The Size and Growth of Firms

Singh, Ajit and Whittington, Geoffrey

University of Cambridge, University of Edinburgh

January 1975

The Size and Growth of Firms [1, 2]

AJIT SINGH
University of Cambridge

and

GEOFFREY WHITTINGTON
University of Edinburgh

1. INTRODUCTION

This paper is a sequel to the analysis of the growth process of firms presented in Chapters 4 and 5 of our book *Growth, Profitability and Valuation* [17]. The relationship between size and growth of firms is explored using a more comprehensive set of data than was used in the book. In particular, the book was based on data relating to individual quoted companies in the UK in only three large industries, whereas, in this paper, we extend the analysis to cover all major industrial groups in manufacturing, construction and distribution.

The relationship between size and growth of firms, and particularly stochastic models based on the Law of Proportionate Effect or Gibrat's Law, have previously been studied by a number of economists other than ourselves (see among other [5, 6, 7, 8, 12, 15 and 18]). Relatively few of these studies have used individual industry data. Industry is, however, an important variable, because the characteristics of the average firm vary significantly and systematically between industries (see [16]). Furthermore, none of the previous studies is based on as comprehensive a set of data as is the analysis presented below. The availability of data on such an extensive scale, showing the growth experience over the period 1948-60 of nearly 2000 individual firms, divided into 21 industrial groups, yields some interesting insights which have not been possible in earlier studies. It also leads to the revision of some important conclusions of our own previous study and of other similar studies.

2. SIZE AND GROWTH: SOME THEORETICAL CONSIDERATIONS

The relationship between the size and growth of firms in economic theory has traditionally involved the twin notions of the optimum size of the firm and industrial equilibrium. The former has been rigorously restated recently [21] in terms of certain propositions of organization theory, which has the additional advantage of being applicable to multi-product firms. However, even if there is an optimum size of the firm, the traditional theory unfortunately gives very little guidance as to the exact nature of the relationship between size and growth which one would expect to observe among a *cross-section* of firms, except in the trivial case of static equilibrium where by definition no firm would grow. Clearly, a cross-sectional relationship between the two variables will be observed only if some or all firms are not in equilibrium; its specific form would depend upon the causes of disequilibrium and the assumed speed of adjustment towards equilibrium.

Nevertheless, if all firms within an industry are assumed to face the same *U*-shaped long run average cost curve as postulated in traditional theory, it can be argued that one

[1] *First version received December 1972; final version accepted February 1974 (Eds.).*

[2] The results reported in this paper follow from a project carried out at the Department of Applied Economics, Cambridge, with the aid of a grant from the SSRC. The authors are grateful to Dr Hendry and the anonymous referees for their constructive criticism of an earlier draft of this paper.

would expect to observe a negative relationship between firm size and growth among a cross-section of firms in the industry. This is because the large firms are assumed to be at or near their optimum size and would therefore have to grow very little and might even shrink if they exceeded optimum size. The small firms would be furthest below the optimum size and would need to grow at a faster rate to achieve this size.

In recent years, a number of economists have approached the problem of size and growth in a theoretical framework rather different from that of the traditional theory (see [11]). They argue that, in a modern corporation characterized by a divorce of ownership from control, salaried managers will be less interested in maximizing the profits (or stock market valuation) of the firm than in maximizing the rate of growth, to the extent that these two objectives conflict. It is further suggested that there is no limit to the absolute *size* of the firm as such, but that there does exist a limit to its *growth rate* per unit time. This particular framework indicates a positive relationship between size and growth on a cross-sectional basis. This is because, *ceteris paribus*, the larger the size of the firm, the more will it be expected to be managerially dominated, and the more, therefore, will it wish to grow, compared to a small firm which is likely to be owner-controlled and therefore less interested in growth *per se*.

The above two approaches in terms of economic theory clearly do not produce many testable hypotheses, but there also exists another way of looking at the relationship between size and growth which is a great deal more promising. In this view, growth is regarded as a purely stochastic phenomenon resulting from the cumulative effect of the chance operation of a large number of forces acting independently of each other. The economic motivation for this conception may be expressed as follows. The chances of growth or shrinkage of individual firms will depend on their profitability as well as on many other factors which in turn will depend on the quality of the firm's management, the range of its products, availability of particular inputs, the general economic environment, etc. During any particular period of time, some of these factors would tend to increase the size of the firm, others would tend to cause a decline, but their combined effect would yield a probability distribution of the rates of growth (or decline) for firms of each given size. It is commonly asserted that this probability distribution is the same for all size-classes of firms. This is the well-known Law of Proportionate Effect [LPE], which has attracted a great deal of attention in the literature, and which, in its strong form, simply says that the probability of a firm growing at a given proportionate rate during any specified period of time is independent of the initial size of the firm. Thus, if the size of the ith firm at time " t " is denoted by S_{it}, the Law of Proportionate Effect asserts that

$$S_{it}/S_{i,t-1} = \varepsilon_{it}, \qquad \ldots(1)$$

where ε_{it} is a random variable distributed independently of $S_{i,t-1}$.

Apart from yielding many precisely testable hypotheses which will be described later, the LPE has some important economic implications. First, like the managerialist approach discussed above, the Law implies that there is no optimum size of the firm, although, unlike that approach, it does not imply that size and growth should be positively related. Secondly, in its strongest form, it suggests that the rate of growth of the firm in one period has no influence on its growth in the subsequent periods.[1] Thirdly, in its strong form, stated in equation (1), the Law implies increasing industrial concentration in a *constant* population of firms over time. This is intuitively obvious, and is easily demonstrated by the application of the Central Limit Theorem to $\log \varepsilon_{it}$. If the assumptions of the Central Limit Theorem are met the variance of $\log S_{it}$ will increase proportionately through time, and as $t \to \infty$ it will become infinite (cf. [8]). However, if the LPE does not operate in the strong form stated above [9], or if it is assumed that there is a particular way in which

[1] This assumption enables the LPE to be treated as a first-order Markov process. However, a less stringent version of this Law, which does not require serial independence of growth rates, can also be incorporated in a stochastic model [8]. See further Section 7 below.

firms enter or leave the population [15], there need not be increasing industrial concentration over time.

Furthermore, the stochastic processes derived from the LPE are broadly able to explain the observed size distributions of firms, which from widely different populations have been known to approximate the Pareto or log-normal distributions. The usual economic theories of the firm outlined above yield no predictions about the precise form of the size distribution of firms, but the LPE, with suitable modifications, does generate distributions of the type which are observed. For instance, it can be shown that if firm growth is governed by the LPE, as given in equation (1), the size distribution of firms would tend towards a log-normal distribution [18].

In view of its important implications, the empirical sections which follow will examine directly the validity of the LPE. It asserts two immediately testable hypotheses concerning the cross-section relationship between the size and growth of firms: (*a*) that firms of different size-classes have the same average proportionate growth rate; and (*b*) that the dispersion of growth rates about the common mean is the same for all size-classes. Both (*a*) and (*b*) are necessary conditions for the validity of the Law in its most stringent form, but they are not sufficient since the Law suggests that the entire distribution of growth rates should be the same for the firms of different sizes. There is another implication of the strongest form of the Law which can also be directly tested, namely: (*c*) that the rate of growth of the firm in one period should be independent of its growth rate in subsequent periods, i.e. there should be no serial correlation in firm growth rates. If this were not so, it might be expected, *ceteris paribus*, that opening size and subsequent growth would be related because both are related to past growth.[1] Hypothesis (*c*) is of considerable economic interest and deserves to be examined in its own right.

3. EMPIRICAL RESULTS: HYPOTHESES (*a*) AND (*b*)

In this section, we shall test hypotheses (*a*) and (*b*) by comparing the means and standard deviations of growth rates of firms in different size-classes. The tests are based on data pertaining to all UK companies in Manufacturing, Construction, Distribution and Miscellaneous Services which had a quotation on the stock market and which survived over the period 1948-60 or over either of the two shorter periods 1948-54 and 1954-60. The same data, but confined to only four manufacturing industries (Food, Non-electrical Engineering, Clothing and Footwear, and Tobacco), were used in [17], to which the reader is referred for a precise definition of the population of companies studied, for a full account of the nature and limitations of the date, and for a discussion of different measures of " size " and " growth ". We simply note that size is measured here by the balance-sheet value of the firm's net assets; " growth " of net assets is not corrected for changes in the price level. Another measure of growth (of physical assets) is also used; it measures the increase in the fixed tangible assets of the firm, and is also based on balance-sheet values, with no correction for inflation. In the context of the discussion of the LPE, it must be emphasized this study is confined to quoted companies.

3.1. *Average Growth Rate by Opening Size-Class*

Table I gives the means and standard deviations of growth rates (per annum) of the surviving firms in different size-classes over the period 1948-60, and over each of the two sub-periods 1948-54 and 1954-60 respectively, for all 21 industries together. Tables for individual industries are not given here to save space.[2] Firms are arranged by " opening size ", i.e. size at the beginning of each of the relevant periods, and a geometric scale has been used for division into size-classes.

[1] The caveat is important: the effect of positive serial correlation could be offset by other economic factors which tended to cause a negative relationship between opening size and growth.

[2] Copies of all unpublished tables are available from the authors.

TABLE I

Growth of net assets by opening size-class: all 21 industries together: 1948-60, 1948-54, and 1954-60

Opening size-class (£000's)	Period								
	1948-60			1948-54			1954-60		
	n	*m*	*s*	*n*	*m*	*s*	*n*	*m*	*s*
1 < 250	483	6·1	6·6	676	5·7	7·2	409	6·4	12·0
2 < 500	464	6·5	5·2	587	6·9	6·6	454	5·8	8·1
3 < 1000	389	7·0	5·3	492	7·2	7·0	506	6·2	7·3
4 < 2000	271	7·2	5·4	330	7·5	6·2	433	6·6	7·1
5 < 4000	167	7·9	5·4	203	7·5	6·6	247	7·2	6·3
6 > 4000	181	8·4	4·9	201	8·5	5·9	315	8·5	6·1
All companies	1955	6·9	5·7	2489	6·9	6·8	2364	6·6	8·2
Welch-Aspin test[1]	1<3, 1<4, 1<5, 1<6, 2<4, 2<5, 2<6, 3<5, 3<6, 4<6			1<2, 1<3, 1<4, 1<5, 1<6, 2<6, 3<6, 4<6			1<6, 2<5, 2<6, 3<5, 3<6, 4<6, 5<6		

Note to Table I

n = Number of observations.
m = Mean annual growth rate in percentage points.
s = Standard deviation, uncorrected for degree of freedom.

The table shows an almost systematic positive relationship between size and average growth rate in all three periods. The Welch-Aspin test [1], which does not assume equal variances in the two populations, was used to test the significance of the observed differences between the mean growth rates in the various size-classes; the results of the test are reported in the last row of Table I. Thus we find that, for the period 1948-60, of the 15 possible pairs of size-class means for which significant differences could have been found, the differences in the mean growth rates were significant at the 10 per cent level in 10 cases; for 1948-54 they were significant in 8 cases and for 1954-60 they were significant in 7 cases. These results suggest on the whole a significant, though not a strong, positive association between firm size and average growth rate.

The above conclusion must be treated with caution since it could have arisen solely from aggregating industries with very different distributions of sizes and growth rates of firms. It is, therefore, necessary to examine the relationship between the two variables in the individual industries. The tables for the individual industries showed that because of the relatively small number of observations in each size-class, the differences between the mean growth rates were significant in comparatively few cases (120 out of 675 possible cases). Nevertheless, in a large majority of industries, firms in the larger size-classes showed, on the whole, higher average growth than firms in the smaller size-classes. The weight of the entire evidence, both for the individual industries and for the aggregate of industries, points to the conclusion that there is a weak positive relationship between size and growth. At the very least, given the evidence, the hypothesis of positive association cannot be rejected.

This conclusion was confirmed by the results of the regression analysis (not reported here) in which we fitted the following simple model to the cross-section of firms in each industry, for each period.

$$\text{Growth} = a + b \cdot \log \text{Opening Size} + e. \qquad \ldots(2)$$

[1] The Welch-Aspin test section reports those differences between pairs of size-class means which are statistically significant at the 10 per cent level, using a two-tailed test.

We must therefore conclude that hypothesis (*a*) of the LPE is not supported by the data. The evidence of the regression analysis as well as of the distribution of growth rates by size-class clearly suggests a mildly positive relationship between the variables.

3.2 *The Standard Deviation of Growth by Size Class*

Table I shows that the standard deviation of growth rates declines with an increase in firm size in all three periods. An approximate statistical test [1] was used to examine the hypothesis of homogeneity in the dispersion of growth rates of firms in different size-classes. The hypothesis was decisively rejected at the 1 per cent level in each time-period.

In order to check the possibility of aggregation bias, the relationship between the two variables was examined in the individual industries. In *most* industries, the standard deviation of growth rates in the largest size-classes was less than that observed in the smallest size-classes. The homogeneity hypothesis was rejected at the 5 per cent level in 17 out of 20 industries [2] for the period 1954-60, in 12 industries for the period 1948-54 and in 10 industries for the period 1948-60. The balance of the whole evidence clearly indicates that the second prediction of the LPE, that the dispersion of growth rates in different size-classes is the same, should also be rejected. The evidence against the second prediction is considerably stronger than that against the first.

It is not at all surprising that this hypothesis is rejected. Indeed, from an economic point of view, one would expect that large firms would tend to have more uniform growth rates than small firms. This is because large firms are likely to be more diversified and this would allow them to offset an adverse growth rate in one market against a good performance in another. If a large company was merely a group of smaller subsidiary companies operating independently in different markets, so that the growth rates of subsidiary companies were independent of each other, then an elementary statistical theorem shows that the standard deviation of the holding company's growth rates would be inversely proportional to the square root of its size. In fact, in none of the individual industry groups nor in " all industries " together (see Table I) does the standard deviation decline with an increase in the size of the firm as rapidly as is required by this theorem. This merely confirms the common-sense view that a large firm cannot be viewed as an aggregation of independent smaller firms: the performance of different parts or divisions of the firm are not totally unrelated to each other (cf. [14]).

4. THE REGRESSION OF LOGARITHMS OF CLOSING SIZE ON LOGARITHMS OF OPENING SIZE

Another way of testing whether or not the requirements of the LPE are met is to study the relationship between the logarithms of firm sizes at the beginning and at the end of a period. If the LPE is valid, then following from equation (1) there will be a systematic relationship between the two variables, which would be reflected by the parameters of the equation:

$$\log S_{it} = a_t + b \,.\log S_{i,t-1} + \log \varepsilon_{it}, \qquad \ldots(3)$$

where $\log \varepsilon_{it}$ is a homoscedastic random variable with zero mean. When $b = 1$ and the variance of $\log \varepsilon_{it}$ is in fact constant, this will mean that for all firms, irrespective of size, the average and variance of the logarithms of proportionate growth are the same, i.e. the two basic requirements of the LPE are met. If, however, $b > 1$, it is easy to show that the large firms will grow proportionately faster and the dispersion in the size of firms will increase. If " b " is less than 1, the smaller firms will tend to grow proportionately faster,

[1] The test used was that given in Table 31 of *Biometrika Tables for Statisticians*, Vol. 1, supplemented in marginal cases by the M-test (*Biometrika Tables*, Table 32).

[2] Only 20 of the 21 industrial groups could be tested because the small number of companies in Tobacco (Industry 14) made it unsuitable for testing.

which will tend to reduce the degree of dispersion, although ε_{it} may be powerful enough to outweigh this effect.

The results obtained by fitting equation (3) by least squares to the cross-section of firms in each industry for the whole period 1948-60 are given in Table II. The corresponding tables for the two sub-periods are omitted to save space. The most striking feature of the regression results is that " b " exceeds 1 in almost every industry over each of the three time-periods considered. It is true that " b " significantly (5 per cent level) exceeds 1 in only a few individual industries, but in view of the fact that it is almost always greater than unity and significantly exceeds one for all industries together, this confirms the conclusion that the data reject the first essential requirement of the LPE. The evidence in favour of rejecting this requirement is stronger on the basis of equation (3) than on the basis of regression equation (2), used earlier.

TABLE II

Regression results: whole period, 1948-60

Equation: log Closing Size = $a+b$.log Opening Size+ε

Industry		a	b	SE (b)	$\bar{R}^2$
Bricks, Pottery, etc.	(01)	0·14	1·11†	0·05	0·85
Chemicals and Allied Industries	(02)	−0·10	1·15†	0·06	0·81
Metal Manufacture	(03)	0·95*	1·02	0·04	0·89
Non-electrical Engineering	(04)	0·99*	1·02	0·03	0·82
Electrical Engineering	(05)	1·17*	1·01	0·05	0·85
Vehicles	(06)	−0·00	1·14	0·09	0·74
Metal Goods n.e.s.	(07)	0·77*	1·03	0·06	0·73
Cotton and Man-made Fibres	(08)	0·56	1·02	0·05	0·89
Woollen and Worsted	(09)	0·18	1·09	0·07	0·81
Hosiery, etc.	(10)	0·67*	1·01	0·05	0·82
Clothing and Footwear	(11)	−0·28	1·14	0·08	0·72
Food	(12)	0·67*	1·03	0·05	0·86
Drink	(13)	−0·09	1·09†	0·04	0·88
Tobacco	(14)	2·42	0·83	0·33	0·38
Paper, Printing, etc.	(15)	0·62*	1·04	0·04	0·85
Leather, etc.	(16)	0·17	1·08	0·06	0·70
Construction	(17)	0·53	1·07	0·10	0·72
Wholesale Distribution	(18)	0·75*	0·98	0·05	0·71
Retail Distribution	(19)	0·71*	1·04	0·05	0·79
Entertainment and Sport	(20)	−0·27	1·08	0·04	0·92
Miscellaneous Services, etc.	(21)	0·52*	1·01	0·03	0·87
All industries		0·41	1·06†	0·01	0·82

Notes: * "a" coefficient significantly different from zero at the 5 per cent level.
† "b" coefficient significantly different from one at the 5 per cent level. (t-tests).
Natural logarithms and the pre-1958 2-digit Standard Industrial Classification were used.

There are, however, two qualifications to this conclusion which should be considered. Firstly, the values of " b " may be subject to an upward bias if the assets of large firms are more likely to have been revalued than those of small firms. There is some evidence [17, pp. 90-92] that this was in fact the case during the period studied. However, the conclusions of Section 3 are not affected by this qualification, since the measure of proportionate growth used there excludes the effects of revaluation. Secondly, in view of the results of Section 3, it is most likely that the error term in equation (3) is heteroscedastic. This would affect the efficiency of the estimates reported in Table II, but the estimates would still be unbiased. (See, however, Section 5 below.)

Since b typically exceeds 1, it must imply increased industrial concentration amongst continuing companies (as measured by the variance of log S_t) in 1960 compared with either 1948 or 1954. The restriction to continuing companies is important since concentration amongst all firms over a period of time is not merely a function of the growth process of the existing firms, but also of the nature and the rate of new entry and exit from the company population. The problem of " births " and " deaths " will be dealt with in Section 6, but it is sufficient to note here that the variance of log size did increase in almost every industry and for all industries together over the periods observed.[1]

One interesting insight into the concentration process is that the " b " coefficient seems to be highest in those industries in which the average rate of growth of firms was lowest, i.e. within the relatively stagnant industries the large firms tended to grow faster. The Spearman rank correlation coefficients, r_s, between the mean industry growth rates and the " b " coefficients, ranked across 21 industries are as follows:

1948-54 $-0{\cdot}553$*; 1954-60 $-0{\cdot}574$*; 1948-60 $-0{\cdot}198$

* indicates significantly different from 0 at the 10 per cent level.

The above result, which implies a negative association between *changes* in industrial concentration and industry growth rates, is not surprising from an economic point of view. Although the question has not before been considered in empirical studies of the LPE, there is some evidence both from UK [4] and US [19] which supports this conclusion.

5. THE PERSISTENCE OF GROWTH

In this section we shall investigate whether firms which had high (or low) growth rates over one six-year period (1948-54) also tended to have high (or low) growth rates in the subsequent six-year period (1954-60). It will be recalled that the LPE in the strongest form discussed in Section 1 (a first-order Markov process) implied no serial correlation between firm growth rates. On the other hand, many of the recent economic models of firm growth [11] are " steady state " models in which firms are assumed to choose long-run stable growth paths depending on their respective utility functions and their resource and other constraints. These models suggest a high degree of persistence in the growth rates of firms. It is important to find out which of these two views is more in accord with the empirical evidence.

This problem was investigated by means of regression analysis. Denoting the proportionate growth per annum of the ith firm over the period 1954-60 by g_{it},over the period 1948-54 by $g_{i,t-1}$, the following regression equation was fitted by least squares to the cross-section of firms in each industry separately and in all industries together:

$$g_{it} = a + b \,.\, g_{i,t-1} + e_{it} \qquad \ldots(4)$$

The results, which are not given here, indicate that there is a definite tendency for the relative growth rates of individual firms to persist: the " b " coefficient is positive in almost all the individual industries and in " all industries " together; it is also significantly (5 per cent level) different from zero in many individual industries and for all industries together. On the other hand, since the values of R^2 are uniformly low (about $0{\cdot}05$), the past growth record of the firm cannot be regarded as a good predictor of its future growth. Furthermore, although the " b " coefficient is greater than zero, it is always considerably below 1 (on average about $0{\cdot}3$), which implies a tendency for firm growth rates to regress over time towards the mean growth rate of the industry.[2]

To overcome the problem of extreme values as well as of possible non-linearity, rank correlation analysis was used to supplement the results of the regression analysis.

[1] It must be emphasized that there is no particular virtue in the variance of log size as a measure of concentration, and we recognize all its defects from an economic point of view [2]. We are using it here merely as one possible indicator of the degree of industrial concentration.

[2] It should also be noted that any measurement errors in the observation of size in 1954 will cause b to be biased downwards.

The rank correlation coefficients were positive in *every* industry, and although they were relatively small, they were statistically significant at the 5 per cent level in 17 out of 21 industries. These results thus provide even stronger evidence that there is a definite but relatively small degree of persistence in the growth rates of firms, where growth is measured in terms of net assets.[1] It is, however, important to remember that the persistence of growth has been studied here only over a subsequent six-year period. One would expect to observe a stronger persistence in the growth rates of firms over shorter time-periods and less persistence if time-spans of much longer than 6 years were examined.

In view of the observed persistence in growth and the mildly positive relationship between size and growth, it is possible that the estimates of the regression coefficients in equations (3) and (4) above, particularly the " b " coefficient in (3), may have an upward bias due to omitted variables, i.e. it is possible that the weak positive association between growth and opening size was due to the fact that both growth and opening size are positively associated with past growth, rather than to a direct structural relationship. To check for this bias, the following regression equation which approximately encompasses both (3) and (4) was also estimated.[2]

$$\log (S_{it}/S_{i,t-1}) = a + b \cdot \log S_{i,t-1} - c \cdot \log (S_{i,t-1}/S_{i,t-2}) + v_{it}, \qquad \text{...(5)}$$

where t, $t-1$ and $t-2$ refer to 1960, 1954 and 1948 respectively. The regression results showed that the positive association between size and growth was very much weaker when the influence of past growth was removed, i.e. the coefficient " b " was much lower in equation (5) than in equation (3). For example, when the equation was estimated across the pooled population for all industries, " b " was 0·04 in equation (3) and 0·01 in equation (5), and only the former was significantly different from zero at the 5 per cent level. The positive serial correlation of growth rates was, on the other hand, reduced only slightly by allowing for the influence of opening size.

We conclude that a large proportion of the positive relationship between size and growth is due to the positive serial correlation of growth rates. This does not affect our conclusion that the Law of Proportionate Effect is contradicted by the observed relationship between growth and opening size, but it does draw attention to the probability that serial correlation of growth rates is the main cause of this result.

6. BIRTHS AND DEATHS

The empirical analysis of the previous sections has been confined to surviving companies. In this section we present the results of a limited analysis of companies which " died " (i.e. disappeared from the population) or which were " born " (i.e. added to the population) during some of the periods considered, in order to give a rough indication of the size and impact of the birth and death process.

The analysis of births and deaths is based on a restrictive range of companies; companies which were born in 1948-54 and subsequently continued throughout the period 1954-60, and companies which died in 1954-60 and had previously continued throughout 1948-54. In spite of its limited scope, this analysis revealed three important features of the birth and death processes during the periods examined. Firstly, it was found that although most of the births occur in the smaller size-classes, a considerable number occur in all size-classes. This pattern of births is contrary to that assumed in some recent stochastic models of firm growth [15, 7], which hypothesize that the birth of companies occurs only in the smallest size-class. The wide range of size of births is partly due to the

[1] When growth is measured in terms of "physical" assets rather than net assets, the observed persistence in growth rates was lower.

[2] We are grateful to a referee for suggesting this equation. One could obviously attempt more complicated autoregressive schemes, but since we are only testing the LPE in its strongest form which precludes any serial correlation in growth rates, and in view of the data available to us, we have confined ourselves to a second-order autoregressive equation above.

fact that *quoted* companies are often in existence as unquoted companies for a number of years before achieving a quotation and so being born into our population. It is also due to the fact that new quoted companies are often formed as a result of mergers: in such a case, the new company will be as large as the sum of its component companies.

Secondly, it was found that most of the deaths also occur in the smaller size-classes and that the incidence of deaths declines systematically with an increase in firm size. In fact, when the largest size-class (firms with net assets of greater than £4 million) is further subdivided, it is found that the incidence of deaths declines much more sharply with an increase in the size of the firms. Thus, there is a negative, non-linear relationship between firm size and the probability of death (cf. [17, p. 89] and [16, ch. 2]). It is important to note in this context that if the LPE is thought to apply to *all* firms and not just the *surviving* firms, the incidence of death (or very high negative growth rate) should be independent of firm size. This is clearly not the case.

Thirdly, we studied the net impact of births and deaths on the company population. When the " net change " due to births and deaths in the number of companies in each size-class was considered, it showed that there was a proportional loss of companies which was spread more or less evenly over all size-classes except the smallest, in which there was a much higher net loss of companies. The next highest net loss occurred in the largest size-class, due to the low birth rate in that size-class. This suggests that the concentration index, as measured by the variance of log size, might have decreased because the extreme sizes tended to disappear most. An examination of changes in the dispersion of logarithms of firm sizes for the various populations showed that the impact of births and deaths caused the dispersion to decrease in 12 out of the 21 individual industries and when all industries are considered together. On the other hand, we find that for the continuing populations of firms the concentration index increased in 20 of our 21 industrial groups, which is not surprising in view of our results in Sections 3 and 4. It should, however, be noted that it is rather artificial to separate the impact of births and deaths from the effects of the growth process in this manner. As will be discussed in Section 7, take-overs are the major cause of " death " and are also a major means by which the surviving companies grow.

7. SUMMARY, CONCLUSIONS AND ECONOMIC IMPLICATIONS

We summarize below the main stylized facts about the growth process of firms which have emerged from our analysis of the records of nearly 2000 UK quoted companies over the period 1948-60. These facts are as follows:

(1) Among the surviving firms, there is a mildly positive relationship between size and growth. The larger size-classes tend to have a somewhat higher mean growth rate than firms in the smaller size-classes. This finding revises a major conclusion of our previous study [17] and that of many of the other studies in this area; that mean growth rates are much the same for firms of various sizes. [17] was, however, based on an analysis of quoted companies in only three large industry groups; it is the extension of the analysis to all 21 industries which has enabled us to establish that there does exist a definite, albeit weak, positive relationship between size and growth.

(2) The dispersion of growth rates declines with an increase in firm size. The large firms do not, however, experience as high a degree of uniformity in their growth rates as would be compatible with the view that the typical large firm is merely an aggregation of typical independent small firms. This conclusion confirms the findings of [17].

(3) Firms which have an above (or below) average growth rate over one 6-year period also tend to have an above (or below) average growth rate in the subsequent 6-year period. The evidence from this study indicates a definite, but relatively small, degree of persistence in firm growth rates over the period examined. This finding confirms and reinforces a

tentative conclusion of [17]. It should be emphasized that the persistence of growth rates has been examined here only over a 6-year period. The degree of persistence of growth is likely to be greater over shorter time-periods and it may disappear altogether if a time span of much longer than 6 years is considered.

(4) The persistence of growth rates through time ((3) above) is a major cause of the positive association between size and growth ((1) above).

(5) A limited analysis of " births " and " deaths " has shown: (*a*) that, although the incidence of births declines with an increase in firm size, a considerable number of births occur in all size classes; (*b*) that the incidence of deaths also declines with an increase in firm size, and amongst the largest firms alone, it declines more sharply as the size of the firm increases—thus, there is a non-linear negative relationship between size and the probability of death; (*c*) that the net impact of the birth and death process was, in a majority of the industries studied, to reduce the index of concentration, as measured by the dispersion of the logarithms of firm sizes.

There are two essential points to note about the findings (1) to (5) above. First, it is important that none of these findings is unacceptable from the economic point of view; the discussion in Sections 2 to 6 has shown that all of them have a good economic explanation. The observed weak positive relationship between size and growth is compatible with the emphasis of contemporary theories on the *growth* of the firm, although they are not consistent with the traditional static concept of the optimal *size* of the firm. On the other hand, the fact that the degree of persistence in growth rates is small, argues against the analysis of the firm growth process in terms of the steady-state long-run growth models of the kind used in these theories (cf. [21]).

Secondly, we have seen that all these findings, particularly (1) to (3) above, are incompatible with the LPE in its strong form discussed in Section 2. There are, however, many stochastic models of firm growth which incorporate some weaker versions of this law and which generate skew distributions broadly similar to that of the size of firms. The question arises as to whether the stylized facts discovered in this study are compatible with any of these other models.

The most promising models in this context are those of Ijiri and Simon [7] and [8], which explicitly incorporate serial correlation in firm growth rates. These models are *broadly* compatible, particularly with respect to the surviving firms, with the first three stylized facts outlined above (i.e. those pertaining to the means and the standard deviations of growth rates and to the observed persistency in growth). It might be argued that the Law of Proportionate Effect is an " extreme hypothesis ", in the sense used by L. J. Savage, and one might therefore ignore the mildly positive relationship between size and growth [7]. The major weakness of these models lies, however, in their treatment of births and deaths, particularly the latter. [8] does not deal with the problem at all whereas [7], although it specifically considers new entry in the lowest size class, ignores " mergers or decreases in sizes of individual firms ".

However, mergers and take-overs are not only quantitatively very important, they make the births and deaths of firms a far more complicated process than that incorporated in other stochastic economic models such as those of income or wealth: in particular, the death of a firm by take-over implies a substantial increment to the growth of the acquiring firm. The importance of amalgamations is indicated by the fact that they are overwhelmingly the largest single cause of death of firms quoted on the stock market and exert an increasingly important influence both on the growth of firms and their size distribution. During the years 1954-60, mergers and take-overs accounted for nearly 80 per cent of deaths of companies quoted on the UK stock markets [16]. Furthermore, mergers are an important cause of the birth of very large quoted companies. The incidence of deaths for the UK quoted firms since the middle 50's has been more than 3 per cent a year [3, 16], an historically unprecedently high rate.

Mergers and take-overs are also known to possess certain other characteristics, e.g.

both in the UK and the USA, merger activity has taken place in irregular, long-term (a decade or longer) waves [10]. There is certainly no reason to believe that the probabilities of firm disappearance through take-over remain constant through time. It seems obvious, therefore, that before one can consider "steady state" distributions of firms in any economically meaningful sense, stochastic models of firm growth and size must pay adequate attention to the salient features of merger and take-over activity. These features are discussed more fully in [16].

Finally, it is interesting to compare the " stylised facts " about the growth process of firms with those concerning the relationship between the size and profitability of firms. An investigation [20] of the relationship between long-run (6-year or 12-year average) profitability and size for the data examined in this study, revealed, among other things: (i) that average profitability declines slightly with size, (ii) that the standard deviation of profitability also declines, but relatively more sharply, with an increase in firm size. Furthermore, it was found (iii) that the persistence in the average profitability of firms was much higher than that observed for their growth rates.

A comparison of these facts with those outlined earlier for the relationship between size and growth has implications for the relationship between the growth and profitability of firms. These were discussed in [17] and will be examined further in a subsequent paper.

REFERENCES

[1] Aspin, A. A. and Welch, B. L. " Tables for use in Comparisons whose Accuracy involves Two Variances, Separately Estimated ", *Biometrika*, **36** (1949), 290-296.

[2] Blair, J. M. " Statistical Measures of Concentration in Business ", *Bulletin of the Oxford Institute of Statistics*, **18** (1956), 351-372.

[3] Department of Trade and Industry. *A Survey of Mergers, 1958-1968* (London: HMSO, 1970).

[4] George, K. D. " Changes in British Industrial Concentration, 1951-58 ", *Journal of Industrial Economics* (July 1967).

[5] Hart, P. E. and Prais, S. J. " The Analysis of Business Concentration: a Statistical Approach ", *Journal of the Royal Statistical Society* A, **119** (1956), 150-181.

[6] Hymer, S. and Pashigian, P. " Firm Size and Rate of Growth ", *Journal of Political Economy*, **70** (1962), 556-569.

[7] Ijiri, Y. and Simon, H. A. " Business Firm Growth and Size ", *American Economic Review*, **54** (1964), 77-89.

[8] Ijiri, Y. and Simon, H. A. " A Model of Business Firm Growth ", *Econometrica*, **35** (1967), 348-355.

[9] Kalecki, M. " On the Gibrat Distribution ", *Econometrica*, **13** (1945), 161-170.

[10] Markham, J. W. " Survey of the Evidence and Findings on Mergers ", in *Business Concentration and Price Policy* (Princeton University Press, 1955), pp. 141-182.

[11] Marris, R. L. and Wood, A. J. B. (eds.). *The Corporate Economy* (London: Macmillan and Co., 1971).

[12] Samuels, J. M. " Size and Growth of Firms ", *Review of Economic Studies*, **32** (1965), 105-112.

[13] Simon, H. A. " On a Class of Skew Distribution Functions ", *Biometrika*, **42** (1955), 425-440.

[14] Simon, H. A. " Comment: Firm Size and Rate of Growth ", *Journal of Political Economy*, **72** (1964), 81-82.

[15] Simon, H. A. and Bonini, C. P. " The Size Distribution of Business Firms ", *American Economic Review*, **48** (1958), 607-617.

[16] Singh, A. *Take-overs: Their Relevance to the Stock Market and the Theory of the Firm* (London: Cambridge University Press, 1971).

[17] Singh, A. and Whittington, G. *Growth, Profitability and Valuation* (London: Cambridge University Press, 1968).

[18] Steindl, J. *Random Processes and the Growth of Firms* (London: Charles Griffin & Co., 1965).

[19] " Industrial Structure and Competition Policy ", in *Studies by the Staff of the Cabinet Committee on Price Stability* (Washington, 1969).

[20] Whittington, G. *The Prediction of Profitability, and Other Studies of Company Behaviour* (London: Cambridge University Press, 1971).

[21] Williamson, O. E. *Corporate Control and Business Behavior* (Englewood Cliffs: Prentice-Hall, 1970).