A Comment on ”Cycles and Instability in a Rock-Paper-Scissors Population Game: A Continuous Time Experiment”

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Abstract

The authors (Cason, Friedman and Hopkins, *Review of Economic Studies*, 2014) claimed that *control treatments (using simultaneous matching in discrete time) replicate previous results that exhibit weak or no cycles*. After correcting two mathematical mistakes in their cycles tripwire algorithm, we study the cycles by scanning the tripwire in the full strategy space of the games and we find significant cycles that were omitted by the authors. So we suggest that all of the treatments exhibit significant cycles.

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Keywords: experiments, learning, cycles, mixed equilibrium, discrete time

The existence of cycles in mixed equilibrium games has been a cutting edge question in the field crossing game theory [1] and evolutionary game theory [2] for decades. It is a milestone contribution of Cason, Friedman and Hopkins [3] that they confirmed the existence of cycles by quantitative measurement in control experiments, but we can hardly agree with one of their conclusions in their latest article.

In their article [3], they claimed that control treatments (using simultaneous matching in discrete time) replicate previous results that exhibit *"weak or no cycles"*, which depended on the result —— the stable discrete time treatments (SD) do not exhibit clear cyclical behavior indicated by *CRI's not significantly different from 0* (see their Table 3).

Two mathematical mistakes in the measurement algorithm were found after we checked their result above.[1] Then we corrected the mistakes (see Appendix). Using the refined measurement, if the start point (α, β) of the tripwire for counting cycles (see Fig. 1) was set at $(\frac{1}{4}, \frac{1}{4})$ —— Nash equilibrim (NE) of the games as [3], CRI of SD will still not be significantly different from 0 (see up panel in Table 1 and compare it with their Table 3). So, for SD, we agree that there are only weak or no cycles *around NE.*

However, if we set (α, β) at (0.23, 0.26) for SD-Mixed and at (0.22, 0.40) for SD-Pure and use the CRI measurement as criterion still,[2] we can find that CRI of both SD-Mixed and SD-Pure are *significantly different from 0* , indicating that there are indeed cyclical behaviors (see low panel in Table 1).

Meanwhile, if we use the accumulated counting number of cycles C as the index (see the Eq.(2) in [4] for detailed explanation) instead of CRI, we also find that all $\bar{C}$ (of the experimental blocks) for the treatments are significantly different from 0 (see Table 2). This, again, indicates the existence of cycles.

So, for SD, we doubt the validity of their conclusion that there are *"weak or no cycle"*. We suggest that the treatments (using simultaneous matching in discrete time) do exhibit significant cycles rather than *"weak or no cycles"*.[3]

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[1] We thank the authors' [3] confirmation on this point during the ESA-NA (2014) Conference.

[2] Our pilot results suggest that using angular motion of the transits (e.g. the θ in Eq.(10) in [4]) is an efficient way of observing cycles too. We hope to return to this issue in a future study.

[3] We would like to point out the followings. (1) The start point (α, β) for the two SD games is close to the actual mean observed value of the aggregated social strategy, which seems to tell us that cycles are actually around the mean observed value instead of NE. (2) Their treatments of RPS games are economical since all they need are 6 groups of 8 subjects and a period of time of 15 minutes. If cycle could be obtained, such treatments could be great exemplified classroom experiments for teaching evolutionary game theory.

Game Condition	(α, β)	*Number of Counter-Clock -wise Transits*	*Number of Clockwise Transits*	*Cycle Rotation Index* (CRI)	p-value
S Continuous-Instant	$(\frac{1}{4}, \frac{1}{4})$	25.2	5.5	0.65*	0.028
S Continuous-Slow	$(\frac{1}{4}, \frac{1}{4})$	9.4	0.8	0.87*	0.027
S Discrete-Mixed (SD-Mixed)	$(\frac{1}{4}, \frac{1}{4})$	2.3	1.2	0.38	0.116
S Discrete-Pure (SD-Pure)	$(\frac{1}{4}, \frac{1}{4})$	1.0	1.0	0.13	0.753
U_a Continuous-Instant	$(\frac{1}{4}, \frac{1}{4})$	31.9	0.9	0.94*	0.027
U_a Continuous-Slow	$(\frac{1}{4}, \frac{1}{4})$	8.2	0.0	1.00*	0.014
U_a Discrete-Mixed	$(\frac{1}{4}, \frac{1}{4})$	2.3	0.2	0.79*	0.027
U_a Discrete-Pure	$(\frac{1}{4}, \frac{1}{4})$	1.9	0.3	0.79*	0.027
U_b Continuous-Slow	$(\frac{1}{4}, \frac{1}{4})$	0.3	8.5	-0.93*	0.028
S Discrete-Mixed (SD-Mixed)	(0.23, 0.26)	2.3	1.1	0.41*	0.035
S Discrete-Pure (SD-Pure)	(0.22, 0.40)	4.1	3.3	0.14*	0.028

Table 1 Mean Transits and CRI (Update Table 3 in [3] with Refined Measurement).
*Denotes CRI Index is significantly (p-value $< 5\%$) different from 0 according to 2-tailed Wilcoxon test.

Game Condition	(α, β)	B_1 †	B_2	B_3	B_4	B_5	B_6	$\bar{C}$ ‡	p-value
S Continuous-Instant	$(\frac{1}{4}, \frac{1}{4})$	128	53	120	74	58	157	98.3*	0.028
S Continuous-Slow	$(\frac{1}{4}, \frac{1}{4})$	49	47	45	55	24	38	43.0*	0.028
S Discrete-Mixed (SD-Mixed)	$(0.23, 0.26)$	3	0	3	2	9	18	5.8*	0.035
S Discrete-Pure (SD-Pure)	$(0.22, 0.40)$	6	4	2	4	8	1	4.2*	0.027
U_a Continuous-Instant	$(\frac{1}{4}, \frac{1}{4})$	158	114	218	195	175	70	155.0*	0.028
U_a Continuous-Slow	$(\frac{1}{4}, \frac{1}{4})$	45	53	48	39	28	34	41.2*	0.028
U_a Discrete-Mixed	$(\frac{1}{4}, \frac{1}{4})$	9	16	10	6	6	17	10.7*	0.027
U_a Discrete-Pure	$(\frac{1}{4}, \frac{1}{4})$	5	11.5	1.5	13	5.5	11	7.9*	0.028
U_a Discrete-Pure	$(\frac{1}{4}, \frac{1}{4})$	-52	-29	-43	-49	-40	-33	-41.0*	0.028

Table 2: The accumulated counting number of cycles (C) in each block. † B indicates block, number $1 \sim 6$ indicates the block number in related treatment. ‡ $\bar{C}$ indicates the mean accumulated counting number of cycles of an experimental block. ***Denotes C index is significantly (p-value $< 5\%$) different from 0 according to 2-tailed Wilcoxon test.

Appendix

CRI was defined as [3] CRI=$\frac{CCT-CT}{CCT+CT}$, and CCT and CT can be interpreted as follows. Supposing the Poincare section ("tripwire") is the segment between P_c:≡(α, β) as shown in Fig. A1 (referring to the Fig. 1 in [3]) and P_e=$(\alpha, 0)$, and a transit is a directional segment from state (x_1, y_1) observed at t to state (x_2, y_2) at $(t + 1)$. These two segments could cross at X as

$$X :\equiv (X_x, X_y) = \left(\alpha, y_1 + \frac{(y_2 - y_1)(\alpha - x_1)}{x_2 - x_1}\right). \quad (1)$$

Accordingly, $CCT = \sum_{C_t>0} C_t$ and $CT = \sum_{C_t<0} |C_t|$ in which C_t value of the transit (at time t) should be[4]

Condition 1	$C_t = 0$	if $X \notin (P_c, P_e] \cup x_2 = x_1$
Condition 2	$C_t = 1$	if $X \in (P_c, P_e] \cap x_2 > \alpha > x_1$
	$C_t = -1$	if $X \in (P_c, P_e] \cap x_2 < \alpha < x_1$
Condition 3	$C_t = \frac{1}{2}$	if $X \in (P_c, P_e] \cap x_2 > x_1 \cap (x_1 = \alpha \cup x_2 = \alpha)$
	$C_t = -\frac{1}{2}$	if $X \in (P_c, P_e] \cap x_2 < x_1 \cap (x_1 = \alpha \cup x_2 = \alpha)$

At the same time, the accumulated counting number of cycles is C:=$\sum C_t$ (exactly the same as Eq.(2) in [4]). According to [4], when C serves as an index, the criterion for testing the existence of cycles is that: if C is significantly different from 0, then there are cycles; otherwise, no cycles.

There are two mathematical mistakes in the measurement algorithm (see Result_3.do in their paper's supplement). Mistake-1: For Condition 2, a necessary condition for $C_t = \pm 1$ was wrongly set as $\frac{y_1+y_2}{2} < \beta$. Mistake-2: For Condition 3, C_t was wrongly set as 0.

Game Condition	(α, β)	*Algorithm refined* (before/after)	*Index chosed* CRI/C	*Mean Index value*	p-value
S Discrete-Pure (SD-Pure)	(0.22, 0.40)	before	CRI	0.07	0.173
	(0.22, 0.40)	before	C	2.17	0.113
	(0.22, 0.40)	after	CRI	0.14*	0.028
	(0.22, 0.40)	after	C	4.20*	0.027

Table A1: Explanation for Necessity of Correction for the Measurement Algorithm.
*Denotes CRI Index is significantly (p-value < 5%) different from 0 according to 2-tailed Wilcoxon test.

These mistakes need to be corrected. Table A1 exhibits both results, algorithm not refined and refined. Obviously, without refined algorithm we cannot find cycles even if we set (α, β) at $(0.22, 0.40)$. On the contrary, with the refined algorithm, the existence of cycles can be verified by both CRI and C. This comparison implies that the algorithm correction is necessary.

Programs for the replication of the results in this paper are provided in supplements.

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[4]For a careful description of this measurement, see the Eq.(2) for the accumulated counting number C for cycles in [4]. We thank the authors' [3] confirmation on this point.

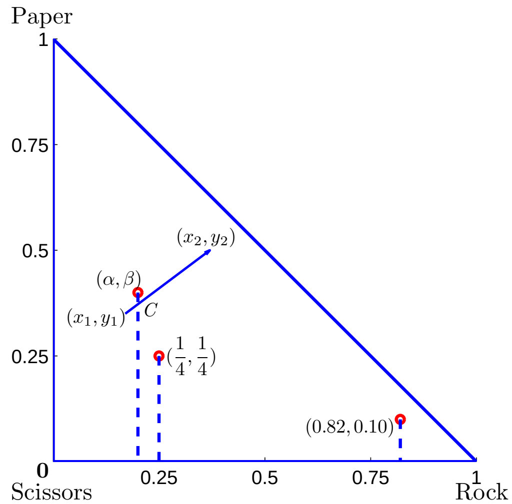

Figure A1: Illustration of the tripwire for counting cycles (Poincare section) [3]. A tripwire for counting cycles is a segment constructed between the reference point (α, β) and the simplex edge, as the vertical dashed line extending below the point (α, β). The reference point of the right tripwire should be at (α, β)=(0.82, 0.10), while the authors [3] set it at $(\alpha, \beta) = (\frac{1}{4}, \frac{1}{4})$ (the Nash equilibrium), the reference point of the tripwire in the middle, and thus discovering weak or no cycles for stable discrete time treatments (SD). The arrow indicates a transit from (x_1, y_1) to (x_2, y_2), crossing the tripwire from the left side.

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