long-run moving trend inflation are taken into account, a purely forward-looking model approximates the short-run dynamics of inflation quite well. Woodford (2007) considers that Cogley and Sbordone (2005) present an alternative interpretation of the apparent need for lagged inflation terms in the NKPC (see also Hornstein, 2007). Indeed, data on inflation in most industrial economies show high levels of volatility and a transition from high inflation to low inflation in the 1980s, which strongly implies the existence of trends in inflation (e.g., Stock and Watson, 2006; Sbordone, 2007). Ascari (2004) argues that disregarding trend inflation is very far from being an innocuous assumption and that the results obtained by models log-linearized around a zero inflation steady state are misleading (see also Bakhshi et al., 2003). These studies suggest that the puzzle of inflation persistence in the NKPC will be solved by incorporating trend inflation into the NKPC. However, if we proceed further in this research direction, another serious theoretical problem arises, that is, the lack of a micro-foundation of trend inflation. Can trend inflation be explained as a consequence of rational agents' optimizations? Why do monetary policymakers often allow upward trends in inflation? This paper presents a micro-foundation of trend inflation.

The fiscal theory of the price level (FTPL) argues that a problem with conventional inflation theory is that it largely neglects the importance of the government's borrowing behavior in inflation dynamics (e.g., Leeper, 1991; Sims, 1994, 1998, 2001; Woodford, 1995, 2001; Cochrane, 1998a, 1998b, 2005). The FTPL implies that, if a government's borrowing behavior is well modeled, the mechanism of severely deviated inflation paths can be explained without assuming ad hoc frictions or irrationality. In this paper, this possibility is explored and a model of trend inflation that is firmly based on a micro-foundation is constructed (see Harashima, 2008b). The model indicates that trend inflation accelerates or decelerates if the time preference rates of the government and the representative household are heterogeneous.

Another important factor in the Phillips curve that should also be carefully examined is the nature of output gaps. In the NKPC, output gaps are assumed to be generated only by frictions. Without frictions, no output gaps can exist because, if an economy is under full price flexibility, its equilibrium output level is always sustained. However, this New Keynesian explanation has not generally been regarded as sufficiently successful, because price rigidity has been criticized for its fragile theoretical (micro-) foundation and its inability to explain the persistent nature of inflation. As shown above, Mankiw (2001) severely criticized the NKPC. This criticism implies that there will be other sources of output gaps. In this paper, I consider another source of output gaps that are generated even under full price flexibility (see Harashima, 2012, 2013a, 2013b). Rational agents will usually not allow Pareto inefficiency (e.g., output gaps) to remain for a long period; it will disappear soon after it is generated under full price flexibility. However, an exception is possible because a Nash equilibrium can conceptually coexist with Pareto inefficiency. If a Nash equilibrium that consists of strategies that generate Pareto inefficient payoffs is rationally selected, rigidity-like phenomena may be observed. This paper shows that a Nash equilibrium consisting of strategies of choosing a Pareto inefficient transition path of consumption to the steady state (hereafter called a "Nash equilibrium of a Pareto inefficient path") is generated even in a frictionless economy if—and probably only if—the rate of time preference shifts. An essential reason for the generation of this path is that households are intrinsically risk averse and not cooperative. In a strategic environment, this generates the possibility that, if consumption needs to be substantially and discontinuously increased to keep Pareto optimality, a non-cooperative household's strategy to deviate from the Pareto optimal path gives a higher expected utility than the strategy of choosing the Pareto optimal path.

The above-mentioned two factors (a model of trend inflation and a mechanism of output gaps under full price flexibility) are considered in analyses of monetary policies, and an alternative model of the Phillips curve is constructed. In contrast to the NKPC, both factors are fully based on micro-foundations. Comparisons between this new model and the NKPC indicate that the role of frictions has been overestimated and that frictions are less important than has been thought.

The paper is organized as follows. In Section 2, I construct a model of trend inflation that assumes an economically Leviathan government, in which the government and the representative household behave in purely forward-looking manners and achieve simultaneous optimization. Section 3 shows that a Nash equilibrium of a Pareto inefficient path is rationally generated when the time preference rates of risk-averse and non-cooperative households shift. In Section 4, a new model of the Phillips curve is constructed and compared with the NKPC. Finally, I offer concluding remarks in Section 5.

2 TREND INFLATION

2.1 The model of trend inflation[1]

2.1.1 The government

2.1.1.1 The government budget constraint

The government budget constraint is

$$\dot{B}_t = B_t i_t + G_t - X_t - \vartheta_t \quad ,$$

where B_t is the nominal obligation of the government to pay for its accumulated bonds, i_t is the nominal interest rate for government bonds, G_t is the nominal government expenditure, X_t is the nominal tax revenue, and ϑ_t is the nominal amount of seigniorage at time t. The tax is assumed to be lump sum, the government bonds are long term, and the returns on the bonds are realized only after the bonds are held during a unit period (e.g., a year). The government bonds are redeemed in a unit period, and the government successively refinances the bonds by issuing new ones at each time t. Let $b_t = \frac{B_t}{P_t}$, $g_t = \frac{G_t}{P_t}$, $x_t = \frac{X_t}{P_t}$, and $\varphi_t = \frac{\vartheta_t}{P_t}$, where P_t is the price level

[1] The model of the optimal trend inflation in this paper is based on the inflation model in Harashima (2007). Harashima (2004b, 2008a, 2013a) are also related to the model and analyses in this paper..

at time t. Let also $\pi_t = \frac{\dot{P}_t}{P_t}$ be the inflation rate at time t. By dividing by P_t, the budget constraint is transformed to $\frac{\dot{B}_t}{P_t} = b_t i_t + g_t - x_t - \varphi_t$, which is equivalent to

$$\dot{b}_t = b_t i_t + g_t - x_t - \varphi_t - b_t \pi_t = b_t (i_t - \pi_t) + g_t - x_t - \varphi_t \quad . \tag{1}$$

Because the returns on government bonds are realized only after holding the bonds during a unit period, investors buy the bonds if $\bar{i}_t \geq E_t \int_t^{t+1} (\pi_s + r_s) ds$ at time t, where $\bar{i}_t$ is the nominal interest rate for bonds bought at t and r_t is the real interest rate in markets at t. Hence, by arbitrage, $\bar{i}_t = E_t \int_t^{t+1} (\pi_s + r_s) ds$ and if r_t is constant such that $r_t = r$ (i.e., if it is at steady state), then

$$\bar{i}_t = E_t \int_t^{t+1} \pi_s \, ds + r \quad .$$

The nominal interest rate $\bar{i}_t = E_t \int_t^{t+1} \pi_s \, ds + r$ means that, during a sufficiently small period between t and $t + dt$, the government's obligation to pay for the bonds' return in the future increases not by $dt\,(\pi_t + r)$ but by $dt\left(E_t \int_t^{t+1} \pi_s \, ds + r \right)$. If π_t is constant, then $E_t \int_t^{t+1} \pi_s \, ds = \pi_t$ and $\bar{i}_t = \pi_t + r$, but if π_t is not constant, these equations do not necessarily hold.

Since bonds are redeemed in a unit period and successively refinanced, the bonds the government is holding at t have been issued between t - 1 and t. Hence, under perfect foresight, the average nominal interest rate for all government bonds at time t is the weighted sum of $\bar{i}_t$ such that

$$i_t = \int_{t-1}^{t} \bar{i}_s \left(\frac{\overline{B}_{s,t}}{\int_{t-1}^{t} \overline{B}_{v,t} \, dv} \right) ds = \int_{t-1}^{t} \int_{s}^{s+1} \pi_v \, dv \left(\frac{\overline{B}_{s,t}}{\int_{t-1}^{t} \overline{B}_{v,t} \, dv} \right) ds + r \quad ,$$

where $\overline{B}_{s,t}$ is the nominal value of bonds at time t that were issued at time s. If the weights $\frac{\overline{B}_{s,t}}{\int_{t-1}^{t} \overline{B}_{v,t} \, dv}$ between t - 1 and t are not so different from each other, then approximately $i_t = \int_{t-1}^{t} \int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds + r$. To be precise, if the absolute values of π_s for $t-1 < s \leq t+1$ are sufficiently smaller than unity, the differences among the weights are negligible and then approximately

$$i_t = \int_{t-1}^{t} \int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds + r \tag{2}$$

(see Harashima, 2008). The average nominal interest rate for the total government bonds, therefore, develops by $i_t = \int_{t-1}^{t} \int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds + r$. If π_t is constant, then $\int_{t-1}^{t} \int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds$ $= \pi_t$; thus, $i_t = \pi_t + r$. If π_t is not constant, however, the equations $\int_{t-1}^{t} \int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds = \pi_t$ and $i_t = \pi_t + r$ do not necessarily hold.

2.1.1.2 An economically Leviathan government

Under a proportional representation system, the government represents the median household whereas the representative household from an economic perspective represents the mean household.[2] Because of this difference, they usually have different preferences. To account for this essential difference, a Leviathan government is assumed in the model.[3] There are two extremely different views regarding government's behavior in the literature on political economy: the Leviathan view and the benevolent view (e.g., Downs 1957; Brennan and Buchanan 1980; Alesina and Cukierman 1990). From an economic point of view, a benevolent government maximizes the expected economic utility of the representative household, but a Leviathan government does not. Whereas the expenditure of a benevolent government is a tool used to maximize the economic utility of the representative household, the expenditure of a Leviathan government is a tool used to achieve the government's own policy objectives.[4] For example, if a Leviathan government considers national security to be the most important political issue, defense spending will increase greatly, but if improving social welfare is the top political priority, spending on social welfare will increase dramatically, even though the increased expenditures may not necessarily increase the economic utility of the representative household.

Is it possible, however, for such a Leviathan government to hold office for a long period? Yes, because a government is generally chosen by the median of households under a proportional representation system (e.g., Downs 1957), whereas the representative household usually presumed in the economics literature is the mean household. The economically representative household is not usually identical to the politically representative household, and a majority of people could support a Leviathan government even if they know that the government does not necessarily pursue only the economic objectives of the economically representative household. In other words, the Leviathan government argued here is an economically Leviathan government that maximizes the political utility of people, whereas the conventional economically benevolent government maximizes the economic utility of people. In addition, because the politically and economically representative households are different (the median and mean households, respectively), the preferences of future governments will also be similarly different from those of the mean representative household. In this sense, the current and future governments presented in the model can be seen as a combined government that goes on indefinitely; that is, the economically Leviathan government always represents the median representative household.

The Leviathan view generally requires the explicit inclusion of government expenditure, tax revenue, or related activities in the government's political utility function (e.g., Edwards and Keen 1996). Because an economically Leviathan government derives political utility from expenditure for its political purposes, the larger the expenditure is, the happier the Leviathan government will be. But raising tax rates will provoke people's antipathy, which

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[2] See the literature on the median voter theorem (e.g., Downs 1957). Also see the literature on the delay in reforms (e.g., Alesina and Drazen 1991).

[3] The most prominent reference to Leviathan governments is Brennan and Buchanan (1980).

[4] The government behavior assumed in the fiscal theory of the price level reflects an aspect of a Leviathan government. Christiano and Fitzgerald (2000) argue that non-Ricardian policies correspond to the type of policies in which governments are viewed as selecting policies and committing themselves to those policies in advance of prices being determined in markets.

increases the probability of being replaced by the opposing party that also nearly represents the median household. Thus, the economically Leviathan government regards taxes as necessary costs to obtain freedom of expenditure for its own purposes. The government therefore will derive utility from expenditure and disutility from taxes. Expenditure and taxes in the political utility function of the government are analogous to consumption and labor hours in the economic utility function of the representative household. Consumption and labor hours are both control variables, and as such, the government's expenditure and tax revenue are also control variables. As a whole, the political utility function of economically Leviathan government can be expressed as $u_G(g_t, x_t)$.[5] In addition, it can be assumed on the basis of previously mentioned arguments that $\frac{\partial u_G}{\partial g_t} > 0$ and $\frac{\partial^2 u_G}{\partial g_t^2} < 0$, and therefore that $\frac{\partial u_G}{\partial x_t} < 0$ and $\frac{\partial^2 u_G}{\partial x_t^2} > 0$.[6] An economically Leviathan government therefore maximizes the expected sum of these utilities discounted by its time preference rate under the constraint of deficit financing.

2.1.1.3 The optimization problem

The optimization problem of an economically Leviathan government is

$$Max\ E\int_0^\infty u_G(g_t, x_t)\exp(-\theta_G t)dt$$

subject to the budget constraint

$$\dot{b}_t = b_t(i_t - \pi_t) + g_t - x_t - \varphi_t \quad , \tag{3}$$

where u_G is the constant relative risk aversion utility function of the government, θ_G is the government's rate of time preference, and E is the expectation operator. All variables are expressed in per capita terms, and population is assumed to be constant. The government maximizes its expected political utility considering the behavior of the economically representative household that is reflected in i_t in its budget constraint.

2.1.2 Households

The economically representative household maximizes its expected economic utility. Sidrauski (1967)'s well-known money in the utility function model is used for the optimization problem. The representative household maximizes its expected utility

$$E\int_0^\infty u_P(c_t, m_t)\exp(-\theta_P t)dt$$

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[5] It is possible to assume that governments are partially benevolent. In this case, the utility function of a government can be assumed to be $u_G(g_t, x_t, c_t, l_t)$, where c_t is real consumption and l_t is the leisure hours of the representative household. However, if a lump-sum tax is imposed, the government's policies do not affect steady-state consumption and leisure hours. In this case, the utility function can be assumed to be $u_G(g_t, x_t)$.

[6] Some may argue that it is more likely that $\frac{\partial u_G}{\partial x_t} > 0$ and $\frac{\partial^2 u_G}{\partial x_t^2} < 0$. However, the assumption used is not an important issue here because $-x_t\left[\frac{\partial u_G(g_t, x_t)}{\partial x_t}\right]^{-1}\frac{\partial^2 u_G(g_t, x_t)}{\partial x_t^2}\frac{\dot{x}_t}{x_t} = 0$ at steady state, as will be shown in the solution to the optimization problem later in the paper. Thus, the results are not affected by which assumption is used.

subject to the budget constraint

$$\dot{a}_t = (r_t\, a_t + w_t + \sigma_t) - [c_t + (\pi_t + r_t) m_t] - g_t \ ,$$

where u_P and θ_P are the utility function and the time preference rate of the representative household, c_t is real consumption, w_t is real wage, σ_t is lump-sum real government transfers, m_t is real money, $a_t = k_t + m_t$, and k_t is real capital. It is assumed that $r_t = f'(k_t)$, $w_t = f(k_t) - k_t f'(k_t)$, $u_P' > 0$, $u_P'' < 0$, $\frac{\partial u_P(c_t, m_t)}{\partial m_t} > 0$, and $\frac{\partial^2 u_P(c_t, m_t)}{\partial m_t^2} < 0$, where $f(\cdot)$ is the production function. Government expenditure (g_t) is an exogenous variable for the representative household because it is an economically Leviathan government. It is also assumed that, although all households receive transfers from a government in equilibrium, when making decisions, each household takes the amount it receives as given, independent of its money holdings. Thus, the budget constraint means that the real output $f(k_t)$ at any time is demanded for the real consumption c_t, the real investment $\dot{k}_t$, and the real government expenditure g_t such that $f(k_t) = c_t + \dot{k}_t + g_t$. The representative household maximizes its expected economic utility considering the behavior of government reflected in g_t in the budget constraint. In this discussion, a central bank is not assumed to be independent of the government; thus, the functions of the government and the central bank are not separated. This assumption can be relaxed, and the roles of the government and the central bank are explicitly separated in Section 2.2.

Note that the time preference rate of government (θ_G) is not necessarily identical to that of the representative household (θ_P) because the government and the representative household represent different households (i.e., the median and mean households, respectively). In addition, the preferences will differ because (1) even though people want to choose a government that has the same time preference rate as the representative household, the rates may differ owing to errors in expectations (e.g., Alesina and Cukierman 1990); and (2) current voters cannot bind the choices of future voters and, if current voters are aware of this possibility, they may vote more myopically as compared with their own rates of impatience in private economic activities (e.g., Tabellini and Alesina 1990). Hence, it is highly likely that the time preference rates of a government and the representative household are heterogeneous. It should be also noted, however, that even though the rates of time preference are heterogeneous, an economically Leviathan government behaves based only on its own time preference rate, without hesitation.

2.1.3 The simultaneous optimization

First, I examine the optimization problem of the representative household. Let Hamiltonian H_P be $H_P = u_P(c_t, m_t)\exp(-\theta_P\, t) + \lambda_{P,t}[r_t a_t + w_t + \sigma_t - c_t - (\pi_t + r_t) m_t - g_t]$, where $\lambda_{P,t}$ is a costate variable, c_t and m_t are control variables, and a_t is a state variable. The optimality conditions for the representative household are;

$$\frac{\partial u_P(c_t, m_t)}{\partial c_t} \exp(-\theta_P\, t) = \lambda_{P,t} \ , \tag{4}$$

$$\frac{\partial u_P(c_t, m_t)}{\partial m_t} \exp(-\theta_P\, t) = \lambda_{P,t}(\pi_t + r_t) \ , \tag{5}$$

$$\dot{\lambda}_{P,t} = -\lambda_{P,t}\, r_t \quad , \tag{6}$$

$$\dot{a}_t = (ra_t + w_t + \sigma_t) - [c_t + (\pi_t + r_t)m_t - g_t] \quad , \tag{7}$$

$$\lim_{t\to\infty} \lambda_{P,t}\, a_t = 0 \quad . \tag{8}$$

By conditions (4) and (5), $\left[\frac{\partial u_P(c_t, m_t)}{\partial c_t}\right]^{-1} \frac{\partial u_P(c_t, m_t)}{\partial m_t} = \pi_t + r_t$, and by conditions (4) and (6),

$$-c_t\left[\frac{\partial u_P(c_t, m_t)}{\partial c_t}\right]^{-1} \frac{\partial^2 u_P(c_t, m_t)}{\partial c_t^2}\frac{\dot{c}_t}{c_t} + \theta_P = r_t \quad . \tag{9}$$

Hence,

$$\theta_P = r_t = r \tag{10}$$

at steady state such that $\dot{c}_t = 0$ and $\dot{k}_t = 0$.

Next, I examine the optimization problem of the economically Leviathan government. Let Hamiltonian H_G be $H_G = u_G(g_t, x_t)\exp(-\theta_G t) + \lambda_{G,t}[b_t(i_t - \pi_t) + g_t - x_t - \varphi_t]$, where $\lambda_{G,t}$ is a costate variable. The optimality conditions for the government are;

$$\frac{\partial u_G(g_t, x_t)}{\partial g_t}\exp(-\theta_G t) = -\lambda_{G,t} \quad , \tag{11}$$

$$\frac{\partial u_G(g_t, x_t)}{\partial x_t}\exp(-\theta_G t) = \lambda_{G,t} \quad , \tag{12}$$

$$\dot{\lambda}_{G,t} = -\lambda_{G,t}(i_t - \pi_t) \quad , \tag{13}$$

$$\dot{b}_t = b_t(i_t - \pi_t) + g_t - x_t - \varphi_t \quad , \tag{14}$$

$$\lim_{t\to\infty} \lambda_{G,t}\, b_t = 0 \quad . \tag{15}$$

Combining conditions (11), (12), and (13) and equation (2) yields the following equations:

$$-g_t\left[\frac{\partial u_G(g_t, x_t)}{\partial g_t}\right]^{-1} \frac{\partial^2 u_G(g_t, x_t)}{\partial g_t^2}\frac{\dot{g}_t}{g_t} + \theta_G = i_t - \pi_t = r_t + \int_{t-1}^{t}\int_{s}^{s+1} \pi_\upsilon\, d\upsilon\; ds - \pi_t \tag{16}$$

and

$$-x_t\left[\frac{\partial u_G(g_t, x_t)}{\partial x_t}\right]^{-1} \frac{\partial^2 u_G(g_t, x_t)}{\partial x_t^2}\frac{\dot{x}_t}{x_t} + \theta_G = i_t - \pi_t = r_t + \int_{t-1}^{t}\int_{s}^{s+1} \pi_\upsilon\, d\upsilon\; ds - \pi_t \quad . \tag{17}$$

Here, $g_t \left[\frac{\partial u_G(g_t, x_t)}{\partial g_t} \right]^{-1} \frac{\partial^2 u_G(g_t, x_t)}{\partial g_t^2} \frac{\dot{g}_t}{g_t} = 0$ and $x_t \left[\frac{\partial u_G(g_t, x_t)}{\partial x_t} \right]^{-1} \frac{\partial^2 u_G(g_t, x_t)}{\partial x_t^2} \frac{\dot{x}_t}{x_t} = 0$ at steady state such that $\dot{g}_t = 0$ and $\dot{x}_t = 0$; thus,

$$\theta_G = r_t + \int_{t-1}^{t} \int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds - \pi_t \quad . \tag{18}$$

Hence, by equation (10),

$$\int_{t-1}^{t} \int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds = \pi_t + \theta_G - \theta_P \tag{19}$$

at steady state such that $\dot{g}_t = 0$, $\dot{x}_t = 0$, $\dot{c}_t = 0$, and $\dot{k}_t = 0$.[7]

Equation (19) is a natural consequence of simultaneous optimization by the economically Leviathan government and the representative household. If the rates of time preference are heterogeneous between them, then

$$i_t - r = \int_{t-1}^{t} \int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds \neq \pi_t \quad .$$

This result might seem surprising because it has been naturally conjectured that $i_t = \pi_t + r$. However, this is a simple misunderstanding because π_t indicates the instantaneous rate of inflation at a point such that $\pi_t = \frac{\dot{P}_t}{P_t}$, whereas $\int_{t-1}^{t} \int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds$ roughly indicates the average inflation rate in a period. Equation (19) indicates that π_t develops according to the integral equation $\pi_t = \int_{t-1}^{t} \int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds - \theta_G + \theta_P$. If π_t is constant, the equations $i_t = \pi_t + r$ and $\int_{t-1}^{t} \int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds = \pi_t$ are true. However, if π_t is not constant, the equations do not necessarily hold. Equation (19) indicates that the equations $i_t = \pi_t + r$ and $\int_{t-1}^{t} \int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds = \pi_t$ hold only in the case where $\theta_G = \theta_P$ (i.e., a homogeneous rate of time preference). It has been previously thought that a homogeneous rate of time preference naturally prevails; thus, the equation $i_t = \pi_t + r$ has not been questioned. As argued previously, however, a homogeneous rate of time preference is not usually guaranteed.

2.1.4 The law of motion for trend inflation

Equation (19) indicates that inflation accelerates or decelerates as a result of the government and the representative household reconciling the contradiction in heterogeneous rates of time preference. If π_t is constant, the equation $\pi_t = \int_{t-1}^{t} \int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds$ holds; conversely, if $\pi_t \neq \int_{t-1}^{t} \int_{s}^{s+1} \pi_\upsilon \, d\upsilon \; ds$, then π_t is not constant. Without the acceleration or deceleration of inflation,

[7] If and only if $\theta_G = -\frac{g_t - x_t - \varphi_t}{b_t}$ at steady state, then the transversality condition (15) $\lim_{t \to \infty} \lambda_{G,t} \, b_t = 0$ holds. The proof is shown in Harashima (2008b).

therefore, equation (19) cannot hold in an economy in which $\theta_G \neq \theta_P$. In other words, it is not until $\theta_G \neq \theta_P$ that inflation can accelerate or decelerate. Heterogeneous time preferences ($\theta_G \neq \theta_P$) bend the path of inflation and enables inflation to accelerate or decelerate. The difference of time preference rates ($\theta_G - \theta_P$) at each time needs to be transformed to the accelerated or decelerated inflation rate π_t at each time.

Equation (19) implies that inflation accelerates or decelerates nonlinearly in the case in which $\theta_G \neq \theta_P$. For a sufficiently small period dt, π_{t+1+dt} is determined with π_s $(t-1 < s \leq t+1)$ that satisfies $\int_{t-1}^{t}\int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds - \pi_t = \theta_G - \theta_P$, so as to hold the equation $\int_{t}^{t+dt}\int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds = \int_{t-1}^{t-1+dt}\int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds + \pi_{t+dt} - \pi_t$. A solution of the integral equation (19) for given θ_G and θ_P is

$$\pi_t = \pi_0 + 6(\theta_G - \theta_P)t^2 \quad . \tag{20}$$

Generally, the path of inflation that satisfies equation (19) for $0 \leq t$ is expressed as

$$\pi_t = \pi_0 + 6(\theta_G - \theta_P)\exp\left[z_t \ln(t)\right] \quad , \tag{21}$$

where z_t is a time dependent variable. The stream of z_t is various depending on the boundary condition, i.e., the past and present inflation during $-1 < t \leq 0$ and the path of inflation during $0 < t \leq 1$ that is set to make π_0 satisfy equation (19). However, z_t has the following important property. If π_t satisfies equation (19) for $0 \leq t$, and $-\infty < \pi_t < \infty$ for $-1 < t \leq 1$, then

$$\lim_{t \to \infty} z_t = 2 \quad .$$

Proof is shown in Harashima (2008b). Any inflation path that satisfies equation (19) for $0 \leq t$ therefore asymptotically approaches the path of equation (20). The mechanism behind the law of motion for inflation (equation [20]) is examined more in detail in Harashima (2008b).

2.1.5 The optimal trend inflation

The trend inflation should be consistent with equation (21). The discrete-time version of equation (21) is

$$\pi_t^T = \pi_\varphi^T + 6(\theta_G - \theta_P)\exp\left[z_t \ln(t - \varpi)\right] \tag{22}$$

and equivalently

$$\pi_{t+1}^T = \pi_t^T + 6(\theta_G - \theta_P)\left\{\exp\left[z_{t+1} \ln(t - \varpi + 1)\right] - \exp\left[z_t \ln(t - \varpi)\right]\right\} \tag{23}$$

where π_t^T is the trend component in inflation in period t, and $\varpi \; (\leq t)$ is the period when the latest shock on θ_G occurred. It will be explained later in Section 2.2 that θ_G should be time-variable and shocks on θ_G play an important role in inflation dynamics. When a shock on θ_G occurs and the value of θ_G is changed in period ϖ, the trend inflation need be adjusted to be consistent with the new value of θ_G for the new initial period ϖ. The value of z_t is determined by the mechanism explained in Section 2.1.2. Equations (22) and (23) are used in the model as the trend component in inflation.

2.2 The central bank

In Section 2.1, central banks are not explicitly considered because they are not assumed to be independent of governments. However, in actuality, central banks are independent organizations in most countries even though some of them are not sufficiently independent. Furthermore, in the conventional inflation model, it is the central banks that control inflation and governments have no role in controlling inflation. Conventional inflation models show that the rate of inflation basically converges at the target rate of inflation set by a central bank. The target rate of inflation therefore is the key exogenous variable that determines the path of inflation in these models.

Both the government and the central bank can probably affect the development of inflation, but they would do so in different manners, as equation (21) and conventional inflation models indicate. However, the objectives of the government and the central bank may not be the same. For example, if trend inflation is added to conventional models by replacing their aggregate supply equations with equation (21), inflation cannot necessarily converge at the target rate of inflation because another key exogenous variable (θ_G) is included in the models. A government makes inflation develop consistently with the equation (21), which implies that inflation will not necessarily converge at the target rate of inflation. Conversely, a central bank makes inflation converge at the target rate of inflation, which implies that inflation will not necessarily develop consistently with equation (21). That is, unless either θ_G is adjusted to be consistent with the target rate of inflation or the target rate of inflation is adjusted to be consistent with θ_G, the path of inflation cannot necessarily be determined. Either θ_G or the target rate of inflation need be an endogenous variable. If a central bank dominates, the target rate of inflation remains as the key exogenous variable and θ_G should then be an endogenous variable. The reverse is also true.

A central bank will be regarded as truly independent if θ_G is forced to be adjusted to the one that is consistent with the target rate of inflation set by the central bank. For example, suppose that $\theta_G > \theta_P$ and a truly independent central bank manipulates the nominal interest rate. Here,

$$i_t = \int_{t-1}^{t} \int_{s}^{s+1} \pi_v \, dv \, ds + r = \theta_G + \pi_t \tag{24}$$

at steady state such that $\dot{g}_t = 0$, $\dot{x}_t = 0$, $\dot{c}_t = 0$, and $\dot{k}_t = 0$ by equations (1), (7), and (13). If the accelerating inflation rate is higher than the target rate of inflation, the central bank can raise the nominal interest rate from $i_t = \theta_G + \pi_t$ (equation (24)) to

$$i_t = \theta_G + \pi_t + \psi$$

by positive ψ by intervening in financial markets to lower the accelerating rate of inflation. In this case, the central bank keeps the initial target rate of inflation because it is truly independent. The government thus faces a rate of increase of real obligation that is higher than θ_G by the extra rate ψ.[8] If the government lowers θ_G so that $\theta_G < \theta_P$ and inflation stops accelerating,

[8] The extra rate ψ affects not only the behavior of government but also that of the representative household, in which

the central bank will accordingly reduce the extra rate ψ. If, however, the government does not accommodate θ_G to the target rate of inflation, the extra rate ψ will increase as time passes because of the gap between the accelerating inflation rate and the target rate of inflation widens. Because of the extra rate ψ, the government has no other way to achieve optimization unless it lowers θ_G to one that is consistent with the target rate of inflation. Once the government recognizes that the central bank is firmly determined to be independent and it is in vain to try to intervene in the central bank's decision makings, the government would not dare to attempt to raise θ_G again anymore.

Equation (22) implies that a government allows inflation to accelerate because it acts to maximize its expected utility based only on its own preferences. A government is hardly the only entity that cannot easily control its own preferences even when these preferences may result in unfavorable consequences. It may not even be possible to manipulate one's own preferences at will. Thus, even though a government is fully rational and is not weak, foolish, or untruthful, it is difficult for it to self-regulate its preferences. Hence, an independent neutral organization is needed to help control θ_G. Delegating the authority to set and keep the target rate of inflation to an independent central bank is a way to control θ_G. The delegated independent central bank will control θ_G because it is not the central bank's preference to stabilize the price level—it is simply a duty delegated to it. An independent central bank is not the only possible choice. For example, pegging the local currency with a foreign currency can be seen as a kind of delegation to an independent neutral organization. In addition, the gold standard that prevailed before World War II can be also seen as a type of such delegation.

Note also that the delegation may not be viewed as bad from the Leviathan government's point of view because only its rate of time preference is changed, and the government can still pursue its political objectives. One criticism of the argument that central banks should be independent (e.g., Blinder 1998) is that, since the time-inconsistency problem argued in Kydland and Prescott (1977) or Barro and Gordon (1983) is more acute with fiscal policy, why is it not also necessary to delegate fiscal policies? An economically Leviathan government, however, will never allow fiscal policies to be delegated to an independent neutral organization because the Leviathan government would then not be able to pursue its political objectives, which in a sense would mean the death of the Leviathan government. The median household that backs the Leviathan government, but at the same time dislikes high inflation, will therefore support the delegation of authority but only if it concerns monetary policy. The independent central bank will then be given the authority to control θ_G and oblige the government to change θ_G in order to meet the target rate of inflation.

Without such a delegation of authority, it is likely that generally $\theta_G > \theta_P$ because θ_G represents the median household whereas θ_P represents the mean household. Empirical studies indicate that the rate of time preference negatively correlates with permanent income (e.g., Lawrance 1991), and the permanent income of the median household is usually lower than that of the mean household. If generally $\theta_G > \theta_P$, that suggests that inflation will tend to accelerate unless a central bank is independent. The independence of the central bank is therefore very important in keeping the path of inflation stable.

Note also that the forced adjustments of θ_G by an independent central bank are exogenous shocks to both the government and the representative household because they are

the conventional inflation theory is particularly interested. In this sense, the central bank's instrument rule that concerns and simultaneously affects both behaviors of the government and the representative household is particularly important for price stability.

planned solely by the central bank. When a shock on θ_G is given, the government and the representative household must recalculate their optimal paths including the path of inflation by resetting θ_G, π_t, and φ in equation (22).

3 OUTPUT GAPS

3.1 Model with non-cooperative households [9]

This section examines another source of output gaps other than frictions. A Ramsey type growth model with non-cooperative households is constructed to examine economic fluctuations.

3.1.1 The shock

The model describes the utility maximization of households after an upward time preference shock. This shock was chosen because it is one of the few shocks that result in a Nash equilibrium of a Pareto inefficient path. Another important reason for selecting an upward time preference shock is that it shifts the steady state to lower levels of production and consumption than before the shock, which is consistent with the phenomena actually observed in a recession.

Although the rate of time preference is a deep parameter, it has not been regarded as a source of shocks for economic fluctuations, possibly because the rate of time preference is thought to be constant and not to shift suddenly. There is also a practical reason, however. Models with a permanently constant rate of time preference exhibit excellent tractability (see Samuelson, 1937). However, the rate of time preference has been naturally assumed and actually observed to be time-variable. The concept of a time-varying rate of time preference has a long history (e.g., Böhm-Bawerk, 1889; Fisher, 1930). More recently, Lawrance (1991) and Becker and Mulligan (1997) showed that people do not inherit permanently constant rates of time preference by nature and that economic and social factors affect the formation of time preference rates. Their arguments indicate that many incidents can affect and change the rate of time preference throughout a person's life. For example, Parkin (1988) examined business cycles in the United States, explicitly considering the time-variability of the time preference rate, and showed that the rate of time preference was as volatile as technology and leisure preference.

3.1.2 Households

Households are not intrinsically cooperative. Except in a strict communist economy, households do not coordinate themselves to behave as a single entity when consuming goods and services. The model in this paper assumes non-cooperative, identical, and infinitely long living households and that the number of households is sufficiently large. Each of them equally maximizes the expected utility

$$E_0 \int_0^\infty \exp(-\theta t) u(c_t) dt \quad ,$$

subject to

$$\frac{dk_t}{dt} = f(A, k_t) - \delta k_t - c_t \quad ,$$

where y_t, c_t, and k_t are production, consumption, and capital per capita in period t, respectively;

[9] The model in Section 3 is based on the model by Harashima (2012). See also Harashima (2004a, 2013b, 2013c).

A is technology and constant; u is the utility function; $y_t = f(A, k_t)$ is the production function; $\theta(>0)$ is the rate of time preference; δ is the rate of depreciation; and E_0 is the expectations operator conditioned on the agents' period 0 information set. y_t, c_t, and k_t are monotonously continuous and differentiable in t, and u and f are monotonously continuous functions of c_t and k_t, respectively. All households initially have an identical amount of financial assets equal to k_t, and all households gain the identical amount of income $y_t = f(A, k_t)$ in each period. It is assumed that $\frac{du(c_t)}{dc_t} > 0$ and $\frac{d^2u(c_t)}{dc_t^2} < 0$; thus, households are risk averse. For simplicity, the utility function is specified to be the constant relative risk aversion utility function

$$u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma} \qquad \text{if } \gamma \neq 1$$
$$u(c_t) = \ln(c_t) \qquad \text{if } \gamma = 1 \ ,$$

where γ is a constant and $0 < \gamma < \infty$. In addition, $\frac{\partial f(A, k_t)}{\partial k_t} > 0$ and $\frac{\partial^2 f(k_t)}{\partial k_t^2} < 0$. Both technology ($A$) and labor supply are assumed to be constant.

The effects of an upward shift in time preference are shown in Figure 1. Suppose first that the economy is at steady state before the shock. After the upward time preference shock, the vertical line $\frac{dc_t}{dt} = 0$ moves to the left (from the solid vertical line to the dashed vertical line in Fig. 1). To keep Pareto efficiency, consumption needs to jump immediately from the steady state before the shock (the prior steady state) to point Z. After the jump, consumption proceeds on the Pareto efficient saddle path after the shock (the posterior Pareto efficient saddle path) from point Z to the lower steady state after the shock (the posterior steady state). Nevertheless, this discontinuous jump to Z may be uncomfortable for risk-averse households that wish to smooth consumption and not to experience substantial fluctuations. Households may instead take a shortcut and, for example, proceed on a path on which consumption is reduced continuously from the prior steady state to the posterior steady state (the bold dashed line in Fig. 1), but this shortcut is not Pareto efficient.

Choosing a Pareto inefficient consumption path must be consistent with each household's maximization of its expected utility. To examine the possibility of the rational choice of a Pareto inefficient path, the expected utilities between the two options need be compared. For this comparison, I assume that there are two options for each non-cooperative household with regard to consumption just after an upward shift in time preference. The first is a jump option, *J*, in which a household's consumption jumps to Z and then proceeds on the posterior Pareto efficient saddle path to the posterior steady state. The second is a non-jump option, *NJ*, in which a household's consumption does not jump but instead gradually decreases from the prior steady state to the posterior steady state, as shown by the bold dashed line in Figure 1. The household that chooses the *NJ* option reaches the posterior steady state in period $s(\geq 0)$. The difference in consumption between the two options in each period t is $b_t (\geq 0)$. Thus, b_0 indicates the difference between Z and the prior steady state. b_t diminishes continuously and becomes zero in period s. The *NJ* path of consumption (c_t) after the shock is monotonously continuous and differentiable in t and $\frac{dc_t}{dt} < 0$ if $0 \leq t < s$. In addition,

$$
\begin{aligned}
&\bar{c} < c_t < \hat{c}_t && \text{if } 0 \le t < s \\
&c_t = \bar{c} && \text{if } 0 \le s \le t \quad ,
\end{aligned}
$$

where $\hat{c}_t$ is consumption when proceeding on the posterior Pareto efficient saddle path and $\bar{c}$ is consumption in the posterior steady state. Therefore,

$$
\begin{aligned}
&b_t = \hat{c}_t - c_t > 0 && \text{if } 0 \le t < s \\
&b_t = 0 && \text{if } 0 \le s \le t \quad .
\end{aligned}
$$

It is also assumed that, when a household chooses a different option from the one the other households choose, the difference in the accumulation of financial assets resulting from the difference in consumption (b_t) before period s between that household and the other households is reflected in consumption after period s. That is, the difference in the return on financial assets is added to (or subtracted from) the household's consumption in each period after period s. The exact functional form of the addition (or subtraction) is shown in Section 3.1.4.

3.1.3 Firms

Unutilized products (b_t) are eliminated quickly in each period by firms because holding b_t for a long period is a cost to firms. Elimination of b_t is accomplished by discarding the goods or preemptively suspending production, thereby leaving some capital and labor inputs idle. However, in the next period, unutilized products are generated again because the economy is not proceeding on the Pareto efficient saddle path. Unutilized products are therefore successively generated and eliminated. Faced with these unutilized products, firms dispose of the excess capital used to generate b_t. Disposing of the excess capital is rational for firms because the excess capital is an unnecessary cost, but this means that parts of the firms are liquidated, which takes time and thus disposing of the excess capital will also take time. If the economy proceeds on the *NJ* path (that is, if all households choose the *NJ* option), firms dispose of all of the remaining excess capital that generates b_t and adjust their capital to the posterior steady-state level in period s, which also corresponds to households reaching the posterior steady state. Thus, if the economy proceeds on the *NJ* path, capital k_t is

$$
\begin{aligned}
&\bar{k} < k_t \le \hat{k}_t && \text{if } 0 \le t < s \\
&k_t = \bar{k} && \text{if } 0 \le s \le t \quad ,
\end{aligned}
$$

where $\hat{k}_t$ is capital per capita when proceeding on the posterior Pareto efficient saddle path and $\bar{k}$ is capital per capita in the posterior steady state.

The real interest rate i_t is

$$
i_t = \frac{\partial f(A, k_t)}{\partial k_t} \quad .
$$

Because the real interest rate equals the rate of time preference at steady state, if the economy proceeds on the *NJ* path,

$$
\begin{aligned}
&\tilde{\theta} \le i_t < \theta && \text{if } 0 \le t < s \\
&i_t = \theta && \text{if } 0 \le s \le t \quad ,
\end{aligned}
$$

where $\tilde{\theta}$ is the rate of time preference before the shock and θ is the rate of time preference after the shock. i_t is monotonously continuous and differentiable in *t* if $0 \le t < s$.

3.1.4 Expected utility after the shock

The expected utility of a household after the shock depends on its choice of the *J* or *NJ* path. Let *Jalone* indicate that the household chooses option *J*, but the other households choose option *NJ*; *NJalone* indicate that the household chooses option *NJ*, but the other households choose option *J*; *Jtogether* indicate that all households choose option *J*; and *NJtogether* indicate that all households choose option *NJ*. Let *p* ($0 \le p \le 1$) be the subjective probability of a household that the other households choose the *J* option (e.g., *p* = 0 indicates that all the other households choose option *NJ*). With *p*, the expected utility of a household when it chooses option *J* is

$$E_0(J) = pE_0(Jtogether) + (1-p)E_0(Jalone) \quad , \tag{25}$$

and when it chooses option *NJ* is

$$E_0(NJ) = pE_0(NJalone) + (1-p)E_0(NJtogether) \quad , \tag{26}$$

where $E_0(Jalone)$, $E_0(NJalone)$, $E_0(Jtogether)$, and $E_0(NJtogether)$ are the expected utilities of the household when choosing *Jalone*, *NJalone*, *Jtogether*, and *NJtogether*, respectively. Given the properties of *J* and *NJ* shown in Sections 3.1.2 and 3.1.3,

$$\begin{aligned} E_0(J) = {} & pE_0\left[\int_0^s \exp(-\theta t) u(c_t + b_t) dt + \int_s^\infty \exp(-\theta t) u(\hat{c}_t) dt\right] \\ & + (1-p)E_0\left[\int_0^s \exp(-\theta t) u(c_t + b_t) dt + \int_s^\infty \exp(-\theta t) u(\bar{c} - \bar{a}) dt\right] \quad , \end{aligned} \tag{27}$$

and

$$\begin{aligned} E_0(NJ) = {} & pE_0\left[\int_0^s \exp(-\theta t) u(c_t) dt + \int_s^\infty \exp(-\theta t) u(\hat{c}_t + a_t) dt\right] \\ & + (1-p)E_0\left[\int_0^s \exp(-\theta t) u(c_t) dt + \int_s^\infty \exp(-\theta t) u(\bar{c}) dt\right] \quad , \end{aligned} \tag{28}$$

where

$$\bar{a} = \theta \int_0^s b_r \exp \int_r^s i_q \, dq \, dr \quad , \tag{29}$$

and

$$a_t = i_t \int_0^s b_r \exp \int_r^s i_q \, dq \, dr \quad , \tag{30}$$

and the shock occurred in period *t* = 0. Figure 2 shows the paths of *Jalone* and *NJalone*. Because there is a sufficiently large number of households and the effect of an individual household on the whole economy is negligible, in the case of *Jalone*, the economy almost proceeds on the *NJ* path. Similarly, in the case of *NJalone*, it almost proceeds on the *J* path. If the other households choose the *NJ* option (*Jalone* or *NJtogether*), consumption after s is constant as $\bar{c}$ and capital is adjusted to $\bar{k}$ by firms in period s. In addition, a_t and i_t are constant after s such that a_t equals $\bar{a}$ and i_s equals θ, because the economy is at the posterior steady state. Nevertheless, during the transition period before s, the value of i_t changes from the value of the prior time preference rate to that of the posterior rate. If the other households choose option J (*NJalone* or *Jtogether*), however, consumption after s is $\hat{c}_t$ and capital is not adjusted to $\bar{k}$ by firms in period s and remains at $\hat{k}_t$.

As mentioned in Section 3.1.2, the difference in the returns on financial assets for the household from the returns for each of the other households is added to (or subtracted from) its consumption in each period after period s. This is described by a_t and $\bar{a}$ in equations (27) and (28), and equations (29) and (30) indicate that the accumulated difference in financial assets resulting from b_t increases by compound interest between the period r to s. That is, if the household takes the *NJalone* path, it accumulates more financial assets than each of the other J households, and instead of immediately consuming these extra accumulated financial assets after period s, the household consumes the returns on them in every subsequent period. If the household takes the *Jalone* path, however, its consumption after s is $\bar{c} - \bar{a}$, as shown in equation (27). $\bar{a}$ is subtracted because the income of each household, $y_t = f(A, k_t)$, including the *Jalone* household, decreases equally by b_t. Each of the other *NJ* households decreases consumption by b_t at the same time, which compensates for the decrease in income; thus, its financial assets (i.e., capital per capita; k_t) are kept equal to $\hat{k}_t$. The *Jalone* household, however, does not decrease its consumption, and its financial assets become smaller than those of each of the other *NJ* households, which results in the subtraction of $\bar{a}$ after period s.

3.2 Pareto inefficient transition path [10]

3.2.1 Rational Pareto inefficient path

3.2.1.1 Rational choice of a Pareto inefficient path

Before examining the economy with non-cooperative households, I first show that, if households are cooperative, only option J is chosen as the path after the shock because it gives a higher expected utility than option *NJ*. Because there is no possibility of *Jalone* and *NJalone* if households are cooperative, then $E_0(J) = E_0(Jtogether)$ and $E_0(NJ) = E_0(NJtogether)$. Therefore,

$$
\begin{aligned}
& E_0(J) - E_0(NJ) \\
&= E_0\left[\int_0^s \exp(-\theta t) u(c_t + b_t) dt + \int_s^\infty \exp(-\theta t) u(\hat{c}_t) dt\right] - E_0\left[\int_0^s \exp(-\theta t) u(c_t) dt + \int_s^\infty \exp(-\theta t) u(\bar{c}) dt\right] \\
&= E_0\left\{\int_0^s \exp(-\theta t)[u(c_t + b_t) - u(c_t)] dt + \int_s^\infty \exp(-\theta t)[u(\hat{c}_t) - u(\bar{c})] dt\right\} > 0
\end{aligned}
$$

because $c_t < c_t + b_t$ and $\bar{c} < \hat{c}_t$.

Next, I examine the economy with non-cooperative households. First, the special case with a utility function with a sufficiently small γ is examined.

Lemma 1: If $\gamma(0 < \gamma < \infty)$ is sufficiently small, then $E_0(Jalone) - E_0(NJtogether) > 0$.
Proof: $\lim_{\gamma \to 0}[E_0(Jalone) - E_0(NJtogether)]$

[10] The idea of a rationally chosen Pareto inefficient path was originally presented by Harashima (2004b).

$$
\begin{aligned}
&= E_0\int_0^s \exp(-\theta t)\lim_{\gamma\to 0}[u(c_t+b_t)-u(c_t)]dt + E_0\int_s^\infty \exp(-\theta t)\lim_{\gamma\to 0}[u(\bar{c}-\bar{a})-u(\bar{c})]dt \\
&= E_0\int_0^s \exp(-\theta t)b_t\,dt - E_0\int_s^\infty \exp(-\theta t)\bar{a}\,dt \\
&= E_0\int_0^s \exp(-\theta t)b_t dt - E_0\theta\left[\int_0^s\left(b_r \exp\int_r^s i_q\,dq\right)dr\right]\int_s^\infty \exp(-\theta t)dt \\
&= E_0\int_0^s \exp(-\theta t)b_t dt - E_0\exp(-\theta s)\int_0^s\left(b_r \exp\int_r^s i_q\,dq\right)dr \\
&= E_0\exp(-\theta s)\int_0^s b_t\left\{\exp[\theta(s-t)]-\exp\int_t^s i_q\,dq\right\}dt > 0 \quad ,
\end{aligned}
$$

because, if $0 \le t < s$, then $i_t < \theta$ and $\exp[\theta(s-t)] > \exp\int_t^s i_q\,dq$. Hence, because $\exp[\theta(s-t)] > \exp\int_t^s i_q\,dq$, $E_0(Jalone) - E_0(NJtogether) > 0$ for sufficiently small *γ*. ■

Second, the opposite special case (i.e., a utility function with a sufficiently large *γ*) is examined.

Lemma 2: If $\gamma(0<\gamma<\infty)$ is sufficiently large and if $0 < \lim_{\gamma\to\infty}\frac{\bar{a}}{\bar{c}} < 1$, then $E_0(Jalone)-E_0(NJtogether) < 0$.

Proof: Because $0 < b_t$, then

$$
\lim_{\gamma\to\infty}\frac{1-\gamma}{\bar{c}^{\,1-\gamma}}[u(c_t+b_t)-u(c_t)] = \lim_{\gamma\to\infty}\left[\left(\frac{c_t+b_t}{\bar{c}}\right)^{1-\gamma} - \left(\frac{c_t}{\bar{c}}\right)^{1-\gamma}\right] = 0
$$

for any period $t(<s)$. On the other hand, because $0 < \bar{a}$, then for any period $t(<s)$, if $0 < \lim_{\gamma\to\infty}\frac{\bar{a}}{\bar{c}} < 1$,

$$
\lim_{\gamma\to\infty}\frac{1-\gamma}{\bar{c}^{\,1-\gamma}}[u(\bar{c}-\bar{a})-u(\bar{c})] = \lim_{\gamma\to\infty}\left[\left(1-\frac{\bar{a}}{\bar{c}}\right)^{1-\gamma} - 1\right] = \infty \quad .
$$

Thus,

$$
\begin{aligned}
&\lim_{\gamma\to\infty}\frac{1-\gamma}{\bar{c}^{\,1-\gamma}}[E_0(Jalone) - E_0(NJtogether)] \\
&= \lim_{\gamma\to\infty}\frac{1-\gamma}{\bar{c}^{\,1-\gamma}}\int_0^s \exp(-\theta t)\lim_{\gamma\to\infty}[u(c_t+b_t)-u(c_t)]dt \\
&+ \lim_{\gamma\to\infty}\frac{1-\gamma}{\bar{c}^{\,1-\gamma}}\int_s^\infty \exp(-\theta t)\lim_{\gamma\to\infty}[u(\bar{c}-\bar{a})-u(\bar{c})]dt \\
&= 0 + \infty > 0 \quad .
\end{aligned}
$$

Because $\frac{1-\gamma}{\bar{c}^{\,1-\gamma}} < 0$ for any $\gamma(1<\gamma<\infty)$, then if $0 < \lim_{\gamma\to\infty}\frac{\bar{a}}{\bar{c}} < 1$, $E_0(Jalone) - E_0(NJtogether)$

< 0 for sufficiently large $\gamma(<\infty)$. ■

The condition $0 < \lim_{\gamma\to\infty}\frac{\bar{a}}{\bar{c}} < 1$ indicates that path *NJ* from c_0 to $\bar{c}$ deviates sufficiently from the posterior Pareto efficient saddle path and reaches the posterior steady state $\bar{c}$ not taking much time. Because steady states are irrelevant to the degree of risk aversion (γ), both c_0 and $\bar{c}$ are irrelevant to γ.

By Lemmas 1 and 2, it can be proved that $E_0(Jalone) - E_0(NJtogether) < 0$ is possible.

Lemma 3: If $0 < \lim_{\gamma\to\infty}\frac{\bar{a}}{\bar{c}} < 1$, then there is a $\gamma^*(0 < \gamma^* < \infty)$ such that if $\gamma^* < \gamma < \infty$, $E_0(Jalone) - E_0(NJtogether) < 0$.

Proof: If $\gamma(>0)$ is sufficiently small, then $E_0(Jalone) - E_0(NJtogether) > 0$ by Lemma 1, and if $\gamma(<\infty)$ is sufficiently large and if $0 < \lim_{\gamma\to\infty}\frac{\bar{a}}{\bar{c}} < 1$, then $E_0(Jalone) - E_0(NJtogether)$ < 0 by Lemma 2. Hence, if $0 < \lim_{\gamma\to\infty}\frac{\bar{a}}{\bar{c}} < 1$, there is a certain $\gamma^*(0 < \gamma^* < \infty)$ such that, if $\gamma^* < \gamma < \infty$, then $E_0(Jalone) - E_0(NJtogether) < 0$. ■

However, $E_0(Jtogether) - E_0(NJalone) > 0$ because both *Jtogether* and *NJalone* indicate that all the other households choose option *J*; thus, the values of i_t and k_t are the same as those when all households proceed on the posterior Pareto efficient saddle path. Faced with these i_t and k_t, deviating alone from the Pareto efficient path (*NJalone*) gives a lower expected utility than *Jtogether* to the *NJ* household. Both *Jalone* and *NJtogether* indicate that all the other households choose option *NJ* and i_t and k_t are not those of the Pareto efficient path. Hence, the sign of $E_0(Jalone) - E_0(NJtogether)$ varies depending on the conditions, as Lemma 3 indicates.

By Lemma 3 and the property $E_0(Jtogether) - E_0(NJalone) > 0$, the possibility of the choice of a Pareto inefficient transition path, that is, $E_0(J) - E_0(NJ) < 0$, is shown.

Proposition 1: If $0 < \lim_{\gamma\to\infty}\frac{\bar{a}}{\bar{c}} < 1$ and $\gamma^* < \gamma < \infty$, then there is a $p^*(0 \le p^* \le 1)$ such that if $p = p^*$, $E_0(J) - E_0(NJ) = 0$, and if $p < p^*$, $E_0(J) - E_0(NJ) < 0$.

Proof: By Lemma 3, if $\gamma^* < \gamma < \infty$, then $E_0(Jalone) - E_0(NJtogether) < 0$ and $E_0(Jtogether)$ $- E_0(NJalone) > 0$. By equations (25) and (26),

$$E_0(J) - E_0(NJ) = p[E_0(Jtogether) - E_0(NJalone)] + (1-p)[E_0(Jalone) - E_0(NJtogether)] .$$

Thus, if $0 < \lim_{\gamma\to\infty}\frac{\bar{a}}{\bar{c}} < 1$ and $\gamma^* < \gamma < \infty$, $\lim_{p\to 0}[E_0(J) - E_0(NJ)] = E_0(Jalone) - E_0(NJtogether) < 0$ and $\lim_{p\to 1}[E_0(J) - E_0(NJ)] = E_0(Jtogether) - E_0(NJalone) > 0$. Hence, by the intermediate value theorem, there is $p^*(0 \le p^* \le 1)$ such that if $p = p^*$, $E_0(J) - E_0(NJ) = 0$ and if $p < p^*$, $E_0(J) - E_0(NJ) < 0$. ■

Proposition 1 indicates that, if $0 < \lim_{\gamma\to\infty} \frac{\bar{a}}{\bar{c}} < 1$, $\gamma^* < \gamma < \infty$, and $p < p^*$, then the choice of option *NJ* gives the higher expected utility than that of option *J* to a household; that is, a household may make the rational choice of taking a Pareto inefficient transition path. The lemmas and proposition require no friction, so a Pareto inefficient transition path can be chosen even in a frictionless economy. This result is very important because it offers counter-evidence against the conjecture that households never rationally choose a Pareto inefficient transition path in a frictionless economy.

3.2.1.2 Conditions for a rational Pareto inefficient path

The proposition requires several conditions. Among them, $\gamma^* < \gamma < \infty$ may appear rather strict. If γ^* is very large, path *NJ* will rarely be chosen. However, if path *NJ* is such that consumption is reduced sharply after the shock, the *NJ* option yields a higher expected utility than the *J* option even though γ is very small. For example, for any $\gamma(0 < \gamma < \infty)$,

$$
\begin{aligned}
&\lim_{s\to 0}\frac{1}{s}\left[E_0(Jalone) - E_0(NJtogether)\right] \\
&= \lim_{s\to 0}\frac{1}{s}\int_0^s \exp(-\theta t)\left[u(c_t + b_t) - u(c_t)\right]dt + \lim_{s\to 0}\frac{1}{s}\int_s^\infty \exp(-\theta t)\left[u(\bar{c}-\bar{a}) - u(\bar{c})\right]dt \\
&= u(c_0 + b_0) - u(c_0) - \frac{1}{\theta}\lim_{s\to 0}\frac{u(\bar{c}) - u(\bar{c} - s\theta b_0)}{s} = u(c_0 + b_0) - u(c_0) - b_0\frac{du(\bar{c})}{d\bar{c}} \\
&= \frac{(c_0 + b_0)^{1-\gamma} - c_0^{1-\gamma}}{1-\gamma} - b_0\bar{c}^{-\gamma} = \bar{c}^{-\gamma}\left\{\bar{c}^{\gamma}\left[\frac{(c_0 + b_0)^{1-\gamma}}{1-\gamma} - \frac{c_0^{1-\gamma}}{1-\gamma}\right] - b_0\right\} < 0 \quad ,
\end{aligned}
$$

because $\lim_{\gamma\to 1}\bar{c}^{\gamma}\left[\frac{(c_0+b_0)^{1-\gamma}}{1-\gamma} - \frac{c_0^{1-\gamma}}{1-\gamma}\right] = \bar{c}\left[\ln(c_0+b_0) - \ln(c_0)\right] = \bar{c}\ln\left(1+\frac{b_0}{c_0}\right) < b_0$ and $\lim_{\gamma\to\infty}\bar{c}^{\gamma}\left[\frac{(c_0+b_0)^{1-\gamma}}{1-\gamma} - \frac{c_0^{1-\gamma}}{1-\gamma}\right] = \lim_{\gamma\to\infty}\bar{c}^{\gamma}c_0^{1-\gamma}\left[\frac{\left(1+\frac{b_0}{c_0}\right)^{1-\gamma} - 1}{1-\gamma}\right] = 0$ because $\bar{c} < c_0$. That is, for each combination of path *NJ* and γ, there is $s^*(>0)$ such that, if $s < s^*$, then $E_0(Jalone) - E_0(NJtogether) < 0$.

Consider an example in which path *NJ* is such that b_t is constant and $b_t = \bar{b}$ before s (Figure 3); thus, $E_0\int_0^s b_t = s\bar{b}$. In this *NJ* path, consumption is reduced more sharply than it is in the case shown in Figure 2. In this case, because $\bar{a} > E_0\theta\int_0^s b_t = \theta s\bar{b}$, $0 < \gamma$, and $c_s < c_t$ for $t < s$, then $E_0\int_0^s \exp(-\theta t)\left[u(c_t + b_t) - u(c_t)\right]dt < E_0\int_0^s \exp(-\theta t)dt\left[u(c_s + \bar{b}) - u(c_s)\right] = E_0\frac{1-\exp(-\theta s)}{\theta}\left[u(c_s + \bar{b}) - u(c_s)\right]$, and in addition, $E_0\int_s^\infty \exp(-\theta t)\left[u(\bar{c}-\bar{a}) - u(\bar{c})\right]dt = E_0\int_s^\infty \exp(-\theta t)dt\left[u(\bar{c}-\bar{a}) - u(\bar{c})\right] = E_0\frac{\exp(-\theta s)}{\theta}\left[u(\bar{c}-\bar{a}) - u(\bar{c})\right] < E_0\frac{\exp(-\theta s)}{\theta}\left[u(\bar{c}-\theta s\bar{b}) - u(\bar{c})\right]$.

Hence,

$$
\begin{aligned}
&E_0(Jalone) - E_0(NJtogether) \\
&= E_0\int_0^s \exp(-\theta t)[u(c_t + b_t) - u(c_t)]dt + E_0\int_s^\infty \exp(-\theta t)[u(\bar{c} - \bar{a}) - u(\bar{c})]dt \\
&< E_0 \frac{1-\exp(-\theta s)}{\theta}\left[u\left(c_s + \bar{b}\right) - u(c_s)\right] + E_0 \frac{\exp(-\theta s)}{\theta}\left[u\left(\bar{c} - \theta s\bar{b}\right) - u(\bar{c})\right] \\
&= E_0 \frac{1-\exp(-\theta s)}{\theta}\left\{\left[u\left(c_s + \bar{b}\right) - u(c_s)\right] - \frac{\exp(-\theta s)}{1-\exp(-\theta s)}\left[u(\bar{c}) - u\left(\bar{c} - \theta s\bar{b}\right)\right]\right\}
\end{aligned}
$$

As γ increases, the ratio $\dfrac{u\left(c_s + \bar{b}\right) - u(c_s)}{u(\bar{c}) - u\left(\bar{c} - \theta s\bar{b}\right)}$ decreases; thus, larger values of s can satisfy $E_0(Jalone) - E_0(NJtogether) < 0$. For example, suppose that $\bar{c}$ = 10, c_s = 10.2, $\bar{b}$ = 0.3, and θ = 0.05. If $\gamma = 1$, then s^* = 1.5 at the minimum, and if $\gamma = 5$, then s^* = 6.8 at the minimum. This result implies that, if option *NJ* is such that consumption is reduced relatively sharply after the shock (e.g., $b_t = \bar{b}$) and $p < p^*$, option *NJ* will usually be chosen. Choosing option *NJ* is not a special case observed only if γ is very large, but option *NJ* can normally be chosen when the value of γ is within usually observed values. Conditions for generating a rational Pareto inefficient transition path therefore are not strict. In a recession, consumption usually declines sharply after the shock, which suggests that households have chosen the *NJ* option.

3.3 Nash equilibrium

3.3.1 A Nash equilibrium consisting of *NJ* strategies

A household strategically determines whether to choose the *J* or *NJ* option, considering other households' choices. All households know that each of them forms expectations about the future values of its utility and makes a decision in the same manner. Since all households are identical, the best response of each household is identical. Suppose that there are $H\ (\in N)$ identical households in the economy where *H* is sufficiently large (as assumed in Section 3.1). Let $q_\eta\ (0 \le q_\eta \le 1)$ be the probability that a household $\eta\ (\in H)$ chooses option *J*. The average utility of the other households almost equals that of all households because *H* is sufficiently large. Hence, the average expected utilities of the other households that choose the *J* and *NJ* options are E_0(*Jtogether*) and E_0(*NJtogether*), respectively. Hence, the payoff matrix of the *H*-dimensional symmetric mixed strategy game can be described as shown in Table 1. Each identical household determines its behavior on the basis of this payoff matrix.

In this mixed strategy game, the strategy profiles

$$(q_1,q_2,\ldots,q_H) = \{(1,1,\ldots,1),\ (p^*, p^*,\ldots, p^*),\ (0,0,\ldots,0)\}$$

are Nash equilibria for the following reason. By Proposition 1, the best response of household η is *J* (i.e., q_η = 1) if $p > p^*$, indifferent between *J* and *NJ* (i.e., any $q_\eta \in [0,1]$) if $p = p^*$, and *NJ* (i.e., q_η = 0) if $p < p^*$. Because all households are identical, the best-response correspondence of each household is identical such that q_η = 1 if $p > p^*$, [0,1] if $p = p^*$, and 0 if $p < p^*$ for any household $\eta \in H$. Hence, the mixed strategy profiles (1, 1,…,1), ($p^*, p^*,\ldots, p^*$), and (0,0,…,0) are the intersections of the graph of the best-response correspondences of all households. The Pareto efficient saddle path solution (1,1,…,1) (i.e., *Jtogether*) is a pure strategy Nash equilibrium, but a Pareto inefficient transition path (0,0,…,0) (i.e., *NJtogether*) is also a pure strategy Nash equilibrium. In addition, there is a mixed strategy Nash equilibrium $(p^*, p^*, ..., p^*)$.

3.3.2 Selection of equilibrium

Determining which Nash equilibrium, either *NJtogether* (0,0,…,0) or *Jtogether* (1,1,…,1), is dominant requires refinements of the Nash equilibrium, which necessitate additional criteria. Here, if households have a risk-averse preference in the sense that they avert the worst scenario when its probability is not known, households suppose a very low *p* and select the *NJtogether* (0,0,…,0) equilibrium. Because

$$
\begin{aligned}
& E_0(Jalone) - E_0(NJalone) \\
&= E_0\left\{\int_0^s \exp(-\theta t)\left[u(c_t + b_t) - u(c_t)\right]dt + \int_s^\infty \exp(-\theta t)\left[u(\bar{c} - \bar{a}) - u(\hat{c}_t + a_t)\right]dt\right\} \\
&< E_0\left\{\int_0^s \exp(-\theta t)\left[u(c_t + b_t) - u(c_t)\right]dt + \int_s^\infty \exp(-\theta t)\left[u(\bar{c} - \bar{a}) - u(\bar{c})\right]dt\right\} \\
&= E_0(Jalone) - E_0(NJtogether) < 0 \ ,
\end{aligned} \tag{31}
$$

by Lemma 3, *Jalone* is the worst choice in terms of the amount of payoff, followed by *NJtogether*, and *NJalone*, and *Jtogether* is the best. The outcomes of choosing option *J* are more dispersed than those of option *NJ*. If households have a risk-averse preference in the above-mentioned sense and avert the worst scenario when they have no information on its probability, a household will prefer the less dispersed option (*NJ*), fearing the worst situation that the household alone substantially increases consumption while the other households substantially decrease consumption after the shock. This behavior is rational because it is consistent with preferences. Because all households are identical and know inequality (31), all households will equally suppose that they all prefer the less dispersed *NJ* option; therefore, all of them will suppose a very low *p*, particularly $p = 0$, and select the *NJtogether* (0,0,…,0) equilibrium, which is the Nash equilibrium of a Pareto inefficient path. Thereby, unlike most multiple equilibria models, the problem of indeterminacy does not arise, and "animal spirits" (e.g., pessimism or optimism) are unnecessary to explain the selection.

3.4 Amplified generation of unutilized resources

A Nash equilibrium of a Pareto inefficient path successively generates unutilized products (b_t). They are left unused, discarded, or preemptively not produced during the path. Unused or discarded goods and services indicate a decline in sales and an increase in inventory for firms. Preemptively suspended production results in an increase in unemployment and idle capital. As a result, profits decline and some parts of firms need to be liquidated, which is unnecessary if the economy proceeds on the *J* path (i.e., the posterior Pareto efficient path). If the liquidation is implemented immediately after the shock, b_t will no longer be generated, but such a liquidation would generate a tremendous shock. The process of the liquidation, however, will take time because of various frictions, and excess capital that generates b_t will remain for a long period. During the period when capital is not reduced to the posterior steady-state level, unutilized products are successively generated. In a period, b_t is generated and eliminated, but in the next period, another, new, b_t is generated and eliminated. This cycle is repeated in every period throughout the transition path, and it implies that demand is lower than supply in every period. This phenomenon may be interpreted as a general glut or a persisting disequilibrium by some definitions of equilibrium. That is, b_t is another source of output gaps than frictions.

2.5 Time preference shock as the exceptional shock

Not all shocks result in a Nash equilibrium of a Pareto inefficient path. If anything, this type of shock is limited because such a shock needs to force consumption to fluctuate very jaggedly to maintain Pareto efficiency. A Pareto inefficient path is preferred, because these substantially jagged fluctuations can be averted. An upward time preference shock is one shock that necessitates a substantially jagged fluctuation as shown in Figure 1. Other examples are rare because shocks that do not change the steady state (e.g., monetary shocks) are not relevant. One other example is technology regression, which would move the vertical line $\frac{dc_t}{dt} = 0$ to the left in Figure 1 and necessitate a jagged consumption path to keep Pareto efficiency. In this sense, technology and time preference shocks have similar effects on economic fluctuations. However, a technology regression also simultaneously moves the curve $\frac{dk_t}{dt} = 0$ downwards, and accordingly, the Pareto efficient saddle path also moves downwards. Therefore, the jagged consumption is smoothed out to some extent. As a result, the substantially jagged consumption that can generate a recession would require a large-scale, sudden, and sharp regression in technology, which does not seem very likely. An upward time preference shock, however, only moves the vertical line $\frac{dc_t}{dt} = 0$ to the left.

In some macro-economic models with multiple equilibria, changing equilibria may necessitate substantially jagged consumption to keep Pareto optimality. There are many types of multiple equilibra models that depend on various types of increasing returns, externalities, or complementarities, but they are vulnerable to a number of criticisms (e.g., insufficient explanation of the switching mechanism; see, e.g., Morris and Shin, 2001). Examining the properties, validity, and plausibility of each of these many and diverse models is beyond the scope of this paper.

4 PHILLIPS CURVE

4.1 *Models of the Phillips curve*

4.1.1 Trend inflation and inflation

The micro-foundation of trend inflation discussed in Section 2 indicates that inflation π_t is a function of trend inflation π_t^T, in particular such that

$$\pi_t = \pi_t^T + v_{1,t} \quad , \tag{32}$$

where $v_{1,t}$ is a variable that represents factors other than trend inflation in period t. Equation (32) indicates that the aggregate supply equation (the Phillips curve) is modeled as a variable moving around a trend and occasionally diverting from the trend because of other factors.

4.1.2 Output gaps and inflation

Section 2 shows that shifts in θ_P change the path of trend inflation (π_t^T) unless θ_G is immediately changed in the same direction and by the same magnitude as θ_P. Usually θ_G will not change immediately after a shift in θ_P, so the path of trend inflation will usually change after a shift in θ_P. Hence, π_t changes as θ_P shifts; thus, π_t is a function of θ_P such that

$$\pi_t = h_\theta(\theta_{P,t}) \quad . \tag{33}$$

In Section 3, I showed that outputs y_t fluctuate with shifts of θ_P and unutilized resources (b_t) are generated. The unutilized resources indicate the existence of output gaps. The output gaps x_t can be described as follows:

$$x_t = \ln(y_t) - \ln(\tilde{y}_t) \ ,$$

where $\tilde{y}_t$ is y_t at the steady state or on the saddle path. Let also

$$x_{b,t} = \ln(y_t) - \ln(y_t + b_t) \ .$$

That is, $x_{b,t}$ is the output gap generated owing to b_t in period t and is a part of x_t. Because b_t is a function of θ_P, $x_{b,t}$ is also a function of θ_P such that

$$x_{b,t} = h_b(\theta_{P,t}) \ , \tag{34}$$

where $\theta_{P,t}$ is θ_P in period t. Suppose that $\theta_{P,t}$ is a Markov process and shifts in θ_P occasionally occur. By equations (33) and (34), $x_{b,t}$ will be observed to correlate with π_t such that

$$\pi_t = h_\theta\left[h_b^{-1}(x_{b,t})\right] \ . \tag{35}$$

Equation (35) does not indicate causation; it merely indicates that there is a correlation between π_t and $x_{b,t}$. The causations are described by equations (33) and (34).

There is, however, a conventional correlation between inflation and output gaps, and it is caused by frictions in price flexibility. The output gaps generated by frictions are traditionally thought to be the only sources of output gaps in the NKPC. Suppose that the sources of output gaps are only b_t and frictions. Thus, the output gaps that are generated by frictions in period t are

$$x_{F,t} = \ln(y_t + b_t) - \ln(\tilde{y}_t) \ .$$

That is,

$$x_t = x_{b,t} + x_{F,t} \ .$$

According to the micro-foundation of the NKPC, $x_{F,t}$ is correlated with π_t in a forward-looking manner such that

$$\pi_t = h_F(x_{F,t+i|t-1}) + v_{2,t} \quad \text{for } i = 0,1,2,\ldots.. \tag{36}$$

Where $v_{2,t}$ is a variable that represents factors other than trend inflation and $x_{F,t+i|t-1}$ is the $x_{F,t+i}$ expected in period $t-1$.

4.1.3 Three models of inflation in the aggregate supply equation

Combining equations (32) and (36), inflation can be modeled as

$$\pi_t = \pi_t^T + h_F(x_{F,t+i|t-1}) \qquad \text{for } i = 0,1,2,\ldots. \ .$$

With i.i.d. shocks ε_t, the data generation mechanism of π_t can be modeled as

$$\pi_t = \pi_t^T + h_F\left(x_{F,t+i|t-1}\right) + \varepsilon_{1,t} \quad \text{for } i = 0,1,2,\ldots.. \tag{37}$$

or more simply

$$\pi_t = a_1\pi_t^T + \varphi_1 x_{F,t|t-1} + \varepsilon_{1,t} \quad , \tag{38}$$

where a_1 and φ_1 are constants and expected to be positive, and $\varepsilon_{1,t}$ is an i.i.d. shock in period t. It is important to note that equations (37) and (38) are aggregate supply equations that are firmly constructed on a micro-foundation basis. Another important point is that equations (37) and (38) do not include the correlation indicated by equation (35).

Equations (37) and (38) superficially resemble the pure NKPC and the hybrid NKPC, but they are actually completely different. Typical pure and hybrid NKPCs can be described, respectively, as

$$\pi_t = \varphi_2 x_{F,t|t-1} + \varepsilon_{2,t} \tag{39}$$

and

$$\pi_t = a_3\pi_{t-1} + \varphi_3 x_{F,t|t-1} + \varepsilon_{3,t} \quad , \tag{40}$$

where a_3, φ_2, and φ_3 are constants and expected to be positive, and $\varepsilon_{2,t}$ and $\varepsilon_{2,t}$ are disturbances in period t. That is, a pure NKPC indicates that inflation is a function of $x_{F,t}$ and a hybrid NKPC indicates that inflation is a function of both lagged inflation and $x_{F,t}$. An important difference between equation (38) and equations (39) and (40) is that equation (38) includes trend inflation but the others do not.

Conceptually, most models of NKPC assume that $x_{b,t}$ does not exist and x_t consists only of $x_{F,t}$ as shown in equations (39) and (40), and data of x_t are usually regarded to be identical to those of $x_{F,t}$. However, if $x_{b,t}$ does exist, estimations of equations (39) and (40) using date of x_t as those of $x_{F,t}$ are in reality estimations of the following aggregate supply equations, respectively:

$$\pi_t = \varphi_2 x_{t|t-1} + \varepsilon_{2,t} = \varphi_2\left(x_{F,t|t-1} + x_{b,t|t-1}\right) + \varepsilon_{2,t} \tag{41}$$

and

$$\pi_t = a_3\pi_{t-1} + \varphi_3 x_{t|t-1} + \varepsilon_{3,t} = a_3\pi_{t|t-1} + \varphi_3\left(x_{F,t|t-1} + x_{b,t|t-1}\right) + \varepsilon_{3,t} \quad , \tag{42}$$

where $x_{t|t-1}$ and $x_{b,t|t-1}$ are x_t and $x_{b,t}$ expected in period t–1, respectively. Although conceptually x_t = $x_{F,t}$ in the NKPC, in reality, inflation is a function of x_t (= $x_{F,t}$ + $x_{b,t}$) in estimation models of the pure NKPC and a function of lagged inflation and x_t (= $x_{F,t}$ + $x_{b,t}$) in hybrid NKPC models. Equations (41) and (42) are therefore actual estimation models of the pure NKPC and the hybrid NKPC, respectively. In the following discussion, equation (38) is referred to as Model 1, and equations (41) and (42) are referred to as Models 2 and 3, respectively.

4.1.4 Superiority of Model 1

In the sense that Model 1 is constructed on the basis of purely forward-looking micro-foundations of both trend inflation and friction, it is superior to Model 3 (hybrid NKPC), which lacks a micro-foundation for including lagged inflation. Model 2 (pure NKPC), however, does have a micro-foundation for the friction component, but it is usually empirically rejected, whereas the results of estimates from Model 3 are usually empirically accepted. Model 1 cannot be easily estimated empirically because it is difficult to distinguish between $x_{F,t}$ and $x_{b,t}$ in the data, but it is highly likely that Model 1 would be empirically supported because the trend inflation in Model 1 and the lagged inflation in Model 3 play almost the same role in the estimation of both models. Model 1 is therefore superior to Model 2 in the sense that it would most likely be empirically supported. As a whole, therefore, it is likely that Model 1 is the closest to the true mechanism of the three models.

4.2 Are frictions important?

4.2.1 Inappropriateness of the pure NKPC (Model 2)

Most empirical research has rejected Model 2, and the reason can be understood by comparing Model 2 with Model 1. If Model 1 is the true mechanism, Model 2 will be naturally rejected empirically because the movement of trend inflation (π_t^T) cannot be captured sufficiently only by x_t (= $x_{F,t}$ + $x_{b,t}$), as shown in Section 2. The estimates of φ_2 in Model 2 therefore will be always statistically non-significant as shown in many empirical researches. This result seems natural, because it is well known that inflation is persistent, and output gaps caused by frictions ($x_{F,t}$) cannot, by their nature, be persistent. Trend inflation, however, can be persistent. For these reasons, Model 1 is superior to Model 2.

4.2.2 A problem in hybrid NKPC (Model 3)

Unlike Model 2, Model 3 lacks a micro-foundation, but the results generated from the model match with empirical data. If Model 1 is the true mechanism, it is natural that the results from Model 3 would fit the empirical data. Suppose for simplicity that $z_t = 2$ in equation (2) because $\lim_{t\to\infty} z_t = 2$ as shown in Section 2. By equation (2),

$$\begin{aligned}\pi_t^T &= \pi_{t-1}^T + 6(\theta_G - \theta_P)\left[(t-\varpi)^2 - (t-\varpi-1)^2\right] \\ &= \pi_{t-1}^T + 6(\theta_G - \theta_P)[2(t-\varpi)+1] \end{aligned} \tag{43}$$

for $t \geq s$. Note that equation (1) indicates that the path of π_t^T just after a shift of θ_P is more complex than what is shown in equation (43) because π_t^T is influenced by its past path. Nevertheless, for simplicity, I assume that equation (43) holds even just after a shift in θ_P because $\lim_{t\to\infty} z_t = 2$, and the path of π_t^T will soon approach the path indicated in equation (43).

By combining equation (43) with Model 1,

$$\pi_t = \pi_{t-1} + \varphi_1\left(x_{F,t|t-1} - x_{F,t-1|t-2}\right) + \left(\varepsilon_{1,t} - \varepsilon_{1,t-1}\right) + 6a_1(\theta_G - \theta_P)[2(t-\varpi)+1] \ . \tag{44}$$

Model 1 is transformed to be a function of lagged inflation (π_{t-1}); that is, Model 1 indicates that π_t is auto-correlated, as Model 3 also indicates. Model 3 includes lagged inflation without showing its micro-foundation, but Model 1 provides this micro-foundation and thus validates the inclusion of lagged inflation in an aggregate supply equation.

At the same time, however, Model 1 (equation [44]) indicates that the coefficient of π_{t-1} should be unity. As is well known, estimates of a_3 in Model 3 are usually far less than unity (e.g., 0.5). The reason for the difference is that, although Models 1 and 3 are similar in that they include lagged inflation, other explanatory variables are not the same. The explanatory variables of π_t in Model 1 are $x_{F,t}$ and θ_P, and those in Model 3 are $x_{F,t}$ and $x_{b,t}$. Combining Model 3 with equation (44) (i.e., Model 1) yields

$$(1-a_3)\pi_{t-1}=\varphi_3\left(x_{F,t|t-1}+x_{b,t|t-1}\right)+\varepsilon_{3,t}-\varphi_1\left(x_{F,t|t-1}-x_{F,t-1|t-2}\right)-\left(\varepsilon_{1,t}-\varepsilon_{1,t-1}\right)-6a_1(\theta_G-\theta_P)[2(t-\varpi)+1] \ .$$

Suppose for simplicity that $\varepsilon_{3,t}=\varepsilon_{1,t}=\varepsilon_{1,t-1}=0$ and $x_{F,t|t-1}-x_{F,t-1|t-2}=0$; thus,

$$(1-a_3)\pi_{t-1}+6a_1(\theta_G-\theta_P)[2(t-\varpi)+1]=\varphi_3\left(x_{F,t|t-1}+x_{b,t|t-1}\right)$$

should always hold. If a_3 = 1 as Model 1 indicates,

$$\varphi_3\left(x_{F,t|t-1}+x_{b,t|t-1}\right)=6a_1(\theta_G-\theta_P)[2(t-\varpi)+1]$$

should be always held. Both $\varphi_3\left(x_{F,t|t-1}+x_{b,t|t-1}\right)$ and $6a_1(\theta_G-\theta_P)[2(t-\varpi)+1]$ are negative, and thus estimates of Model 3 for a_3 = 1 can be statistically significant. However, even if $a_3 < 1$,

$$\varphi_3\left(x_{F,t|t-1}+x_{b,t|t-1}\right)=(1-a_3)\pi_{t-1}+6a_1(\theta_G-\theta_P)[2(t-\varpi)+1]$$

can be fallaciously satisfied if $(1-a_3)\pi_{t-1}+6a_1(\theta_G-\theta_P)[2(t-\varpi)+1]<0$ and a larger value of φ_3 is given. In particular, when π_t is low, the probability that $(1-a_3)\pi_{t-1}+6a_1(\theta_G-\theta_P)[2(t-\varpi)+1]$ is negative will be high, and the probability that a_3 is estimated to be far less than unity will be also high. In this case, the estimated value of φ_3 is fallaciously larger than the case of a_3 = 1. On the other hand, when π_t is high, a_3 will be estimated to be close to unity because $(1-a_3)\pi_{t-1}+6a_1(\theta_G-\theta_P)[2(t-\varpi)+1]$ is positive unless a_3 is close to unity. Even if a_3 is estimated to be far less than unity and statistically significant, therefore, Model 1 indicates that this is a fallacious result.

4.2.3 Frictions are less important than previously thought

In addition to erroneously small values of a_3, the fact that estimated values of φ_3 will be fallaciously larger is also important because it indicates that the influence of frictions (φ_3) will also be overestimated in Model 3. Furthermore, another factor influences the overestimation of frictions. φ_3 is the coefficient not of $x_{F,t}$ but of x_t (= $x_{F,t}$ + $x_{b,t}$); thus, φ_3 reflects not only frictions but also b_t. Model 1's micro-foundation indicates that the output gaps caused by time preference shifts ($x_{b,t}$) are irrelevant to the data generation mechanism of π_t. Equation (35) merely indicates that π_t is superficially correlated with $x_{b,t}$, but there is no causation between the two. Because φ_3 reflects both correlations between π_t and $x_{F,t}$ and π_t and $x_{b,t}$, the estimates of φ_3 will be influenced not only by frictions but also by the movement of b_t. With this effect, therefore, the influence of frictions, if it is measured by φ_3, will be overestimated.

The above two factors combined will greatly bias estimates of φ_3 upwards. It is likely therefore that the influence of frictions is largely overestimated if Model 3 is used for the evaluation. This finding has an important implication. Frictions have been regarded as an important factor in economic activities, but their role may be far smaller than has been previously thought. Even though some degree of frictions may actually exist and have real impacts, the results presented here indicate that the importance of frictions should not be exaggerated. Furthermore, this is most likely true not only for inflation but also for more general economic activities. This conclusion seems very natural, because it is highly likely that humans are sufficiently rational and can quickly and fully exploit the opportunities frictions provide and minimize the obstruction caused to economic activities by frictions.

4.3 Monetary policies

Monetary policies have usually been implemented on the basis of Model 2 or Model 3. If monetary policies were to be implemented on the basis of Model 1, then the effects could be different, so monetary policies based on Model 1 are examined in this section.

4.3.1 Aggregate demand equation

An examination of monetary policies requires not only an aggregate supply equation but also an aggregate demand equation. The following is a typical forward-looking New Keynesian aggregate demand equation (e.g., Clarida et al., 1999; Svensson and Woodford, 2003):

$$x_t = x_{t+1|t-1} - \beta_r \left(i_{t|t-1} - \pi_{t+1|t-1} - r \right) + \eta_t \tag{45}$$

where i_t is the nominal interest rate; r is the real interest rate at steady state; β_r is a constant coefficient; and η_t is an i.i.d. shock with zero mean. Equation (45) is obtained under the assumption that x_t is generated only by frictions. In other words, equation (45) assumes that $x_t = x_{F,t}$. However, in Model 1, $x_t = x_{F,t} + x_{b,t}$. Hence, to be consistent with Model 1, equation (45) should be changed to

$$x_{F,t} = x_{F,t+1|t-1} - \beta_r \left(i_{t|t-1} - \pi_{t+1|t-1} - r \right) + \eta_t \quad . \tag{46}$$

Therefore,

$$\ln\left[\exp\left(y_t\right) + b_t\right] - \tilde{y}_t = \ln\left[\exp\left(y_{t+1|t-1}\right) + b_{t+1|t-1}\right] - \tilde{y}_{t+1|t-1} - \beta_r \left(i_{t|t-1} - \pi_{t+1|t-1} - r \right) + \eta_t$$

and thus

$$x_t = x_{t+1|t-1} + \left(x_{b,t} - x_{b,t+1|t-1} \right) - \beta_r \left(i_{t|t-1} - \pi_{t+1|t-1} - r \right) + \eta_t \quad . \tag{47}$$

Equation (47) indicates that x_t is influenced not only by $i_{t|t-1} - \pi_{t+1|t-1} - r$ but also by $x_{b,t} - x_{b,t+1|t-1}$. For example, when $\theta_{P,t}$ shifts upwards, $x_{b,t} - x_{b,t+1|t-1} < 0$ because $x_{b,t} < 0$, $x_{b,t+1|t-1} < 0$ and $\left| x_{b,t} \right| > \left| x_{b,t+1|t-1} \right|$; thus, x_t decreases by $\left| x_{b,t} - x_{b,t+1|t-1} \right|$ even if the effect of friction does not exist. Note that $x_{b,t}$ is an exogenous variable for the central bank.

4.3.2 Monetary policies

4.3.2.1 Utilizing frictions

Conventional monetary policy controls inflation and output gaps by utilizing frictions through the manipulation of nominal interest rates. However, Model 1 indicates that when economic fluctuations are caused by shifts in θ_P, inflation and output gaps are not necessarily controlled by conventional monetary policy because $x_{F,t}$ and $x_{b,t}$ are both generated. For example, when θ_P shifts downwards, $x_{b,t}$ increases and π_t^T also increases. The response of conventional monetary policy is to raise nominal interest rates to make $x_{F,t}$ decrease through equation (46) (the aggregate demand equation) and consequently make π_t decrease through the aggregate supply equation.

This conventional operation focuses only on $x_{F,t}$ and does not consider the effect of the shift of θ_P on π_t^T. Model 1, however, indicates that π_t depends on π_t^T, which is not affected by $x_{F,t}$. Hence, Model 1 indicates that π_t is not necessarily sufficiently controlled through the use of conventional monetary policy. To stabilize π_t by the conventional monetary policy, nominal interest rates should be raised far more than would be done with the conventional policy, at least up to the point where the effect of $x_{F,t}$ on π_t overwhelms the effect of π_t^T on π_t. Even if nominal interest rates are raised to this far higher rate, π_t^T will accelerate unless θ_G is sufficiently reduced, as shown in Section 3. Nominal interest rates therefore should continue to be increased successively and indefinitely to stabilize inflation. Conversely, if θ_P has a large shift upward, the nominal interest rate will have to be reduced to zero (the lower bound of the nominal interest rate) unless θ_G is sufficiently increased. In this case, deflation will accelerate if θ_G is not sufficiently increased.

There is a great deal of evidence, however, that inflation has been stabilized by conventional monetary policy. I explore the possible reasons for this in the following sections.

4.3.2.2 Controlling the government's time preference

Although trend inflation π_t^T cannot be controlled by conventional monetary policy, it can be controlled through other types of monetary policy. The central bank can stabilize π_t^T by controlling the time preference rate of government (θ_G). As shown in Section 2, by manipulating nominal interest rates, the central bank can force the government to change θ_G. If θ_G changes according to the central bank's plan, then π_t^T will eventually stabilize. Model 1 indicates that, if π_t^T is stabilized at the target rate, π_t will also stabilize in the sense that π_t will not accelerate or decelerate and will remain near the target rate. For example, when θ_P shifts downwards and π_t^T begins to accelerate, the central bank should raise nominal interest rates and force the government to lower θ_G to stabilize π_t^T. If θ_G is successfully lowered as planned, π_t^T will stabilize.

Section 2 shows that acceleration and deceleration of trend inflation are caused by the difference between θ_G and θ_P. Therefore, only monetary policy aimed at controlling the government's time preference rate can eventually stabilize inflation in the sense that π_t does not accelerate or decelerate. Conversely, the monetary policy of utilizing frictions plays only a minor role in the process of inflation stabilization.

4.3.2.3 Indistinguishable effects of monetary policies

The monetary policy of utilizing frictions (conventional monetary policy) nevertheless has been regarded as the main player in inflation stabilization because the tools used in both types of monetary policy (utilizing frictions and controlling θ_G) are the same. Both types of policy manipulate nominal interest rates. In addition, the directions of the effects of both policy types are the same; for example, if nominal interest rates are raised, inflation decreases. Hence, the effects of the two types of monetary policy are not easily distinguishable. Even if a central bank consciously implements a monetary policy of utilizing frictions, it automatically also implements the monetary policy of controlling θ_G at the same time. If inflation stabilizes as a result of the operation, the central bank may believe that the monetary policy of utilizing frictions was effective, even though it was the policy of controlling θ_G that was effective. This indistinguishable nature of the effects of the policy therefore will lead to the incorrect belief that the monetary policy of utilizing frictions is very effective for inflation stabilization even when θ_P shifts.

4.3.2.4 Power to control output gaps

The aggregate demand equation that is consistent with Model 1 (equations [46] and [47]) indicates another important nature of monetary policy. Monetary policies, whether utilizing frictions or controlling θ_G, do not have enough power to stabilize output gaps. Because $x_{b,t}$ is exogenously given for the central bank, monetary policies cannot eliminate $x_{b,t}$. By decreasing $|x_{F,t}|$ through equation (46), $|x_t|$ becomes smaller to some extent, but a large $|x_t|$ will continue to exist because $x_{b,t}$ continues to exist. The results from Model 1 indicate that we should not expect to stabilize large output gaps through monetary policies, although small output gaps caused by frictions may be stabilized by them. In contrast, monetary policies—particularly the monetary policy of controlling θ_G —are very effective for stabilizing inflation.

5 CONCLUDING REMARKS

Pure and hybrid NKPCs have been criticized for empirical failures and the lack of micro-foundation, respectively. An alternative approach to the Phillips curve is to focus on trend inflation. In this paper, a micro-foundation of trend inflation is shown. Another important factor in the Phillips curve is the nature of output gaps. In the NKPC, output gaps are assumed to be generated only by frictions, but in this paper another source of output gaps is considered. These output gaps are generated as a Nash equilibrium consisting of strategies of choosing a Pareto inefficient transition path of consumption to the steady state.

The model presented in this paper is superior to the hybrid NKPC because it is constructed on the basis of purely forward-looking micro-foundations of both trend inflation and friction, and it is superior to the pure NKPC in the sense that it can be empirically supported. Comparisons between the new model and both types of NKPC indicate that the role of frictions has been overestimated and that frictions are less important than previously thought. Even though some amount of frictions may actually exist, their importance should not be exaggerated.

The model also indicates that the conventional monetary policy of utilizing frictions cannot necessarily stabilize inflation. In contrast, the monetary policy of controlling θ_G is very effective. A problem is that the effects of both types of monetary policy are not distinguishable. This indistinguishable nature results in the incorrect belief that the monetary policy of utilizing frictions (conventional monetary policy) is very effective for inflation stabilization even when θ_P shifts.

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Figure 1: A time preference shock

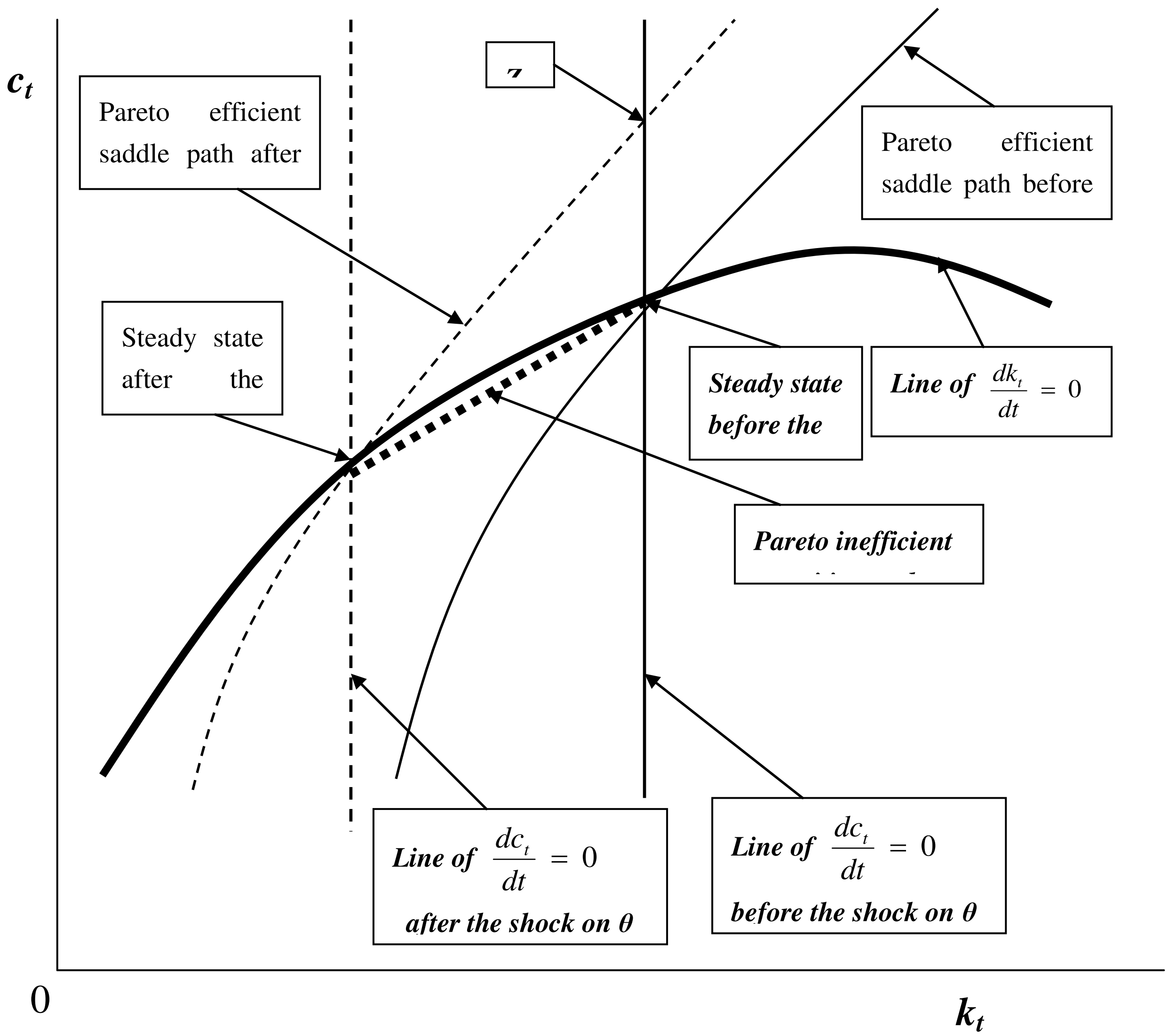

Figure 2: The paths of *Jalone* and *NJalone*

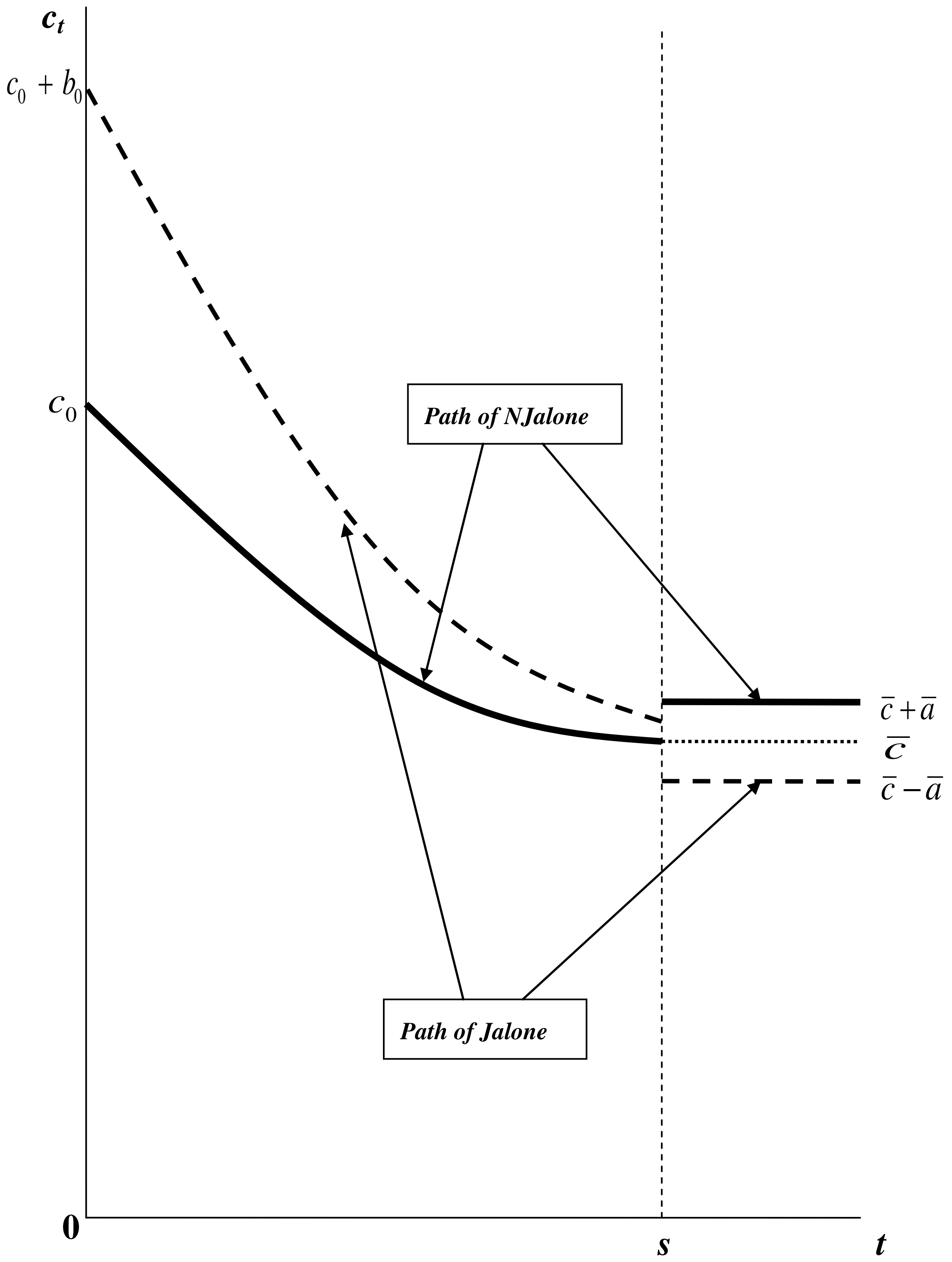

Figure 3: A Pareto inefficient transition path

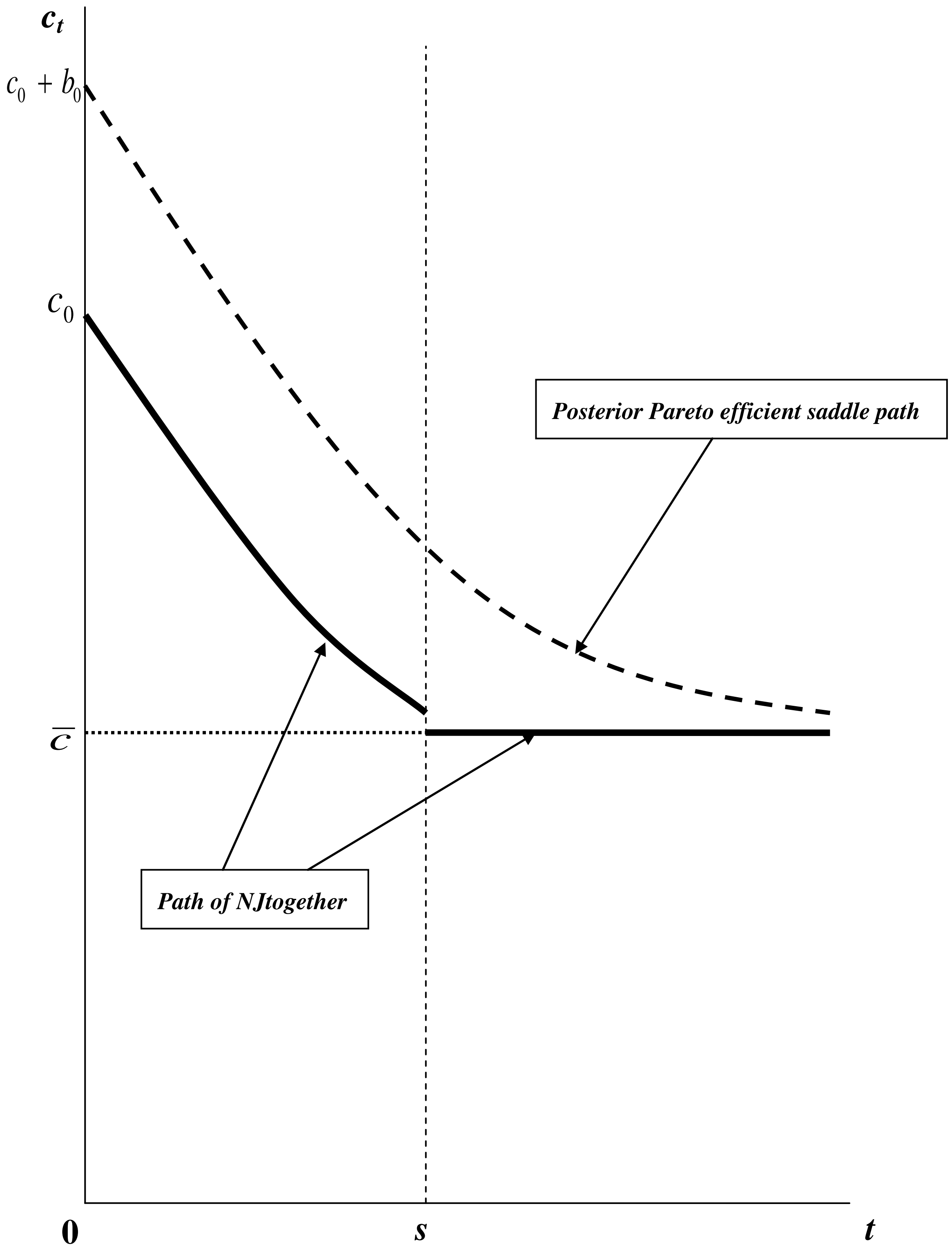

Table 1 The payoff matrix

		Any other household	
		J	***NJ***
Household A	***J***	$E_0(Jtogether), E_0(Jtogether)$	$E_0(Jalone), E_0(NJtogether)$
	NJ	$E_0(NJalone), E_0(Jtogether)$	$E_0(NJtogether), E_0(NJtogether)$