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21 November 2013

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Abstract

This paper presents, discusses and tests a generalized Random Regret Minimization (G-RRM) model. The G-RRM model is created by replacing a fixed constant in the attribute-specific regret functions of the RRM model, by a regret-weight variable. Depending on the value of the regret-weights, the G-RRM model generates predictions that equal those of, respectively, the canonical linear-in-parameters Random Utility Maximization (RUM) model, the conventional Random Regret Minimization (RRM) model, and hybrid RUM-RRM specifications. When the regret-weight variable is written as a binary logit function, the G-RRM model can be estimated on choice data using conventional software packages. As an empirical proof of concept, the G-RRM model is estimated on a stated route choice dataset, and its outcomes are compared with RUM and RRM counterparts.

Keywords: Random Utility Maximization; Random Regret Minimization; Choice model;

Unified approach; Generalized Random Regret Minimization.

Delft, 21 November 2013

1. Introduction

Since the introduction of the Random Regret Minimization model (RRM) for discrete choice analysis (Chorus et al., 2008; Chorus, 2010), it has been acknowledged that the model provides a quite different perspective on choice modeling than does discrete choice analysis' workhorse, the linear-in-parameters Random Utility Maximization (from here on: RUM) model. Particularly, substantial differences have been highlighted in terms of the models' theoretical properties as well as in terms of their empirical outcomes such as choice probability forecasts and elasticities (e.g., Kaplan & Prato, 2012; Thiene et al., 2012; Boeri et al., 2013; Hensher et al., 2013; Beck et al., 2013; Boeri & Masiero, 2014).

Despite – or perhaps because of – these profound differences, there have also been ongoing attempts to combine the RRM model with the RUM model. Two different approaches can be distinguished: a first approach has been to assume that while some attributes of alternatives are processed in a RRM-fashion, others are processed in a RUM fashion. Resulting so-called hybrid RUM-RRM models have been proposed in Chorus et al. (2013), and applied in, for example, de Bekker-Grob and Chorus (2013) and Leong and Hensher (2014). A second approach has been put forward by Hess et al. (2012) and is based on the assumption that while some decision makers base their decisions on RUM-premises (for all attributes), others use RRM-premises (for all attributes). This approach has been recently applied by Hess & Stathopoulos (2014) and Boeri et al. (2014).

In this paper, an approach to combine the two models is put forward which is fundamentally different from the two approaches mentioned above: I formulate a generalized RRM or form here on G-RRM model, which – in terms of choice probability predictions – nests conventional RUM and RRM models (and hybrid RUM-RRM models) as special cases. By doing so, I show that while the two models – RUM and RRM – obviously may differ substantially in terms of their properties and outcomes, they are more related to one another than is usually thought. The generalization consists of replacing a constant with value '1' in the conventional RRM model, with a so-called regret-weight variable. Depending on the value of the regret-weight variable for a particular attribute, the attribute is processed in a RUM-fashion, an RRM-fashion, or in an intermediate RUM-RRM fashion. By parameterizing these regret-weight variables in binary logit form, the G-RRM model can be easily estimated on standard choice data using conventional software packages. An empirical application is provided, to provide a proof of concept. Note throughout the paper, and without loss of general applicability, the focus is on the logit or MNL form of both the RRM and RUM models.

Section 2 presents the G-RRM model, and section 3 presents the empirical proof of concept. Section 4 concludes with a summary of results, and a discussion of potentially fruitful directions for further research.

2. A generalized RRM model (G-RRM)

Since the RRM model has been discussed in detail in a number of previous papers (see for example the papers cited in the introduction), it will be presented here without any accompanying discussion of its model form and properties. The RRM model assumes that decision makers minimize regret when choosing, and that regret of a given alternative is written as follows:

$$RR_i = R_i + \varepsilon_i = \sum_{j \neq i} \sum_m \ln\left(1 + \exp\left[\beta_m \cdot \left(x_{jm} - x_{im}\right)\right]\right) + \varepsilon_i \tag{1}$$

RR_i	denotes the random (or: total) regret associated with a considered alternative i
R_i	denotes the 'observed' regret associated with i
ε_i	denotes the 'unobserved' regret associated with i, its negative being distributed i.i.d. Extreme Value Type I
β_m	denotes the estimable taste parameter associated with attribute x_m
x_{im}, x_{jm}	denote the values associated with attribute x_m for, respectively, the considered alternative i and another alternative j

As has been widely discussed in recent papers, the main contrast between this RRM model and its linear-in-parameters RUM counterpart (written as $U_i = V_i + \varepsilon_i = \sum_m \beta_m \cdot x_{im} + \varepsilon_i$, with ε_i being distributed i.i.d. Extreme Value Type I) lies in the fact that the RRM model features asymmetric and reference-dependent preferences. More specifically, the RRM model postulates that the extent to which a change in an alternative's attribute translates into regret depends on how the alternative performs in terms of the attribute, compared to competing alternatives. The poorer the initial relative performance of the alternative in terms of the attribute, the stronger is the impact of a change in attribute value on regret. This reference dependent asymmetry is a direct result of the convexity of the attribute regret function which includes attributes of competing alternatives: $\ln\left(1 + \exp\left[\beta_m \cdot \left(x_{jm} - x_{im}\right)\right]\right)$. This function is almost horizontal for (large) negative values of $\beta_m \cdot \left(x_{jm} - x_{im}\right)$, implying limited sensitivity to attribute changes in this part of the domain. The function has a slope which approaches β_m for large positive values of $\beta_m \cdot \left(x_{jm} - x_{im}\right)$. The reference dependent asymmetry which is captured in the RRM model, by means of its convex regret function, has been shown to translate into key 'character traits' of

the RRM model such as semi-compensatory behavior and the compromise effect (Chorus & Bierlaire, 2013).

The G-RRM model proposed in this paper replaces the '1' in $\ln\left(1 + \exp\left[\beta_m \cdot \left(x_{jm} - x_{im}\right)\right]\right)$ by a so-called regret-weight variable γ. By varying γ from 0 to 1, and plotting the resulting attribute regret function for $\left(x_{jm} - x_{im}\right)$ ranging from -5 to 5 (keeping β_m fixed at unity), the role of the regret-weight becomes immediately clear; the left hand panel of figure 1 shows the effect on the attribute regret function of a step-wise variation in γ, and the right hand panel shows the effect of a continuous change in γ. Both graphs show that when the regret-weight becomes smaller and starts to approach zero, the reference dependent asymmetry in preferences – which is central to the RRM model and its properties – vanishes. When $\gamma = 0$, there is no asymmetry anymore, implying that the impact on regret of a change in an alternative's attribute is no longer dependent on the alternative's initial performance in terms of the attribute, relative to its competition.
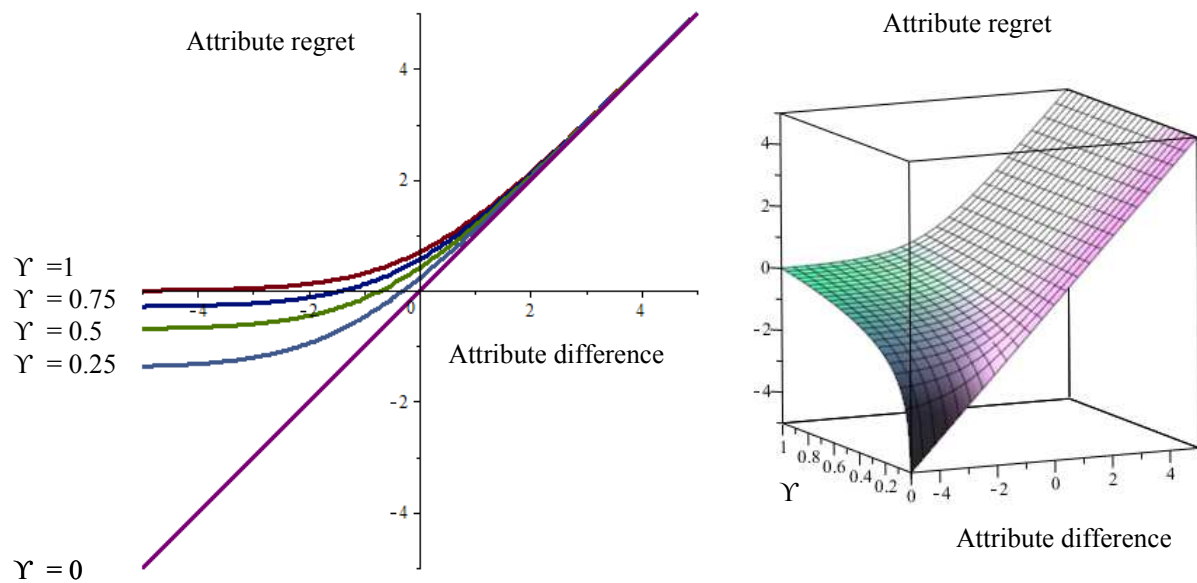

Figure 1: Impact of variation in regret-weight (γ) on attribute regret (β_m=1)

Intuitively, the resulting regret function with symmetric preferences ($\gamma = 0$) looks like the linear function generated by a linear-in-parameters RUM model, and indeed it can be shown that for $\gamma = 0$, the G-RRM model generates the same choice behavior predictions as does a linear-in-parameter RUM model.

Proposition: given a choice set containing J alternatives, choice probabilities generated by a G-RRM model ($\gamma = 0$) with parameters of size β_m equal those generated by a linear-in-parameter RUM model with parameters of size $J \cdot \beta_m$.

Proof: Note first that when $\gamma=0$, $\ln(\gamma + \exp[\beta \cdot (x_j - x_i)]) = \ln(0 + \exp[\beta \cdot (x_j - x_i)]) = \beta \cdot (x_j - x_i)$. Now, the regret of alternative i in the G-RRM model (with $\gamma=0$), given parameters β_m, can be written as:

$$R_i(\beta_m) = \sum_{j \neq i} \sum_m \beta_m (x_{jm} - x_{im}) = \sum_{j \neq i} \sum_m \beta_m x_{jm} - (J - 1) \sum_m \beta_m x_{im}$$

Subtracting from all regrets a constant of size $const = \sum_j \sum_m \beta_m x_{jm}$ gives an adapted regret of size R_i':

$$R_i'(\beta_m) = \sum_{j \neq i} \sum_m \beta_m x_{jm} - \sum_j \sum_m \beta_m x_{jm} - (J - 1) \sum_m \beta_m x_{im}$$

$$= -\sum_m \beta_m x_{im} - (J - 1) \sum_m \beta_m x_{im}$$

$$= -J \cdot \sum_m \beta_m x_{im}$$

By definition, the utility of i, given parameters of size $J \cdot \beta_m$, equals $V_i(J \cdot \beta_m) = \sum_m J \cdot \beta_m x_{im} = J \cdot \sum_m \beta_m x_{im}$. As a consequence, $V_i(J \cdot \beta_m) = -R_i'(\beta_m) = -R_i(\beta_m) + const$. Noting that the constant drops out when taking the difference, this implies that for any alternative j:

$$V_i(J \cdot \beta_m) - V_j(J \cdot \beta_m) = R_j'(\beta_m) - R_i'(\beta_m) = R_j(\beta_m) - R_i(\beta_m).$$

Since choice probabilities are fully determined by differences in utility (regret), this proves that choice probabilities of a linear-in-parameter RUM model conditional on parameters of size $J \cdot \beta_m$ equal those generated by a G-RRM model ($\gamma = 0$) conditional on parameters of size β_m. **QED**

In a practical sense this implies that estimating a G-RRM model (with $\gamma = 0$) gives the same final loglikelihood as a linear-in-parameter RUM model; estimated G-RRM parameters will be J times smaller than their RUM-counterparts. This all in combination establishes that a generalized RRM of G-RRM model which is specified as:

$$RR_i = R_i + \varepsilon_i = \sum_{j \neq i} \sum_m \ln(\gamma_m + \exp[\beta_m \cdot (x_{jm} - x_{im})]) + \varepsilon_i \tag{2}$$

nests – in terms of predicted choice behavior – the conventional RRM model presented in equation (1), and the conventional linear-in-parameter RUM model[1]. The former model is a special case which is obtained when $\gamma_m = \gamma = 1 \; \forall m$, and the latter is a special case which is obtained when $\gamma_m = \gamma = 0 \; \forall m$. Furthermore, hybrid RUM-RRM models of the type Chorus et al. (2013) are also special cases, where $\gamma_m \in \{0,1\} \; \forall m$. When $\gamma_m \in]0,1[\; \forall m$, regret minimization and utility maximization are both co-determinants of choice behavior for the attribute. One more note is in place concerning γ_m: since theoretically speaking γ_m is bounded between 0 and 1,[2] it is pragmatic to parameterize the regret-weight in terms of a binary logit function:

$$\gamma_m = \exp(\delta_m)/(1 + \exp(\delta_m)) \tag{3}.$$

Parameter δ_m in turn can be treated as a dependent variable, with socio-demographic variables and contextual factors as independent variables, to help explain the determinants of regret minimization versus utility maximization behavior for different attributes. For (large) negative values of δ_m, the RUM model is approached for the attribute, and for (large) positive values, the RRM model is approached. When $\delta_m = 0$ (or estimated to be insignificantly different from zero), regret minimization and utility maximization are equally important determinants of choice behavior for the attribute.

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[1] Beyond this equivalence between the G-RRM model with $\gamma = 0$ and the linear-in-parameters RUM model in terms of (rescaled) parameters and choice probabilities, there is also a very close relation between the two models in terms of economic appraisal metrics. More specifically, it can be easily shown that application, based on R'_i, of the RRM-based marginal rate of substitution measure that was proposed in Chorus (2012a) gives the same result as does its linear-in-parameters RUM- counterpart. Furthermore, application, based on R'_i, of the RRM-LogSum that was proposed in Chorus (2012b) gives the same result (though of course with opposite sign) as does the linear-in-parameters RUM-LogSum.

[2] Actually, the G-RRM model can also work with values of γ greater than 1. This leads to a decrease in sensitivity for situations where a competing alternative outperforms the considered one; in fact, the regret function becomes horizontal very quickly after the Y-axis is crossed for $\gamma \gg 1$ (as opposed to gradually for $\gamma = 1$). Also for situations where a competing alternative is outperformed by the considered one, the regret function becomes (much) less sensitive to changes in attribute values, especially when the difference in performance is small. Finally, since by definition the value of the regret function when $\beta_m \cdot (x_{jm} - x_{im}) = 0$ equals $\ln(1+\gamma)$, the resulting regret function is shifted upward for values of $\beta_m \cdot (x_{jm} - x_{im})$ that are either close to (but larger than) zero, equal to, or smaller than zero. The over-all result is a more horizontal (i.e., less sensitive) regret function, which is shifted upwards for large parts of its domain (compared to the situation where $\gamma = 1$). Altogether, the situation where $\gamma > 1$ does not form a particularly intuitive representation of regret.

3. An empirical proof of concept

This section puts the G-RRM model to the test empirically. Importantly, I do not wish to focus on, or draw any conclusions concerning, the performance of the model relative to conventional RRM and RUM models. Not only can such a judgment only be made after many more empirical tests have been performed, but – more importantly – the G-RUM models is not being put forward here as potentially superior (in terms of empirical performance) to one or both of the RRM or linear-in-parameter RUM models. Rather, the model is presented here as a framework to connect the two model types, and as a means to better understand the relation between them. In that sense, the aimed for contribution of this paper is theoretical, more than empirical. Nonetheless, the empirical application of the unified model is of course important for matters of (face) validity.

3.1. The data

The dataset used for empirical testing of the G-RRM model has been used for the estimation of conventional RUM and RRM models in a recently published paper (Chorus, 2012a). The data collection effort focused on route choice behavior among commuters who travel from home to work by car. A total of 550 people were sampled from an internet panel maintained by IntoMart, in April 2011. Sampled individuals were at least 18 years old, owned a car, and were employed. It was taken care of that the sample was representative for the Dutch commuter in terms of gender, age and education level. Of these 550 people, 363 filled out the survey (implying a response rate of 66%). Respondents to the survey were asked to imagine the hypothetical situation where they were planning a new commute from home to work (either because they had recently moved, or because their employer had recently moved, or because they had started a new job). They were asked to choose between three different routes that differed in terms of the following four attributes, with three levels each: average door-to-door travel time (45, 60, 75 minutes), percentage of travel time spent in traffic jams (10%, 25%, 40%), traffic fatalities per year on that route (1, 2, 3), and total costs (€5.5, €9, €12.5). A so-called 'optimal orthogonal in the differences'-design of choice sets resulted in nine choice tasks per respondent (see Figure 2 for an example of such a choice task).

1	Route A	Route B	Route C
Average travel time (minutes)	45	60	75
Percentage of travel time in congestion (%)	10%	25%	40%
Number of traffic fatalities per year	1	2	3
Travel costs (Euros)	€12.5	€9	€5.5
YOUR CHOICE	☐	☐	☐

Figure 2: An example route choice-task

3.2. Empirical analysis

A series of models were estimated using Biogeme (Bierlaire 2003, 2008); estimation results are reported in Table 1: a linear-in-parameters RUM model; a conventional RRM model; a G-RRM model with γ fixed at zero for all attributes; a G-RRM model with generic value for δ, and a G-RRM model with attribute specific values δ_m. Based on the results reported in Table 1, a series of observations can be made.

First, the linear-in-parameters RUM model achieves a slightly higher loglikelihood than does the RRM model. In line with results reported in previous papers, the difference is small. Second, it may be noted that, in line with what was already established theoretically in the proposition, the G-RRM model with regret-weight fixed at zero for all attributes generates i) the same loglikelihood as the linear-in-parameters RUM model, and ii) parameters that are three times smaller than their linear-in-parameters RUM counterparts. Note that as expected, *t*-ratios are the same for the two models. Third, a G-RRM model variant with a generic (across attributes) regret-weight – or, more precisely, with a generic δ – provides a slightly better fit than the RUM model, but this comes at the cost of substantial decreases in *t*-values for the other parameters (suggesting confounding of taste parameters with regret-weight parameters). This model, in sum, seems not to perform well on these data. Fourth, a G-RRM model which allows for attribute-specific values δ_m achieves the best final loglikelihood. Note that a Likelihood ratio test shows that the improvement in fit does not compensate for the loss in degrees of freedom at a 95% significance level.

Table 1: estimation results (robust *t*-values between brackets)

	Linear-in-param. RUM	*Conventional RRM*	*G-RRM* $\gamma = 0$	*G-RRM* δ	*G-RRM* δ_m
Mean travel time	-.0670 (-37.18)	-.0459 (34.84)	-.0223 (-37.18)	-.0280 (-5.04)	-.0225 (-36.86)
% of time spent in traffic jams	-.0320 (-20.74)	-.0212 (19.96)	-.0107 (-20.74)	-.0103 (-5.29)	-.0169 (-21.19)
# traffic fatalities	-.289 (-11.39)	-.189 (-11.40)	-.0964 (-11.39)	-.118 (-5.05)	-.0964 (-11.45)
Travel costs	-.183 (-23.18)	-.115 (-21.99)	-.0609 (-23.18)	-.0743 (-5.47)	-.124 (-22.35)
δ (generic)	-	-	-	-1.23 (-0.97)	-
δ (mean travel time)	-	-	-	-	-6.96 (-2.51)
δ (% of time in traffic jams)	-	-	-	-	0 ns[3]
δ (# traffic fatalities)	-	-	-	-	-6.23 (-10.49)
δ (travel costs)	-	-	-	-	7.10 (7.48)
Number of choices	3897	3897	3897	3897	3897
Null-Loglikelihood	-4281	-4281	-4281	-4281	-4281
Final Loglikelihood	-2847	-2851	-2847	-2846	-2844

However, more interestingly than these differences in final loglikelihood is a discussion of parameter estimates. It turns out that three out of four parameters δ_m are significantly different from zero. Attributes 'mean travel time' and '# traffic fatalities' appear to be processed almost entirely (more precisely, for more than 99%) as a RUM attribute, while attribute 'travel cost' is processed almost entirely (for more than 99%) as an RRM attribute. The insignificance of parameter δ_m for the attribute '% of time in traffic jams' implies that for this attribute, utility maximization and regret-minimization are equally important co-determinants of choice behavior. In line with expectations, taste parameter values (βs) for 'utility-attributes' are very similar to their counterparts in the G-RRM model with $\gamma=0$, while the taste parameter value for the 'regret-

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[3] This parameter was found to be very insignificant (robust *t*-value = 0.64). The model was re-estimated with this parameter fixed at zero, and those results are reported in the Table.

attribute' is of similar magnitude as its RRM counterpart. In combination, these findings suggest that the G-RRM model can be estimated on choice data, and that it provides meaningful outcomes that can be unambiguously interpreted in relation to the outcomes of RUM and RRM models.

4. Conclusions

This paper shows how the canonical linear-in-parameters Random Utility Maximization model, and the more recently proposed Random Regret Minimization model, are more closely connected than one might think when inspecting the often substantial differences between these two models. I show how the two models (and hybrid combinations thereof) can – in terms of choice behavior predictions – be conceived as both originating from a generalized RRM or G-RRM model. This G-RRM model is created by replacing a constant in the conventional RRM model with a regret-weight variable. By parameterizing this regret-weight, the G-RRM model can be estimated empirically on choice data. Although the contribution of this paper is mostly theoretical (i.e. highlighting the connection between the RUM and RRM models), an empirical proof of concept is provided in the form of an empirical estimation of the G-RRM model on a stated route choice dataset. Estimation results are in line with theoretical expectations, and show that the G-RRM model can be used in practice.

A number of directions for further research come to mind: first, it is worthwhile to see how the G-RRM model (and different variants thereof) perform on other datasets. My own experience is that this may vary across datasets in that in a minority of cases convergence issues may arise. This is not entirely unexpected, as *empirical* identification of the regret-weight parameters, simultaneously with taste parameters, may in some cases by complicated due to the fact that both have an impact on the sensitivity of choice probabilities with respect to variation in the corresponding attribute. To what extent this joint impact can be disentangled into a taste component and a regret-weight component, is likely to depend to some extent on the specifics of the dataset at hand. It may be expected that further parameterization of regret-weights can help in this regard: making them dependent on covariates such as socio-demographic variables and/or contextual variables should help identify parameters.

This further parameterization also opens up a more behavior-inspired direction for further research, as it allows the researcher to study what factors may lead to regret minimization (as opposed to utility maximization) behavior. A third avenue for further research is to use the G-RRM model as a precursor to the use of hybrid RUM-RRM models of the type proposed in Chorus et al. (2013). These latter models are more parsimonious than the G-RRM model, as they assume that every attribute is processed either in a utilitarian fashion or in a regret-based fashion. Selecting which attribute is – assumed to be – processed by which decision rule is a cumbersome process (especially when the number of attributes is large), and it also constitutes a slightly ad-

hoc way of model specification. However, by first estimating the G-RRM model one can easily identify which attribute is more likely to be processed as a utilitarian or regret-based attribute. This information can then be used in a second modeling step which consists of specifying the most likely hybrid RUM-RRM model.

Acknowledgement

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