Almost Stochastic Dominance for Risk-Averse and Risk-Seeking Investors

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Abstract

In this paper we first develop a theory of almost stochastic dominance for risk-seeking investors to the first three orders. Thereafter, we study the relationship between the preferences of almost stochastic dominance for risk-seekers with that for risk averters.

Keywords: Almost Stochastic Dominance, expected-utility maximization, risk averters, risk seekers.

JEL Classification: C00, D81, G11.

1 Introduction

There are two major types of persons: risk averters and risk seekers. Markowitz (1952) and Tobin (1958) propose the mean-variance (MV) selection rules for risk averters and risk seekers. Stochastic Dominance (SD) is first introduced in mathematics by Mann and Whitney (1947) and Lehmann (1955). Quirk and Saposnik (1962), Hanoch and Levy (1969), and many others develop the theory of SD related to economics and develop the stochastic dominance rules for risk averters. On the other hand, Meyer (1977), Stoyan (1983), Wong (2007), and many others develop the stochastic dominance rules for risk seekers.

The theory of almost stochastic dominance (almost SD) developed by Leshno and Levy (LL, 2002) plays an important role in several fields, particularly in financial research, and has drawn several important applications; see, for example, Levy (2006, 2009), Bali, et al. (2009), and Levy, et al. (2010). Tzeng et al. (2013) show that the almost second-degree almost SD introduced by Leshno and Levy (2002) does not possess the property of expected-utility maximization. They modify the definition of the almost SD to acquire this property. Nonetheless, Guo, et al. (2013a) have constructed some examples to show that the almost SD definition modified by Tzeng et al. (2013) does not possess any hierarchy property while Guo, et al. (2013) establish necessary conditions for Almost Stochastic Dominance criteria of various orders.

2 Definitions, Notations, Motivation, and Background

Random variables, denoted by X and Y, defined on $\Omega = [a, b]$ are considered together with their corresponding distribution functions F and G, their corresponding probability density functions f and g, and means μ_X and μ_Y, respectively. The following notations will be used throughout this paper:

$$H_j^A(x) = \int_a^x H_{j-1}^A(y)\,dy \quad \text{and} \quad H_j^D(x) = \int_x^b H_{j-1}^D(y)\,dy \quad , \tag{2.1}$$

where $h = f$ or g and $H = F$ or G. In addition, we define

$$\begin{aligned}
&\big|\big|F_n^A(x) - G_n^A(x)\big|\big| = \int_a^b \big|F_n^A(x) - G_n^A(x)\big|dx \ , \\
&\big|\big|F_n^D(x) - G_n^D(x)\big|\big| = \int_a^b \big|F_n^D(x) - G_n^D(x)\big|dx \ , \\
&S_n^A(F,G) = \{x \in [a,b] : G_n^A(x) < F_n^A(x)\} \ , \quad \text{and} \\
&S_n^D(F,G) = \{x \in [a,b] : F_n^D(x) < G_n^D(x)\} \quad \text{for} \quad n = 1,2,3.
\end{aligned} \tag{2.2}$$

We note that the definition of H_i^A can be used to develop the stochastic dominance theory for risk averters (see, for example, Quirk and Saposnik, 1962; Hanoch and Levy, 1969), and thus, we call this type of SD ascending stochastic dominance (ASD) because H_i^A is integrated in ascending order from the leftmost point of downside risk. On the other hand, H_i^D can be used to develop the stochastic dominance theory for risk seekers (see, for example, Hammond, 1974; Li and Wong, 1999), and thus, we call this type of SD descending stochastic dominance (DSD) because H_i^D is integrated in descending order from the rightmost point of upside profit. We first define risk-averse and risk-seeking investors as follows:

Definition 2.1 *For $j = 1,2,3$, U_j^A and U_j^D are sets of utility functions u such that:*

$$\begin{aligned}
U_j^A &= \{u : (-1)^i u^{(i)} \leq 0 \ , \ i = 1, \cdots, j\} \ , \\
U_j^D &= \{u : u^{(i)} \geq 0 \ , \ i = 1, \cdots, j\} \ ,
\end{aligned}$$

where $u^{(i)}$ is the i^{th} derivative of the utility function u.

We call investors the j^{th} order risk averters if their utility functions $u \in U_j^A$ and the j^{th} order risk seekers if their utility functions $u \in U_j^D$. Readers may refer to Menezes, et al. (1980), Post and Levy (2005), Post and Versijp (2007), Fong, et al. (2008), Wong and Ma (2008), and Crainich, et al. (2013) for more properties of the utility functions.

Leshno and Levy (2002) comment that sometimes applying the theory of SD could not draw preferences between two prospects, say X and Y, but most investors will prefer one to the other. They give an example as follows:

Example 2.1 *Suppose that an investor considers two mutually exclusive prospects A and B which involve the same initial investment. Prospect A yields \$900 with a probability of 1/2 and \$100,000 with a probability of 1/2. Prospect B yields \$1,000 with certainty.*

Leshno and Levy (2002) comment that in Example 2.1, A does not dominate B by both MV and SD rules but they believe that almost all investors will choose A. To circumvent the limitation of the MV and SD rules, they introduce the theory of almost SD. Leshno and Levy (2002) and others develop the almost SD rule. We state the almost SD rule developed by Leshno and Levy (2002) and modified by Tzeng et al. (2012) as follows:[1]

Definition 2.2 *Given two random variables X and Y with F and G as their respective distribution functions, for $0 < \epsilon < 1/2$, X is at least as large as Y in the sense of:*

1. *ϵ-almost FASD or ϵ-AFASD, denoted by $X \succeq_{1A}^{almost(\epsilon)} Y$ if and only if*

$$\int_{S_1^A} \left[F_1^A(x) - G_1^A(x)\right] dx \le \epsilon \left|\left|F_1^A(x) - G_1^A(x)\right|\right|,$$

2. *ϵ-almost SASD or ϵ-ASASD, denoted by $X \succeq_{2A}^{almost(\epsilon)} Y$ if and only if*

$$\int_{S_2^A} \left[F_2^A(x) - G_2^A(x)\right] dx \le \epsilon \left|\left|F_2^A(x) - G_2^A(x)\right|\right| \quad and \quad \mu_X \ge \mu_Y \ ,$$

3. *ϵ-almost TASD or ϵ-ATASD, denoted by $X \succeq_{3A}^{almost(\epsilon)} Y$ if and only if*

$$\int_{S_3^A} \left[F_3^A(x) - G_3^A(x)\right] dx \le \epsilon \left|\left|F_3^A(x) - G_3^A(x)\right|\right| \ and \ \ G_n^A(b) \ge F_n^A(b) \ for \ \ n = 2, 3$$

where $S_n^A(F, G)$ and $\left|\left|F_n^A(x) - G_n^A(x)\right|\right|$ for $n = 1, 2, 3$ are defined in (2.2), ϵ-almost FASD, SASD, and TASD stand for ϵ-almost first-, second-, and third-order ASD, respectively.

In Example 2.1, it is clear that most people prefer Prospect A to Prospect B. However, all the traditional MV and SD rules for risk averters cannot be used to determine their

[1] We note that we have modified their notations to distinct them from the notations used for the risk seekers.

domination relationship. Nonetheless, the almost SD rule developed by Leshno and Levy (2002) and Tzeng et al. (2012) could draw preference of A over B. However, though the almost SD rule developed by Leshno and Levy (2002) and others could draw preference for risk averters, but not risk seekers. To complete the theory of almost SD, we will develop the almost stochastic dominance concept for risk seekers. We modify from Example 2.1 to get the following example to illustrate the motivation:

Example 2.2 *Prospect A yields \$1 with a probability of 1/2 and \$100,000 with a probability of 1/2 and Prospect B yields \$99,999 with certainty.*

In Example 2.2, it is clear that most people will prefer Prospect B to Prospect A, no matter whether they are risk averters or risk seekers. Similar to the situation that ASD rule could not draw preference between A and B in Example 2.1, the MV rules for risk seekers and the SD rules for risk seekers could not be used to determine their domination relationship in Example 2.2. Readers may think that this example is not realistic. We note that there are many real examples will draw the same preference as that from Example 2.2. One such example is to invest in the bonds issued by Lehman Brothers (LB) Holdings Inc. before the sub-crime crisis with only one or two percent higher than T-bills. However, investors could end up lose all their investment in the LB bonds because LB goes bankrupt.

Similar to the situation that ASD cannot be used to explain Example 2.1 and Leshno and Levy (2002) develop almost ASD, now, DSD cannot be used to explain Example 2.2, we develop almost DSD, the almost SD rule for risk seekers as shown in the following definition:

Definition 2.3 *Given two random variables X and Y with F and G as their respective distribution functions, for $0 < \epsilon < 1/2$, X is almost at least as large as Y and F is almost at least as large as G in the sense of:*

1. *ϵ-almost FDSD or ϵ-AFDSD, denoted by $X \succeq_{1D}^{almost(\epsilon)} Y$ or $F \succeq_{1D}^{almost(\epsilon)} G$, if and only if*

$$\int_{S_1^D} \left[G_1^D(x) - F_1^D(x)\right] dx \le \epsilon \left|\left|F_1^D(x) - G_1^D(x)\right|\right|,$$

2. ϵ-almost SDSD or ϵ-ASDSD, denoted by $X \succeq_{2D}^{almost(\epsilon)} Y$ or $F \succeq_{2D}^{almost(\epsilon)} G$, if and only if

$$\int_{S_2^D} \left[G_2^D(x) - F_2^D(x)\right] dx \le \epsilon \big|\big|F_2^D(x) - G_2^D(x)\big|\big| \quad and \quad \mu_X \ge \mu_Y \ ,$$

3. ϵ-almost TDSD or ϵ-ATDSD, denoted by $X \succeq_{3D}^{almost(\epsilon)} Y$ or $F \succeq_{3D}^{almost(\epsilon)} G$, if and only if

$$\int_{S_3^D} \left[G_3^D(x) - F_3^D(x)\right] dx \le \epsilon \big|\big|F_3^D(x) - G_3^D(x)\big|\big| \ and \ \ G_n^D(a) \le F_n^D(a) \ for \ \ n = 2, 3$$

where $S_n^D(F, G)$ and $\big|\big|F_n^D(x) - G_n^D(x)\big|\big|$ for $n = 1, 2, 3$ are defined in (2.2), ϵ-almost FDSD, SDSD, and TDSD stand for almost first-, second-, and third-order DSD, respectively.

Now let's turn back to the Example 2.2. As discussed before, B cannot dominate A by SDSD. However, for this example, we can conclude that $B \succeq_{2D}^{almost(\epsilon)} A$. The SD approach is regarded as one of the most useful tools for ranking investment prospects when there is uncertainty, since ranking assets has been proven to be equivalent to expected-utility maximization for the preferences of investors/decision makers with different types of utility functions. It is interesting to examine whether almost SD possesses a property of expected-utility maximization similar to SD. Before we carry on our discussion, we first specify different types of utility functions as shown in the following definition:

Definition 2.4 *For $n = 1, 2,$ and 3, we define*

$$\begin{aligned} U_n^{A*}(\epsilon) &= \left\{u \in U_n^A : (-1)^{n+1} u^{(n)}(x) \le \inf\{(-1)^{n+1} u^{(n)}(x)\}[1/\epsilon - 1] \ \forall x\right\} \ , \\ U_n^{D*}(\epsilon) &= \left\{u \in U_n^D : u^{(n)}(x) \le \inf\{u^{(n)}(x)\}[1/\epsilon - 1] \ \forall x\right\} \ . \end{aligned}$$

We call investors the j^{th} order ϵ-risk averters if their utility functions $u \in U_n^{A*}(\epsilon)$ and the j^{th} order ϵ-risk seekers if their utility functions $u \in U_n^{D*}(\epsilon)$.

3 The Theory

Tzeng et al. (2012) modify the almost SD rule developed by Leshno and Levy (2002) so that the almost SD rule for risk averters possesses the property of expected-utility maximization. In this paper we will show that the almost SD rule for risk seekers also possesses the property of expected-utility maximization. Here, we state both results in the following theorem:

Theorem 3.1 *For* $n = 1, 2,$ *and 3,*[2]

1. $X \succeq_{nA}^{almost(\epsilon)} Y$ *if and only if* $E[u(X)] \geq E[u(Y)]$ *for any* $u \in U_n^{A*}(\epsilon)$*, and*

2. $X \succeq_{nD}^{almost(\epsilon)} Y$ *if and only if* $E[u(X)] \geq E[u(Y)]$ *for any* $u \in U_n^{D*}(\epsilon)$*.*

Now, we turn to examine whether there is any relationship between the almost ASD rule and almost DSD rule. We first show in the following theorem that almost ASD and DSD could be a dual problem:

Theorem 3.2 *For any random variables* X *and* Y *and for* n *=1,2 and 3,*

$$X \succeq_{nA}^{almost(\epsilon)} Y \quad \textit{if and only if} \quad -Y \succeq_{nD}^{almost(\epsilon)} -X \ .$$

We turn to show that sometimes the preference of assets by using almost ASD could be in the same direction as that by using almost DSD but sometimes they are in the opposite direction. We first show in the following theorem for the first order that they are in the same direction:

Theorem 3.3

For any random variables X *and* Y*,*

$$X \succeq_{1A}^{almost(\epsilon)} Y \quad \textit{if and only if} \quad X \succeq_{1D}^{almost(\epsilon)} Y \ .$$

[2]We note that one could easily extend our work to $n > 3$. However, though some studies, see, for example, Eeckhoudt and Schlesinger (2006), Eeckhoudt, et al. (2009), and Denuit and Eeckhoudt (2010), study risk to $n > 3$, most academics and practitioners are only interested in studying the case up to $n = 3$. Thus, we stop at $n = 3$.

Levy and Levy (2002) show that if prospects X and Y have the same finite mean, then sometimes the preference for risk averters and risk seekers could be opposite. Could this property hold for almost SD? We show that this is true as shown in the following theorem:

Theorem 3.4 *If $\mu_X = \mu_Y$, then*

$$X \succeq_{2A}^{almost(\epsilon)} Y \quad \textit{if and only if} \quad Y \succeq_{2D}^{almost(\epsilon)} X \ .$$

As shown later, almost ASD and DSD do not possess the hierarchy property; that is, lower order almost ASD and DSD does not imply higher ones. Nevertheless, in this paper we still recommend not to examine any higher order almost SD (ASD or DSD) if one finds a lower order almost SD (ASD or DSD) relationship. For convenience purpose, we still call, say, almost SASD to be "trivial" almost SASD if one finds there is almost FASD.

We note that in order to have a "non-trivial" second-order almost SD (either ASD or DSD) rule for X and Y, their means must be equal ($\mu_X = \mu_Y$). If their means are not equal, applying Theorem 3.1, we will obtain first-order almost SD rule for both ASD and DSD. Thereafter, applying Theorem 3.3 will conclude that the preferences of X and Y are of the same direction and there are first order almost ASD and DSD between X and Y, not "non-trivial" almost second-order SD.

In addition, Chan, et al. (2012) show that it is possible to have non-trivial third order ASD and DSD between prospects X and Y such that their preferences are the same. Is it possible for the almost SD to have a similar property? In this paper we show that this is possible by showing the following theorem:

Theorem 3.5 *If $\mu_X = \mu_Y$ and $F_3^A(b) = G_3^A(b)$, then $X \succeq_{3A}^{almost(\epsilon)} Y$ if and only if $X \succeq_{3D}^{almost(\epsilon)} Y$.*

We now examine the hierarchy property. To do so, we first discuss the following issue for sets of the utility functions $U_n^{A*}(\epsilon)$ and $U_n^{D*}(\epsilon)$. It is well-known that in Definition 2.1

that $U_{j+1}^A \subset U_j^A$ and $U_{j+1}^D \subset U_j^D$. One may wonder whether there is a similar property for Definition 2.4 that for $\epsilon > 0$, we have $U_{j+1}^{A*}(\epsilon) \subset U_j^{A*}(\epsilon)$ and that $U_{j+1}^{D*}(\epsilon) \subset U_j^{D*}(\epsilon)$. To answer this question, firstly, the ϵ's in these two sets may be different. Secondly, $U_2^{A*}(\epsilon)$ only make constraints on u'', while $U_1^{A*}(\epsilon)$ focus on u'. As we know, higher order derivatives cannot determine lower order ones. Thus, even we have $-u''(x) \leq \inf\{-u''(x)\}[1/\epsilon - 1]\ \forall x$, we may still do not have $u'(x) \leq \inf\{u'(x)\}[1/\epsilon - 1]\ \forall x$. We give the following example for this illustration:

Example 3.1 *consider* $u(x) = 2x - x^2, x \in [0, 1]$. *we can have* $u'(x) = 2 - 2x$ *and* $u''(x) = -2$. *Clearly,* $u \in U_2^{A*}(\epsilon)$, *while it does not belong to* $U_1^{A*}(\epsilon)$ *since* $\inf\{u'(x)\} = 0$.

We note that Theorem 3.1 shows that both almost ASD and DSD rules possess the property of expected-utility maximization. It is well-known that both ASD and DSD possess the hierarchy property (Levy, 1992, 1998) that FASD implies SASD, which, in turn, implies TASD and also FDSD implies SDSD, which, in turn, implies TDSD. Thus, the lowest order of SD relationship is reported and any higher order SD relationship is "trivial" since it can be implied by a lower order SD. For example, if we find X FASD Y, then it is trivial that X SASD Y and X TASD Y. Guo, et al. (2013) find that the almost ASD defined in Definition 2.2 does not possess the hierarchy property such that almost FASD does not imply almost SASD, which also does not imply almost TASD. Similarly, one could easily show that the almost DSD defined in Definition 2.3 does not possess the hierarchy property such that almost FDSD does not imply almost SDSD, which also does not imply almost TDSD. To illustrate the non-hierarchy of almost DSD, we give a simple example to show that almost FDSD does not imply almost SDSD:

Example 3.2 *Prospect A yields \$1 with a probability of 1/2 and \$5 with a probability of 1/2 and Prospect B yields \$3.33 with certainty. It's easy to know that Prospect B dominates A by almost FDSD. Denote the distributions of A and B be* $G(x)$ *and* $F(x)$ *respectively. Note that* $F_2^D(x) - G_2^D(x) = 0.83 - 0.5x$, *if* $1 \leq x \leq 3.33$; *while if* $3.33 < x \leq 5$,

$F_2^D(x) - G_2^D(x) = 0.5(x-5)$. *Then according to the Definition 2.3, we can conclude that Prospect B does not dominate A by almost SDSD.*

4 Illustrations

In this section we will construct some examples to illustrate the theory we have developed in Section 3. We note that all of the examples constructed in this paper could be used to illustrate Theorems 3.1 and 3.2, and thus, we do not discuss the illustration for the assertions in Theorems 3.1 and 3.2.

To illustrate the assertion in Theorem 3.3, we first consider Example 2.1. In this example, since $\min_{F_A}(x) = 900 < \min_{F_B}(x) = 1000$, from the necessary condition of stochastic dominance for risk averters, one could easily check that A does not dominate B by stochastic dominance for risk averters of any order. However, one could easily find that both $A \succeq_{1A}^{almost(\epsilon)} B$ and $A \succeq_{1D}^{almost(\epsilon)} B$ hold at the same time and thus illustrate the assertion in Theorem 3.3. In addition, Example 2.2 can also be used to illustrate the assertion in Theorem 3.3. Some simple computation can lead to conclude both $B \succeq_{1A}^{almost(\epsilon)} A$ and $B \succeq_{1D}^{almost(\epsilon)} A$. The result implies that both first order ϵ-risk averters with utility $u \in U_1^{A*}(\epsilon)$ and first order ϵ-risk seekers with utility $u \in U_1^{D*}(\epsilon)$ will prefer A to B in Example 2.1 and prefer B to A in Example 2.2.

To illustrate the assertion in Theorem 3.4, we use the following example introduced by Levy (2006):

Example 4.1 *Now, Prospect A yields \$1.49 and \$3.51 with equal probabilities of 1/2. Prospect B yields \$1,2,3 and \$4 with equal probabilities of 1/4.*

Levy shows in this example that there is neither FASD nor SASD between A and B. One could also easily verify that there is no FDSD nor SDSD between A and B. Thus, investors with $u \in U_i^A$ or $U_i^D, i = 1, 2$ will be indifferent from A and B. Nonetheless, we observe from this example that A dominates B by ϵ-almost SASD while B dominates A by ϵ-almost SDSD. Thus, second order ϵ-risk averters with utility $u \in U_2^{A*}(\epsilon)$ will prefer A to B while second order ϵ-risk seekers with utility $u \in U_2^{D*}(\epsilon)$ will prefer B to A.

Now we turn to illustrate the assertion in Theorem 3.5 by using the following example:

Example 4.2 *Consider*

$$F(x) = \frac{x+1}{2}, \quad -1 \leq x \leq 1 \quad \textit{and} \quad G(x) = \begin{cases} 0 & -1 \leq x \leq -3/4, \\ x + \frac{3}{4} & -3/4 \leq x \leq -1/4, \\ \frac{1}{2} & -1/4 \leq x \leq 0, \\ x + \frac{1}{2} & 0 \leq x \leq 1/4, \\ \frac{3}{4} & 1/4 \leq x \leq 3/4, \\ x & 3/4 \leq x \leq 1. \end{cases}$$

Notice that both distributions have the same zero mean and $F_3^A(b) = G_3^A(b) = 2/3$. Since $\min_F(x) = -1 < \min_G(x) = -3/4$, F does not dominate G by traditional first-order SD for risk averters. Since $\mu_F = \mu_G = 0$, G does not dominate F by first-order SD for risk seeker either. Moreover, for $x \in [-1, 0], G^{(2)}(x) \leq F^{(2)}(x)$, and $x \in [0, 1], G^{(2)}(x) \geq F^{(2)}(x)$, G does not dominate F by SASD and F does not dominate G by SDSD. Further, we can know that F does not dominate G by ϵ-almost FASD nor ϵ-almost SASD since the statement $\int_{S_n^A} \left[F_n^A(x) - G_n^A(x)\right] dx = 1/2 \big|\big|F_n^A(x) - G_n^A(x)\big|\big|$ holds for $n = 1, 2$. From Theorem 3.2, we can show that G does not dominate F by ϵ-almost FDSD nor ϵ-almost SDSD either. However, for these two prospects, one could easily show that G dominates F by ϵ-almost TASD and ϵ-almost TDSD, which illustrates Theorem 3.5. We note that in this example, G dominates F by both TASD and TDSD.

5 Concluding Remarks

In this paper we first develop a theory of almost stochastic dominance for risk-seeking investors to the first three orders. Thereafter, we study the relationship between the preferences of almost stochastic dominance for risk-seekers with that for risk averters.

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