Does high income inequality signify inequality of opportunities?

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Abstract

It is often presumed that Gini coefficient values taken to reflect high income inequality are largely due to some combination of socioeconomic factors that gives rise to inequality of *opportunities*. We demonstrate, using computer simulations, that practically every Gini value within the entire range observed in state economies can be approximated by at least one of a set of possible models of an economy in which earning is totally due to random factors. Although that clearly does not prove that opportunities are in reality fairly equal, it does suggest that inequality of opportunities is not necessary for high income inequality. At the least, it relegates the burden of proof to whoever ascribes the latter largely to the former.

Economists use the *Lorenz curve*, obtained by plotting cumulative percentage of household income (or wealth[1]) as a function of percentile of income (see Figure 1), to display inequality in distribution of income within a given economy, as well as to compare between different economies the extents of within-economy inequality (see Cowell, 2000; Jenkins & van Kerm, 2009; Schnitzer, 1974).

Insert Figure 1 about here

More formally, the Lorenz curve is a function, F, associating cumulative income (by definition, non-negative) with percentiles. Its derivative function, f, namely the one associating with each percentile its own total income, is *by definition* monotonically increasing.

Let the value of f for any percentile, i, be denoted f_i, and let the value of F for any percentile, i, be denoted F_i.

By definition, for each i>1,

$$f_{i-1} < f_i \quad . \qquad (1)$$

Also, by definition, for each i>1,

$$F_i = \Sigma_{j=1,I} f_j = \Sigma_{j=1,I-1} f_j + f_i = F_{i-1} + f_i \quad . \qquad (2)$$

Hence, for each i>2,

$$F_i = F_{i-1} + f_i \quad \text{and} \quad F_{i-1} = F_{i-2} + f_{i-1} \quad . \qquad (3)$$

By (1) and (3),

$$F_{i-1} - F_{i-2} < F_i - F_{i-1} \quad . \qquad (4)$$

Thus, that Lorenz curves are practically always convex is hardly an empirical phenomenon. Such a curve would coincide with the line of perfect equality only in the extremely unlikely case that incomes were allocated by fiat uniformly between households, irrespective of any characteristic or consideration, which does not happen even in the most egalitarian societies (with the possible exception of some small communes like old-time kibbutzim in Israel). Barring that extreme case, the convexity of the Lorenz curve is actually entailed *by its definition*.

The Attribution fallacy

Since the Lorenz curve is almost necessarily convex, it is the *degree* of convexity that indicates the extent of inequality. The departure of the Lorenz curve from the line of perfect equality is typically measured by the *Gini coefficient*. That measure (see Dalton, 1920) is defined as the ratio of the area between the line of perfect equality and the observed Lorenz curve to the area between the line of perfect equality and the line of perfect inequality, which turns out to equal a half of the mean of absolute differences across all pairs of incomes (Sen, 1997/1973, pp. 30-31).

Although the Gini coefficient relates *just* to inequality of incomes (or of wealth) per se, it is considered quite prevalently as a general measure of *economic inequality* (see Sen, 1997, for a critical commentary). When applied to plotting the distribution of assets, it might also be interpreted as a measure of *social inequality* (see, e.g., Wikipedia entry "Lorenz curve", 2013), which appears to reflect a presumption that inequality of financial resources is due to inequality of opportunities, namely to inequality in social circumstances essential for the *opportunity* to obtain the resources. That presumption should easily be identified as an instance of the fallacy known as *affirming the consequent*: The fact that inequality of opportunities often results in inequality of assets or incomes does not at all imply that any inequality of those sorts must be attributed to inequality of oppurtunities.

A further well-known, if not widespread, belief is that free market economies are inherently biased in favor of the rich, mainly because the opportunity to earn income depends much on the preceding state of income and assets, which by Marxist theory (see Schnitzer, 1974, pp. 14-17), in the case of employers is affected mostly by surplus value, while in the case of employees is limited to the more meager labor income. Consequently, much like stated in Karl Marx' “General law of capitalist accumulation" (Marx, 1965/1867), the capital get increasingly concentrated and centralized (see, e.g., Bauman, 2009; Krugman, 2006).

Do indeed “the rich get richer”, as Scott Fitzgerald’s maxim has it? That must be true to some extent (see review in Björklund & Jäntti, 2009), especially if assets are entirely inherited to offsprings (see results of simulation in Epstein & Axtel, 1996), but probably not to the same extent that critics of free market economy believe (see, e.g., Lebergott, 1976; Tomes, 1981).

Anyhow, deductions about socioeconomic sources of the allocation of income, which in turn might have had some impact on sociological theories and political movements, do *not* squarely follow from the convexity of Lorenz curves. The Lorenz curve in itself is not very informative about social or economic factors to which the income distribution is due. It certainly is quite mute about equality of opportunities. Maintaining that it nonetheless does reflect unequal opportunities to a considerable extent must be due to a kind of attribution fallacy, derived from the false belief that if opportunities were equal, income inequality would be negligible.

To realize in a reasonably compelling manner that such a belief is indeed false, it is useful to inspect imaginary allocation processes that *guarantee* equal opportunities, namely that do not discriminate between individuals by any given property or prior condition. For one, opportunities may be equal when incomes are granted to persons by a fair lottery.

Simple instances of totally random allocation processes

Case a: Incomes from a fixed set are raffled. This condition is met whenever incomes in *any* sort of a known distribution are assigned to persons by means of a random procedure. Consider, for example, a discrete uniform income distribution. A conceivable generating process for such as a distribution may be a fair lottery in which M persons are allocated at random to R income levels with j (=M/R) receivers each (or, with some heavier role of chance, a fair lottery in which each of the M persons has an equal probability of gaining any of the R income levels). Since the process is random, it gives each person a-priori an equal probability to receive each of the alternative income levels. Nonetheless, the Lorenz curve would clearly be considerably convex, because despite the egalitarian *process*, the *outcome* is far from being egalitarian. The obvious reason is that income inequality is imposed by the nature of the process: Income is ordained to vary to a known extent, though everybody is equally likely to earn any income level. Thus, though opportunities are not unequal, incomes *by definition* are.

That process, however, is clearly of quite limited interest, since it does not generate an income distribution by some sort of known economic process that distributes total incoming total resources or redistributes existing ones, rather maps a fixed set of incomes to an equal-size set of persons. That administrative procedure that leaves the distribution intact is obviously not an economic process in any customary sense.

Case b: A number of equal-prize lotteries are administered. Now consider a more interesting a-priori egalitarian process, one that does *not* ordain the extent of inequality, and furthermore does not preclude emergence of equality: Allocating randomly a total sum S (say, \$1,000,000) to a given number of persons, M, (say, 1,000), by means of N (where $M \leq N$) independent lotteries in each of which a single person wins exactly 1/N of the total sum. It is a-priori egalitarian, because everybody starts with 0, has an equal chance to receive the first drawn prize, and then an equal chance, irrespective of previously allocated prizes, to receive any further drawn prize. That could account for generating some of the variance in income, though it certainly cannot realistically generate all of it.

To model the process, note its similarity to a case of succesive runs of a simple experiment having more than two possible results, like spinning a four-sided spinning top (dreidel). The probability, for a composite experiment with N succesive spins, of any specific quadruple of occurrence frequencies of each result, x_i, is given by the multinomial rule:

$$(N! / (x_1!\, x_2!\, x_3!\, x_4!)) \cdot 1/4^N \quad .$$

Allocating money to M persons by N independent lotteries is analogous to recording the results of N spins of an M-faced spinning top. Thus, the probability of any specific allocation (namely, M-tuple of number of prizes won by each person, x_i) is given by the multinomial rule:

$$(N! / (x_1!\, x_2! \ldots\ldots x_M!)) \cdot 1/M^N \quad .$$

The entire set of consequences of applying that composite experiment to any pair of M and N, for any possible M-tuple, can be mapped onto (a) frequency distributions specifying the likelihood for any possible of the M persons to end up with 0, 1, 2, etc prizes, (b) frequency distributions specifying the likelihood for any set of M-tuples sharing the same partition (e.g., (2,1,1), (1,2,1) and (1,1,2)).

To illustrate, in the case of 3 lotteries and 3 possible winners, there are 10 possible allocation results: (3,0,0), (2,1,0), (2,0,1), (1,2,0), (1,0,2), (1,1,1), (0,2,1), (0,1,2), (0,3,0), (0,0,3), with the following respective likelihoods: 1/27, 3/27, 3/27, 3/27, 3/27, 6/27, 3/27, 3/27, 1/27, 1/27. For any possible winner, the Binomial likelihoods of ending up with 0, 1, 2 or 3 prizes are 8/27, 12/27, 6/27, 1/27, respectively. As for frequency distributions, as seen above there are three types of them: (a) 3 permutations of (3,0,0), (b) 6 permutations of (2,1,0), (c) one permutation of one prize to each. Their respective likelihoods are 3/27, 18/27, 6/27.

Similarly, in the case of 4 lotteries and 3 possible winners, the Binomial likelihoods of ending up with 0, 1, 2, 3 or 4 prizes are 16/81, 32/81, 24/81, 8/81, 1/81, respectively. As for frequency distributions, there are four types of them: (a) 3 permutations of (4,0,0), (b) 6 permutations of (3,1,0), (c) 3 permutations of (2,2,0), (d) 3 permutations of (2,1,1), with respective likelihoods of 3/81, 24/81, 18/81, 36/81.

Consider now the somewhat more interesting case of N=10 and M=10. The number of disribution types is too large to report all likelihoods. Suffice it to say that the likelihood of complete equality is .0004, and that when distributions are arranged lexicographically, the median one is (3,2,1,1,1,1,1,0,0,0), which yields a Gini coefficient of .36.

Since calculating by analytic formulae for larger parameter values turned out to be infeasible due to combinatorial explosion, we opted for averaging estimates made by computer simulation.

We varied both M (namely, number of persons in the population) and N/M (namely, lotteries per person), conducted for each combination 100 independent allocations, then averaged statisti of the distributions. The mean Gini coefficients for each of those combinations is presented in Table 1A. The family of averaged Lorenz curves for all three N/M values used and M=10000 is displayed in Figure 2. Note that the outcome is far from producing an equal distribution: For M=10000 (which seems to be about the start of the asymptote), the mean Gini coefficient was found to be ~.28 when N/M is 4 (see Table 1A), and ~.80 when N/M is 0.25.

Of course, with large enough N/M the distribution would tend to equality, which directly follows from the Bernoulli theorem. It is possible to regard that consequence of the Bernoulli theorem as implying that inequality cannot in practice be due to a random allocation process. On the other hand, it is not clear that the number of lotteries within a time frame used to measure income is actually large enough to render inequality negligible.

The concept of lotteries is actually metaphorical, of course: It is meant to stand for fortuitous events each providing some opportunity of considerably improving somebody's lot. Is it clear that the number of such events within the relevant time frame must be more than 4-fold of the number of possible winners? Just to illustrate, imagine we record the yearly household income in a village by the sea, most inhabitants of which find their living by fishing, each using his own small boat. The daily yield per fisherman is regularly pretty meager, but with luck he would catch a really huge fish that he could sell for a very high sum of money. Is it probable that the *average* number of such lucky occasions in a year exceeds 4 (meaning, of course, that the total number of such events across all M fishermen is 4M)? It seems that even 1 is an overestimate. At least, N/M=1 definitely cannot be ruled out.

Anyhow, note that the results in Table 1A are just a first approximation, subject to the assumption that all prizes are equal, made for the sake of simplicity.

When we relaxed the assumption of a uniform prize - not sufficiently realistic even for an utterly simplistic process like the one simulated here, and varied prize magnitude (.1/N .5/N, 1/N, 1.5/N, 1.9/N of the total sum S, with *even* probabilities) for M=10000 the mean Gini coefficient was found to be ~.33 when N/M is 4 (see Table 1B), and ~.86 when N/M is 0.25.

When prize magnitude was varied with a *skewed* set of probabilites (.41, .27, .17, .10, .05, for prize magnitudes 0.488/N, 0.740/N, 1.176/N, 2/N, 4/N of the total sum S, respectively), for M=10000 the mean Gini coefficient was found to be ~.35 when N/M is 4 (see Table 1C). and ~.86 when N/M is 0.25.

We further relaxed another assumption - that there is only one winner in any given lottery. We rather introduced, in addition to variable prize magnitude, the tenet that the prize of a lottery is split equally between a random number of co-winners (drawn, for simplicity, from a discrete uniform distribution with the range 1-4). In that case, for M=10000 the mean Gini coefficient was found to be somewhat smaller,~.26, when N/M is 4 (see Table 1D), and ~.75 when N/M is 0.25.

In sum, over the four versions of the model, the minimal mean Gini coefficient at asymptote ranges between ~.26 and ~.35, the maximal one ranges between ~.75 and ~.83, and the medium one ranges between ~.47 and ~.63.

Insert Table 1 and Figure 2 about here

Catch-as-catch-can allocation processes

A shared feature of the allocation processes described above is that on each allocation event the gain is won by an extremely small subset of the population, typically one person, and at most four. Another feature is that even when the amount of gain, its probability, and/or number of winners are variable across events, any of those parameters is definite for each given event.

Let us now consider another type of random allocation process that though ensuring also equal opportunity to all population members, is much less systematic in its terms of allocation. Before formally defining it, let us illustrate it by an incident that fits that definition: Imagine that an armored car, used for transferring large amounts of money notes, crashes, all the notes in it scatter on the road, and passers-by rush to pick the notes as much as any of them can. Theoretically, each member of the population could attend such an incident, thereby able to share the loot. However, clearly quite a limited number of them happen to actually be there. In addition, the portion of the loot grabbed by any of the passers-by is indefinite, since after a particular note is taken by person *i*, any of the passers-by, including *i* herself, may take any of the yet untaken notes. Furthermore, the amount of notes taken by each of the passers-by depends not only on chance but also on some access parameters (e.g., proximity, physical abilities etc).

This illustration should of course not taken to mean that any CACC process is illegal. It may include, for example, a case in which a number of people, each separately, find on the beach a school of great fish thrown to the shore during a heavy storm.

In our simulation of CACC allocation processes, we varied M (number of persons in the population), N/M (ratio of number of lotteries and number of persons in the population) and m (number of participants in any particular process), conducted for each combination 100 independent allocations, then averaged statisti of the distributions. The mean Gini coefficients for each of those combinations is presented in Table 2. Note that the outcome, much like the outcome of national lotteries, is far from producing an equal distribution. As can be seen, the mean Gini coefficient at asymptote (M=10000) ranges between ~.04 and ~.75. It might be reasonably argued that the high values of N/M (namely, those in which the number of such events equals or even exceeds the number of population members) are very implausible, hence the plausible range is actually between ~.22 and ~.75.

Insert Table 2 about here

In addition, we conducted a simulation of an augmented version of CACC, in which access of participants to the notes was not uniform: 20% of them were 4 times as likely to grab notes as were other participants. The mean Gini coefficients for each of those combinations is presented in Table 3.

Insert Table 3 about here

As can be seen, the mean Gini coefficient at asymptote (M=10000) in that case is somewhat higher: The plausible range is between ~.25 and ~.81.

Finally, we conducted a simulation of a case in which income is determined *both* by equal-prize lotteries and allocation processes of the augmented version of CACC type, in which access of participants to the notes was not uniform: 20% of them were 4 times as likely to grab notes as were other participants. The mean Gini coefficients for each of the comninations of that conjoint allocation regime, only at asymptote (namely M=10000) is presented in Table 4.

Insert Table 4 about here

As can be seen, the mean Gini coefficient at asymptote for the plausible value of N/M ranges in that case between .41 and .48.

Effort-dependent random earning

Clearly, not all incomes can be attributed to luck in winning, without investing much effort, a good that in any given event is unsplittable (or minimaly splittable), or a number of such goods. In the bulk of occasions of income earning, the latter must be due to pedestrian labor rewarded modestly. The next step was to include that chunk of income - *regular earnings* - in the model.

The regular earnings may result from a constant source that may vary in existence and magnitude between individuals (e.g., salary, rent), quite often sufficiently to give rise to substantial income inequality. However, since we were interested in the highly *random* constituent of regular earnings, we modeled only a process in which the *weekly* haul is an aggregate of minimally small gains. We postulated, for the sake of simplicity, the conservative premise that all earners do exactly the same for living, including parameter values. We further postulated a moderately high likelihood of gaining any of the small gains in a unit time that is small enough to enable gaining no more than one in it (such as is the time needed for re-aiming in hunting birds by slingshot). In our model, we set that likelihood to .05, varied gain size G (.5, .75, 1, 1.25, 1.5) with the same probability p (.01) for each gain size value. We operationally defined haul to be the sum of gains in L time units of work in a week. Its magnitude depends on the size of the particular gain on any unit time, hence haul magnitude distribution is multinomial.

We simulated the model with two possible values of L (200, 2000), then calculated the Gini values of yearly income (namely, across 52 weeks). Table 5 presents mean Gini values (over 100 replications) of yearly regular random earnings under all combinations of L and M. As can be seen, the mean Gini value at asymptote ranges between ~.01 and ~.03, much smaller than it is expected to be when income is due to rare chance events. Yet, of course, the values would be higher, if the postulate of perfect equality (between earners) in the parameters of the earning process was relaxed.

Insert Table 5 about here

Finally, we tested cases in which total income over the whole period represented in a Lorenz curve (say, a year) is a sum of the three components presented above: (a) *chance of winning a fast-&-large gain* – the yearly cumulative gain in N ordinary lotteries, as described above, (b) *chance of winning a fast-&-moderate catch-as-catch-can loot* – the yearly cummulative gain in all *catch-as-catch-can* lotteries, as described above, (c) *regular earnings* – the sum of 52 values of weekly income due to routine work, each sampled from the same distribution.

We calculated Gini values in economies that have all three, but to enable inspecting the effects of various weights of the three sources, we added two other parameters – specifying ratio of expected prize in ordinary, equal-prize (EP, for short) lotteries, if won, to mean yearly income (5, 10, 50, 100), or ratio of expected loot in catch-as-catch-can (CACC, for short) lotteries to mean yearly income (1, 10, 100). The mean Gini values, only for M=10000, are presented in Table 6, as a function of ratio of prize in EP lottery to mean yearly income, ratio of prize in CACC lottery to mean yearly income, number of lotteries per person in each of the two types of lotteries, as well as number of time units of work per week in regular earnings.

Insert Table 6 about here

It is noteworthy that in any of the conjunctions of possible earning sources, there is at least one combination of parameter values that yields a considerable (>.39) mean Gini value. The greater the ratio, with respect to mean regular earnings, of the prizes of lotteries, the larger is that maximal mean Gini value. Yet, especially effective is the ratio between the prize of equal-prize lottery and mean regular earnings. Furthermore, when the latter prize is ≥10, it acts to reverse the effect of the corresponding ratio in CACC lotteries. For example, when the ratios of the prize of the equal-size lottery and the prize of the CACC lottery to mean regular earnings are 100 and 1 respectively, maximal mean Gini value is quite substantial - almost .72.

Thus, it would seem unsafe to attribute to social, or to any sort of non-random, factors (much less, exclusively) almost any empirical Gini value of those yearly calculated for countries, were it considered reasonable that their economies were affected by anything akin to the random allocation processes, aside of regular earning, discussed above. How reasonable is that actually? This issue is discussed below.

Are those models instructive?

How relevant are the above models to the economic process by which goods are distributed between members of a community?

The mini-models underlying the simulations above were not of course conceived to model an economy, much less the economy of a modern country. They are rather meant to examine how much inequality could be produced in a regime in which capital is allocated *only* by random processes, which may be used as a sort of baseline inequality for allocations emerging in real economies that are evidently manyfold more complex.

On the other hand, the greater the number of variables added to characterize the random allocation, the closer it comes to a model that traces the outline of an economy in which chance plays a major role. The model might appear to be a caricature of an economy as we now know it. Evidently, present economies do not work that way as a rule, though some part of the variance of household income must be accounted for by fortuitous discoveries or fairly unique events such as inception of bright ideas bearing extraordinarily large economic fruit.

Still, note that very primitive economies, such as ones that existed in stone-age, or even bronze-age, communities, may presumably not be extremely different from that caricature. The stone age hunter must have depended much on his good luck – sometimes a deer, more often a rabbit, quite often nothing at all. Whenever he had some luck in hunting, its contribution to the household welfare was probably much greater than the less fortuitous, yet much smaller, contributions of the gatherers in his family.

Anyhow, it is instructive that in a world in which economies worked that way, considerable inequality would be observed, and the variation of inequality between economies would be due just to variability of M and N (regardless of the total sum of money being distributed!). Conceivably, whatever factors actually produce inequality, or modulate it, in present-day economies mimic in some yet-unknown way the simple random processes described above, at least in their effects. If real values of N/M, as well as of other parameters, were within the range that produces substantial inequality in our model (which there seems to be no way to ascertain, since their model definitions are very hard to operationalize), then inequality would seem to be due largely to natural causes rather than just to sociopolitical structures.

Note, augmenting the models discussed above with sources of variation that are not due to mere chance (e.g., innate individual differences known to affect earning capacity) would not necessarily reduce Gini values, possibly even increase them. All that is not meant to argue that income inequality is mostly due to chance. Yet, it may be enough for shifting the burden of proof to advocates of the stance that it is considerably due to sociopolitical factors.

That does not mean of course that inequality is an inevitable evil, let alone a justified phenomenon, neither that there is little to be done to reduce it. However, realizing how liable inequality is to emerge, even without the structures characetristic of modern, free-market economies, makes one somewhat skeptic of attributing it to any of the latter.

Take, for example, the premise of concentration of capital due to the propensity of earning opportunity to grow with the extent of assets being already held. Since the models simulated here could produce ample inequality in spite of being predicated on *absolutely equal* opportunities throughout the process, one may ponder about the validity of that premise, and at least require very good direct evidence that a *considerable* chunk of inequality is due to concentration of capital.

Possibly, opportunities are not as unequal in our economy as often believed. But even if they are considerably unequal, some sociopolitical factors in a good deal of modern economies (e.g., progressive taxes, welfare policy, common laud of donations to charity, etc.) must be potent enough to compensate for the impact of that inequality.

Either way, it would seem that "the law of Nature" is not more egalitarian than are most economic regimes, capitalism included. More probably, compassionate liberalism mitigates them both (see Piketty & Saez, 2003, 2006): Low levels of inequality are present mostly in states that effectively apply measures resulting in income redistribution (e.g., Sweden, Denmark). Inequality is higher in countries in which the government does less in that respect (e.g., Thailand, United States).

The fact that inequality is particularly high in countries in which central rule is very weak or practically nonexistent (e.g., Haiti, Sierra Leone[3] provides strong evidence of the cardinal role of institutional intervention in curtailing inequality. That indicates that inequality is probably not the product of institutional subjugation of some primitive natural order, as Marxist thought suggests, rather a quite likely outcome within an environment in which individuals seek income independently with minimal cooperation or central intervention.

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Footnotes

1. Below we omit any mentioning of wealth or assets. Everywhere the word “income” is mentioned, the argument applies to wealth as well.

2. Similarly, only a relative poverty rate that is higher than the one generated by random procedures like those may be regarded as reflecting inherent socioeconomic bias. Typically, an index of relative poverty rate is measured as the proportion of households with an income less than a given percentage, say 50%, of the median income. Note that an index defined in that way is sensitive to how income is distributed among low income households, which may be quite weakly related with how it is distributed among households with above-median income. More important, any rise in the median that is due to some beneficial factor not shared by many low income households (e.g., reducing income tax, salary raise for all public sector workers) is bound to effectuate some increase in the poverty rate index.

3. At least, by the time this paper is written.

Table 1: Gini Values for Equal-Prize Lottery

1a: With equal gain

Size of population	Number of lotteries per person: 0.25	1	4
400	*0.798*	*0.522*	*0.274*
2,000	*0.800*	*0.525*	*0.278*
10,000	*0.801*	*0.524*	*0.277*
50,000	*0.801*	*0.524*	*0.277*

1b: With varying gains

Size of population	Number of lotteries per person: 0.25	1	4
400	*0.860*	*0.615*	*0.331*
2,000	*0.862*	*0.616*	*0.332*
10,000	*0.863*	*0.618*	*0.333*
50,000	*0.863*	*0.617*	*0.333*

1c: With varying gains with varying probabilities

Size of population	Number of lotteries per person: 0.25	1	4
400	*0.857*	*0.623*	*0.352*
2,000	*0.862*	*0.627*	*0.353*
10,000	*0.863*	*0.628*	*0.354*
50,000	*0.863*	*0.627*	*0.354*

1d: With varying gains and varying probabilities for 1-4

Size of population	Number of lotteries per person: 0.25	1	4
400	*0.743*	*0.468*	*0.255*
2,000	*0.750*	*0.472*	*0.256*
10,000	*0.750*	*0.473*	*0.256*
50,000	*0.749*	*0.474*	*0.256*

Table 2: Gini Values for "Catch as catch can"

	Size of population							
	400		2,000		10,000		50,000	
Group size	Number of lotteries per person		Number of lotteries per person		Number of lotteries per person		Number of lotteries per person	
10	0.04	*0.744*	0.04	*0.750*	0.04	*0.751*	0.04	*0.751*
	0.2	*0.402*	0.2	*0.408*	0.2	*0.409*	0.2	*0.409*
	1.0	*0.180*	1.0	*0.184*	1.0	*0.186*	1.0	*0.186*
	5.0	*0.080*	5.0	*0.083*	5.0	*0.083*	5.0	*0.083*
50	0.04	*0.449*	0.04	*0.466*	0.04	*0.469*	0.04	*0.469*
	0.2	*0.203*	0.2	*0.214*	0.2	*0.216*	0.2	*0.216*
	1.0	*0.091*	1.0	*0.096*	1.0	*0.097*	1.0	*0.097*
	5.0	*0.039*	5.0	*0.043*	5.0	*0.043*	5.0	*0.044*

Table 3: Gini Values for modified "Catch as catch can"

	Size of population							
	400		2,000		10,000		50,000	
Group size	Number of lotteries per person		Number of lotteries per person		Number of lotteries per person		Number of lotteries per person	
10	0.04	*0.802*	0.04	*0.807*	0.04	*0.808*	0.04	*0.808*
	0.2	*0.481*	0.2	*0.485*	0.2	*0.487*	0.2	*0.487*
	1.0	*0.222*	1.0	*0.226*	1.0	*0.227*	1.0	*0.227*
	5.0	*0.098*	5.0	*0.102*	5.0	*0.102*	5.0	*0.102*
50	0.04	*0.513*	0.04	*0.529*	0.04	*0.532*	0.04	*0.532*
	0.2	*0.242*	0.2	*0.250*	0.2	*0.252*	0.2	*0.252*
	1.0	*0.108*	1.0	*0.113*	1.0	*0.14*	1.0	*0.114*
	5.0	*0.047*	5.0	*0.050*	5.0	*0.051*	5.0	*0.051*

Table 4: Gini Values for modified "Catch as catch can"(CACC)+"Equal Prize lottery"(EP)

	Weight of CACC vs. EP		
Number of lotteries per person	1	10	100
0.25	*0.476*	*0.413*	*0.437*
1	*0.268*	*0.212*	*0.225*
4	*0.141*	*0.107*	*0.113*

Table 5: Gini Values for random regular earnings

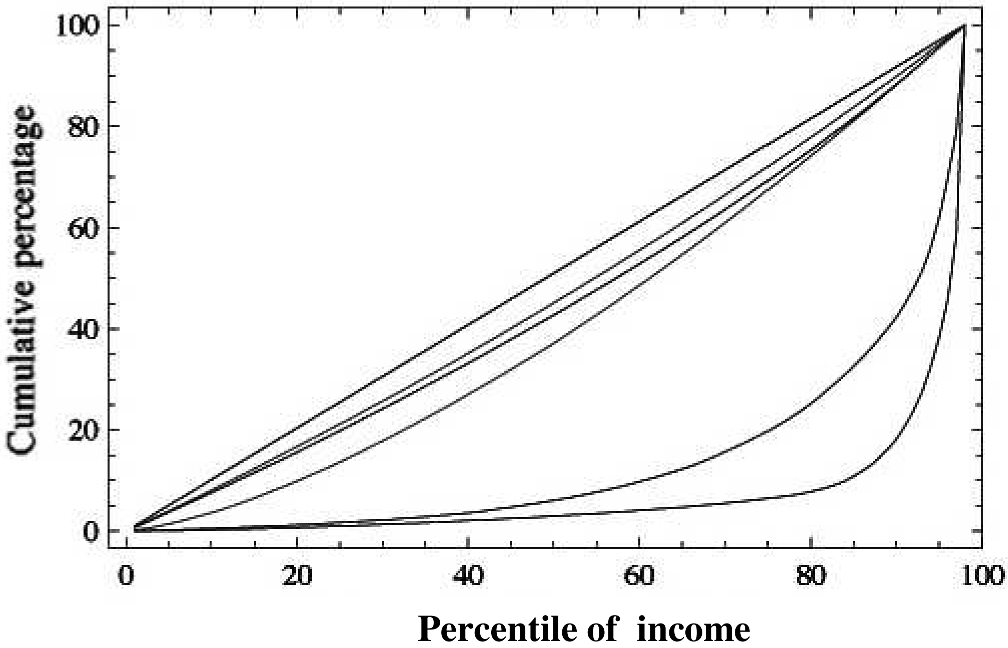
	Number of lotteries	
	200	2000
Size of population		
400	*0.023*	*0.006*
2,000	*0.025*	*0.008*
10,000	*0.026*	*0.008*
50,000	*0.026*	*0.008*

Table 6: Gini Values for combination of all 3 models of earnings

Ratio of EP prize to mean yearly income		Ratio of CACC prize to mean yearly income								
		1			10			100		
		Number of lotteries per person in each of the two types								
	L (Time units of work in regular earnings)	0.25	1	4	0.25	1	4	0.25	1	4
5	200	*.389*	*.341*	*.206*	*.334*	*.211*	*.113*	*.408*	*.215*	*.109*
	2000	*.388*	*.340*	*.205*	*.334*	*.211*	*.113*	*.408*	*.216*	*.109*
10	200	*.509*	*.395*	*.228*	*.397*	*.255*	*.139*	*.399*	*.210*	*.106*
	2000	*.508*	*.395*	*.228*	*.397*	*.255*	*.139*	*.398*	*.210*	*.106*
50	200	*.683*	*.455*	*.250*	*.607*	*.390*	*.213*	*.413*	*.224*	*.115*
	2000	*.684*	*.456*	*.250*	*.607*	*.391*	*.213*	*.412*	*.224*	*.115*
100	200	*.715*	*.464*	*.253*	*.670*	*.427*	*.232*	*.467*	*.266*	*.141*
	2000	*.715*	*.464*	*.253*	*.670*	*.427*	*.232*	*.467*	*.267*	*.140*

Figure Caption

Figure 1. An illustration of a family of Lorenz curves, each plotting cumulative percentage of household income as a function of percentile of income (the diagonal represents the locus of complete equality).


Cumulative percentage
100
80
60
40
20
0
0
20
40
60
80
100
Percentile of income