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Abstract

The economic environment for financial institutions has become increasingly risky. Hence these institutions must find ways to manage risk of which one of the most important forms is interest rate risk. In this paper we use the mean-variance (mean-standard deviation) approach to examine a banking firm investing in risky assets and hedging opportunities. The mean-standard deviation framework can be used because our hedging model satisfies a scale and location condition. The focus of this study is on how interest rate risk affects optimal bank investment in the loan and deposit market when derivatives are available. Furthermore we explore the relationship among the first- and second-degree stochastic dominance efficient sets and the mean-variance efficient set.

JEL classification: G21, G22

Keywords: banking firm, investment, technology, risk, derivatives, hedging, (μ, σ)-preferences, stochastic dominance.

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1. Introduction

In our paper we examine the behavior of a banking firm under risk. The banking firm can hedge its investment risk exposure by trading futures contracts. Interest rate risk is one of the most important forms of risk faced by banks as financial intermediaries. Management of interest rate risk through the selection and monitoring of borrowers and creating diversified loan portfolios has always been one of the predominant challenges in risk management in the banking sector (Freixas and Rochet (2008), Bessis (2009)).

In our model we will use the term interest rate derivatives for both securities originating from loan securitization and financial advanced instruments such as interest rate futures and options. One of our objectives is to examine how the possibility of selling part or the entire uncertain loan portfolio of a bank at a deterministic forward rate will affect the bank's behavior in the deposit and loan market. In general interest rate risk occurs when the bank borrows at fixed interest rates and lends at variables interest rates. The interest rate risk exposure from this maturity mismatch. Interest rate derivatives are financial instruments designed to reduce interest rate risk. Interest rate derivatives can take the form of forwards, options or swaps which may be imbedded in financial assets such as loans and bonds. Interest rate derivatives allow an investor to reduce or eliminate interest rate risk or to buy interest rate risk with the expectation of benefiting from it.

The number of derivatives transactions has increased significantly worldwide in recent years. The main reason for the rise of interest rate derivatives is an increase in dramatically fluctuations in interest rate as in the Asian Financial Crises in 1997, the Russian Financial Crises 1998, the Argentinean Financial Crisis in 2001, Enron in 2002 and the financial crisis in 2007. There are many ways in which financial managers can utilize derivatives. The main applications are hedging, arbitrage and speculation. This paper shall focus on hedging, i.e., the desire of an investor to reduce risks in order to stabilize the income and cash flow streams.

The basic motivation of the study can be interpreted as follows. Banks face risk. If the banking firm does not hedge, there will be some stochastic variability in the cash flows. Random fluctuations in cash flows due to interest rate risk result in variability in the amount of the bank's profits. Variability in profits will generally be undesirable, to the extent that there is risk aversion. As derivatives have the ability to reduce this variability in cash flows, consequently the expected utility of the bank manager can be increased (Wilson (1998), DeRoon et al. (2003)).

This study analyzes the optimal hedging policy of a risk-averse competitive banking firm faced with an exogenous change in interest rate risk and the expected futures prices of derivatives. The two-dimensional mean-standard deviation model are used because this approach provides a clear and straightforward economic intuition for the bank's revision of its optimal hedging policy whenever a parameter of the decision-making process changes.

Schneeweiß (1967) and Meyer (1987) have shown that if all random alternatives to be ranked are equal in distribution to one another, with the exception of scale and location, then any expected utility ranking of all random alternatives can be based solely on the means and standard deviations of the alternatives. Many well-known families of distribution functions, including the bank-hedging model presented in this paper, satisfy this location and scale condition.

The analysis in this paper is based on the concept of (μ, σ)-preferences. The (μ, σ)-criterion on decision-making under uncertainty has experienced growing attention in very recent contributions, see for example, Löffler (1996), Bar-Shira and Finkelshtain (1999), Wong and Li (1999), Ormiston and Schlee (2001), Wagener (2002), Broll, Wahl and Wong (2006). For a (μ, σ) risk-averse bank manager, this study derives a direct relationship between an exogenous change in parameters, the optimal hedge ratio and the elasticity of risk aversion. Furthermore, the relationship between the first- and second-degree stochastic dominance efficient sets and the mean-variance efficient set are explored.

The plan of the paper is as follows. Section 2 presents the model of a competitive banking firm under interest rate risk when derivatives are available. Derivation of the main results will be included. In section 3 the concept of stochastic dominance is introduced to explore the relationship between the first and second-degree stochastic dominance efficient sets and the mean-variance efficient set. Section 4 will be the conclusion.

2. Optimal Hedging and Increase in Risk

Consider a risk-averse bank in a one period framework. The bank is a financial intermediary, taking deposits D and making loans L. By bank's technology it faces operational costs $C(L, D)$ with strictly positive marginal costs: $C_L > 0$ and $C_D > 0$. Equity capital, K, of the bank is assumed to be as given. At the beginning of the period, the bank has the following balance sheet:

$$L + M = K + D,$$

where M is the bank's interbank market position. M can take a positive or a negative sign, implying lending or borrowing in the interbank market at an interest rate r_M which is assumed to be deterministic.

Loans, L, granted by the bank are risky subjected to interest rate fluctuation. A simply situation is that the bank lends at variable interest rate. Another situation is that they the bank issue loans at fix rate in a longer period while the funds are supported by the short term loans which are influenced by the fluctuation of interbank rate. Hence, the repayments of the loans, $\tilde{L}_1$, are uncertain. Therefore the effective rate of return, $\tilde{r}_L = (\tilde{L}_1 - L)/L$, is risky. Deposits issued by the bank have the same maturity as the loans. The bank is a quantity setter in the loan and deposit market where the supply of deposits is perfectly elastic and the deposit rate, r_D, is given. With interest rate risk, the random profit of the bank is defined as

$$\tilde{\Pi} = \tilde{r}_L L + rM - r_D D - C(L, D).$$

Profits consist of the uncertain interest earned on loans plus positive (negative) interest on interbank position minus interest rate paid on deposits and operational costs.

As highlighted in the literature, there are many new instruments in the financial markets today which allow efficient risk management in banking. The creation of such instruments to manage interest rate risk is one of the most important steps towards complete risk-sharing markets. The following section shall analyze the impact of interest rate derivatives on a bank's optimal deposit, loan decisions and risk management.The interest rate derivative trades a risky cash flow into a certain cash flow. The bank can hedge the interest rate risk by taking a short (long) position, i.e., selling (buying) contracts H, in the interest rate derivatives market. The given forward rate is denoted by r_F. It is assumed that there is a positive risk premium in the futures market (backwardation), i.e. $r_F < E(\tilde{r}_L)$.

Substituting the bank's balance constraint and taking into account hedging possibilities will lead to

$$\tilde{\Pi} = (\tilde{r}_L - r)L + (r - r_D)D + r_M K - C(L, D) + (r_F - \tilde{r}_L)H.$$

The bank management is (μ, σ)-risk averse. This means that (i) the agent's preferences can be represented by a two-parameter function $V(\sigma, \mu)$ defined over mean μ and standard deviation σ of the underlying random variable, Y, such that:

$$V(\sigma, \mu) = E[u(Y)] = \int_a^b u(y)\, f_Y(y; \mu, \sigma)\, dy$$

where u is the utility function, Y is an investment or return with mean μ, standard deviation σ and pdf f_Y.

Let the return X be the random variable with zero mean and variance one, with the location-scale family $\mathcal{D}$ generated by X such that

$$\mathcal{D} = \{Y \mid Y = \mu + \sigma X \quad , \quad -\infty < \mu < \infty \quad , \quad \sigma > 0\,\}\,. \tag{1}$$

The expected utility $V(\sigma, \mu)$, see Meyer (1987), for the utility U on the random variable Y can then be expressed as:

$$V(\sigma, \mu) = E[U(Y)] = \int_a^b u(\mu + \sigma x)\, d\,F(x)$$

where $[a, b]$ is the support of X, F is the distribution function of X, and the mean and variance of Y are μ and σ^2 respectively. We note that the requirement of the zero mean and unit variance for X is not necessary. However, without loss of generality, we can make these assumptions as we will always be able to find such a seed random variable in the location-scale family.

For any constant α, the indifference curve drawn on the (σ, μ) plane such that $V(\sigma, \mu)$ is a constant can be expressed as:

$$C_\alpha = \{ \, (\sigma, \mu) \mid V(\sigma, \mu) \equiv \alpha \, \} \, .$$

In the indifference curve, Wong (2006) follows Meyer (1987) to have:

$$V_\mu(\sigma, \mu) \, d\mu + V_\sigma(\sigma, \mu) \, d\sigma = 0$$

or

$$V_\mu(\sigma, \mu) \, \frac{d\mu}{d\sigma} + V_\sigma(\sigma, \mu) = 0$$

where

$$V_\mu(\sigma, \mu) = \frac{\partial \, V(\sigma, \mu)}{\partial \, \mu} = \int_a^b u'(\mu + \sigma x) \, d \, F(x) \tag{2}$$

and

$$V_\sigma(\sigma, \mu) = \frac{\partial \, V(\sigma, \mu)}{\partial \, \sigma} = \int_a^b u'(\mu + \sigma x) \, x \, d \, F(x) \, . \tag{3}$$

He obtained the following propositions:

Proposition 1: *If the distribution function of the return with mean μ and variance σ^2 belongs to a location-scale family and for any utility function u, if $u' > 0$, then the indifference curve C_α can be parameterized as $\mu = \mu(\sigma)$ with slope*

$$S(\sigma, \mu) = -\frac{V_\sigma(\sigma, \mu)}{V_\mu(\sigma, \mu)} \, .$$

In addition,

a. *if $u'' \leq 0$, then the indifference curve $\mu = \mu(\sigma)$ is an increasing function of σ; and*

b. *if $u'' \geq 0$, then the indifference curve $\mu = \mu(\sigma)$ is a decreasing function of σ.*

Proposition 2: *The distribution function of the return with mean μ and variance σ^2 belongs to a location-scale family. For any utility function u with $u' > 0$, we have*

a. *if $u'' \leq 0$, then $\mu = \mu(\sigma)$ is a convex function of σ, and*

b. *if $u'' \geq 0$, then $\mu = \mu(\sigma)$ is a concave function of σ.*

Given (μ, σ)-risk aversion, the decision problem of the bank management reads:

$$\max_{L,D,H} V(\sigma_\Pi, \mu_\Pi),$$

where μ_Π and σ_Π are expected profit and the standard deviation of profit respectively. Examination of the first order necessary conditions for the maximization problem leads to the following proposition:

Proposition 3: *Given an interest rate derivative market and the bank's exposure to interest rate risk as described above, in the optimum: bank can separate its decision on risk management from its decisions on loan and deposit volumes and with backwardation, the bank under-hedges its interest rate risk exposure.*

Proposition 1 is an example of the well-known separation property in the presence of hedging instrument. As a consequence, the bank will choose a volume of loans and deposits as in the case of certainty. Furthermore Proposition 1 is the result of the unbiasedness, i.e., a risk premium leads to an underhedged position.

The elasticity of risk aversion can be derived by characterizing the sensitivity of the hedger against a change in risk. In order to analyze an increase in interest rate risk and the revision of the optimal hedge policy, H, the elasticity of risk aversion is defined as follows:

Definition 1: Given $\sigma > 0$, the elasticity of risk aversion with respect to the standard deviation is

$$\varepsilon_{S,\sigma} := -S_\sigma \frac{\sigma}{S}$$

where $S = -V_\sigma / V_\mu$ and $S_\sigma = \partial S / \partial \sigma$.

Let S be the marginal rate of substitution between μ and σ, thus, S is interpreted as a measure of risk aversion in (σ, μ)-space. The elasticity of risk aversion, $\varepsilon_{S,\sigma}$, is – in absolute value – given by the percentage change in risk aversion divided by the percentage change in risk.

The change in interest rate risk is as follows: $\tilde{r}_L(\gamma) = E(\tilde{r}_L) + \gamma\tilde{\eta}$, where $\tilde{\eta}$ has zero mean and unit standard deviation. Then, increasing γ models an increase in interest rate risk. Substituting $\tilde{r}_L(\gamma)$ for the random variable generates a relationship between optimal hedge amount $H(\gamma)$ and the interest rate risk measured by the standard deviation of $\tilde{r}_L(\gamma)$. Now the following claim can be made:

Proposition 4: *Given backwardation in the derivatives market, when the interest rate risk increases, then the optimal hedge will increase if the elasticity of risk aversion is less than unity; remains unchanged if the elasticity is unity; and decreases, if the elasticity of risk aversion is greater than unity.*

Definition 2: *Given $\sigma > 0$, the elasticity of risk aversion with respect to the expected profit wealth is*

$$\varepsilon_{S,\mu} := -S_\mu \frac{\mu}{S}$$

where $S = -V_\sigma / V_\mu$ and $S_\mu = \partial S / \partial \mu_r$.

Corollary 1: *With an increase in the expected futures rate, μ_r, the hedge ratio will decrease if the elasticity of risk aversion is less than unity; remain unchanged if the elasticity of risk aversion is unity; and increases if the elasticity of risk aversion is greater than unity.*

3. Stochastic Dominance and Mean-Variance Approach

Mean-variance efficient sets have been widely used in both Economics and Finance to analyze how people make their choices among risky assets. Markowitz (1959) demonstrated that if the ordering of alternatives is to satisfy the Von Neumann-Morgenstern (1947) (NM) axioms of rational behavior, only a quadratic NM utility function is consistent with an ordinal expected utility function that depends solely on the mean and variance of the return. Thereafter, Hanoch and Levy (1969) formulated an efficient set definition corresponding to the quadratic utility assumption. Baron (1974) pointed out that even if the return for each alternative has a normal distribution, the mean-variance framework cannot be used to rank alternatives consistent with the NM axioms unless a quadratic NM utility function is specified.

Meyer (1987) extended the mean-variance theory to include the comparison among distributions that differ only by location and scale parameters and to include general utility functions with only convexity or concavity restrictions. Levy (1989) elaborated on Meyer's results to prove that the first- and second-degree stochastic dominance efficient sets are equal to the mean-variance efficient set under certain conditions while Sinn (1990) found that the sign changes of the indifference curve slope depend on the speed of increase in the absolute risk aversion.

Earlier on, this paper has used the mean-variance (or mean-standard deviation) approach to examine a banking firm that is subjected to certain interest rate risk and hedging opportunities for a scale and location family of distributions. Next, it shall explore the linkage of the mean-variance efficient set to both the first- and second-degree stochastic dominance efficient sets and to the utility functions for both non-satiated and risk-averse investors. The definition of the stochastic dominance concept is as follows:

Random variables, denoted by X and Y, defined on Ω are considered together with their corresponding distribution functions F and G, and their corresponding probability density functions f and g, respectively. The following notations will be used throughout this paper:

$$\begin{aligned}
&\mu_F = \mu_X = E(X) = \int_a^b t\,dF(t) \quad , \quad \mu_G = \mu_Y = E(Y) = \int_a^b t\,dG(t) \quad , \\
&h(x) = H_0^A(x) = H_0^D(x) \quad , \quad H_j^A(x) = \int_a^x H_{j-1}^A(y)\,dy \quad , \\
&H_j^D(x) = \int_x^b H_{j-1}^D(y)\,dy \quad j = 2, 3;
\end{aligned} \tag{4}$$

where $h = f$ or g and $H = F$ or G.[1] In (4), $\mu_F = \mu_X$ is the mean of X, whereas $\mu_G = \mu_Y$ is the mean of Y.

We next define the first, second, and third order ascending stochastic dominances which are applied to risk averters; and then define the first, second, and third order descending stochastic dominances which are applied to risk seekers. The following definitions of SD are well-known in SD, see, for example, Sriboonchitta, et al. (2009).

Definition 3: Given two random variables X and Y with F and G as their respective distribution functions, X is at least as large as Y and F is at least as large as G in the sense of:

a. FASD, denoted by $X \succeq_1 Y$ or $F \succeq_1 G$, if and only if $F_1^A(x) \leq G_1^A(x)$ for each x in $[a, b]$,

b. SASD, denoted by $X \succeq_2 Y$ or $F \succeq_2 G$, if and only if $F_2^A(x) \leq G_2^A(x)$ for each x in $[a, b]$,

[1] The above definitions are commonly used in the literature; see for example, Wong and Li (1999), Li and Wong (1999) and Anderson (2004).

c. TASD, denoted by $X \succeq_3 Y$ or $F \succeq_3 G$, if and only if $F_3^A(x) \leq G_3^A(x)$ for each x in $[a, b]$

where FASD, SASD, and TASD stand for first, second, and third order ascending stochastic dominance respectively.

If in addition there exists x in $[a, b]$ such that $F_i^A(x) < G_i^A(x)$ for $i = 1, 2$ and 3, we say that X is large than Y and F is large than G in the sense of SFASD, SSASD, and STASD, denoted by $X \succ_1 Y$ or $F \succ_1 G$, $X \succ_2 Y$ or $F \succ_2 G$, and $X \succ_3 Y$ or $F \succ_3 G$ respectively, where SFASD, SSASD, and STASD stand for strictly first, second, and third order ascending stochastic dominance respectively.

Definition 4: Given two random variables X and Y with F and G as their respective distribution functions, X is at least as large as Y and F is at least as large as G in the sense of:

a. FDSD, denoted by $X \succeq^1 Y$ or $F \succeq^1 G$, if and only if $F_1^D(x) \geq G_1^D(x)$ for each x in $[a, b]$,

b. SDSD, denoted by $X \succeq^2 Y$ or $F \succeq^2 G$, if and only if $F_2^D(x) \geq G_2^D(x)$ for each x in $[a, b]$,

c. TDSD, denoted by $X \succeq^3 Y$ or $F \succeq^3 G$, if and only if $F_3^D(x) \geq G_3^D(x)$ for each x in $[a, b]$,

where FDSD, SDSD, and TDSD stand for first, second, and third order descending stochastic dominance respectively.

If in addition there exists x in $[a, b]$ such that $F_i^D(x) > G_i^D(x)$ for $i = 1, 2$ and 3, we say that X is large than Y and F is large than G in the sense of SFDSD, SSDSD, and STDSD, denoted by $X \succ^1 Y$ or $F \succ^1 G, X \succ^2 Y$ or $F \succ^2 G$, and $X \succ^3 Y$ or $F \succ^3 G$ respectively, where SFDSD, SSDSD, and STDSD stand for strictly first, second, and third order descending stochastic dominance respectively.

We remark that if $F \succeq_i G$ or $F \succ_i G$, then $-H_j^A$ is a distribution function for any $j > i$, and there exists a unique measure μ such that $\mu[a, x] = -H_j^A(x)$ for any $x \in [a, b]$. Similarly, if $F \succeq^i G$ or $F \succ^i G$, then H_j^D is distribution function for any $j > i$. H_j^D and H_j^A are defined in (4).

The SD approach is regarded as one of the most useful tools for ranking investment prospects when there is uncertainty, since ranking assets has been proven to be equivalent to expected-utility maximization for the preferences of investors with different types of utility functions. Before we carry on our discussion further, we first state different types of utility functions as shown in the following definition:

Definition 5:

a. For $n = 1, 2, 3, U_n^A, U_n^{SA}, U_n^D$ and U_n^{SD} are sets of utility functions u such that:

$$\begin{aligned} U_n^A(U_n^{SA}) &= \{u : (-1)^{i+1}u^{(i)} \geq (>)\, 0\,,\; i = 1, \cdots, n\,, \\ U_n^D(U_n^{SD}) &= \{u : u^{(i)} \geq (>)\, 0\,,\; i = 1, \cdots, n\}. \end{aligned}$$

where $u^{(i)}$ is the i^{th} derivative of the utility function u.

b. The extended sets of utility functions are defined as follows:

$$\begin{aligned} U_1^{EA}(U_1^{ESA}) &= \{u : u \textit{ is (strictly) increasing }\}, \\ U_2^{EA}(U_2^{ESA}) &= \{u \textit{ is increasing and (strictly) concave }\}, \\ U_2^{ED}(U_2^{ESD}) &= \{u \textit{ is increasing and (strictly) convex }\}, \\ U_3^{EA}(U_3^{ESA}) &= \{u \in U_2^{EA} : u' \textit{ is (strictly) convex }\}, \textit{and} \\ U_3^{ED}(U_3^{ESD}) &= \{u \in U_2^{ED} : u' \textit{ is (strictly) convex }\}. \end{aligned}$$

Note that in Definition 5 'increasing' means 'nondecreasing' and 'decreasing' means 'nonincreasing'. We also remark that in Definition 5, $U_1^A = U_1^D$ and $U_1^{SA} = U_1^{SD}$. We will use two notations U_1^{ED} and U_1^{ESD} in this paper such that $U_1^{ED} \equiv U_1^{EA}$ and $U_1^{ESD} \equiv U_1^{ESA}$. It is known that u in U_2^{EA}, U_2^{ESA}, U_2^{ED}, or U_2^{ESD} , and u' in U_3^{EA}, U_3^{ESA}, U_3^{ED} or U_3^{ESD} are differentiable almost everywhere and their derivatives are continuous almost everywhere.

An individual chooses between F and G in accordance with a consistent set of preferences satisfying the von Neumann-Morgenstern (1944) consistency properties. Accordingly, F is (strictly) preferred to G, or equivalently, X is (strictly) preferred to Y if

$$\Delta Eu \equiv E[u(X)] - E[u(Y)] \geq 0 (> 0), \tag{5}$$

where $E[u(X)] \equiv \int_a^b u(x)dF(x)$ and $E[u(Y)] \equiv \int_a^b u(x)dG(x)$.

Proposition 5: *Let X and Y be random variables with probability distribution functions F and G respectively. Suppose u is a utility function. For $n = 1, 2$ and 3; we have the following:*

a. *if $X \succeq_n (\succ_n) Y$, then $E[u(X)] \geq (>) E[u(Y)]$ for any u in U such that $U_n^A \subseteq U \subseteq U_n^{SA}$ $(U_n^{EA} \subseteq U \subseteq U_n^{ESA})$.*

b. *if* $X \succeq^n (\succ^n) Y$, *then* $E[u(X)] \geq (>) E[u(Y)]$ *for any* u *in* U *such that* $U_n^D \subseteq U \subseteq U_n^{SD}$ $(U_n^{ED} \subseteq U \subseteq U_n^{ESD})$.

The proof for Proposition 5 can be found in Wong and Li (1999) and the references therein.

The basic principle underlying stochastic dominance is quite straightforward. As an example, suppose that investors attempt to choose between two risky assets, X and Y. Also, suppose that the distributions of returns to assets X and Y are highly complicated, but the return to asset X always exceeds the return to asset Y. In this case, as long as investors are non-satiated, no one will buy asset Y since the investors can always do better by holding asset X. This basically iterates the results of the above proposition with $n = 1$. Similarly, the above proposition with $n = 2$ demonstrates that the non-satiated and risk-averse investors will prefer risky asset X to Y if and only if X stochastically dominates Y in the second order. As Proposition 3 provides the equivalent relationship between stochastic dominance and utility function, a person is thus known as a first order stochastic dominance (FSD) risk investor (or known as a non-satiated investor) if his/her utility function belongs to U_1^E and call a person a second order stochastic dominance (SSD) risk averter (or called as a non-satiated and risk-averse investor) if his/her utility function belongs to U_2^E The preference of random profits in a location-scale family of distributions for the FSD and SSD investors are further explored in the following proposition:

Proposition 6: *For the random profits* $\tilde{\Pi}_1$ *and* $\tilde{\Pi}_2$ *with means* μ_1 *and* μ_2 *respectively such that* $\tilde{\Pi}_1 = p + q\tilde{\Pi}_2$,

a. *if* $p+qy > (\geq) y$ *for all* y, *then* $E[u(\tilde{\Pi}_1)] \geq (>) E[u(\tilde{\Pi}_2)]$ *for any* u *such that* $u \in U_1^{SA}$ *and* U_1^{SD} $(U_1^A$ *and* $U_1^D)$; *and*

b. *if* $0 \leq q \leq 1$ *such that* $p/(1-q) > (\geq)\ \mu_2$, *i.e.,* $\mu_1 > (\geq)\ \mu_2$, *then* $E[u(\tilde{\Pi}_1)] \geq (>) E[u(\tilde{\Pi}_2)]$ *for any* $u \in U$ *such that* $U_2^A \subseteq U \subseteq U_2^{SA}$ $(U_2^{EA} \subseteq U \subseteq U_2^{ESA})$.

c. *if* $0 \leq q < 1$ *such that* $p/(1-q) \leq \mu_2$, *i.e.,* $\mu_2 \geq \mu_1$, *then* $E[u(\tilde{\Pi}_2)] \geq (>) E[u(\tilde{\Pi}_1)]$ *for any* $u \in U$ *such that* $U_2^D \subseteq U \subseteq U_2^{SD}$ $(U_2^{ED} \subseteq U \subseteq U_2^{ESD})$.

Proposition 7: *For the effective rates of returns* $\tilde{r}_1$ *and* $\tilde{r}_2$, $\tilde{\Pi}_i = \tilde{r}_i L + rM - r_D D - C(L, D)$. *For any* $j = 1, 2, 3$,

a. *if* $\tilde{r}_1 \succeq_j \tilde{r}_2$, *then* $E[u(\tilde{\Pi}_1)] \geq (>) E[u(\tilde{\Pi}_2)]$ *for any* u *such that* $u \in U_j^{SA}$ (U_j^A); *and*

b. *if* $\tilde{r}_1 \succeq^j \tilde{r}_2$, *then* $E[u(\tilde{\Pi}_1)] \geq (>) E[u(\tilde{\Pi}_2)]$ *for any* u *such that* $u \in U_j^{SD}$ (U_j^D).

Similarly, Corollary 2 tells us that if two effective rates of returns $\tilde{\Pi}_1$ and $\tilde{\Pi}_2$ satisfy $\tilde{\Pi}_1 = p + q\tilde{\Pi}_2$ with $p + qy > (\geq)\ y$, then the effective rate of return $\tilde{\Pi}_1$ stochastically dominates the effective rate of return $\tilde{\Pi}_2$ and hence $\tilde{\Pi}_1$ is preferred by FSD investors. If the effective rates of returns $\tilde{\Pi}_1$ $\tilde{\Pi}_2$ satisfying the inequality relationship as stated in Part (2) of Corollary 2, then $\tilde{\Pi}_1$ is preferred to $\tilde{\Pi}_2$ for any SSD investor.

In Part (2) of Proposition 4 and Part (2) of Corollary 2, one can easily show that $\tilde{\Pi}_1$ has a bigger mean and smaller variance than $\tilde{\Pi}_2$. Hence, we obtain the following corollary:

Corollary 2: *For the effective rates of returns $\tilde{r}_{L_1}$ and $\tilde{r}_{L_2}$ with means μ_1 and μ_2 and variances σ_1^2 and σ_2^2 respectively such that $\tilde{r}_{L_1} = p + q\tilde{r}_{L_2}$ and $\tilde{\Pi}_i = \tilde{r}_{L_i}L + rM - r_D D - C(L, D)$, if r, r_D and $C(L, D)$ are independent of $\tilde{r}_{L_i}$.*

a. *if $p+qy > (\geq)y$ for all y, then $E[u(\tilde{\Pi}_1)] \geq (>)E[u(\tilde{\Pi}_2)]$ for any u such that $u \in U_1^{SA}$ and U_1^{SD} (U_1^A and U_1^D); and*

b. *if $0 \leq q \leq 1$ such that $p/(1-q) > (\geq)\ \mu_2$, i.e., $\mu_1 > (\geq)\ \mu_2$, then $E[u(\tilde{\Pi}_1)] \geq (>)E[u(\tilde{\Pi}_2)]$ for any $u \in U$ such that $U_2^A \subseteq U \subseteq U_2^{SA}$ ($U_2^{EA} \subseteq U \subseteq U_2^{ESA}$).*

c. *if $0 \leq q < 1$ such that $p/(1-q) \leq \mu_2$, i.e., $\mu_2 \geq \mu_1$, then $E[u(\tilde{\Pi}_2)] \geq (>)E[u(\tilde{\Pi}_1)]$ for any $u \in U$ such that $U_2^D \subseteq U \subseteq U_2^{SD}$ ($U_2^{ED} \subseteq U \subseteq U_2^{ESD}$).*

It is common to compare the assets in the $\sigma - \mu$ indifference curves diagram if one applies the mean-variance (mean-standard deviation) approach. In this connection, we can re-write Proposition 4 and Corollary 2 in the following corollaries to state its relationship in the $\sigma - \mu$ indifference curves diagram:

Corollary 3: *For the random profits $\tilde{\Pi}_1$ and $\tilde{\Pi}_2$ with means μ_1 and μ_2 and variances σ_1^2 and σ_2^2 respectively such that $\tilde{\Pi}_1 = p + q\tilde{\Pi}_2$ in the $\sigma - \mu$ indifference curves diagram,*

a. *if (σ_1, μ_1) is in the north of (σ_2, μ_2), the random profit $\tilde{\Pi}_1$ is preferred to the random profit $\tilde{\Pi}_2$ for any FSD risk investor; and*

b. *if (σ_1, μ_1) is in the north-west of (σ_2, μ_2), the random profit $\tilde{\Pi}_1$ is preferred to the random profit $\tilde{\Pi}_2$ for any SSD risk averter.*

c. *if (σ_1, μ_1) is in the north-east of (σ_2, μ_2), the random profit $\tilde{\Pi}_1$ is preferred to the random profit $\tilde{\Pi}_2$ for any SSD risk seeker.*

d. *if $E\tilde{\Pi}_1 = E\tilde{\Pi}_2 = 0$ and $\tilde{\Pi}_1 = q\tilde{\Pi}_2$ with $0 \leq q < 1$, then SASD risk averters will prefer $E\tilde{\Pi}_1$ while SDSD risk seekers will prefer $E\tilde{\Pi}_2$.*

Corollary 4: *For the effective rates of returns $\tilde{r}_{L_1}$ and $\tilde{r}_{L_2}$ with means μ_1 and μ_2 and variances σ_1^2 and σ_2^2 respectively such that $\tilde{r}_{L_1} = p + q\tilde{r}_{L_2}$ and $\tilde{\Pi}_i = \tilde{r}_{L_i}L + rM - r_D D - C(L, D)$ in the $\sigma - \mu$ indifference curves diagram, if r, r_D and $C(L, D)$ are independent of $\tilde{r}_{L_i}$, then*

a. *if (σ_1, μ_1) is in the north of (σ_2, μ_2), the random profit $\tilde{\Pi}_1$ is preferred to the random profit $\tilde{\Pi}_2$ for any FSD risk investor; and*

b. *if (σ_1, μ_1) is in the north-west of (σ_2, μ_2), the random profit $\tilde{\Pi}_1$ is preferred to the random profit $\tilde{\Pi}_2$ for any SSD risk averter.*

c. *if (σ_1, μ_1) is in the north-east of (σ_2, μ_2), the random profit $\tilde{\Pi}_1$ is preferred to the random profit $\tilde{\Pi}_2$ for any SSD risk seeker.*

d. *if $E\tilde{\Pi}_1 = E\tilde{\Pi}_2 = 0$ and $\tilde{\Pi}_1 = q\tilde{\Pi}_2$ with $0 \le q < 1$, then SASD risk averters will prefer $E\tilde{\Pi}_1$ while SDSD risk seekers will prefer $E\tilde{\Pi}_2$.*

The above corollaries provide an easy way to compare different random profits and compare different the effective rates of returns. One can simply plot their means and standard deviations in the $\sigma - \mu$ indifference curves diagram. Those in the north are preferred to those in the south for any FSD risk investor; and those in the north-west are preferred to those in the south-east for any SSD risk averter.

At last, we develop 2 propositions for the convex combination of risks or profits as shown in the following:

Proposition 8: *Let $n \ge 2$. If $\tilde{\Pi}_1, \cdots, \tilde{\Pi}_n$ are n independent and identically distributed random profits, then we have*

$$\text{a.} \qquad \frac{1}{n}\sum_{i=1}^{n}\tilde{\Pi}_i \succeq_2 \sum_{i=1}^{n}\lambda_i\tilde{\Pi}_i \succeq_2 \tilde{\Pi}_i \qquad \textit{for any} \quad (\lambda_1, \cdots, \lambda_n) \in \Lambda_n \,, \qquad \textit{and}$$

$$\text{b.} \qquad \tilde{\Pi}_i \succeq^2 \sum_{i=1}^{n}\lambda_i\tilde{\Pi}_i \succeq^2 \frac{1}{n}\sum_{i=1}^{n}\tilde{\Pi}_i \qquad \textit{for any} \quad (\lambda_1, \cdots, \lambda_n) \in \Lambda_n \,,$$

where $\Lambda_n = \{(\lambda_1, \cdots, \lambda_n) : \lambda_i \ge 0$ for $: i = 1, \cdots, n,$ and $\sum_{i=1}^{n}\lambda_i = 1\}$.

4. Concluding Remarks

In the paper, analyses have been conducted on optimal hedging of a competitive banking firm where interest rate risks in the economy are tradable on a financial risk-sharing market. Specifically, it has been shown that a (μ, σ)-risk-averse bank management revises its optimal hedging policy according to its preference, i.e., its intensity of risk-aversion. The elasticity of risk aversion determines whether or not a bank management decreases or increases the optimum hedge ratio when parameters of the decision-making process changes.

Furthermore, reviews have been made on the mean-variance (mean-standard deviation) approach together with extensions on the results derived from the literature. Comments have been made on the finding in the literature which states that the first- and second-degree stochastic dominance efficient sets are equal to the mean-variance efficient set.

One could extend the theory developed in this paper to study the preferences for other types of investors, for example, investors with S-shaped and reverse S-shaped utility functions (Broll, Egozcue, Wong, and Zitikis, 2010; Egozcue, Fuentes García, Wong, and Zitikis, 2011; Fong, Lean, and Wong, 2008; Wong and Chan, 2008). Extension also include to study comparison of convex combination of assets (Egozcue and Wong, 2010). One could also adopt behaviorial economics (Lam, Liu and Wong, 2010, 2011) and other measures like value at risk (Ma and Wong, 2010) to study the issue. In addition, one could check whether the market for different assets are efficient (Chan, de Peretti, Qiao, and Wong, 2011; Fong, Wong, and Lean, 2005; Lean, McAleer, and Wong, 2010).

Lastly, we only develop the theory to compare the preference of the assets. We have not mentioned which statistics could be used to make such comparison. Readers may consider apply the SD test developed by Bai, Li, Liu, and Wong (2011) and others to make statistical comparison for SD. One could apply the statistics developed by Bai, Liu and Wong (2009), Bai, Wang and Wong (2011), and Leung and Wong (2008) and others to make statistical comparison for MV.

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