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25 November 2013

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Abstract

We study the optimal production of a competitive risk-averse firm under price uncertainty. We suppose that the firm is also regret-averse. For example, if market prices ex post turn out to be very high the firm might regret not producing more. If it turns out that the price is low the firm might regret an over-production. We find that optimal output under regret aversion might be higher than under risk aversion. We also prove that optimal production could increase or decrease when the regret-averse coefficient increases. In general, we show that the regret-avers firm tend to hedge their bets, taking into account the possibility that their decisions may turn out to be ex post not optimal. These predictions can help explain the fact the price uncertainty has not such an extreme impact than those would be derived from pure risk-averse preferences.

JEL classification: D03, D21

Keywords: Firm, decision making, price uncertainty, regret aversion, risk aversion

Acknowledgments This research is partially supported by Technische Universität Dresden, University of Montevideo, Universität Augsburg, Research Grants Council of Hong Kong, and Hong Kong Baptist University. The fourth author would like to thank Professors Robert B. Miller and Howard E. Thompson for their continuous guidance and encouragement.

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Abstract

We study the optimal production of a competitive risk-averse firm under price uncertainty. We suppose that the firm is also regret-averse. For example, if market prices ex post turn out to be very high the firm might regret not producing more. If it turns out that the price is low the firm might regret an over-production. We find that optimal output under regret aversion might be higher than under risk aversion. We also prove that optimal production could increase or decrease when the regret-averse coefficient increases. In general, we show that the regret-avers firm tend to hedge their bets, taking into account the possibility that their decisions may turn out to be ex post not optimal. These predictions can help explain the fact the price uncertainty has not such an extreme impact than those would be derived from pure risk-averse preferences.

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1 Introduction

The purpose of this paper is to examine the impact of regret aversion on optimal production of a competitive risk-averse firm facing price uncertainty. The main assumption of our study is that firms avoid the unfavorable consequences of experiencing an outcome that is worse that the best that could have been achieved had the market price been known in advance. We introduce a modification of the objective function of the firm by introducing a two-attribute utility function of the firm. We also introduce a regret coefficient which put a linear weight on the regret component of the utility function. If this coefficient is zero, the firm is a standard risk-averse expected utility maximizer.

The firm faces price uncertainty and possesses preferences exhibiting not only risk aversion but also regret aversion. Regret aversion indicates the firm's desire to avoid consequences wherein the firm appears to have made ex-post suboptimal decisions even though

those decisions are ex-ante optimal based on the information available at that time. In the literature regret is defined as the disutility arising from not having chosen the ex-post optimal alternative. The economic literature provides an axiomatic foundation of regret theory, which is supported by extensive experimental studies that document regret-averse preferences among individuals.

In this paper, we follow the regret-theoretical approach of Braun and Muermann (2004) to characterize the a competitive production firm's regret-averse preferences by a utility function that includes disutility from having chosen ex-post suboptimal alternatives. The firm can hedge against the possibility of regret when the realized market price turns out to be high. The regret-averse firm as such has greater ex-ante incentive to produce more than the purely risk-averse firm. To gain more insights, we show discuss some numerical results. In general, we show that firms with regret-aversion would tend to 'hedge their bets', taking into account the possibility that their production or investment decisions may turn out to be ex post not optimal. Regret aversion as such plays a crucial role in determining the firm's optimal production.

The rest of this paper is organized as follows. Section 2 delineates the regret-theoretical model of an international firm facing exchange rate uncertainty, where the firm is not only risk averse but also regret averse. Section 3 examines the firm's incentive to export to the foreign country. Section 4 derives the firm's optimal production and export decisions. Section 5 offers a two-state example. The final section concludes.

The rest of the paper is organized as follows. In section 2, we introduce the model of a competitive firm under price risk, risk aversion and regret aversion. We derive our main results. In section 3 we present some numerical examples. The final section concludes.

2 The regret–averse firm

In this study we extend the literature by assuming the following two-attribute regret-averse utility function u on the profits Π for the regret-averse competitive firm

$$u(\Pi) = v(\Pi) - k\, g\left[v(\Pi^{\max}) - v(\Pi)\right] \tag{1}$$

The first attribute accounts for risk aversion and is characterized by the firm's utility function v to with $v' > 0$ and $v'' < 0$. The second attribute relates to the fact that the firm is concerned about the prospect of regret. The function g indicates the regret-averse attribute in which $g(0) = 0$ and, for any $Q > 0$, $g'(Q) > 0$ and $g''(Q) > 0$. The parameter $k \geq 0$ measures the weight of the regret attribute relative to the first risk-aversion attribute; Π is the firm's profit function defined as $\tilde{\Pi} = \tilde{p}Q - C(Q)$. and the ex-post optimal profit, denoted by $\Pi^{\max} = pQ^*(p) - C(Q^*(p))$, is the optimal profit if there were no price uncertainty.

Note that the function v is essentially what Bernoulli and Marshall describe as the psychological experience of pleasure associated with the satisfaction of desire. The utility function v satisfying $v' > 0$ and $v'' < 0$ is also called the Bernoulli utility function. However, the two-attribute regret-averse utility function u defined in (1) suggests that the pleasurable psychological experience of having Π will depend not only on $v(\Pi)$ but also on the nature of $v(\Pi^{\max}) - v(\Pi)$. Possessing the maximum profit $\Pi^{\max}$ is the most desirable rather than having Π and the individual may experience regret. One may reflect on how much better one's position would have been had one chosen differently, and this reflection may reduce the pleasure that one derives from Π. One could also view possessing the maximum profit $\Pi^{\max}$ as rejoicing, the extra pleasure associated with knowing that, as matters have turned out, one has taken the best decision.

Thus, the two-attribute regret-averse utility function u defined in (1) incorporates the concepts of both regret and rejoicing. To formulate the sensation of regret and rejoicing in this way is to assume that the degree to which a person experiences these sensations depends only on the utility associated with the two consequences in question: 'what is' and

'what might have been.' The regret-averse attribute g with $g(0) = 0$ and, for any $x > 0$, $g'(Q) > 0$ and $g''(Q) > 0$ indicates that the more pleasurable the consequence that might have been, the more regret and the less rejoicing will be experienced.

We further assume that the regret-averse firm obtains the optimal output Q^* by maximizing the following expected utility of profits and regret

$$\max_{Q} \mathrm{E}\left\{v(\Pi) - kg\left[v(\Pi^{\max}) - v(\Pi)\right]\right\} \ . \tag{2}$$

There are many advantages to using this modeling setting. First this setting covers both the theory for risk-averse competitive firms and others when $k = 0$ and the theory for regret-averse competitive firms when $k > 0$.

We introduce the regret-averse attribute, g, and the weight of the regret attribute, k, so that we can study the behavior of regret-averse competitive firms with different values of g and/or k: the higher the value of k and/or g', the stronger the attitude of regret. Nevertheless, the utility function u proposed in the literature could not be used to study the behavior of different types of regret-averse competitive firms. Thus, our regret-averse function defined in (1) is a more specific and tractable regret-averse function.

In our modeling setting, we allow random prices to be any random variable. Because of the above advantages, our model setting allows us to make comparative statics of the optimal output by varying the regret term as well as g and v, but the model developed by Paroush and Venezia cannot. Last, we conclude that our model setting has greater appeal to intuition than the one developed by Paroush and Venezia. Before we further develop our model, we first solve the maximization problem in (2) to obtain its first-order condition such that

$$\mathrm{E}\left\{(1 + k\, g'([v(\Pi^{\max}) - v(\Pi(x))]))v'[\Pi(Q)](\tilde{p} - C'(Q))\right\}. \tag{3}$$

By the assumptions of g, v, and C, the second-order condition is satiesfied.

Under risk aversion we know that $E(\tilde{p})$ could be less than $C'(Q^*)$ if we apply the theory developed by Paroush and Venezia (1979), which some economists believe is not reasonable. Thus, our first objective is to see whether the more specific and tractable regret-averse function stated in (1) could be used to circumvent the limitation. We find that our proposed regret-averse function stated in (1) could be used for this purpose as stated in the following theorem

Propositon 1 For any regret-averse firm with utility function u defined in (1) that will maximize the expectation of u and face an uncertain price, it will choose an optimal output Q^* such that the expected price, $E(\tilde{p})$, exceeds the marginal costs, $C'(Q^*)$, i.e. $E(\tilde{p}) > C'(Q^*)$.

One question is whether this optimal output increases or decreases as the regret-averse attribute varies. One way to answer this question is to study the comparative statics of this optimal output when the regret parameter k changes, as we do in the following

Proposition 2 Under the conditions and assumptions stated before, we have

$$E(\tilde{p}) - C'(Q^*) \geq \frac{\text{cov}[-\tilde{p}, v'(\Pi(Q^*))]}{Ev'(\Pi(Q^*)}, \quad \frac{dQ^*}{dk} \leq 0.$$
$$E(\tilde{p}) - C'(Q^*) \leq \frac{\text{cov}[-\tilde{p}, v'(\Pi(Q^*))]}{Ev'(\Pi(Q^*)}, \quad \frac{dQ^*}{dk} \geq 0.$$

As we have shown in the above theorems optimal output can go either way when the second attribute varies. Importantly, this condition does not directly depend on the second attribute, which accounts for the feeling of regret. The intuition behind this result is as follows: By concavity of v, the sign of $\text{cov}[-\tilde{p}, v'(\Pi(Q^*))]$ is positive. It means that if the difference between the certain price and the marginal cost is large enough, then the regret factor would lower the optimal output. In this case, the regret factor amplifies the decline in the optimal product. For the second case, the analysis is the opposite.

3 Some illustrations

We construct an example to illustrate some of our findings.

Example 1 We assume $C(Q) = Q^2$, $u(Q) = \sqrt{x}$, and the price of output, p, to be a random variable with $p = \$50$ and $p = \$10$ of equal probability so that its expected value $E(\tilde{p}) = \$30$ and the firm maximizes

$$\max_Q \mathrm{E}(pQ - Q^2)^{1/2} = \max_Q \left\{ 0.5\sqrt{(10Q - Q^2)} + 0.5\ \sqrt{(50Q - Q^2)} \right\} \ .$$

Its first-order condition is

$$\frac{10Q - 2Q}{4\sqrt{(10Q - Q^2)}} + \frac{50Q - 2Q}{4\sqrt{(50Q - Q^2)}} = 0 \ .$$

Thus, the optimum is at $Q^* = 8.33$ when $C'(Q^* = 8.33) = 2 \cdot 8.33 = 16.66 < 30 = E(\tilde{p})$. Example 1 supports the finding that an optimal output Q^* characterized by marginal cost $C'(Q^*)$ is less than the expected price $\mathrm{E}(\tilde{p})$. We turn to constructing an example to illustrate that $\mathrm{E}(\tilde{p})$ could be less than $C'(Q^*)$ based on the theory developed by Paroush and Venezia (1979)

Example 2 We define the utility function of the firm in the theory under uncertainty for regret-averse competitive firms developed by Paroush and Venezia (1979) to be a function of both profits $\Pi(p, Q)$ and regret $R(p, Q)$ such that

$$u[\Pi(Q, p), R(Q, p)] = \sqrt{\Pi(Q, p)} - R(Q, p)^{1.1} \ , \tag{4}$$

where $R(Q, p) = \Pi^{\max} - \Pi(Q, p)$. We note that this utility function satisfies all the assumptions required, for example, $u_\Pi > 0$ and $u_R < 0$. In addition, we assume that price fits the following distribution

price	probability
20	98%
48	1%
50	1%

(5)

Thus, we have $\mathrm{E}(\tilde{p}) = 20.58$. We further assume that the cost function is the same as the one used in Example 2 such that $C(Q) = Q^2$. We first examine the situation under certainty in which the firm will obtain optimal output by equating its marginal revenue to its marginal costs such that $2Q = 20.58$ and thus we have $Q = 10.29$ and the optimal output of the firm is 10.29 if we assume that price is certain, that is, $p = 20.58$. Under the assumption of utility function defined in above and that price follows the distribution defined in the table the firm will maximize

$$\begin{aligned}\max_{Q} \mathrm{E}[u(\Pi)] &= 0.98\left[\sqrt{20Q-Q^2}-\left(100-20Q+Q^2\right)^{1.1}\right] \\ &\quad +0.01\left[\sqrt{48Q-Q^2}-\left(576-48Q+Q^2\right)^{1.1}\right] \\ &\quad +0.01\left[\sqrt{50Q-Q^2}-\left(625-50Q+Q^2\right)^{1.1}\right]. \end{aligned} \tag{6}$$

Maximizing yields $Q = 10.52$, which is larger than that under the certainty theory ($Q = 10.29$). At last, we construct the following example to illustrate Proposition 1 and 2 for the theory of the behavior of regret-averse firms developed in this study.

Example 3 Let $v(Q) = \sqrt{Q}$, $C(Q) = Q^2$, $g(Q) = Q^{1.5}$, and $p = 50$ or $p = 10$ with equal probability. Under the theory developed in this article, regret-averse firms will produce the optimal output by maximizing the following:

$$\begin{aligned}\max_{Q} \mathrm{E}[u(\Pi)] &= \max_{Q}\left\{\frac{1}{2}\left[\sqrt{50Q-Q^2}-k\left(25-\sqrt{50Q-Q^2}\right)^{1.5}\right]\right. \\ &\quad \left.+\frac{1}{2}\left[\sqrt{10Q-Q^2}-k\left(5-\sqrt{10Q-Q^2}\right)^{1.5}\right]\right\}. \end{aligned}$$

We have the following: if $k = 0$, then $Q^* = 8.33$ (risk-averse case), if $k = 0.5$, then $Q^* = 8.83$, and if $k = 1$, then $Q^* = 8.92$. The above findings show that the optimal output under the regret-averse theory developed in this study is still less than that under the certainty theory, in which we get $Q = 10.29$; optimal production varies when k changes. From the above, we have $dQ^*/dk > 0$.

4 Concluding remarks

We introduce a more specific and tractable regret-averse function into the theory of the firm to anticipate feelings of regret and rejoicing. This model yields a range of inferences in the theory of production consistent with the behavior of regret-averse firms. For instance, we have shown that a regret-averse firm with an uncertain output price would choose an optimal output that is greater than under pure risk-aversion. We have also shown that this optimal output could increase or decrease as the regret-averse coefficient varies. In general, we claim that firms with regret-averse preferences would hedge their bets, taking into account the possibility that their decisions may turn out to be ex post suboptimal. In comparison with pure risk aversion our derived results are 'less extreme' when regret aversion is taken into account.

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