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1 April 2011

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First Version: April 2011
This Version: November 2013

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Abstract

We describe a set of model-dependent statistical benchmarks that can be used to estimate and evaluate dynamic models of firms' investment and financing. The benchmarks characterize the empirical counterparts of the models' policy functions. These empirical policy functions (EPFs) are intuitively related to the corresponding model, their features can be estimated very easily and robustly, and they describe economically important aspects of firms' dynamic behavior. We calculate the benchmarks for a traditional trade-off model using Compustat data and use them to estimate some of its parameters. We present two Monte Carlo exercises, one that shows EPF-based estimation has lower average bias and lower variance than traditional moment-based estimation and another that shows EPF-based tests are better at detecting misspecification.

Keywords: trade-off model,structural models of capital structure, estimation of dynamic models, indirect inference, model evaluation

JEL Classification Codes: C14, C52, C61; G31, G32

1 Introduction

A large literature in finance and economics studies dynamic models of the firm, in which agents, period by period, optimally choose optimal decisions as a function of the current state of their environment.[1] While these sophisticated dynamic programming problems are analytically complex and often only have approximate numerical solutions, this general research endeavor is promising: investing and financing are intrinsically dynamic problems that can only have a quantitatively satisfactory representation in a dynamic model. Moreover, dynamic models allow researchers to extract a wealth of time series and cross sectional predictions with which to compare model and data. This richness allows researchers to discipline dynamic models more than static models. In general, this discipline is useful because it allows the evaluation of different models' ability to match the data, and ultimately, to establish the quantitatively better theoretical bases for understanding firms' behavior.

However, an issue of concern in the financing and investment literature is that there is no agreement on the right predictions to use for disciplining these quantitative dynamic models. Different researchers make different choices about which features of the data to consider, as exemplified in Table 1, which lists two sets of benchmarks used in the literature. The result is a wealth of different models, all of which claim that they successfully explain the data, a situation that puts in question the scientific value of this type of research. A methodology in which models are not falsifiable is not useful. This paper argues that, instead of arbitrary features of the data, for each model there is a natural set of statistics to be used for estimation and evaluation. Moreover, these statistics are, to a large extent, common across existing specifications of investment and financing problems. Therefore, they can be thought of as benchmarks that dynamic capital structure models should aim

[1] For a comprehensive review, see Strebulaev and Whited (2012).

to match. While we provide a number of these benchmarks for a particular capital structure model, similar objects are easily calculated for any model. In this sense, we provide one practical method to answer the typical "Which Moments to Match?" question of Gallant and Tauchen (1996).

We derive these benchmarks from the model's policy functions, which describe the optimal response of the firm (the control variables) as a function of its environment (the state variables). Our central argument is that the empirical counterpart of these policy functions should be informative in the evaluation and estimation of a dynamic model. Therefore, we develop a method for using robust estimates of the main features of these empirical policy functions (EPFs) to estimating the parameters of dynamic models. Of course, there are many ways to characterize an empirical relationship. In this paper, we focus mainly on the first-order features of these EPFs, namely their slopes. Among these features, we focus on statistically robust estimators, which insures that we are looking at representative features of the data.

To estimate our EPFs we use only the within-firm variation of each of the relevant variables. This practice is consistent with a typical model setup in which firms are ex-ante homogeneous. The use of within-firm variation implies that we do not target the mean values of any of the state or control variables. Instead, we use their variation across time and states of nature. While it is possible that, for a particular model, there is a unique feature of the data (such as the mean of a variable) that can help estimate its parameters, our argument here is that a common set of benchmarks that captures the first order features of the empirical policy functions should be a starting point with which to evaluate these models.

Using EPFs as benchmarks for estimating and evaluating models confers several advantages over the practice of using arbitrary moments to estimate models. First, these benchmarks capture an economically important fraction of the variance of the control variables in the data. For example, out of the total dispersion of the investment to capital ratio in the data, around 20% can be captured by the slope of the EPFs. Thus, our benchmarks are describing a large fraction of what firms actually do.

Second, these benchmarks are intrinsically related to the economic dynamics of the model. The point of writing dynamic models is that we have dynamic data to test them against and therefore powerful tests of these models need to involve facts that describe firms' dynamics. In contrast to this principle, models in the literature are often estimated or tested with respect to sample averages, such as mean leverage or mean cash holdings.

The third argument is that these quantities are often used already in some structural estimations, so they are not large departures from tradition. Instead what we propose in this paper is a transparent, simple, robust method for choosing and computing benchmarks that can potentially let us compare one model to another meaningfully.

We conduct several Monte Carlo exercises to gauge the finite-sample performance of estimators that use empirical EPFs, and we compare these estimators to ones that use traditional moments. We find two important results. First, estimation with these EPF benchmarks generates parameter estimates that are less variable and have less bias than those generated with moments. Second, estimation with these EPF benchmarks leads to specification tests that have more power to reject misspecified models.

Although our paper is clearly related to the many applied papers that have used simulated method of moments to estimate the parameters of dynamic models, it is more closely related to a set of papers that deal with estimation of dynamic oligopoly games in industrial organization. For example, Rust (1994) describes a set of methods for solving and estimating Markov Decision Processes and Eckstein and Wolpin (1989) provide a survey of estimation methods for discrete choice models. Aguirregabiria and Mira (2007) and

Bajari, Benkard, and Levin (2007) describe two-step algorithms for estimating a dynamic game under particular assumptions about the game's equilibrium. The estimation method proposed by Bajari et al. (2007) is closely related to the one in this paper and focuses on estimating the policy functions as the essential quantities to input into a simulated minimum distance estimator. Our paper stands apart from this literature because our methods are "full-solution" methods that actually match theoretical and estimated policy functions.

This paper proceeds as follows: Section 2 of the paper presents a generic model and detailed steps for benchmark calculation, a measure of the explanatory power of a model, and a brief description of indirect inference and bootstrapping as applied here; section 3 presents a generic dynamic trade-off model of capital structure, along with its benchmarks, and a comparison of indirect inference performed with traditional moments and with our benchmarks; section 5 presents two Monte Carlo exercises, the first, a parameter recovery experiment that shows our benchmarks generate parameter estimates with lower volatility and bias, and the second that shows that our benchmarks are better at detecting misspecification; section 6 concludes.

2 Benchmarks and Estimation

2.1 Generic Model

The models of the firm that we consider can be described generically in terms of a Bellman equation:

$$V(x) = \max_{h} \left\{ D(x,h) + \beta \mathbb{E}\left(V\left(x'\right) \mid x, h\right) \right\}, \tag{1}$$

in which x is an $M = N + K$ vector of state variables, with a prime indicating tomorrow and no prime indicating today. This vector can contain endogenous state variables that can be directly manipulated by the N vector of control variables, h, as well as K exogenous stochastic state variables, which follow a Markov process. The control and state variables are linked via a law of motion given by

$$x' \equiv g(x, h) \tag{2}$$

$V(x)$ represents the market value of the firm's equity, $D(x, h)$ represents payments to or from shareholders, β is the discount factor and $\mathbb{E}(\cdot \mid x, h)$ is the expectation with respect to the Markov transition function, given x and h. For example, the variable x typically contains the firm's capital, leverage, and a profitability shock. The vector of control variables typically contains investment and debt issuance. The firm observes x and then maximizes the present discounted value of the sum of current and future dividends by setting the control variable h optimally during each period.

In this generic setup, the solution of the model consists of the optimal policy function $H(x)$ and the value function of the firm $V(x)$. The policy and value functions satisfy the following system:

$$H(x) = \underset{h}{\operatorname{argmax}} \left\{ D(x, h) + \beta \mathbb{E} \left(V\left(x'\right) \mid x, h \right) \right\} \tag{3}$$

and

$$V(x) = D(x, H(x)) + \beta \mathbb{E} \left(V\left(x'\right) \mid x, H(x) \right) \tag{4}$$

For simplicity, we assume that all of the state variables are observable. As we show below, however, in many of the cases in which some of the stochastic state variables are unobservable, the policy function can be expressed in terms of observable transformations of the state variables.

2.2 Benchmarks

Given the process for exogenous variables, the policy function $H(x)$ characterizes the solution of the model. It is the main object that translates the assumptions of the model into a functional prediction about the firm's actions in different situations. Therefore, a direct, simple and theoretically motivated way to evaluate a dynamic model is to evaluate its ability to replicate the firms' observed policies. In this paper we argue that a good approach to doing this is to characterize firms' policy functions empirically, and then use this characterizations as the objectives in structural model estimation and evaluation. This section describes one robust way to characterize the first-order and the second-order features of these objects, i.e. their slope and convexity.

Empirical Policy Functions

In order to obtain empirical policy function estimates it is necessary to have estimates of the state in which firms are and of the policies that they follow in each state. We use a simple process, inspired in the portfolio formation frequently used in the asset pricing literature, to summarize these objects in a few key numbers.

The empirically estimated equivalent of the function $H(x)$ is an estimated function $\hat{H}$ such that:

$$\hat{H}(x) = \left(\hat{H^1}(x), \hat{H^2}(x), \ldots, \hat{H^N}(x)\right). \tag{5}$$

where $\hat{H^n}(x)$ represents the average or the median behavior along control dimension n for firms that observe a value x for the state variable. A typical procedure is to use regression analysis to estimate H as a linear function of x, however that procedure is not robust to the presence of outliers or to the possibility of nonlinearity. Therefore we characterize $H\hat{(x)}$ by splitting the state space into percentile bins for each of the state variables and then estimating the median choice for the control variables for all firms within a particular bin as described in detail below.

Step 1: Demean each state and control variable at the firm level. This step is important because the models described and exemplified above are typically models where firms are ex-ante homogenous. Also, we are concerned mainly with the dynamics of the different variables, as opposed to any cross-sectional variation. Label the demeaned state variables as $x_{i,t}$ and control variables as $h_{i,t}$ for each firm-period observation.

Step 2: Generate B equally spaced bins across each of the state variables of the model. For example, if we use five binds, we define the bins as the 0%–20%, 20%–40%, 40%–60%, 60%–80% and 80%–100% percentiles of each state variable and label them from 1 to 5. Classify each observation of each state variable as belonging into one variable-specific bin. This classification is done independently for each variable, that is, it is non-sequential sorting. Each firm-period observation is therefore given a classification as $b(x_{i,t}) = (b_1, b_2, \ldots, b_M)$, with each b_m representing state variable m's bin classification for that firm in that period. Similarly, classify the control variables in bins $c(h_{i,t}) = (c^1, c^2, \ldots, c^N)$.

Step 3: Estimate the policy function for control variable n as functions of state variable m as the median observed choice within each of the (composite) bins:

$$\hat{H}^n_m(B) = \text{median}_{\{i,t|b_m(x_{i,t})=B\}}(h^n_{i,t}). \tag{6}$$

Step 4: Finally, characterize the features of $H\hat{(}x)$ by comparing the policies over bins across each state variable. Here, we consider both slope and curvature. The first-order features (the slope) of the policy function can be characterized as the ratio of the difference between the median policies in the first and last bins to the ratio of the medians of the state variable in those bins:

$$\beta^n_m(1,B) = \frac{\hat{H}^n_m(B) - \hat{H}^n_m(1)}{\tilde{x}\mid_{b_m(x)=B} - \tilde{x}\mid_{b_m(x)=1}}, \tag{7}$$

in which $\tilde{x}\mid_{b_m(x)=k}$ represents the median value of the state variable x in the k^{th} bin of that variable.

The second order features (the curvature) of the policy function can be characterized as the ratio of the change in slope over the state variable to the range of that particular state variable.

$$\Gamma^n_m(1,B) = \frac{\beta^n_m(B/2,B) - \beta^n_m(1,B/2)}{(\tilde{x}\mid_{b_m(x)=B} - \tilde{x}\mid_{b_m(x)=1})/2} \tag{8}$$

2.3 Potential Explanatory Power

An important question in evaluating a dynamic model of the firm is how much of the actual behavior of the firm can we expect to rationalize. We define a robust measure of this quantity as follows:

$$PE^n_m = \frac{\tilde{h^n_{i,t}}\mid_{i,t|b_m(x_{i,t})=B} - \tilde{h^n_{i,t}}\mid_{i,t|b_m(x_{i,t})=1}}{\tilde{h^n_{i,t}}\mid_{i,t|c^n(h_{i,t})=B} - \tilde{h^n_{i,t}}\mid_{i,t|c^n(h_{i,t})=1}} \tag{9}$$

where $\tilde{h^n_{i,t}}\mid_{i,t|c^n(h_{i,t})=k}$ represents the median of the n^{th} control variable in the k^{th} quintile of the empirical h^m distribution and $\tilde{h^n_{i,t}}\mid_{i,t|b_m(x_{i,t})}= k$ represents the median value of h^n for data observations where the m^{th} state variable is in the k^{th} quintile of the empirical x_m distribution. A large value of this ratio suggests that a large fraction of the total variance of the control variable h^n in the data can potentially be explained by a model in which h^n is the control variable and x_m is the state variable.

2.4 Benchmark's Variance-Covariance

The statistics described above are not traditional moments in the sense of being sample averages of a function of each observation. Therefore, the standard methods to obtain estimates of the variance and covariance of an estimate or vector of estimates are not useful in this case. This section describes how we estimate the variance and covariance of the statistics described above.[2]

2.5 Bootstrapping

One way to obtain a measure of the variance of the estimators described above is through bootstrapping. The bootstrapping procedure uses the variation of the estimator across different artificial samples as a measure of the volatility of the estimator. These artificial samples are drawn from the original sample and used to recalculate the estimator. The estimation is then performed for a J bootstrapped samples. The variation of the estimator in the population is then estimated to be similar to the variation across the different estimates obtained in this way.

We perform bootstrapping on our estimators by sampling with replacement, taking the time series of each firm as one element of the sample. The estimate of the variance of an estimator $\hat{\theta}$ is

$$V(\hat{\theta}) = (1/J)\sum_{j=1}^{J}\Big(\theta s_j - \bar{\hat{\theta}})'(\theta s_j - \bar{\hat{\theta}}\Big) \tag{10}$$

where is θs_j is the estimator evaluated with the sample s_j and $\hat{\theta}$ is the mean over $j \in \{1, \ldots, J\}$ of θs_j.

[2] An alternative method is the use of influence functions, as described in the appendix.

2.6 Indirect Inference

Once the above benchmarks are calculated we can use them to estimate a model through the indirect inference procedure in Smith (1993) and Gourieroux, Monfort, and Renault (1993).We now briefly describe the procedure, as it applies to our policy function benchmarks. First, we now explicitly allow the policy and value functions, $h = H(x, \theta)$ and $V(x, \theta)$, to depend on a vector of structural model parameters, θ. These parameters include such quantities as the curvature of a production function, the variance of an exogenous state variable, or a cost of external finance. On an intuitive level, it might be desirable to estimate θ via maximum likelihood. However, for most dynamic models of the firm, a closed-form likelihood is not available. Indirect inference fills this gap by using an auxiliary model, with its own parameter vector, b. This auxiliary model should ideally capture important features of the data. The goal is then to estimate θ by minimizing the distance between the parameter vector of the auxiliary model, b, estimated with a real data set and the same parameter parameter vector estimated with data simulated from a model.

Let y_T be a real data matrix of length T, Without loss of generality, the parameters of the auxiliary model can be represented as the solution to the maximization of a criterion function

$$b_T = \underset{b}{\operatorname{argmax}}\, J_T\left(y_T, b\right),$$

The simulation of data from the model in section 2.1 proceeds as follows. First, given θ, pick a starting value for x, x_0. Next, update x_0 by by using the policy function to generate h, and then using the law of motion to generate x', and so on. Let y_t^s be the resulting simulated data matrix of length T from simulation s, $s = 1, \ldots, S$. For each of these data sets, estimate b^s by maximizing an analogous criterion function

$$b_T^s(b) = \arg\max_b J_T(x_T^s, b^s(b)),$$

Note that the $b_T^s(b)$, as explicit functions of the structural parameters, θ.

The indirect estimator of b is then defined as the solution to the minimization of

$$\begin{aligned}
\hat{\theta} &= \arg\min_b \left[b_T - \frac{1}{S}\sum_{h=1}^{S} b_T^s(\theta) \right]' \hat{W}_T \left[b_T - \frac{1}{S}\sum_{h=1}^{S} b_T^s(\theta) \right] & (11) \\
&\equiv \arg\min_b \hat{G}_T' \hat{W}_T \hat{G}_T, & (12)
\end{aligned}$$

in which $\hat{W}_T$ is a positive definite matrix that converges in probability to a deterministic positive definite matrix W. We use the the optimal weight matrix, which is the inverse of the covariance matrix of θ described in Section 2.5.

The indirect estimator is asymptotically normal for fixed S. Define $J \equiv \text{plim}_{T\to\infty}(J_T)$. Then

$$\sqrt{N}\left(\hat{\theta} - \theta\right) \xrightarrow{d} \mathcal{N}\left(0, \text{avar}(\hat{\theta})\right)$$

where

$$\text{avar}(\hat{\theta}) \equiv \left(1 + \frac{1}{S}\right) \left[\frac{\partial J}{\partial b \partial \theta'} \left(\frac{\partial J}{\partial b} \frac{\partial J'}{\partial b} \right)^{-1} \frac{\partial J}{\partial \theta \partial b'} \right]^{-1}.$$

The technique provides a test of the overidentifying restrictions of the model, with

$$\frac{TS}{1+S} \hat{G}_T' \hat{W}_T \hat{G}_T$$

converging in distribution to a χ^2, with degrees of freedom equal to the dimension of θ minus the dimension of b.

3 Trade-Off Model

In this section we show the results of an indirect inference structural estimation using EPFs, and compare the results to one that uses traditional moments. We simulate data from a trade-off model and then perform indirect inference on data simulated by that model. We then describe the ability of the two indirect inference procedures to recover the parameters of the original model. We compare the two estimations in terms of average bias and estimator volatility. The rest of this section describes an example of a model in this literature. It is a simplified version of Hennessy and Whited (2005):

The firm's cash flow

We consider a firm that uses capital, K_t to generate operating income according to $A_t K_t^{\alpha}$, where K_t is capital, $0 < \alpha < 1$ is a parameter that governs returns to scale, and A_t is a productivity shock. The productivity shock, A_t, is lognormally distributed and follows the process given by:

$$\ln(A_t) = \bar{A} + \rho \ln(A_{t-1}) + \sigma \varepsilon_t, \qquad \varepsilon_t \sim \mathcal{N}(0,1) \tag{13}$$

The firm's cash flow (E_t) is its operating income plus its net debt issuance ΔB_t, minus its net expenditure on investment, $C(I_t)$, and minus its interest payments on debt, $B_t r_t^B$:

$$E_t = A_t K_t^{\alpha} - C(I_t) + \Delta B_t - B_t r_t^B (B_t, K_t) \tag{14}$$

In 14, I_t is defined by a standard capital stock accounting identity:

$$K_{t+1} \equiv K_t (1 - \delta) + I_t \tag{15}$$

Similarly, net debt issuance, ΔB_t is defined by

$$B_{t+1} \equiv B_t + \Delta B_t \tag{16}$$

Dividends and equity issuance

The firm's dividends and equity issuance are defined in terms of the firm's cash flows. A positive cash flow implies the firm's optimal decision is to pay dividends ($D_t = E_t$) to its stockholders, a negative cash flow implies that the firm's optimal decision is to set dividends at 0 and instead obtain funds from the equity market ($X_t = -E_t$).

$$E_t > 0 \quad \Rightarrow \quad D_t = E_t, X_t = 0 \tag{17}$$

$$E_t \leq 0 \quad \Rightarrow \quad D_t = 0, X_t = (1+\lambda)(-E_t), \tag{18}$$

in which λ stands for the proportional cost of issuing equity.

Real sector frictions

The firm faces a set of frictions. Consistent with most of the literature it faces convex costs of investment as well as some degree of investment irreversibility. The firm's cost of investment function includes the cost of purchasing the capital, plus a convex cost of investment and plus an investment irreversibility term.

$$C(I_t) = I_t + \gamma K_t (I_t/K_t)^2 - \xi K_t \mathcal{I}(I_t < 0), \tag{19}$$

where γ represents the magnitude of convex costs of investment, ξ represents the magnitude of fixed costs of investment, and $\mathcal{I}(I_t > 0)$ represents an identifier function that is 1 if $I_t > 0$ and 0 otherwise.

Interest rate on debt

The interest rate on debt is a risk free rate plus a 'risk premium', an increasing, convex function of leverage:

$$r_t^B = R^{rf} + R^{rp}(B_t/K_t)^2 \tag{20}$$

The firm's optimization problem

The firms problem is to maximize the discounted value of dividends for current owners of the firm. With this objective the firm chooses investment and net debt issuance. The resulting Bellman equation is then given by:

$$V(K_t, B_t, A_t) = \max_{I_t, \Delta B_t} \left\{ D_t \mathcal{I}_{E_t>0} + (1+\lambda) X_t \mathcal{I}_{E_t<0} + \mathbb{E}_t V(K_{t+1}, B_{t_1}, A_{t+1}), \right\}, \qquad (21)$$

subject to (15) and (16). Here, $\mathcal{I}$ is an indicator function.

State space and control space transformations

In the model above, K_t, A_t and B_t are the state variables and I_t and ΔB_t are the control variables. However, A_t is an unobservable shock. Therefore, to estimate the parameters of this model, we need to work with observable transformations of the state and control variables. Further, because the *levels* of the *real* K_t and B_t are only defined up to a constant of proportionality defined by an arbitrary price index. We therefore work with variables in ratio form. We use the investment rate I_t/K_t and the debt issuance rate $\Delta B_t/K_t$ as the control variables of the model. Similarly, we use profitability, $\Pi_t \equiv A_t K_t^{\alpha}/K_t$, and leverage, $L_t = B_t/K_t$, as the state variables of the model.[3]

4 Benchmarks for Trade-Off Model

4.1 Data

We draw our sample of firms from the Compustat database from 1962 to 2012. We screen the sample as follows. The firm must have a CRSP share code of 10 or 11. We then drop all firms with fewer than two years of data, or that belong to the financial (SIC code 6) or regulated (SIC code 49) sectors. Finally, we observations in which any of the

[3]Note that, as in most capital structure and investment models, once A_t or Π_t as defined above are given, the firms Value V or Tobin's Q (V_t/K_t) are *not* state variables of the model.

variables we use are missing. We define the following variables to be used in the rest of the analysis. Book Leverage is (DLC +DLTT)/AT, Profitability is OIBDP/GPPE, Investment is CAPX/GPPE, and Debt Issuance is Δ (DLC + DLTT)/AT.

4.2 Summary Statistics

Table 2 presents the summary statistics of the state and control variables of the model as defined above. Note that the values are consistent with other similar studies. The zero value for the 10th percentile of investment rates and the 50th percentile of debt issuance is consistent with research on lumpiness in the investment and financing literature.

The variation of the state variables is an essential quantitative feature of the data. As the summary statistics show, the bottom quintile of profitability is about 16% lower than the mean, while the top quintile is about 13% higher, so that the 5–1 inter-quintile range is about 29% of average profitability. With respect to leverage it shows that the inter-quintile range is about 62% of total assets, evenly distributed above and below the mean.

4.3 First-Order Benchmarks

Table 3 describes the estimates of the first-order features (i.e the slopes) of the firm's policy functions. It shows the result of grouping each firm year observation according to the value of productivity and leverage into quintiles and then obtaining the median policy of firms in these bins in terms of investment and debt issuance. The main results are the policy function slopes in the last column of the table. It shows that each percentage point of leverage reduces investment by 0.32 percentage points, and reduces debt issuance by 0.17 percentage points. Also, it shows that each percentage point of profitability increases investment by 1.26 percentage points and increases debt issuance by 0.07 percentage points. The standard errors presented in this table suggest these figures are estimated with high precision, with all of the standard errors lying below 10% of the estimated value.

4.4 Second-Order Benchmarks

Table 4 describes the estimates of the second order features in the data. It shows that investment is concave in leverage, that is, investment is more affected by debt at high leverage levels. In contrast, debt issuance policy is convex in leverage, that is, it is *less* sensitive to leverage at high leverage levels. Also, investment convex in profitability, that is, it is more sensitive to profitability at high profitability levels. Finally, it shows that the slopes of debt increase relative to profitability are not significantly different at high and low profitability levels. Note that for brevity, we do not use these benchmarks in the estimation exercises that follow.

4.5 Covariance Matrix of the Policy Function Benchmarks

The indirect inference procedure requires, ideally, that one use the inverse of the variance-covariance matrix of the policy function slopes, $\hat{W}_T$, to normalize the distance between the real-data and simulated policy function slopes. As described in section 2.4 above, one way to estimate this matrix is through bootstrapping, which is what we do in this paper. Below we compare the performance of a traditional moments-based estimation with an estimation based on our policy function benchmarks. As a first step, Tables 6 and 7 describe the bootstrap-estimated variance-covariance matrices for both the moments and policy functions we use. The main feature of this table is that the traditional moments are substantially less variable than the EPF benchmarks. This result makes sense in that fewer observations are used in the estimation of the empirical policy functions.

4.6 Estimation Results

Table 8 describes the results of using the two different sets of empirical benchmarks to estimate the parameters of the Trade-Off model. It shows that the results are substantially different depending on the benchmark used. The EPF-based benchmarks result in a higher estimate of capital adjustment costs, a lower estimate of the credit risk growth coefficient and of the fixed cost of investment. The minimum value of the distance function is much larger in the case of the EPF model. We interpret this difference as implying that the EPF-based benchmarks are more powerful at detecting model misspecification.

Table 9 displays the results of the indirect inference estimation of the trade-off model from the point of view of the value of the different benchmarks with each of the two estimated sets of parameters. It shows that, as expected, the EPF-based inference generates a model whose benchmarks are closer to the EPF-benchmarks in the data, and vice versa, the TM-based inference generates a model whose benchmarks that are closer to the TM benchmarks in the data.

Moreover, this table lets us highlight one of the key advantages of focusing on EPF-based benchmarks. The advantage is that the the distance between the EPF-based benchmarks in the data and in the estimated model have a clear, useful interpretation, in contrast to the same distance between TM-benchmarks. Comparing the first and third row of the second set of columns of the table shows that in the estimated Trade-Off model a unit of leverage increase leads to 0.421 units decrease in the investment rate, compared to 0.321 in the data, a unit of leverage decreases debt issuance by 0.444 units, compared to 0.174 in the data, a unit of profitability increases investment by 1.013 units compared to 1.258 in the data and a unit of profitability increases debt issuance by 0.034 compared to 0.074 in the data. In other words, the Trade-Off model yields an investment and debt issuance that are too sensitive to leverage, and not sensitive enough to profitability. These conclusions are a useful way to contrast the model with the data from the point of view of capital structure theory.

Finally, comparing the first and second lines of this table suggest that although the Indirect Inference estimation of the Trade-Off model with TM-based benchmarks is able to generate TM-benchmarks that are close in the data and in the model, the estimated model is not able to replicate the main quantitative relationships between variables in the model. Investment and debt issuance are an order of magnitude too sensitive to leverage and debt issuance is an order of magnitude to sensitive to profitability. While the EPF-basd estimation leads TM-based benchmarks that are also very much off the mark, it is not clear how to interpret these distances meaningfully.

4.7 Potential Explanatory Power of the Trade-Off Model

Relative Variation: State and Control Variables

An essential measure of whether our theories explain the data is one that compares the relative variation of explanatory and dependent variables. The figures in table 5 show this comparison. These are one set of benchmarks for dynamic model evaluation: they describe the relative variation in the data between state variables and control variables. Under this measure a quantitatively good dynamic model of investment and debt issuance that as a function of changing profitability and current leverage is one that replicates the relative variation in investment and debt issuance with respect to leverage and profitability.

Table 5 describes this relative variation. It shows that the variation range of investment along the leverage state variable is 13.1% of the total variation range of investment. Similarly the variation of debt issuance along the leverage state variable is equivalent to 16.3% of the total variation of debt issuance. Along the profitability dimension the values are 25%

for investment and 3.7% for debt issuance. These figures provide a measure of how much of the variation in each of these variables we could expect a dynamic model with leverage and profitability as state variables to explain.

5 Monte Carlo Exercises

This section describes the results of a set of Monte Carlo experiments that put to test the intuition described above. These results are important in the sense that they show that in a practical sense, estimation using EPF-based benchmarks produces better results than estimation using traditional moments. We design these experiments as follows. Each Monte Carlo is based on 100 simulated data sets. Each data set has a length of 50 and a cross-sectional size of 3,000. These dimensions are roughly the average time-series and cross-sectional dimensions of our real data set. We create our simulated samples as follows. First, we choose values for three key parameters: the variable and fixed costs of adjustment of the capital stock, $\gamma = 1$ and $\xi = 0.03$, and the premium on debt financing, $R^{rp} = 2$. To keep the Monte Carlo tractable, we treat the curvature of the profit function, α, and the parameters that govern the process for A_T, σ and ρ, as known, setting them to 0.8, 0.15, and 0.65, respectively.

5.1 Parameter Recovery

Table 10 describes the mean and the variance of two sets of statistics that can be used to estimate a structural model. The first one is a set of traditional moments. The second set corresponds to the slopes of the empirical policy functions as described above. The statistics come from simulated data from the model above. The first column of the table shows the simulated means of the different benchmarks. The second and third columns describe the standard deviation of both sets of of statistics, which we compute in two ways. The first calculation is simply the standard deviation across the different Monte Carlo trials. The second measure comes from simulating the model once and then taking bootstrapping the benchmark by sampling with replacement over the firms in the sample. This exercise suggests that, in this example, bootstrapping provides very good estimates of the variance of the benchmark, which gives us confidence in the bootstrap estimates of the empirical variance covariance matrices given before.

Table 10 describes a set of empirical estimates that we use for structural estimation. It consists of two panels, each describing the means and two estimates of the standard error of that mean. The first standard error estimate, in column 2, consists of the error calculated as the standard deviation of the mean observed for different repetitions of model simulations. The second estimate, in column 2, uses bootstrapping calculating a set of means through sampling from individual firms in the first simulation. The table shows that, in the model, the firm level bootstrapping estimate of the standard error of the mean provides a good approximation to standard deviation of the estimate across different simulations.

Table 11 shows the results of a Monte Carlo experiment estimating the trade-off model described above using both the traditional moment-based and EPF benchmarks for estimation. Both methods are able to get close to the parameter values of the model. However, the estimation where direct inference uses EPF-type statistics performs better than the one where the same procedure uses traditional moment statistics. The EPF based estimation results in significantly lower bias in two out of three parameters, but most importantly in dramatically reduced variance of the estimators. The variance reduction is of the order of 80%.

5.2 Power of Specification Tests

The last comparison we perform between estimation with traditional moments and estimation with EPF-based moments focuses on mis-specification tests. An important concern in economics is our ability to tell whether a model fits the data well enough or not. One such test is a misspecification test. In the case of indirect inference, an available test is obtained by a comparison of the distance between the estimated parameters of the auxiliary model in the data and in the model with its theoretical distribution. Under conditions specified in Gourieroux et al. (1993) the objective function described in equation 11 has a well defined Ξ^2 distribution.

In order to asses the power of different benchmarks to detect mis-specification we perform a Monte Carlo experiment. The experiment consists of simulating data from a know model and then using simulated data from this known model to estimate the parameters of two models: a well specified model and a misspecified model. We perform both of these estimations twice, first using traditional moments as benchmarks and second with the EPF-based benchmarks. We then compare the rejection power of tests based on each one.

The misspecified model is similar to the one in section 3, but where the smooth risk premium function has been substituted with a collateral constraint. In other words, the interest rate equation, 20 in section 3, has been substituted for

$$r_t^B = R^{rf} IB_t < \lambda K_t + R^{rf} IB_t >= \mu K_t$$

i.e. a flat interest equal to the risk free rate as long as book leverage is below μ and a 100% interest rate otherwise. In the mis-specified model estimation, we estimate μ instead of estimating R^{rp}.

Table 12 presents the results of the exercise described above. It shows that while indirect inference of a well-specified model yields similar values for the objective function for both the TM and EPF benchmarks, this is not the case for a mis-specified model. We interpret this result as implying that the EPF benchmarks are (much) better at detecting mis-specification without rejecting the true model more often.

6 Conclusion

We describe a set of benchmarks that we use for the quantitative evaluation of dynamic corporate finance models. The benchmarks are a small set of numbers that characterize the empirical counterparts of the policy functions from these models, which provide the optimal firm policies, given the current state of the firm. We then describe a simple set of steps to calculate these EPF-based benchmarks. We argue that these benchmarks are intuitive, robust and theoretically motivated.

We then calculate these benchmarks for a typical dynamic capital structure model, and use them to estimate the model. We also estimate the model using traditional moments. We confirm that, in the estimation of the model, the choice of benchmarks is important. Different benchmarks generate different parameter estimates when using them for indirect inference.

We then show in first a Monte Carlo exercise that estimation with the proposed benchmarks results in less biased and less variable estimates of the parameters of the model, relative to a standard moments-based estimation. In a second Monte Carlo exercise, we show that a specification test based on our proposed benchmarks has more power than the same test based on traditional moments. It rejects the misspecified model more often, even though it rejects the correctly-specified model just as often.

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Table 1: Examples of matching statistics used in the capital structure literature

Hennessy and Whited (2005)	**Nikolov and Schmid (2012)**
Average investment/assets	Average of investment
Variance of investment/assets	Variance of investment
Average EBITDA/assets	Serial correlation of cash
Average debt-assets ratio(net of cash)	Average leverage
Average equity issuance/assets	Variance of cash
Frequency of equity issuance	Serial correlation of investment
Investment-q sensitivity	Average of Tobin's q
Debt-q sensitivity	Serial correlation of Tobin's q
Serial correlation of income/assets	Average cash
Std. Dev. of shock to income/assets	Serial correlation of leverage
	Covariance of cash and investment

Benchmarks correspond to Table II in Hennessy and Whited (2005) and to Tables 3 and 4 in Nikolov and Schmid (2012).

Table 2: Summary Statistics: State Variables

	mean	p50	sd	p10	p90
Market Leverage	0.26	0.18	0.53	0.00	0.67
Book Leverage	0.28	0.24	0.30	0.00	0.62
Profitability	0.08	0.10	0.19	−0.06	0.23
Market to Book	12.13	3.60	54.88	1.06	24.29
Observations	232033				

This table describes summary statistics for the state variables in this paper. Each row is calculated using the observations for which the variable is available, and so the sample is different from row to row.

Table 3: Empirical Policy Function Slopes

	Control Variable Bin 1	Control Variable Bin 5	State Variable Range	**1 to 5 Slope**
β_L^i	0.017	-0.059	0.237	**-0.320**
	(0.002)	(0.002)	(0.003)	**(0.013)**
β_L^{di}	0.007	-0.034	0.237	**-0.174**
	(0.001)	(0.001)	(0.003)	**(0.007)**
β_Π^i	-0.092	0.059	0.120	**1.258**
	(0.002)	(0.002)	(0.001)	**(0.027)**
β_Π^{di}	-0.009	0.000	0.120	**0.074**
	(0.001)	(0.000)	(0.001)	**(0.005)**

Columns 1 and 2 describe the median of the policy function observations at time t for those firms classified in the first and 5th quintile of the state variable at time $t-1$. Columns 3 and 4 describe the median of the state variable at time $t-1$ in the 1st and 5th quintile bins for that variable. Column 5 describes the slope of the policy function calculated as (Control Bin 5- Control Bin 1)/(State Bin 1 - State Bin 5). Standard errors are calculated with bootstrapping, by sampling at the firm level. β_L^i = slope of investmtent policy over leverage state, β_L^{di} = slope of debt increase policy over leverage state, β_Π^i = slope of investment policy over profitability state, and β_Π^{di} = slope of debt increase policy over profitability state. Data are from Compustat from 1987 to 2012. Variables are demeaned at the firm level. Variable definitions are available in the appendix.

Table 4: Table of Empirical Policy Function Convexities

	Slope		State Variable	**Convexity**
	Bin 1-3	Bin 3-5	Range	
Γ_L^i	-0.316	-0.324	0.118	**0.061**
	(0.019)	(0.024)	(0.001)	**(0.293)**
Γ_L^{di}	-0.298	-0.059	0.118	**-2.024**
	(0.011)	(0.008)	(0.001)	**(0.113)**
Γ_Π^i	1.155	1.377	0.060	**-3.716**
	(0.042)	(0.037)	(0.001)	**(0.978)**
Γ_Π^{di}	0.071	0.079	0.060	**-0.126**
	(0.008)	(0.013)	(0.001)	**(0.323)**

Columns 1 and 2 describe the slope of the policy function approximated from the medians in the state variable quintiles 1 to 3 and 3 to 5. Columns 3 describes the corresponding range for the state variable. Column 4 describes the convexity of the policy function calculated as (Slope 3-5- Slope 1-3)/(State Var. Range). Standard errors are calculated with bootstrapping, by sampling at the firm level. Γ_L^i = convexity of investmtent policy over leverage state, Γ_L^{di} = convexity of debt increase policy over leverage state, Γ_Π^i = convexity of investment policy over profitability state, and Γ_Π^{di} = convexity of debt increase policy over profitability state. Data is from Compustat from 1987 to 2012. Variables are demeaned at the firm level. Variable definitions are available in the appendix.

Table 5: Range of Control Variable Variation in the Data

Control Variable	Total Range	Range Over State Variable	PE_{SV}^{CV}
State Variable: Leverage			
Investment	0.60	-0.077	**0.131**
Debt Increase	0.25	-0.041	**0.163**
State Variable: Profitability			
Investment	0.60	0.152	**0.251**
Debt Increase	0.25	0.009	**0.037**

The first column describes the q5-q1 range of the control variable. The second column describes the range of the same variable over the q5-q1 range of the corresponding state variable, leverage(L) in the top panel and Profitability(Π) in the second one. The third column describes the ratio between these two.

Table 6: Variance-Covariance Matrix, Traditional Moments

	$\mu(L)$	$\sigma^2(L)$	$\mu(i)$	$\sigma^2(i)$
$\mu(L)$	32.1			
$\sigma^2(L)$	73.2	1003.00		
$\mu(i)$	0.691	8.63	3.18	
$\sigma^2(i)$	-0.141	0.06	0.99	1.05

Figures describe the variance-covariance matrix of the vector of statistics listed in the first column, estimated using bootstrapping over firms in the sample. All variables are multiplied by 10 million. Data is from Compustat. Variable definitions are in the appendix.

Table 7: Variance-Covariance Matrix, EFP based benchmarks

	β^i_L	β^{di}_L	β^i_Π	β^i_Π
β^i_L	106			
β^{di}_L	56.9	459		
β^i_Π	-7.67	-20.3	639	
β^{di}_Π	6.69	41.1	127	205

Figures describe the variance covariance matrix of the vector of statistics listed in the first column, estimated using bootstrapping over firms in the sample. All variables are multiplied by 10 million. Data is from Compustat. Variable definitions are in the appendix.

Table 8: Structural Estimates of Trade-Off model Parameters with Different Benchmarks

Benchmark type	Estimated Parameters γ	R^{rp}	ξ	Minimum of Objective
Traditional moments	-0.028	0.0525	0.046	16859.2
Empirical policy functions	1.031	0.658	0.035	28281.3

Coefficients obtained through indirect inference on the three parameters, Adjustment Costs, Credit Risk and Fixed Costs. Indirect inference performed by minimizing the (inverse covariance matrix weighted) distance of the simulated values of each set of benchmarks from the corresponding values found in Compustat. γ is the cost of capital adjustment parameter, R^{rp} is the size parameter for the risk premium function, and ξ is the parameter of the non-convex costs of capital adjustment.

Table 9: Benchmark Values at Indirect Inference Estimates, Trade-Off Model

	Traditional moments				Empirical policy functions			
	$\mu(L)$	$\sigma^2(L)$	$\mu(i)$	$\sigma^2(i)$	β^i_L	β^{di}_L	β^i_Π	β^{di}_Π
Data	0.285	0.088	0.095	0.101	-0.322	-0.174	1.258	0.074
Tradtional moments	0.235	0.013	0.135	0.098	-2.655	-1.963	1.733	0.678
Empirical policy functions	0.077	0.000	0.104	0.010	-0.421	-0.444	1.013	0.034

Value of traditional moment benchmarks and EPF-based benchmarks at Indirect Inference estimates of the Trade-Off model parameters. The first column describes the source of the benchmarks: Traditional Moment based Indirect Inference (TM), Empirical Policy Function based Indirect Inference (EPF), or data.

Table 10: Measures of the Standard Error of Benchmarks, Simulated Data

Benchmark	$\hat{\mu}$	$SE(\hat{\mu})$	B.S $SE(\hat{\mu})$
Traditional moments			
$\mu(i)$	10.42%	0.112%	0.241%
$\sigma^2(i)$	0.84%	0.009%	0.009%
$\mu(L)$	4.3%	0.023%	0.039%
$\sigma^2(L)$	0.024%	0.002%	0.001%
Empirical policy functions			
β^i_L	-4.53	0.322	0.510
β^{di}_L	-0.88	0.035	0.034
β^i_Π	0.95	0.012	0.009
β^{di}_Π	0.05	0.027	0.029

This table describes different measures of estimate standard errors. The means, sample volatility estimate, and bootstrapping estimate over repetitions are calculated with 30 repetitions of the model simulation. The bootstrapping within one repetition estimate is calculated by bootstrapping across firms in a single repetition of the simulation. i stands for the investment rate I/K, , di stands for debt issuance rate DI/K, Π stands for profitability $AK^{\alpha-1}$, and L stands for leverage.$\mu(\cdot)$ stands for the mean, $\sigma^2(\cdot)$ stands for the variance and β^n_m stands for the first order characterization of the policy for control variable n as a function of state variable m.

Table 11: Comparison of Model Estimation with Different Matching Objectives

Model/Estimation	Estimates		
	γ	R^{rp}	ξ
Parameter values	1.00	2.00	0.03
EPF-based estimation			
Mean Estimate	1.11	1.90	0.0286
Average Bias	0.11	-0.10	-0.0014
S.E.	0.0064	0.0639	0.0002
TM-based estimation			
Mean Estimate	0.72	2.02	0.0373
Average Bias	-0.28	0.02	0.0073
S.E.	0.0353	0.0728	0.0015

Monte Carlo mean and standard error of Indirect Inference parameter estimates obtained with two sets of benchmarks, and corresponding model parameters.

Table 12: Comparison of Mis-specification Detection Power

Estimation	Objective Function
Correct Model, TM-based estimation	15.5
Correct Model, EPF-based estimation	11.9
Mis-specified Model, TM-based estimation	233.8
Mis-specified Model, EPF-based estimation	1274.5

Column 1 describes the model that was estimated. 'Correct model' implies that the same model was used to produce the data and to estimate the parameters. 'Mis-specified model' implies that the alternative model was used to estimate the parameters. The average of the objective function is taken over 30 observations in each of which a different random parameter was chosen for the original model, a different random sequence was used for simulating the original model, a different random starting point was used for the Indirect Inference search, and a different random sequence was used for simulation in the Indirect Inference simulations.