Income Distribution and Macroeconomics

Galor, Oded and Zeira, Joseph

Brown University, Hebrew University

1 July 1988

Online at https://mpra.ub.uni-muenchen.de/51644/
MPRA Paper No. 51644, posted 26 Nov 2013 07:21 UTC
Income Distribution and Macroeconomics

Working Paper No. 89-25

Oded Galor* and Joseph Zeira†

July, 1988

Revised, September, 1989†

*Brown University.
†The Hebrew University of Jerusalem and Brown University.
†We are indebted to Giora Hanoch, Zvi Hercowitz, Paul Krugman, Greg Mankiw, Dani Tsiddon and Itzhak Zilcha for helpful comments on the first draft of this paper. We are also grateful for participants of the Hurwitz Conference on Economic Development in Tel-Aviv University, 1989, of the Macroeconomic Workshops in LSE and in the Hebrew University of Jerusalem and of the Research Seminar in the Bank of Israel.
Abstract

This paper analyzes the role of income distribution in macroeconomic analysis. The study demonstrates that the long-run equilibrium depends on the initial distribution of income. In accordance with empirical evidence concerning the correlation between income distribution and output, an economy that is characterized by a relatively equal distribution of wealth is likely to be wealthier in the long run. The study may, therefore, provide an additional explanation for the persistent differences in per-capita output across countries. Furthermore, the paper may shed light on cross-countries differences in macroeconomic adjustment to aggregate shocks.

*Journal of Economic Literature Classification Numbers:* 033, 110.

*Key Works:* Wealth distribution, income distribution, growth, capital market imperfections.
I. Introduction

Recent attempts to explain the sustained cross-country differences in per capita output and growth rates have focused on the role of international differences in the formation of knowledge in the presence of externalities and increasing returns to scale (e.g., Romer (1986), Lucas (1988)). These studies abstracted, however, from the significant role that income distribution may play in the explanation of the phenomenon, conducting the analysis within a growth model characterized by a representative agent.\(^1\)

Empirical evidence suggests that income distribution may indeed provide at least a partial explanation for the cross-country differences in per-capita output. Kravis (1960) established a correlation between income distribution and output levels, demonstrating that income is more equally distributed within wealthier countries. An analysis of recent statistics provided by the annual reports of the World Bank, as presented in Table 1, suggests that the observations made by Kravis are still prevalent.

Table 1: Income Distribution and Per-Capita Output

<table>
<thead>
<tr>
<th>Income Type</th>
<th>Lowest 20 percent</th>
<th>Second Quintile</th>
<th>Third Quintile</th>
<th>Fourth Quintile</th>
<th>Highest 20 percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low income economies</td>
<td>5.1</td>
<td>8.7</td>
<td>13.2</td>
<td>19.8</td>
<td>53.2</td>
</tr>
<tr>
<td>Lower middle income</td>
<td>4.4</td>
<td>8.3</td>
<td>12.8</td>
<td>20.4</td>
<td>54.1</td>
</tr>
<tr>
<td>Upper middle income*</td>
<td>4.0</td>
<td>8.6</td>
<td>13.6</td>
<td>21.3</td>
<td>52.5</td>
</tr>
<tr>
<td>High income economies</td>
<td>6.5</td>
<td>12.3</td>
<td>17.4</td>
<td>23.7</td>
<td>40.1</td>
</tr>
</tbody>
</table>

*Excluding Hungary and Yugoslavia


Exploring the time path of income distribution along a development path, Kuznets (1955) suggested that in the early stages of development income inequality rises whereas in the mature stages income inequality declines. This controversial hypothesis has been confirmed by Summers, Kravis and Heston (1984), who analyzed the time patterns of inequality across countries, demonstrating that during the period 1950-1980 income inequality declined sharply within industrialized countries, declined moderately within middle income nations while increasing somewhat across low income countries. Furthermore, Lindert and Williamson (1985) suggested that the British pattern of development supports the original findings of Kuznets. The correlation between income distribution and output was largely interpreted as a reflection of the implications of economic growth for income distribution. Nevertheless, reverse causality, from income distribution to output as well as mutual implications are consistent with the existing empirical evidence. The theory presented in this paper will hinge upon the latter interpretation.

This paper establishes a theoretical linkage, between income distribution and aggregate economic activity as is reflected by investment, output, and growth. The analysis indicates that wealth distribution plays an important role in the evolution of the economy over time. An economy that is characterized by a relatively equal distribution of wealth is likely to be wealthier in the long run, in accordance with the described empirical observations. Furthermore, the study suggests that wealth distribution is significant in the determination of macroeconomic adjustment to aggregate supply shocks, demand shocks and technological innovations. It is shown that the structural component of the adjustment process (i.e., the shift of labor from the unskilled to the skilled sector) is larger the more equal income distributed.

The paper develops a dynamic general equilibrium model characterized by overlapping-generations with intergenerational altruism. The economy consists of one good that

\[\text{The model shares some of the features developed by Louy (1981) who explored the creation of}\]
can be produced by either a skill-intensive or an unskilled-intensive process. The aggregate endowment of labor is exogenously given, whereas the decomposition of the labor force between skilled and unskilled labor is endogenously determined. Individuals live two periods. In the first they may acquire education or work as unskilled labor and in the second they are employed as skilled or unskilled labor according to their education level. Individuals are characterized by identical preferences over consumption and bequest. They may differ, however, in their inheritance of capital endowments.

It is shown that due to capital market imperfections (associated with enforcement costs) which result in a higher interest rate for borrowers than for lenders, the acquisition of skills is limited to those individuals who inherit a sufficiently high level of wealth. Thus, the distribution of wealth determines the aggregate level of skilled and unskilled labor and consequently the aggregate level of output.

The economy is characterized by multiple steady-state equilibria as a result of non-convexities in the production of human capital. Wealth distribution, therefore, carries long-run as well as short-run implications. The historical distribution of wealth determines the dynamic evolution of the economy, the long-run decomposition of the labor force between skilled and unskilled labor, and consequently the long-run output. If the marginal productivity of unskilled labor is constant, the economy is segmented into rich dynasties in which generation after generation invests in human capital, and poor dynasties in which each generation works as unskilled workers. However, variable marginal productivity of unskilled labor permits a greater mobility across sectors and consequently less rigid segmentation between rich and poor dynasties.

Given that ability and preferences are identical across individuals, increasing returns to scale and capital market imperfections are essential for the existence of a wealth distribution in an economy characterized by a random assignment of ability to individuals. In our model, however, individuals are identical in their ability as well as their preferences and a nondegenerate wealth distribution is due to multiple long-run equilibria created via capital market imperfections and non-convexities in production.
nondegenerate wealth distribution in the long run. In the absence of each of these components the steady-state equilibrium is unique and the wealth of all dynasties converge in the long run, resulting in a degenerate wealth distribution. The role of income distribution in the analysis of macroeconomic activity and in the explanation of long-run differences in output levels across countries, is therefore, eliminated.

The assertion that wealth distribution is highly relevant for macroeconomic analysis is quite novel.\(^3\) Neoclassical growth theory which forms the foundation for macroeconomic analysis abstracts from the role of wealth distribution. In the framework of a representative infinitely lived agent heterogeneity of any nature is ignored by definition, whereas in the standard overlapping-generations model heterogeneity of individuals is permitted solely across generations. A notable exception is the recent study by Murphy, Shleifer and Vishny (1988) which formalize the hypothesis of development economists concerning the central role played by income distribution in industrialization and economic development. To our judgment this highly relevant relationship between income distribution and macroeconomics will attract further studies.

The paper is organized as follows. Section 2 presents the basic model. Section 3 describes the short-run equilibrium where wealth distribution affects output and investment. Section 4 examines the long-run equilibrium where economic inequality can be persistent. Section 5 extends the basic model to incorporate variable wages for unskilled workers. Section 6 studies the relationship between wealth distribution and national income. Section 7 examines the adjustment to aggregate shocks. Section 8 offers some concluding remarks and the appendix examines the robustness of the results under a different specification of the bequest motive.

\(^3\)A random examination of several graduate macroeconomic textbooks reveals that income distribution is not even mentioned (e.g., Blanchard and Fischer (1989), Sargent (1987)). The same pattern exists in undergraduate texts.


2. The Basic Model

Consider a small open economy in a one-good world. The good can be used for either consumption or for investment. The good can be produced by two technologies, one which uses skilled labor and capital and the other using unskilled labor only. Production in the skilled labor sector is described by:

\[ Y_t^s = F(K_t, L_t^s), \]  

where \( Y_t^s \) is output in this sector at time \( t \), \( K_t \) is the amount of capital and \( L_t^s \) is skilled labor input. \( F \) is a concave production function with constant returns to scale. It is assumed that investment in human capital and in physical capital is made one period ahead of time. For the sake of simplicity it is assumed that there are no adjustment costs to investment and no depreciation of capital. Production in the unskilled labor sector is described by:

\[ Y_t^n = w_n \cdot L_t^n, \]  

where \( Y_t^n \) and \( L_t^n \) are output and unskilled labor input respectively, and \( w_n > 0 \) is the marginal productivity in this sector.

Individuals in this economy live two periods each in overlapping generations. They can either work as unskilled in both periods of life or invest in human capital when young and be skilled workers in the second period of life. The amount of investment in human capital is \( h > 0 \). An individual supplies one unit of labor in each of the working periods. Note that the fixed amount of investment implies that there is a region of
increasing returns to scale.\(^4\)

Each individual has one parent and one child and hence there is no population growth. In each generation there is a continuum of individuals of size \(L\). In the world we describe people care about their children and hence they leave them bequests. It is also assumed, for the sake of simplicity, that people consume in second period of life only. Formally, we assume that an individual derives utility both from consumption in second period life and from bequest to offspring.\(^5\)

\[
  u = \alpha \log c + (1 - \alpha) \log b,  
\]

where \(c\) is consumption in second period, \(b\) is bequest, and \(0 < \alpha < 1\). Notice that all individuals are born with the same potential abilities and with the same preferences. They differ only in the amounts they inherit from their parents.

Capital is assumed to be perfectly mobile so that both firms and individuals have free access to the international capital markets. The world rate of interest is fixed and equal to \(r > 0\). Individuals can lend any amount at this rate. As for borrowing, we assume that a borrowing individual can evade debt payments, by moving to other places etc., but this activity is costly. Lenders can avoid such defaults by keeping track of borrowers, but such precautionary measures are costly as well. Assume that if lenders invest an amount \(z\) at keeping track of a borrower, this borrower can still evade the lenders but only at a cost of \(\beta z\), where \(\beta > 1\). As is later shown in the paper, these inspection costs create a capital market imperfection, where individuals can borrow only at an interest rate higher than \(r\).

Unlike individuals, firms are unable to evade debt payment, due to obvious rea-

---

4 The exact specification of the increasing returns to scale does not affect the main results of the paper.

5 An alternative way to model the bequest motive is to assume that individuals draw utility from their offspring’s utility, as in Barro (1974). We show in the appendix that this specification leads to similar results.
sons as immobility, reputation, etc. Hence, firms can borrow at the lenders interest rate \( r \). Due to the absence of adjustment costs to investment, and to the fact that the number of skilled workers is known one period in advance, the amount of capital in the skilled labor sector is adjusted each period so that:

\[
F_K(K_t, L_t') = r. \tag{4}
\]

Hence, there is a constant capital-labor ratio in this sector, which determines the wage of skilled labor \( w_s \). This wage \( w_s \) depends on \( r \) and on technology only, and is constant through time.

We further assume that both labor markets and the good market are perfectly competitive.

3. Wealth Distribution and Short-Run Equilibrium

Let us first examine the capital market equilibrium for individual borrowers. It is clear that lenders to individuals must have positive costs of keeping track of each borrower, since otherwise everyone defaults. Hence, the individual must borrow at a rate higher than \( r \), to cover these tracking costs. An individual who borrows an amount \( d \) pays an interest rate \( i_d \) which covers lenders interest rate and lenders costs \( z \), assuming that competitive financial intermediation operates on zero profits:

\[
d \cdot i_d = d \cdot r + z. \tag{5}
\]

Lenders choose \( z \) to be high enough to make evasion disadvantageous:

\[
d(1 + i_d) = \beta z. \tag{6}
\]
These two conditions determine $i_d$, which is equal to:

$$i_d = i = \frac{1 + \beta r}{\beta - 1} > r.$$  

(7)

Borrowers rate of interest $i$ is, therefore, independent of the amount borrowed $d$. This result can be intuitively explained as follows: the larger the amount borrowed the greater the incentive to default and hence there is a larger effort by lenders to keep track. Hence, these costs are not fixed but are increasing with $d$.

We now turn to describe individual optimal decisions with regard to investment in human capital and to bequest to child. Let us consider an individual who inherits an amount $x$ in first period of life. If this individual decides to work as an unskilled worker and not invest in human capital, his (her) lifetime utility is:

$$U_n(x) = \log[(x + w_n)(1 + r) + w_n] + \epsilon;$$

(8)

where:

$$\epsilon = \alpha \log \alpha + (1 - \alpha) \log(1 - \alpha).$$

This unskilled worker is a lender who leaves a bequest of size:

$$b_n(x) = (1 - \alpha)[(1 + r)(x + w_n) + w_n].$$

(9)

An individual with inheritance $x \geq h$, who invests in human capital, is a lender with utility:

$$U_s(x) = \log[w_s + (x - h)(1 + r)] + \epsilon.$$

(10)

and a bequest of:
\[ b_t(x) = (1 - \alpha)[w_t + (x - h)(1 + r)]. \tag{11} \]

An individual who invests in human capital but has inheritance \( x \) smaller than \( h \) is a borrower, with lifetime utility:

\[ U_t(x) = \log[w_t + (x - h)(1 + i)] + \epsilon. \tag{12} \]

and a bequest of:

\[ b_t(x) = (1 - \alpha)[w_t + (x - h)(1 + i)]. \tag{13} \]

It is clear that if \( w_t - h(1 + r) < w_n(2 + r) \) all individuals prefer to work as unskilled. Since this is a case with limited interest we assume that:\textsuperscript{6}

\[ w_t - h(1 + r) \geq w_n(2 + r). \tag{14} \]

Hence, as investment in human capital pays back more than unskilled labor, lenders prefer to invest in human capital as is seen from equations (8) and (10). Borrowers invest in human capital as long as \( U_t(x) \geq U_n(x) \), that is as long as:

\[ x \geq f = \frac{1}{i + r}[w_n(2 + r) + h(1 + i) - w_t]. \tag{15} \]

Individuals who inherit an amount \( x \) smaller than \( f \) would prefer not to invest in human capital but work as unskilled. Education is, therefore, limited to individuals with high enough initial wealth, due to a higher interest rate for borrowers.

\textsuperscript{6}If (14) does not hold, individuals all over the world prefer to work as unskilled. Hence, there is no capital and an excess supply of loans prevails. This drives the world rate of interest down until (14) is satisfied. Hence (14) is a reasonable assumption.
Notice that the economy is Pareto-inefficient since not all individuals invest in human capital, even if such an investment raises net income. This Pareto-inefficiency is a result of a capital market imperfection, caused by costs of keeping track of individual borrowers.  

The amount an individual inherits in first period of life, therefore, fully determines his (her) decisions whether to invest in human capital or work as unskilled, and how much to consume and to bequeath to a child. Let \( D_t \) be the distribution of inheritances by individuals born in period \( t \). This distribution satisfies:

\[
\int_0^\infty dD_t(x_t) = L_t. \tag{16}
\]

The distribution \( D_t \), therefore, fully determines economic performance in period \( t \). It determines the amount of skilled labor and unskilled labor:

\[
L'_t = \int_f^\infty dD_t(x_t) \quad \text{and} \quad L''_t = \int_0^f dD_t(x_t), \tag{17}
\]

respectively. It, therefore, determines output as well.

Thus, the distribution of wealth in the economy has a strong effect on the macroeconomic equilibrium, on labor supply, on output and on investment. Note that this effect is mainly due to the capital market imperfection, as credit is not available to all. But this effect of wealth distribution can be of economic interest only if an unequal distribution of wealth can persist in the long run as well and not be a temporary phenomenon. In the next section we see that this is indeed the case, under the assumptions of imperfect capital markets and increasing returns to scale.

Due to this Pareto-inefficiency there is room for government intervention, that raises both income and welfare. An example for such a policy can be a subsidy to education, which reduces \( h \) and is financed by a tax on skilled labor from next period on. Under such a policy the government taxes all investors in human capital and avoids the costs of keeping track on each borrower, which lenders have to do.
4. The Dynamics of Wealth Distribution

The distribution of wealth not only determines equilibrium in period $t$, but also determines next period distribution of inheritances $D_{t+1}$:

$$
\begin{align*}
\mathcal{D}_{t+1} = \begin{cases} 
    b_n(x_t) = (1 - \alpha)[(x_t + w_n)(1 + r) + w_n], & \text{if } x_t < f \\
    b_i(x_t) = (1 - \alpha)[w_i + (x_t - h)(1 + i)], & \text{if } f \leq x_t < h \\
    b_s(x_t) = (1 - \alpha)[w_s + (x_t - h)(1 + r)], & \text{if } h \leq x_t.
\end{cases}
\end{align*}
$$

(18)

In order to illustrate the dynamic evolution of income distribution through time we present in Figure 1 the curves $b_n$ and $b_s$, which describe the dynamic relationships between inheritance and bequest for unskilled and skilled workers, respectively. Notice that $f$ is determined by the intersection of $b_n$ and $b_i$.

Individuals who inherit less than $f$ work as unskilled and so are their descendants in all future generations. Their inheritances converge to a long-run level $\bar{\omega}_n$:

$$
\bar{\omega}_n = \frac{1 - \alpha}{1 - (1 - \alpha)(1 + r)} w_n(2 + r).
$$

(19)

Individuals who inherit more than $f$ invest in human capital but not all their descendants will remain in the skilled labor sector in future generations. The border line passes at $g$ in Figure 1:

$$
g = \frac{(1 - \alpha)(h(1 + i) - w_s)}{(1 + i)(1 - \alpha) - 1}.
$$

(20)
Individuals who inherit less than $g$ in period $t$ may invest in human capital, but after some generations their descendants become unskilled workers and their inheritances converge to $\bar{x}_n$. Individuals who inherit more than $g$ invest in human capital and so do their descendants, generation after generation. Their bequests converge to $\bar{x}$:

$$\bar{x} = \frac{1 - \alpha}{1 - (1 - \alpha)(1 + r)} [w, - h(1 + r)].$$ \hspace{1cm} (21)

Thus, dynasties in this economy are concentrated in the long run in two groups: rich dynasties, where generation after generation invests in human capital, and poor ones, where generation after generation are unskilled workers.

It is time to discuss the assumptions hidden in Figure 1. The slopes of $b_n$ and $b_s$ at $\bar{x}_n$ and $\bar{x}$, respectively, are lower than one. This means that $\alpha$ and $r$ satisfy:

$$(1 - \alpha)(1 + r) < 1.$$ \hspace{1cm} (22)

This assumption guarantees that the process of bequest from generation to generation is stable and does not explode. Notice also that the $b_s$ curve is drawn in its steep part with a slope higher than one, implicitly assuming:

$$(1 - \alpha)(1 + r) = \frac{\beta}{\beta - 1}(1 + r)(1 - \alpha) - 1.$$ \hspace{1cm} (23)

Assuming the contrary leads to long-run distributions of labor which are all concentrated in either the unskilled labor sector or in the skilled sector. Since this is both unrealistic and not very interesting we restrict ourselves to the case described by (23). It is also assumed in Figure 1 that $g$ lies between $\bar{x}$, and $\bar{x}_n$, for similar reasons.
The dynamic evolution of the economy can be deduced from the individual dynamics, which are presented in Figure 1. The economy converges to a long-run equilibrium in which the population is divided into two groups: skilled workers with wealth $\bar{x}$, and unskilled workers with wealth $\bar{x}_n$. The relative size of these two groups depends on the initial distribution of wealth since the long-run number of unskilled workers $L^u_\infty$ is equal to $L^g_t$, where:

$$L^g_t = \int_0^\infty dD_t(\bar{x}_t).$$  \hspace{1cm} (24)

The long-run level of average wealth is:

$$\bar{x}, - \frac{L^g_t}{L}(\bar{x}, - \bar{x}_n),$$  \hspace{1cm} (25)

which is decreasing with $L^g_t/L$.

Hence, the long-run levels of income and wealth are positively related to the initial number of individuals who inherit more than $g$. Thus, an economy which is initially poor, ends up poor in the long run as well. An economy which is initially rich and its wealth is distributed among many, ends up rich. But an economy with a large initial amount of wealth, which is held by few, ends up poor in the long run.

The long-run equilibrium, therefore, depends in this model on the initial distribution of wealth and is, therefore, historically dependent. There are multiple long-run equilibria and the specific equilibrium the economy converges to is determined by the initial distribution of wealth.\textsuperscript{8} Note that the existence of two equilibria for each dynasty, in points A and B in Figure 1, is a result of the increasing returns to scale assumption, but it is due credit constraints that dynasties can end up in either of these equilibria. Since if capital markets have been perfect, all individuals would have invested in human capital.

\textsuperscript{8}These results resemble some of Lucas' (1985) results in a model of growth with an externality to investment in human capital.
The basic model, which is described and analyzed in Sections II - IV, therefore, demonstrates that the distribution of wealth has significant effects on macroeconomic variables such as investment, output and growth.

5. Variable Wages

In this section we extend the basic model developed so far to include variable wages for unskilled workers. This extension has two intentions. The first is to make the model more realistic, as it introduces greater mobility between sectors and relaxes the strong segmentation between dynasties in the basic model. The second intention is to enable us to analyze additional issues, such as the correlation between wealth and equality and the reaction of the economy to aggregate shocks.

Let us, therefore, assume that production by unskilled labor involves another factor of production, land and natural resources, which is in a fixed amount. Marginal productivity of unskilled labor is no longer constant but variable.

Let us, therefore, consider the same small open economy described in Section II with only one change: The marginal productivity of unskilled labor is no longer fixed but is a diminishing function of the number of unskilled workers \( L^n_t \). This marginal productivity is described by:

\[
w^n_t = P(L^n_t), \quad P'(L^n_t) < 0,
\]

where \( P \) is also the inverse of the demand for unskilled labor. We also assume, for the sake of simplicity, that the unskilled work in their first period of life only.

The supply of unskilled workers is determined by the amount of individuals who prefer not to invest in human capital:
\[ L_t^e = \int_0^{f(w^n_t)} dD_t(x_t), \]  

where \( f(w^n_t) \) is the threshold level for investment in human capital:

\[ f(w^n_t) = \frac{1}{1 - \frac{1}{i - r}} [w^n(1 + r) + h(1 + i) - w]. \]

At a wage level \( w^n_t = w^*/(1 + r) \) all individuals are indifferent between investing in human capital and working as unskilled and the supply curve become flat. Figure 2 presents the demand \( P_t \), supply \( S_t \), and equilibrium in the unskilled labor market. Notice that the equilibrium determines the wage of unskilled, the number of unskilled workers and hence the number of investors in human capital.

It is clear from Figure 2 that the equilibrium depends on the distribution of inheritances \( D_t \). If there are many individuals with small inheritances the unskilled wage \( w^n_t \) is low, while if a large number of individuals have high inheritances, \( w^n_t \) is high. In the next section we show that the historically given distribution of wealth \( D_t \) affects the equilibrium not only in the short run, but in the long run as well.

6. Wealth Distribution and National Income

The dynamics of the economy in the extended model can be analyzed by use of Figure 3, which describes individual bequest dynamics. As in Figure 1, \( b_t \) describes the bequests of investors in human capital while \( b_n \) describes bequests of unskilled workers. Figure 3 differs from Figure 1 in one respect: the \( b_n \) line is no longer fixed but shifts with \( w^n_t \), which is endogenous.

The dynamics of the economy depend on the initial equilibrium and hence on the initial distribution of wealth. Let us define an economy as “developed” if enough individuals have high inheritances, so that the equilibrium wage \( w^n_t \) is high enough so
that \( f(w^n_t) > g \). Such a case is described by the \( b^n_n \) curve in Figure 3. It can be shown that an economy is developed if and only if \( w^n_t > w_g \), where \( w_g \) is defined by:

\[
 w_g = \frac{1}{1 + r} \frac{\alpha + \alpha r - r}{\alpha + \alpha i - i} [w, - h(1 + i)]. \tag{29}
\]

Similarly an economy is defined as "less developed" if \( w^n_t \leq w_g \).

Figure 3 describes how much each individual inherits in period \( t + 1 \). If the economy is less developed bequests are described by \( b^n_1 \) and \( b^n_2 \). and this curve, therefore, determines next period supply of labor \( S_{t+1} \). It can be shown that \( S_{t+1} \) is rotated relative to \( S_t \) around \( w_g \), as described in Figure 4.

Hence, the wage of unskilled workers falls and \( b^n_1 \) shifts downward. This process continues and the economy converges to the long-run equilibrium at point \( A \) in Figure 4, where the wage is \( w^n_{\infty} \), the number of unskilled workers is \( L^n_{\infty} \) and \( S^n_{\infty} \) is the corresponding supply curve. The long-run wealth of the unskilled is the corresponding \( \bar{x}_n \) and their position is given by point \( A \) in Figure 3. Notice that the long-run number of unskilled workers \( L^n_{\infty} \) equals precisely the number of those who inherit less than \( g \) in the initial period: \( L^n_{\infty} = L^n_f \). Thus, long-run equilibrium and income distribution depend on the initial distribution of wealth. Assume now that the economy is developed and the initial wage is higher than \( w_g \), as described by \( b^n_2 \) in Figure 3. It can be shown that in this case the supply curve in the next period shifts everywhere to the left. Hence, the wage rate rises: \( w^n_{t+1} > w^n_t \), as is shown in Figure 5.

This process continues until equilibrium is reached at \( B \), where the wage rate is equal to \( w_s/(1 + r) - h \), and \( b^n_n \) coincides with \( b^n_2 \). This is, therefore, an egalitarian long-run equilibrium, where net life-time incomes of skilled workers and of unskilled workers are equal.

The long-run economic dynamics of countries in this model, therefore, crucially
depend on the number of individuals who inherit less than \( g \) in period \( t \), \( L_t^g \). It can be shown that a country is developed if and only if \( P(L_t^g) > w_g \).

We can now summarize the results of this section in the following theorem:

**Theorem 1.** If an economy satisfies: \( 0 < g < \bar{w} \), its dynamics depend on the number of individuals who inherit less than \( g \) in period \( t \), \( L_t^g \):

(a) A less developed economy, where \( P(L_t^g) \leq w_g \), converges to an unequal distribution of income, where:

\[ w^*_{\infty} < w/(1 + r) - h. \]

(b) A rich economy, where \( P(L_t^g) > w_g \), converges toward an equal distribution of income, where:

\[ w^*_{\infty} = w/(1 + r) - h. \]

If \( g \leq 0 \) all countries are developed and converge to an equal distribution of income, while if \( \bar{w} \leq g \) all are poor and converge to an unequal distribution. Since we are interested in a situation where income distribution varies across economies we concentrate on the more interesting case and assume that: \( 0 < g < \bar{w} \). It can be shown that \( g < \bar{w} \) iff \( (1 - \alpha)w > h \) and \( g > 0 \) iff \( h(1 + i) > w \).

In this section we, therefore, show that wealth and equality are highly correlated and affect one another. On the one hand, countries with greater income per capita have a more equal distribution of income. On the other hand, countries with a more equal initial distribution of wealth grow more rapidly and have a higher income level in the long run. These results shed a new light on the empirical findings on income distribution across countries. They conform with the empirical finding that income is more equally distributed in developed than in less-developed countries.\(^9\)

---

7. Aggregate Shocks and Income Distribution

In this section we continue to study the extended model, where wages are variable, in order to examine how the economy reacts to aggregate shocks. Let us first consider an adverse supply shock to productivity in the unskilled workers sector, namely a reduction in \( P \).

Let us assume for simplicity that the shock is unanticipated and that the economy is already in long-run equilibrium at the time of the shock. Consider first the case of the equal distribution equilibrium. All individuals inherit \( \pi \), and are indifferent between investing in human capital and working as unskilled. Hence, the supply curve is infinitely elastic, like \( S_1 \) in Figure 6. The shock has no effect on the wage of unskilled workers but only on their number. The economy moves from \( A_1 \) to \( B_1 \) and all the adjustment to the shock is through a structural shift, from unskilled to skilled labor, by increased investment in human capital.

Notice that in the period of the shock income falls, due to a sudden increase in investment in human capital, but this is temporary. Net discounted income is the same as before and the equal distribution economy suffers no utility loss as a result of the shock.

Consider now an economy with unequal distribution of income — a less-developed country. If the economy is not very poor, as described by the \( S_2 \) supply curve in Figure 6, the supply shock both lowers \( w^* \) but also increases investment in human capital, as described by the shift from \( A_2 \) to \( B_2 \). But \( B_2 \) is a temporary equilibrium and the long-run equilibrium is at \( C_2 \), since the new investors in human capital will leave bequests which are less than what they have inherited. Hence, the long-run adjustment to the shock is only through a decline in wages of unskilled workers, while the number of skilled workers in unchanged. If the economy is even poorer, as described by the
S_{3} supply curve, there is not even a temporary rise in investment in human capital, and the economy immediately moves from A_{3} to B_{3}. Hence, economies with unequal income distribution suffer a permanent income and utility loss as a result of the shock, and their distribution of income becomes even more unequal.

Hence, we can conclude that economies with more equal distribution of income adjust better, with smaller income losses, to macroeconomic shocks than economies with highly unequal distribution of income. The intuitive reason for that is that the larger the wealth of an unskilled worker, the easier it is for his/her offspring to shift to the other sector if wages fall.

This analysis can shed some light on the long-run implications of the supply shocks in the seventies. Most studies of these events have concentrated on short-run adjustments to the shocks through wage reductions. Thus, Bruno and Sachs (1985) stress the role of real wage flexibility as an important factor in a country's adjustability to such shocks. In this section we look more into the long-run adjustment to a shock via investment in human capital and structural change in the economy. In the long run it is the initial distribution of income which determines how the economy adjusts to the shock, as described above. It may, therefore, be interesting to examine the patterns of structural adjustments to the supply shocks of the seventies in developed vs. less-developed countries.

Let us now consider another type of aggregate shocks, namely a technological innovation in the skilled labor sector which raises the wage level of skilled workers \( w_{s} \). For the sake of simplicity we assume that the economy is in a long-run equilibrium when the change occurs. The patterns of adjustment to the technological innovation are presented in Figure 7.

The technological change raises the bequests of skilled workers and shifts the \( b_{s} \) curve from \( b_{s}^{1} \) to \( b_{s}^{2} \). Consider first a developed country with an equal distribution of income. Such an economy moves in the long run from A to B in Figure 7. In such an
economy unskilled workers wage rises in order to remain equal to the higher net income of skilled workers. Investment in human capital in such an economy is increased since the number of unskilled workers is reduced. Both income and wealth increase.

Consider now a country with unequal distribution of income. Assume first that the country is not too poor, with unskilled workers concentrated before the change at a point C in Figure 7, where \( g_2 < \bar{x}_n \leq g_1 \). In this case, the technological innovation pushes this economy across the threshold to converge to the equal distribution of income, at point B. In such an economy there is a rise in wages in both sectors and a vast investment in human capital. Imagine now that the economy is very poor and unskilled workers are concentrated at a point D, where \( \bar{x}_n < g_2 \), before the innovation. In this case, the wage and number of unskilled workers remain the same and there is no change in investment in human capital. In fact, the only change in such a poor economy is a rise of income and wealth of skilled workers, while the income gap in the society increases.

Thus, the way an economy adjusts to a technological improvement also depends on the initial distribution of income. In countries with fairly equal distribution the rise in skilled labor wage attracts more people to invest in human capital, since they have large enough initial wealth. In poor countries with a very unequal distribution of income there will be no increase in investment in human capital and the economic benefits from the innovation are limited.

8. Concluding Remarks

This paper analyzes the role of income distribution in macroeconomic analysis. The study demonstrates, that in the face of capital market imperfection and increasing returns the scale, the long-run equilibrium depends on the initial distribution of
income. In accordance with empirical evidence concerning the correlation between income distribution and output, the paper shows that in theory an economy that is characterized by a relatively equal distribution of wealth is likely to be wealthier in the long run. The study may, therefore, provide an additional explanation for the persistent differences in per-capita output across countries. Furthermore, the paper may shed light on cross-countries differences in macroeconomic adjustment to aggregate shocks, emphasizing the role of income distribution.
Appendix

As stressed in the paper, the two major assumptions of the model are increasing returns to scale and capital market imperfection. In this appendix we show that the way the bequest motive is modeled in the paper is not crucial to our main results, and they hold under another specification as well. In our paper we assume that individuals’ utility depends on the size of the bequest they leave to heirs. This specification is used in many other works, such as Louri (1981), Karni and Zilcha (1988), as well as others. But there is another way to model bequests, following Barro (1974), which is to assume that individuals’ utility is affected by the utility of heirs. We next show that even under this specification our main results hold.

Let us, therefore, consider exactly the same economy, which is described in the basic model in Section II, with one difference only — utility of an individual is described by:

\[ U = \log C + \beta \cdot U_{\text{off}}, \quad (A1) \]

where \( C \) is consumption in the second period of life, \( \beta \) is a subjective rate of time preference and \( U_{\text{off}} \) is utility of offspring. For simplicity, let us assume that the subjective discount rate, which is identical for all individuals, is equal to the lenders’ interest rate, \( r \):

\[ \beta = 1/(1 + r). \quad (A2) \]

Hence, the optimizing individual can be viewed as maximizing on all future actions of the dynasty, and this optimum is:

\[ \max \sum_{t=1}^{\infty} \frac{1}{(1 + r)^{t-1}} \log C_t, \quad (A3) \]
such that:

\[ C_t = (x_t - h)(1 + r) + w_t - x_{t+1}, \quad \text{if} \quad x_t > h, \]

or:

\[ (x_t - h)(1 + i) + w_t - x_{t+1}, \quad \text{if} \quad x_t < h, \]

or:

\[ (x_t + w_n)(1 + r) + w_n - x_{t+1}, \]

where \( x_t \) is the amount of inheritance of an individual who consumes at \( t \).

The optimal solution for (A3) is quite complicated, due to the discreteness of the problem, but we can reach some conclusions. It is easy to verify that an individual who inherits an amount \( x_t \geq h \) will invest in human capital, leave \( x_t \) to his/her offspring, and so will all following generations. Each will consume

\[ C_t = w_t(1 + r) + rx_t, \quad \text{for all} \ t \]  \quad \text{(A4)}

and they all will have the same amount of wealth, \( x_t \).

Regarding an individual who plans to be unskilled, he or she and all following generations also leave a bequest which is equal to the amount he/she inherited and all consume

\[ C_t = w_n(1 + r) + w_n + rx_t, \quad \text{for all} \ t. \]  \quad \text{(A5)}

The question arises whether there are such dynasties, that prefer to remain unskilled in all generations, or there will be some saving even when \( x_1 < h \), until some generation reaches \( h \), and meanwhile be unskilled or borrow at a high interest rate. The answer is that there is a possibility that dynasties with little wealth will remain unskilled forever. In order to see this, let us consider the discounted sum of consumption in three cases.

a. In case of remaining unskilled:
b. In case of being unskilled one period and then having $x_2 - h$:

$$SC_n(x_1) = \frac{1 + r}{r} [w_n(1 + r) + w_n + r x_1].$$  \hspace{2cm} (A6)

c. In case of borrowing one period and then having $x_2 = h$:

$$SC_i(x_1) = w_n(1 + r) + w_n + \frac{1}{r}(w_n - h) - [h - x_1(1 + r)].$$  \hspace{1cm} (A7)

Notice that $SC_n$ and $SC_i$ are upper limits, since in most cases individuals will have to borrow more than once, or work as unskilled more than one generation, or do both.

At $x_1 = 0$ we have:

$$SC_n(0) = \frac{1 + r}{r} [w_n(1 + r) + w_n],$$
$$SC_i(0) = w_n(1 + r) + w_n + \frac{1}{r}(w_n - h),$$
$$SC_i(0) = \frac{1 + r}{r}(w_n - h) - h(i + 1).$$ \hspace{2cm} (A9)

Hence, if $h$ and $i$ are high, and $w_n$ high enough, we can have

$$SC_n(0) > SC_i(0) \text{ and } SC_n(0) > SC_i(0),$$ \hspace{1cm} (A10)

and an individual who inherits nothing will be unskilled as will he all his progeny. This holds at an interval of $x_1 = 0$ as well.

We can, therefore, conclude that there is a threshold level, $\bar{x} < h$, such that individuals who inherit less than $\bar{x}$ will remain unskilled with wealth $\bar{x}$, they and their offsprings, forever. Individuals who inherit more than $\bar{x}$ will save, invest in human capital and end up at a level of wealth $h$ or more. Thus, the main results of the paper hold under the Barro specification as well.
References


Figure 5

Figure 6