Fuzzification and Defuzzification Process in Genetically Evolved Fuzzy Cognitive Maps (GEFCMs)

Mateou, N. H. and Andreou, A. S. and Zombanakis, George A.

University of Cyprus, University of Cyprus, Bank of Greece

14 July 2004

Fuzzification and Defuzzification Process in Genetically Evolved Fuzzy Cognitive Maps (GEFCMs)

N.H. MATEOU[1], A.S. ANDREOU[1] and G.A. ZOMBANAKIS[2]
[1] Dept. of Computer Science
University of Cyprus
75 Kallipoleos str., CY1678 Nicosia, CYPRUS
[2] Research Department
Bank of Greece
21 Panepistimiou str., Athens 10250, GREECE

Abstract: This paper describes the fuzzification and defuzzification process in the framework of hybrid systems comprising Fuzzy Cognitive Maps (FCMs) and Genetic Algorithms (GAs). More specifically, it provides a stepwise methodology for fuzzification and defuzzification aiming at both an improved approach of the human reasoning pattern and an increase of the decision-making potentials. The fuzzification process is primarily based on producing fuzzy information provided by a group of experts. Each concept is analyzed into trapezoidal membership functions of either fixed or variable widths, with these intervals labeled and stored for the defuzzification process later on, during which the levels are matched according to the membership functions of each concept. The defuzzification process is more complicated than the fuzzification one and consists of four basic iterative stages: The Iteration, the Max-Min Average Computation, the Categorization and, finally, the Realization Inference Stage.

*Key-Words :*Fuzzification, Defuzzification, Fuzzy Cognitive Map, Evolutionary Computing, Decision-Making

1. Introduction

Decision-making is a task of critical importance. There is a wide variety of difficulties that decision makers face when approaching significant, real-world systems under uncertainty: for example decision makers have to face the increased complexity which characterizes the interrelation of the various dynamic components (concepts) of a certain problem encountered. When it comes to requiring numerical data, these may be hard to trace or unreliable while formulating a mathematical model may be difficult, costly, and even impossible. This means that efforts to communicate an understanding of the system and propose policies will have to rely on natural language arguments in the absence of formal models.

The effort to cope with such difficulties started during the seventies, when Axelrod [2] described the cognitive maps in the shape of directed, inter connected and bilevel valued graphs, using them in decision theory applied to the politicoeconomic field [2]. In 1986, Kosko extended Axelrod's graphs to the fuzzy model [13], which became thus FCMs [9], originally proposed as a means of explaining political decision-making processes. What a FCM does, in fact, is allow the policy maker to perform a qualitative simulation through scenario analysis in which arguments and assumptions become explicit relieved of any traces of ornamental rhetoric. In fact, policy proponents can publish a model of the system under discussion and illustrate their case using simulation experiments. The next step involves simulating different scenarios [8] by asking the model to reach a desirable activation level for a certain concept that the policy–maker focuses on. The Genetically Evolved Fuzzy Cognitive Maps (GEFCM) model calculates the new optimal weight matrix, which is then used by the GEFCM model to recalculate the new activation levels of the concepts [3,4].

The fuzzification process [10,11] is based on producing fuzzy information provided by a group of experts, each concept analyzed into trapezoidal membership functions of fixed or variable widths. Each of these intervals are labeled and stored for the defuzzification process later on [10,11]. For each domain expert consulted, their activation levels and weight values are entered and normalized based on their respective ranking [14]. The defuzzification procedure takes place, when the levels are matched according to the membership functions of each concept. This process is more complicated than the fuzzification and consists of four basic iterative stages. To begin with, the Iteration stage involves the determination of the initial levels of activation for each concept on the FCM and the calculation of the final activation levels. The next step computes the minimum, maximum and average values for each concept of this matrix [7], with the levels matched according to their membership functions. The third stage matches the average, minimum and maximum values for each concept derived during fuzzification to find the interval these three parametres fall into. The last stage is the inference stage in which following the creation of hypothetical scenaria, the Fuzzy Knowledge Base [5,6] is used to determine the context in which the target activation level will be realized.

The rest of the paper is organized as follows: Section 2 provides a brief description of the theoretical background of FCMs and GEFCMs. Sections 3 and 4 describe the fuzzification and defuzzification process respectively while section 5 draws the conclusions and suggests further research steps.

2. Introduction to FCM and to GEFCM

The combination of Fuzzy Logic and Neural Networks [1], which have been developed in the world of soft computing [17], creates models that emulate reasoning and the decision-making process using fuzzy causal relationship [7,9]. The flexibility of such models is improved by allowing for a variety of activation levels (ALs) of each concept. Allowing the various activation levels to vary allows the policy maker to consider a wide variety of scenarios depending on the extent to which a variable or concept is active in affecting other variables or concepts of the model. This network of concepts and activation levels composes the so-called Certainty Neuron Fuzzy Cognitive Maps (CNFCM) that have developed to a reliable technique used in strategy selection and evaluation of possible solutions in view of complicated decision–making problems [15,16].

The standard procedure followed in such cases requires first that we model the current situation of the problem in focus, via a FCM. We then identify the main variables or concepts of the model and the causal relationships between them. The "degree of causality" values in the connecting edges indicate the degree to which one concept affects another. Values can range from –1, indicating a strong negative impact, through 0, or no impact, to +1, a strong positive impact. The next step involves testing the model by allowing it to "run" until it stabilises at a certain equilibrium state the validity of which is assessed based on the degree to which it reflects the complication of the problem under study in its current state. Once a satisfactory performance of the model is assessed we can then proceed with performing a scenario-based analysis to reach conclusions that can assist the decision making process. This analysis involves two steps: The dynamic analysis phase and the genetically evolved analysis phase.

2.1 Dynamic analysis phase

The dynamic analysis phase aims at simulating the impact of various political decisions reflected in a number of scenarios upon the key variables or concepts of the problem under consideration.

A FCM works in discrete steps [10]. When a strong positive correlation exists between the current state of a concept and that of another concept in a preceding period, we say that the former exercises a positive influence on the latter, this indicated by a positively weighted arrow directed from the causing to the influenced concept. By contrast, when a strong negative correlation exists, it reveals the existence of a negative causal relationship indicated by an arrow charged with a negative weight. Once the activation levels of each of the system nodes as well as the weighted arrows are set to a specific value based on experts' assessment, the system is free to interact [14]. This interaction continues until the model either reaches a stable equilibrium, or presents a limit cycle or, even, a chaotic behaviour.

The introduction of Certainty Neuron Fuzzy Cognitive Maps (CNFCMs) [15,16] in 1997 provided additional fuzzification to FCMs, by allowing for various activation levels of each concept between the two extreme cases, i.e. activation or not. . More specifically, an updating function f() was used to revise the original certainty factor of a concept following the input of new evidence that revised the analyst's previous assessment which relied on that original certainty factor.

The updating function of a CNFCM is given in equation (1) as follows:

$$A_i^{t+1} = f\left(S_i^t A_i^t\right) - d_i \; A_i^t \qquad (1)$$

where $$S_i^t = \sum_{\substack{j=1 \\ j \neq i}}^{n} A_j^t w_{ij} \qquad (2)$$

and A_i is the activation level of concept C_i at times *(t+1)* or *(t)*.

Equation (2) is the sum of the weighted influences that concept C_i receives at time step t from the rest of the concepts in the model, d_i is a decay factor and $f()$ is the function used for the aggregation of certainty factors [15,16]:

$$fm(A_i^t, S_i^t) = \begin{cases} A_i^t + S_i^t(1-A_i^t) = A_i^t + S_i^t - S_i^t A_i^t, if\ A_i^t \geq 0, S_i^t \geq 0 \\ A_i^t + S_i^t(1+A_i^t) = A_i^t + S_i^t + S_i^t A_i^t, if\ A_i^t < 0, S_i^t < 0, |A_i^t|, |S_i^t| \leq 1 \\ \left(A_i^t + S_i^t\right) \Big/ \left(1 - \min\left(\left|A_i^t\right|, \left|S_i^t\right|\right)\right), otherwise \end{cases} \quad (3)$$

It is important to point out via equation (3) that the external influence can affect the activation of a concept just to a certain degree:

2.2 Genetically evolved analysis phase

This phase starts with calculating the activation levels when the FCM model is at equilibrium using the initial weight matrix. The next stage involves simulating different scenarios by asking the model to reach a target activation level for a certain concept in focus. The GEFCM model calculates the new optimal weight matrix, which is then used by the FCM model to recalculate the new activation levels of the concepts involved. The recalculation of the weights constitutes the most important difference between the GEFCM and the simple FCM.

It is obvious, therefore, that the essence of the Genetically Evolved Fuzzy Cognitive Map (GEFCM) model lies with tracing the optimal weight matrix corresponding to a desired activation level for a given concept as computed by a simple CNFCM model. The primary objective of the GEFCM has been to overcome the main weakness of the CNFCM model, namely the inability to recalculate the weights corresponding to each concept every time a new strategy is adopted.

More specifically, the Genetic Algorithm (GA) evolves a population of individuals [12], each of which consists of a weight matrix describing the degree of causal relationships between the concepts participating in the map. The activation level of a certain concept in focus denoted by $A_{d,i}$ is used to calculate the fitness of each individual-weight matrix WM_i according to the following function:

$$\text{fitness}(WM_i) = 1/(1-\text{abs}(A_{d,i} - \text{mean}_{50}(A_{a,i})) \quad (4)$$

where $A_{d,i}$ is the target (desired) value of the activation level for the concept in focus C_i and mean50($A_{a,i}$) is the mean value of the last fifty actual activation levels of concepts C_i as these are computed by the CNFCM (t variable in equation (3)).

The importance of the decision-making process is underlined by the fact that the decisions taken will not rely exclusively on the initial weight matrix, but in addition on the optimal weights that lead a concept to be activated to a certain predefined degree. In other words, decision-makers are able to consider hypothetical cases reflected through the introduction of a target activation level for a certain variable or concept in the model and then examine how this change has affected the weights and activation levels of the remaining concepts. Based on this information, the decision maker is then able to take decisions leading to the desired simulated solution.

3. Fuzzification Process

The fuzzification process consists of two basic steps. During the first step the interval of each concept is analyzed into trapezoidal membership functions, as shown in Figure 1. Since the concept activation levels fall in the range between -1 and +1, the concept intervals themselves must also fall in this range. The minimum and maximum number of intervals in our model is two and eight respectively, having a fixed width or variable length, as shown in figures 2 and 3.

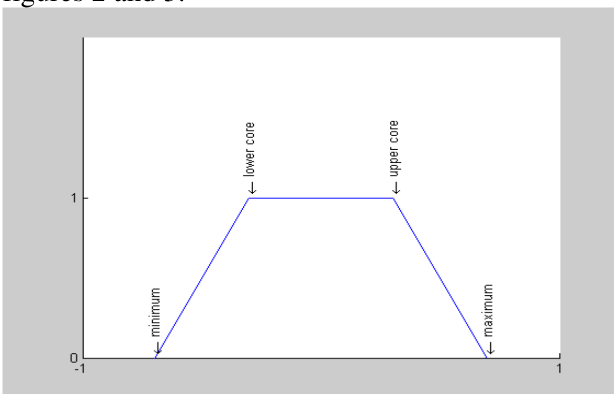


Fig. 1. The trapezium formed by the interval limits and overlap percentage

Figure 2, in particular, shows how the fuzzification of three crisp values causes the distribution of the variables according to a certain profile that reflects the problem under study. It is interesting to point out that this distribution produces two overlapping areas, an outcome which has been regarded as rather common and even desirable on certain occasions. In such a case the problem arising when values that fall within an overlapping area must be allocated is handled during the defuzzification process.

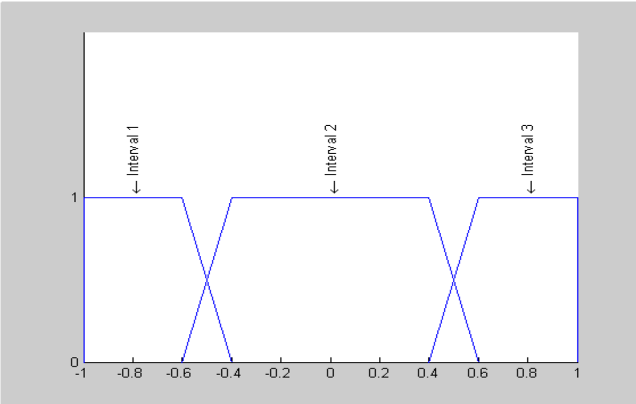


Fig. 2. A concept with 3 membership functions of variable width

In Fuzzy Cognitive Maps the term set consists of specific linguistic variables describing the activation levels of the concepts participating in the model. These variables are linked to specific values within the range [–1, +1]. The number of linguistic variables depends on the complexity of the real-world problem described by the model and the desired model accuracy. The general structure of the fuzzification of six crisp variables describing the activation levels is given in figure 3.

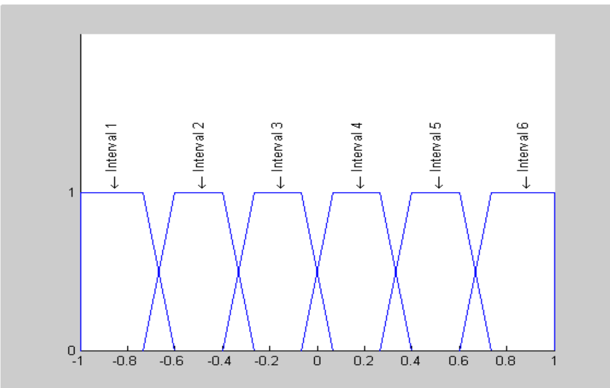
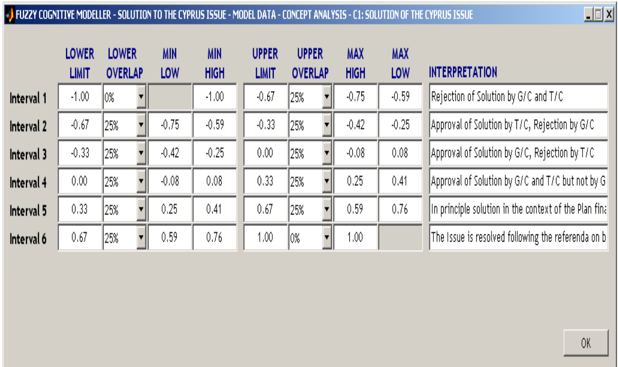


Fig. 3. A concept with 6 membership functions

The first interval begins at -1 and the last interval ends at +1. A software tool intended to provide experts with aid in planning, decision-making and problem-solving processes was built. The user – the decision maker in this case – is then required to feed in the limits as well as the percentage of overlap between these limits at each interval forming the trapezium as shown in figure 3.



FUZZY COGNITIVE MODELLER - SOLUTION TO THE CYPRUS ISSUE - MODEL DATA - CONCEPT ANALYSIS - C1: SOLUTION OF THE CYPRUS ISSUE

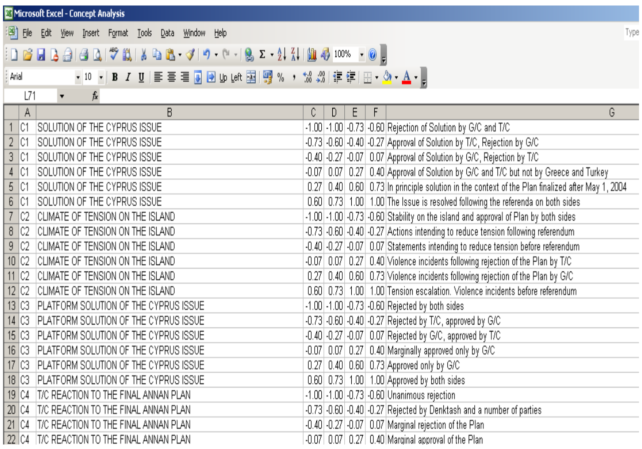
	LOWER LIMIT	LOWER OVERLAP	MIN LOW	MIN HIGH	UPPER LIMIT	UPPER OVERLAP	MAX HIGH	MAX LOW	INTERPRETATION
Interval 1	-1.00	0%		-1.00	-0.67	25%	-0.75	-0.59	Rejection of Solution by G/C and T/C
Interval 2	-0.67	25%	-0.75	-0.59	-0.33	25%	-0.42	-0.25	Approval of Solution by T/C, Rejection by G/C
Interval 3	-0.33	25%	-0.42	-0.25	0.00	25%	-0.08	0.08	Approval of Solution by G/C, Rejection by T/C
Interval 4	0.00	25%	-0.08	0.08	0.33	25%	0.25	0.41	Approval of Solution by G/C and T/C but not by G
Interval 5	0.33	25%	0.25	0.41	0.67	25%	0.59	0.76	In principle solution in the context of the Plan fina
Interval 6	0.67	25%	0.59	0.76	1.00	0%	1.00		The Issue is resolved following the referenda on b

OK

Fig. 4. A concept associated with a given name corresponding to linguistic variable.

Each interval is then given a name, corresponding to a certain linguistic variable as shown in figure 4 and is subsequently stored in a fuzzy knowledge base in order to be used in the defuzzification process.

Building a fuzzy knowledge base is the second step of the fuzzification process. This is a very complicated task requiring occasional adjustment of information, especially in cases of complex applications. The integration of a Fuzzy Knowledge Base (FKB) to GEFCMs aims at facing this task by encoding the domain experts' assessment concerning a given real-world problem and representing this knowledge in a graphical representation language. More specifically, the linguistic sample is encoded directly in a numerical matrix using an uncertainty fuzzy distribution and is subsequently reduced to a scalar form. As shown in figure 5 this linguistic matrix reflects the quantization levels of the input and output spaces, and the number of fuzzy set values assumed by the fuzzy variables.



Microsoft Excel - Concept Analysis

	A	B	C	D	E	F	G
1	C1	SOLUTION OF THE CYPRUS ISSUE	-1.00	-1.00	-0.73	-0.60	Rejection of Solution by G/C and T/C
2	C1	SOLUTION OF THE CYPRUS ISSUE	-0.73	-0.60	-0.40	-0.27	Approval of Solution by T/C, Rejection by G/C
3	C1	SOLUTION OF THE CYPRUS ISSUE	-0.40	-0.27	-0.07	0.07	Approval of Solution by G/C, Rejection by T/C
4	C1	SOLUTION OF THE CYPRUS ISSUE	-0.07	0.07	0.27	0.40	Approval of Solution by G/C and T/C but not by Greece and Turkey
5	C1	SOLUTION OF THE CYPRUS ISSUE	0.27	0.40	0.60	0.73	In principle solution in the context of the Plan finalized after May 1, 2004
6	C1	SOLUTION OF THE CYPRUS ISSUE	0.60	0.73	1.00	1.00	The Issue is resolved following the referenda on both sides
7	C2	CLIMATE OF TENSION ON THE ISLAND	-1.00	-1.00	-0.73	-0.60	Stability on the island and approval of Plan by both sides
8	C2	CLIMATE OF TENSION ON THE ISLAND	-0.73	-0.60	-0.40	-0.27	Actions intending to reduce tension following referendum
9	C2	CLIMATE OF TENSION ON THE ISLAND	-0.40	-0.27	-0.07	0.07	Statements intending to reduce tension before referendum
10	C2	CLIMATE OF TENSION ON THE ISLAND	-0.07	0.07	0.27	0.40	Violence incidents following rejection of the Plan by T/C
11	C2	CLIMATE OF TENSION ON THE ISLAND	0.27	0.40	0.60	0.73	Violence incidents following rejection of the Plan by G/C
12	C2	CLIMATE OF TENSION ON THE ISLAND	0.60	0.73	1.00	1.00	Tension escalation. Violence incidents before referendum
13	C3	PLATFORM SOLUTION OF THE CYPRUS ISSUE	-1.00	-1.00	-0.73	-0.60	Rejected by both sides
14	C3	PLATFORM SOLUTION OF THE CYPRUS ISSUE	-0.73	-0.60	-0.40	-0.27	Rejected by T/C, approved by G/C
15	C3	PLATFORM SOLUTION OF THE CYPRUS ISSUE	-0.40	-0.27	-0.07	0.07	Rejected by G/C, approved by T/C
16	C3	PLATFORM SOLUTION OF THE CYPRUS ISSUE	-0.07	0.07	0.27	0.40	Marginally approved only by G/C
17	C3	PLATFORM SOLUTION OF THE CYPRUS ISSUE	0.27	0.40	0.60	0.73	Approved only by G/C
18	C3	PLATFORM SOLUTION OF THE CYPRUS ISSUE	0.60	0.73	1.00	1.00	Approved by both sides
19	C4	T/C REACTION TO THE FINAL ANNAN PLAN	-1.00	-1.00	-0.73	-0.60	Unanimous rejection
20	C4	T/C REACTION TO THE FINAL ANNAN PLAN	-0.73	-0.60	-0.40	-0.27	Rejected by Denktash and a number of parties
21	C4	T/C REACTION TO THE FINAL ANNAN PLAN	-0.40	-0.27	-0.07	0.07	Marginal rejection of the Plan
22	C4	T/C REACTION TO THE FINAL ANNAN PLAN	-0.07	0.07	0.27	0.40	Marginal approval of the Plan

Fig. 5. Linguistic Fuzzy knowledge Base

4. Defuzzification Process

As we have already pointed out the defuzzification process is more complicated than the fuzzification one and consists of four basic iterative stages. These steps are described in this section while examples of the results of defuzzification process are shown in table 1.

Stage 1. Iteration

- Determination of the initial levels of activation for each concept of the FCM.
- Calculation of the final (baseline) activation levels by running the model for a certain number of iterations and subsequent evaluation of the results derived.
- It is interesting to point out that the iterative procedure is useful in this case since each concept is treated individually. Thus, following one hundred iterations, the results are stored in an m-by-n matrix where m is the number of concepts and n is the number of iterations remaining after the final iteration. We consider that the model is stabilized after one hundred iterations

Stage 2. Max-Min and Mean Computation

- The next step uses this matrix to compute, the minimum, maximum and average values for each concept, while the various levels are matched according to the membership functions of each concept.
- Running the model may lead to three possible outcomes: equilibrium, limit cycle or chaos.
- A concept reaches equilibrium if the absolute difference of Max-Min value is 0.01 or lower in the interval between 0 and 1.
- A concept is classified as a limit cycle in cases in which the absolute difference of Max-Min value is greater than 0.01 but less than 0.75.
- A concept is treated as displaying a chaotic behaviour if the absolute difference of Max-Min value is greater than 0.75.

Stage 3. Categorization

- The average, minimum and maximum values are matched depending on the interval these three parametres fall into as a result of the fuzzification process. In cases of equilibrium and limit cycle, if the average value of the concept falls in just one interval then the concept has a confidence rate of 100% and the final level is assigned the meaning of that interval. Whenever the average value falls within two adjacent intervals, the algorithm retains the interval with the highest confidence rate, with the meaning of that interval assigned to the final level. In the case of chaos, by contrast, no meaning can be given to the final level.

Stage 4. Realization-Inference

- This last stage involves the implementation of a number of hypothetical scenaria with target values set for each activation level and the GECNFCM used to drive the activation of the concept of interest to the desired level.
- If the target is attained then the Fuzzy Knowledge Base is employed to determine the context in which the target activation level is realized.

ID	MIN.	MAX.	AVER.	FINAL	CONF.	ANALYSIS RESULT	BEHAVIOUR
C1	-0,57	-0,57	-0,57	-0,57	100,00	Approval of Solution by T/C, Rejection by G/C	EQUILIBRIUM
C2	-0,01	-0,01	-0,01	-0,01	57,60	Statements reducing tension before referendum	EQUILIBRIUM
C3	-0,79	-0,79	-0,79	-0,79	100,00	Rejected by both sides	EQUILIBRIUM
C4	0,41	0,41	0,41	0,41	100,00	Approved by the majority of the parties	EQUILIBRIUM
C5	0,43	0,43	0,43	0,43	100,00	Approved by the majority of the parties	EQUILIBRIUM
C6	-0,68	-0,68	-0,68	-0,68	58,83	Rejection by both sides/ Greece and Turkey	EQUILIBRIUM
C7	-0,79	-0,79	-0,79	-0,79	100,00	Unanimous rejection	EQUILIBRIUM
C8	-0,87	-0,87	-0,87	-0,87	100,00	Unanimous rejection by all parties	EQUILIBRIUM
C9	0,39	0,39	0,39	0,39	93,36	Pressure on the T/C and the Turkish	EQUILIBRIUM
C10	-0,70	-0,70	-0,70	-0,70	78,32	Full membership of Cyprus freezes	EQUILIBRIUM
C11	-0,54	-0,54	-0,54	-0,54	100,00	No support to Turkish full membership	EQUILIBRIUM
C13	0,72	0,72	0,72	0,72	92,72	Support of the full membership of Turkey	EQUILIBRIUM

Table 1. Defuzzification analysis results

5. Conclusions

This paper described the process of fuzzification and defuzzification in a GEFCM models and demonstrated how this facilitates the decision-making process. The proposed method is simple and straightforward, based on well-known aspects of fuzzy sets and systems. The Fuzzification process comprises two sequential stages. The first one involves the identification of the interval for each concept and its mapping via trapezoidal membership functions. The second step undertakes the construction of a knowledge base according to the classification of each concept and its link to a linguistic variable. The defuzzification process is broken down to four sequential stages involving the iteration, max-min average computation, Categorization and realization inference.

The process of fuzzification and defuzzification in Hybrid models is expected to contribute to the effectiveness of decision-making by allowing the analyst more degrees of freedom. This will certainly contribute to the flexibility of the method since the policy maker will be able to determine the activation level of a certain concept achieved with a set of weights given by the GEFCM. Based on this information, the decision-maker will be able to retrieve the results of the scenario and interpret them with the aid of the Fuzzy Knowledge Base at a descriptive, linguistic level. The information thus derived can then be used to plan strategic and tactical moves aiming at strengthening or weakening selected concepts depending on the final activation levels that the model has suggested.

References:

[1] Aleksander I, Morton H, *An Introduction to Neural Computing*, 1st edn Int. Th. Comp. Press, London, 1995

[2] Axelrod R, *Structure of Decision, The Cognitive Maps of Political Elite*, 1st edn Princeton University Press, 1976

[3] Andreou, A.S., Mateou N.H. and Zombanakis, G.A,Evolutionary Fuzzy Cognitive Maps: A Hybrid System for Crisis Management and Political Decision-Making. *Proceedings of the Computational Intelligent for Modeling, Control & Automation CIMCA*, Vienna, 2003, pp.732-743

[4] Andreou, A.S., Mateou N.H. and Zombanakis, G.A., “Soft Computing for Crisis Management and Political Decision Making: The Use of Genetically Evolved Fuzzy Cognitive Maps” *Soft Computing Journal*, forthcoming

[5] Boose J.H., Rabid acquisition and combination of knowledge from multiple experts in the same domain, *Future computational Systems,* Vol.1, 1986, pp.191-216.

[6] Carlsson C, Fuller R, Adaptive Fuzzy Cognitive Maps for Hyperknowledge Representation in Strategic Formation Process. *Proceedings of the International Panel Conference on Soft and Intelligent Computing*,Budapest, 1996, pp. 43-53

[7] Cox E. *The Fuzzy Systems Handbook - A Practitioners Guide to Building Using and Maintaining Fuzzy Systems*, 1st edn. Academic Press Incorporation, London, 1994

[8] Godet M, *Scenario and Strategic Management*, 1st edn. Butterworths Scientific, London, 1987

[9] B. Kosko (1986) Fuzzy Cognitive Maps. International Journal of Man-Machine Studies, Vol. 24, 1986,pp. 65-75

[10] Kosko B, *Neural Networks and Fuzzy Systems, A dynamic systems approach to Machine Intelligence*, 2nd edn.,Prentice Hall, London,1992

[11] Kosko B, *Fuzzy Thinking, the New Science of fuzzy logic*, 2nd edn. Harper Collins, London, 1995

[12] Michalewicz Z, *Genetic Algorithms + Data Structures = Evolution Programs*, 1st edn. Springer Berlin Heidelberg, 1994

[13] Pelaez CE, Bowles JB, Using Cognitive Maps as a System Model for Failure Modes and Effects Analysis. *Information Science*: 88, 1996,pp. 177-199

[14] Taber WR, Siegel M, Estimation of Expert Weights and Fuzzy Cognitive Maps. *1st IEEE International Conference on Neural Networks* Vol.2:, 1987,pp.319-325

[15] Tsadiras AK, Margaritis KG, Using Certainly Neurons in Fuzzy Cognitive Maps. *Neural Network World,* Vol.6, 1996, pp.719-728

[16] Tsadiras AK, Kouskouvelis I, Margaritis KG, Making Political Decision using Fuzzy Cognitive Maps: The FYROM crisis. *Proceedings of the 8th Panhellenic Conference on Informatics,* Vol.1, 2001, pp.501-510

[17] Zadeh LA, *An introduction to fuzzy logic applications in intelligent systems*, 1st edn. Kluwer Academic Publisher, Boston, 1992