Moment Conditions for Almost Stochastic Dominance

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Moment Conditions for Almost Stochastic Dominance*

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Abstract: This study establishes necessary conditions for Almost Stochastic Dominance criteria of various orders. These conditions take the form of restrictions on algebraic combinations of moments of the probability distributions in question. The relevant set of conditions depends on the relevant order of ASD but not on the critical value for the admissible violation area. These conditions can help to reduce the information requirement and computational burden in practical applications. A numerical example and an empirical application to historical stock market data illustrate the moment conditions. The first four moment conditions in particular seem appealing for many applications.

Keywords: decision theory; utility theory; stochastic dominance; necessary conditions; moments.

JEL Classification: C00, D81, G11.

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1. Introduction

Leshno and Levy (2002) introduce the choice criteria of first-order and second-order Almost Stochastic Dominance (ASD) to improve the discriminating power and robustness of the classical SD rules by focusing on *most* rather than *all* admissible risk preferences. Tzeng *et al*. (2013) revise the second-order ASD rule to improve the correspondence with expected utility theory and develop higher-order ASD rules that impose additional preference assumptions.

This study derives a set of necessary conditions for ASD criteria that are formulated in terms of restrictions on algebraic combinations of moments of the probability distributions in question. These conditions are useful when distribution information is incomplete, for example, limited to a set of descriptive statistics. The conditions can also be used as stopping rules to reduce the computational burden, which is particularly relevant for mathematical programming applications (for example, Kuosmanen, 2004) and statistical re-sampling methods (for example, Linton *et al*, 2005).

Section 2 summarizes a set of known moment conditions based on Jean (1980, 1984) that apply for classical SD rules of any order. Section 3 shows that subsets of these moment conditions apply also to ASD citeria, depending on the relevant order of ASD. The relevant conditions apply for *any* critical value for the violation area, which seems a useful feature given the ambiguity surrounding the relevant specification. In Section 4, a simple numerical example illustrates the differences between the various sets of moment conditions. Section 5 applies the moment conditions to a standard data set from the empirical asset pricing literature. Section 6 concludes with a discussion of the first four moment conditions, which appear relevant for a wide range of applications.

2. Necessary Conditions for SD

Suppose that prospects X and Y have distribution functions F and G defined on support $\Omega \coloneqq [a,b]$ and n-th order non-central moments $M_F^{(n)} \coloneqq \mathbb{E}_F[z^n]$ and $M_G^{(n)} \coloneqq \mathbb{E}_G[z^n]$, $n \in \mathbb{Z}_+$. For $H = F, G$ and $n \in \mathbb{Z}_{++} \coloneqq \mathbb{Z}_+ \backslash \{0\}$, the n-th order integrated CDF is given by

$$H^{(1)}(z) \coloneqq H(z) \quad n = 1; \tag{1a}$$

$$H^{(n)}(z) \coloneqq \int_a^z H^{(n-1)}(x)dx = \frac{1}{(n-1)!}\mathbb{E}_H[(z-x)_+^{n-1}] \quad n \geq 2. \tag{1b}$$

DEFINITION 1: *Prospect X dominates prospect Y by N-th order Stochastic Dominance,* or $X \succ_N Y,\ N \in \mathbb{Z}_{++}$, *if*

$$F^{(N)}(z) \leq G^{(N)}(z) \ \ \forall z \in [a,b]; \tag{2a}$$

$$F^{(n)}(b) \leq G^{(n)}(b) \ \ \forall n \in \mathbb{Z}_+ : 2 \leq n \leq N-1. \tag{2b}$$

Alternative definitions require strict inequality for some or all values $z \in [a,b]$ in (2a) and/or (2b). We ignore this issue here, because the needed adjustments appear obvious, and, furthermore, the differences between weak and strict inequalities are not noticeable in typical applications.

The economic interpretation of the SD criteria can be illustrated using the following monotonic utility functions:

$$\mathcal{U}_N := \left\{u \in \mathcal{C}^N : \Omega \to \mathbb{R}; (-1)^{n+1}\, u^{(n)}(x) \geq 0 \ \ n = 1, \cdots, N\right\}. \tag{3}$$

Prospect X dominates prospect Y by N-th order SD if and only if X achieves a higher expected utility than Y for any admissible utility function: $\mathbb{E}_F[u(x)] \geq \mathbb{E}_G[u(x)]$ for all $u \in \mathcal{U}_N$.

There exist several well-established necessary conditions for SD criteria; see, for example, Jean (1980, 1984) and Levy (2006). For our purposes, it is relevant to consider the following conditions on algebraic combinations of non-central moments:

$$X \succ_N Y \Longrightarrow F^{(n)}(b) \leq G^{(n)}(b) \ \ \forall n \in \mathbb{Z}_{++}. \tag{4}$$

$$H^{(n)}(b) = \frac{(-1)^{n-1}}{(n-1)!}\sum_{m=0}^{n-1}\binom{n-1}{m}(-b)^{n-m-1}M_H^{(m)} \ \ H = F, G; n \in \mathbb{Z}_{++}. \tag{5}$$

These moment conditions apply for *any* order of SD, due to the hierarchical relation $X \succ_N Y \Rightarrow X \succ_n Y, n > N$.

Although we formulate the necessary conditions in terms of raw, non-central moments, equivalent (but less compact) formulations in terms of central moments can be obtained by using the following relation:

$$C_H^{(n)} \coloneqq \mathbb{E}_H\left[\left(z - M_H^{(1)}\right)^n\right] = \sum_{m=0}^{n} \binom{n}{m} (-1)^{n-m} M_H^{(m)} \left(M_H^{(1)}\right)^{n-m} \quad H = F, G; n \in \mathbb{Z}_{++}. \tag{6}$$

3. Necessary Conditions for ASD

Tzeng *et al.* (2013) propose the following revision and extension of the original definition by Leshno and Levy (2002):

DEFINITION 2: *For a given critical value* $0 < \epsilon < 1/2$, *prospect* X *dominates prospect* Y *by N-th order Almost Stochastic Dominance, or* $X \succ_N^{almost(\epsilon)} Y$, $N \in \mathbb{Z}_{++}$, *if*

$$\int_{S_N} \left(F^{(N)}(z) - G^{(N)}(z)\right) dz \leq \epsilon \int_a^b \left|F^{(N)}(z) - G^{(N)}(z)\right| dz; \tag{7a}$$

$$F^{(n)}(b) \leq G^{(n)}(b) \quad \forall n \in \mathbb{Z}_+ : 2 \leq n \leq N; \tag{7b}$$

$$S_N \coloneqq \left\{z \in [a, b] : F^{(N)}(z) > G^{(N)}(z)\right\}. \tag{8}$$

ASD can be interpreted in terms of the following restricted utility functions:

$$\mathcal{U}_N^*(\varepsilon) := \left\{u \in \mathcal{U}_N : (-1)^{N+1} u^N(x) \leq \inf_{x \in \Omega} \ (-1)^{N+1} u^N(x) \left[\frac{1}{\varepsilon} - 1\right]\right\} \quad 0 < \epsilon < 1/2. \tag{9}$$

Compared with the classical utility functions $\mathcal{U}_N$, these functions have a limited relative range for the N-th order derivative. Prospect X dominates prospect Y by N-th order ASD if and only if $\mathbb{E}_F[u(x)] \geq \mathbb{E}_G[u(x)]$ for all $u \in \mathcal{U}_N^*(\varepsilon)$.

The relevant specification of the critical value ϵ is a topic of ongoing research; see, for example, Levy *et al.* (2010). The relevant value seems to depend on the relevant order of ASD (*N*) and the relevant domain of outcomes $[a, b]$ and therefore cannot be tabulated for once and for all.

Clearly, SD arises as the limiting case of ASD for $\epsilon \downarrow 0$ and SD is a sufficient condition for ASD: $X \succ_N Y \Rightarrow X \succ_N^{almost(\epsilon)} Y,\ 0 < \epsilon < 1/2$. Interestingly, certain subsets of the SD moment conditions (4) also represent necessary conditions for ASD:

PROPOSITION 1 (MOMENT CONDITIONS FOR ASD): $X \succ_N^{almost(\epsilon)} Y;\ 0 < \epsilon < 1/2 \Rightarrow$

$$F^{(N+1)}(b) < G^{(N+1)}(b); \tag{10a}$$

$$F^{(n)}(b) \leq G^{(n)}(b) \quad \forall n \in \mathbb{Z}_+ : 2 \leq n \leq N. \tag{10b}$$

PROOF: For $0 < \epsilon < 1/2$, $X \succ_N^{almost(\epsilon)} Y \Rightarrow$ (7a)

$$\Rightarrow \int_{S_N} \left(F^{(N)}(z) - G^{(N)}(z)\right) dz < \frac{1}{2}\int_a^b \left|F^{(N)}(z) - G^{(N)}(z)\right| dz \tag{11a}$$

$$\Leftrightarrow \frac{1}{2}\int_{S_N} \left(F^{(N)}(z) - G^{(N)}(z)\right) dz < \frac{1}{2}\int_{S_N^C} \left(G^{(N)}(z) - F^{(N)}(z)\right) dz \tag{11b}$$

$$\Leftrightarrow \int_a^b \left(F^{(N)}(z) - G^{(N)}(z)\right) dz < 0 \tag{11c}$$

$$\Leftrightarrow F^{(N+1)}(b) < G^{(N+1)}(b). \tag{11d}$$

This derivation uses the decomposition

$$\int_a^b \left|F^{(N)}(z) - G^{(N)}(z)\right| dz = \int_{S_N} \left(F^{(N)}(z) - G^{(N)}(z)\right) dz + \int_{S_N^C} \left(G^{(N)}(z) - F^{(N)}(z)\right) dz$$

based on the complement set $S_N^C \coloneqq \left\{z \in [a,b] : F^{(N)}(z) \leq G^{(N)}(z)\right\}$. Combining (7b) and (11d) completes the proof. ■

Clearly, the first $N-1$ moment conditions ($n = 2, \cdots, N$) follow directly from the definition of ASD (7b). The N-th moment condition (10a) follows in a subtle way from the limiting case of ASD condition (7a) as $\epsilon \uparrow \frac{1}{2}$. For this limiting case, the N-th order derivative $u^N(x)$ in (9) approaches a constant value and the admissible utility functions approximate N-th order polynomials:

$$\mathcal{U}_N^*\left(\frac{1}{2}\right) := \left\{u \in \mathcal{U}_N : u(x) = a_0 + \sum_{n=1}^{N} a_n x^n\right\}. \tag{12}$$

Since $\mathcal{U}_N^*\left(\frac{1}{2}\right) \subset \mathcal{U}_N^*(\epsilon)$ for any $0 < \epsilon < 1/2$, the N-th moment condition is a necessary condition for *any* critical value ϵ, which seems a useful property given the existing ambiguity surrounding the appropriate specification.

It follows from the ASD moment conditions (10a)-(10b) that Fishburn's (1980) Theorem for SD applies also for ASD:

COROLLARY 1 ('FISHBURN IS STILL ALIVE'):

$$X \succ_N^{almost(\epsilon)} Y;\ 0 < \epsilon < 1/2 \Longrightarrow (-1)^{n-1}\left(M_F^{(n)} - M_F^{(n)}\right) > 0 \tag{13}$$

for the smallest $n \leq N$: $M_F^{(n)} \neq M_G^{(n)}$.

PROOF: Using (5), we can formulate (10a)-(10b) as follows:

$$\frac{(-1)^N}{N!}\sum_{m=0}^{N}\binom{N}{m}(-b)^{N-m}\left(M_F^{(m)} - M_G^{(m)}\right) < 0; \tag{14a}$$

$$\frac{(-1)^{n-1}}{(n-1)!}\sum_{m=0}^{n-1}\binom{n-1}{m}(-b)^{n-m-1}\left(M_F^{(m)} - M_G^{(m)}\right) \leq 0 \quad \forall n \in \mathbb{Z}_+ : 2 \leq n \leq N-1. \tag{14b}$$

It follows from (14a) that $(-1)^{n-1}\left(M_F^{(n)} - M_F^{(n)}\right) > 0$ for at least some $n \leq N$. It follows from (14b) that $(-1)^{k-1}\left(M_F^{(k)} - M_G^{(k)}\right) \geq 0$ if $M_F^{(n)} = M_G^{(n)}$ for $n < k \leq N$, and hence $(-1)^{k-1}\left(M_F^{(k)} - M_G^{(k)}\right) > 0$ if we also assume $M_F^{(k)} \neq M_G^{(k)}$. ■

For example, if X and Y have the same arithmetic mean ($M_F^{(1)} = M_G^{(1)}$) and the same second-order moment ($M_F^{(2)} = M_G^{(2)}$), then third-order ASD, or $X \succ_3^{almost(\epsilon)} Y$, requires a higher third-order moment ($M_F^{(3)} > M_G^{(3)}$).

Whereas the first N moment conditions for SD ($n = 2, \cdots, N+1$) carry over to N-th order ASD, the higher-order moment conditions ($n \geq N+2$) do not carry over. For example, whereas $F^{(2)}(b) \leq G^{(2)}(b)$ and $F^{(3)}(b) \leq G^{(3)}(b)$ are necessary conditions for both $X \succ_2 Y$ and $X \succ_2^{almost(\epsilon)} Y$, $F^{(4)}(b) \leq G^{(4)}(b)$ applies only to $X \succ_2 Y$ and not to $X \succ_2^{almost(\epsilon)} Y$. This result is related to the absence of hierarchy between ASD relations of different order: $X \succ_N^{almost(\epsilon)} Y \nRightarrow X \succ_n^{almost(\epsilon)} Y$, $n > N$, as pointed out by Guo *et al.* (2013). Another consequence of the reduced set of moment conditions is that Jean's (1980, 1984) SD conditions for the geometric mean and harmonic mean, which are based on the infinite series of all moments (including $n \geq N+2$), do not apply for ASD.

4. Numerical example

Let $\Omega = [0,1]$ and $F(z) = \begin{cases} \frac{1}{2} & 0 \leq z < 1 \\ 1 & z = 1 \end{cases}$ and $G(z) = \begin{cases} 0 & 0 \leq z < \frac{1}{4} \\ 1 & z \geq \frac{1}{4} \end{cases}$. In this case, we have $F^{(2)}(z) = \frac{1}{2}z$ and $G^{(2)}(z) = z - \frac{1}{4}$. Violations of the SSD rule $X \succ_2 Y$ $(N = 2)$ occur on the interval $S_2 = \left[0, \frac{1}{2}\right]$, and the violation area is $\int_{S_2} \left(F^{(2)}(z) - G^{(2)}(z)\right) dz = \frac{1}{32}$, or one-third of the total area $\int_0^1 \left|F^{(2)}(z) - G^{(2)}(z)\right| dz = \frac{3}{32}$. Consequently, $X \nsucc_2 Y$ and $X \succ_2^{almost(\epsilon)} Y$ for $\frac{1}{3} \leq \epsilon < \frac{1}{2}$. Clearly, $M_F^{(1)} = M_F^{(2)} = M_F^{(3)} = M_F^{(4)} = \frac{1}{2}$; $M_G^{(1)} = \frac{1}{4}$; $M_G^{(2)} = \frac{1}{16}$; $M_G^{(3)} = \frac{1}{64}$; $M_G^{(4)} = \frac{1}{256}$. Confirming the moment conditions for SD (4) and ASD (10a)-(10b), these numbers obey

$$F^{(2)}(b) \leq G^{(2)}(b) \Leftrightarrow M_F^{(1)} \geq M_G^{(1)}; \tag{15}$$

$$F^{(3)}(b) \leq G^{(3)}(b) \Leftrightarrow 2b\left(M_F^{(1)} - M_G^{(1)}\right) \geq \frac{1}{2}\left(M_F^{(2)} - M_G^{(2)}\right). \tag{16}$$

In addition, consistent with $X \nsucc_2 Y$ and $X \succ_2^{almost(\epsilon)} Y \nRightarrow F^{(4)}(b) \leq G^{(4)}(b)$, the numbers do *not* obey

$$F^{(4)}(b) \leq G^{(4)}(b) \Leftrightarrow b^2\left(M_F^{(1)} - M_G^{(1)}\right) + \frac{1}{3}\left(M_F^{(3)} - M_G^{(3)}\right) \geq b\left(M_F^{(2)} - M_G^{(2)}\right); \tag{17}$$

$$\begin{aligned} F^{(5)}(b) \leq G^{(5)}(b) \Leftrightarrow & \frac{2}{3}b^3\left(M_F^{(1)} - M_G^{(1)}\right) + \frac{2}{3}b\left(M_F^{(3)} - M_G^{(3)}\right) \\ & \geq b^2\left(M_F^{(2)} - M_G^{(2)}\right) + \frac{1}{6}\left(M_F^{(4)} - M_G^{(4)}\right). \end{aligned} \tag{18}$$

In this example, one of the possible outcomes for prospect X is a value of zero and hence the geometric mean equals zero and the harmonic mean is not defined. If we replace this outcome with a small positive constant, then prospect X has smaller geometric and harmonic means than prospect Y, which illustrates that Jean's (1990, 1984) conditions for second-order SD do not carry over to second-order ASD.

5. Empirical application

Table I applies the moment conditions (10a)-(10b) to a well-known data set of ten stock portfolios that are formed based on individual stocks' market capitalization of equity (ME). This data set is of particular interest because a wealth of empirical research, starting with Banz (1981), suggests that small-cap stocks earn a return premium that seems to defy rational explanation. Bali *et al.* (2009) present another financial application of ASD.

We apply the moment conditions (10a)-(10b) of order $N = 1, \cdots, 6$ to the empirical distribution of each of the 90 paired combinations (X, Y) of two distinct portfolios $(X \neq Y)$. Every cell in the table shows the highest order (N) of the moment conditions that are satisfied for a given combination. For example, micro-cap portfolio ME1 dominates mega-cap portfolio ME10 by the first three moments but not by the fourth moment.

Naturally, half of the combinations (45) pass the first moment condition on the means (15). The second-order moment conditions (15)-(16) eliminate five more combinations and leave 40 for further analysis. The triplet (15)-(17) reduce the number of potential dominance relations to 21, and the quadruple (15)-(18) leave 12 out of 90. Four combinations pass five moment conditions and only a single combination (X=ME3 and Y=ME2) obeys all six moment conditions.

[Insert Table I about here]

Since the moment conditions are necessary for SD of any order (see (4)), it is clear that 89 out of 90 combinations do not show any classical dominance relation. Applying (2a)-(2b) for various orders to the single remaining combination, we find that ME3 dominates ME2 by fifth-order SD. The lack of SD relations contrasts sharply with the prevailing evidence that small-caps outperform large-caps and casts doubt on the classical SD rules.

Table II summarizes the results of ASD analysis. In this case, the moment conditions are used both to reduce the number of pairwise comparisons and to determine the relevant orders of ASD for every comparison. For every paired combination that passes the M-th order moment conditions, we perform tests for ASD (7a)-(7b) for all orders $N \leq M$ and with critical value $\epsilon = 0.05$. Every cell in the table shows the lowest order of ASD that applies for a given combination. For example, micro-cap portfolio ME1 dominates mega-cap portfolio ME10 by third-order ASD but not by second-order ASD. The analysis is carried out using the Linear Programming test for pairwise ASD in Post and Kopa (2013, Eq. (15), (19), (20)).

The ASD tests reveal as much as nine second-order ASD relations and 12 third-order ASD relations. Notably, stocks in the three top size segments appear dominated by stocks in lower size segments, consistent with the prevailing empirical evidence. The violations of the classical SD criteria appear relatively small and concentrated in the left tail of the distribution where the data set is sparse.

[Insert Table II about here]

6. Concluding Remarks

In practical applications, the relevant choice criteria naturally depend on the specific choice problem, the complexity of the analytical tasks and the available computing platform. Nevertheless, in our research experience, the first four moment conditions, (15)-(18), stand out as particularly relevant for a wide range of applications. Proposition 1 implies that these (but not higher-order) moment conditions are necessary for fourth-order (but not lower-order) ASD for any critical value (ϵ).

The maintained assumptions about the utility functions $\mathcal{U}_4$ (monotonicity, risk aversion, prudence and temperance) are generally accepted as minimal regularity conditions for well-behaved preferences. Relaxing these assumptions often results in a substantial loss of discriminating power. Furthermore, additional restrictions on the fifth and higher derivatives are more debatable and often do not substantially improve the power.

In addition, there exists a broad consensus about the need to exclude pathological utility functions in order to make SD analysis more powerful and robust. ASD was developed for this purpose, but the implementation can be computationally demanding (particularly for large-scale optimization and simulation tasks) and the selection of the critical value introduces an element of ambiguity. The ASD moment conditions reduce the computational burden and the sensitivity to the assumed critical value.

Finally, the quadruple (15)-(18) appears remarkably effective for detecting violations of fourth-order ASD, witness, for example, the large number of eliminations in our empirical application. This effectiveness seems related to the flexibility of the admissible preference structures. The four moment conditions represent fourth-order ASD for the limiting case of $\epsilon \uparrow \frac{1}{2}$, or a quartic utility function (see (12)). The quartic function can give a fourth-order approximation to more general utility functions in $\mathcal{U}_4$ and has more flexibility than the celebrated mean-variance approximation of Levy and Markowitz (1979).

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Table I: Moment conditions

We analyze ten stock portfolios that are formed based on individual stocks' market capitalization of equity (ME). Monthly value-weighted returns from July 1926 to December 2012 come from Kenneth French' data library. Returns are in excess of the one-month US government bond index from Ibbotson and Associates. We apply the moment conditions (10a)-(10b) of order $N = 1, \cdots, 6$ to the empirical distribution of each of the 90 paired combinations (X, Y) of two distinct portfolios $(X \neq Y)$. We set the domain $\Omega \coloneqq [a, b]$ equal to the sample range of returns across all ten portfolios. Every cell shows the highest order (N) of the moment conditions that are satisfied for a given combination. A higher order is represented by a darker shade of grey.

		Prospect X									
		ME1	2	3	4	5	6	7	8	9	ME10
Prospect Y	ME1										
	2	3		6							
	3	2									
	4	2	1	2							
	5	2	1	2	2						
	6	2	1	2	2	1					
	7	2	1	2	2	2	3				
	8	2	2	2	3	3	4	4			
	9	2	2	3	3	3	4	4	5		
	ME10	3	2	3	4	4	4	4	5	5	

Table II: Almost Stochastic Dominance tests

We analyze ten stock portfolios that are formed based on individual stocks' market capitalization of equity (ME). Monthly value-weighted returns from July 1926 to December 2012 come from Kenneth French' data library. Returns are in excess of the one-month US government bond index from Ibbotson and Associates. For every paired combination that passes the M-th order moment conditions (see Table I), we perform tests for ASD (7a)-(7b) for all orders $1 \leq N \leq M$. The critical value for the violation area equals $\epsilon = 0.05$. Every cell shows the lowest order (N) of ASD that applies for a given combination. A higher order is represented by a darker shade of grey. The tests are performed using the Linear Programming formulation of ASD in Post and Kopa (2013, Eq. (15), (19), (20)).

		Prospect X									
		ME1	2	3	4	5	6	7	8	9	ME10
	ME1										
	2	3		3							
	3										
	4										
Prospect Y	5										
	6										
	7						2				
	8				3	3	2	2			
	9			3	3	3	2	2	3		
	ME10	3		3	3	3	2	2	2	2	