Inefficiency and Sustainability

Bazhanov, Andrei

Queen's University, Kingston, Canada

30 November 2013

Inefficiency and Sustainability[†]

Andrei Bazhanov

Queen's University, Kingston, ON, Canada, K7L 3N6, Tel.: 1-613-533-3320

Email: bazhanov@yahoo.com

ABSTRACT

This paper examines the effects of ignored inefficiencies on the reliability of sustainability indicators and effectiveness of investment in resource-based economies. A model of a social planner does not include some phenomena that may influence the path of utility. These unspecified phenomena may cause inefficiency of the economy. In order to simulate this natural discrepancy between theory and real life, this study assumes that the planner applies the policies developed for an efficient (undistorted) model, whereas the real economy is distorted by some neglected effects that can influence utility, production, the balance equation, and the dynamics of the natural reserve. The resulting inefficiency affects the dependence of current utility change on investment. The analysis shows that, for sustainability in the presence of inefficiency, first, changes in institutions and in the patterns of resource extraction may become more important than investments; and secondly, it is preferable to underextract a natural resource under uncertainties in production possibilities and damages from economic activities. An inadequate accounting system, misestimated production possibilities, and insecure property rights are considered as examples of disregarded inefficiencies.

Keywords Dynamic inefficiency · Investment · Natural resource · Sustainability

---

[†] **Abbreviations:** CUC – Current utility change; GI – Genuine investment; HR – Hotelling rule; DHS – Dasgupta-Heal-Solow

JEL Classification O13 · O47 · Q32 · Q38

1. Introduction

Sustainability of real economies is always evaluated in the presence of uncertainties in future production possibilities and in various distortions such as institutional imperfections, economic wastefulness, and damages from economic activities to utility and production. These uncertainties lead to errors in the decisions of a social planner and cause deviations from an efficient and optimal path of economic development. Due to imperfections in information and institutions, inefficiencies always exist in real economies,[1] and it is important to learn how they may affect the reliability of sustainability indicators and sustainability policies developed for simplified models.

The literature on sustainability evaluation of resource-based economies offers an expression, called genuine (net) saving or genuine investment (GI), which is a weighted sum of changes in capital assets that are essential for sustainability. GI is considered sometimes as an indicator of sustainable development. The assets in GI include various forms of human, man-made, and natural capital. The coefficients (weights), multiplied by the changes in these assets, are called the accounting or shadow prices. These prices show how a unit of change in the corresponding asset affects a value that is used to measure sustainability. There are both theoretical and practical difficulties in providing such a definition of this value that can be used as a reliable practical indicator of sustainability of a real economy. These difficulties are connected mostly with the uncertainties in the future paths of the economy and discussed, e. g., in Solow (1993), Dasgupta and Mäler (2000), Asheim et al. (2003), Martinet (2007), and Cairns (2008).[2] Despite all the difficulties, the studies of sustainability of a resource economy provide a number

---

[1] "In the real world environmental externalities are not always internalised. This is one of many causes that prevent market economies from being fully efficient" (Asheim et al., 2003). See also Stiglitz (1991).

[2] See also the reviews in the Handbook of Environmental Accounting (2010) edited by T. Aronsson and K.-G. Löfgren.

of models, methods of solution and interesting insights on the behaviour of this economy under various conditions.

The current paper extends the existing literature by offering an approach for estimating the (possible) effects of inefficiencies that result from ignoring by a planner some phenomena that are essential for the path of utility. These phenomena are called “distortions” in the paper because they are not included into a simplified model. Distortions lead to inefficiencies if and only if a planner, maximizing social welfare, does not take them into account. Hence, technically, in this paper, inefficient economy arises from optimization under imperfections (Lemma 1).

In order to use this approach in practice, some specific forms of distortions should be assumed. Then various scenarios of economic development will show possible damages to utility from ignoring these distortions. This approach is different from the regular construction of optimistic-pessimistic scenarios and comparison of the paths of development indicators assuming that all essential for these indicators phenomena are included into the model.

The paper shows how a natural discrepancy between a model and real life results in inefficiency and, in some cases, unsustainability of the economy. Moreover, in an inefficient unsustainable economy, a feasible investment providing a non-declining path of utility may not exist. In this case, the inefficiency must be reduced first, e. g., by the correction of the institutions or the accounting system.

The idea of developing sustainability indicators stems from the result of Hartwick (1977). The “invest resource rent” rule (zero GI or Hartwick rule) addresses the problem formulated in Solow (1974) for the Dasgupta-Heal-Solow (DHS)[3] model of a maximin-optimal resource-based economy satisfying the standard Hotelling rule (HR) as a necessary condition of dynamic efficiency. For this model, zero GI with resource depletion measured in competitive prices leads

---

[3] This model with the Cobb-Douglas production function, which includes a nonrenewable resource as a factor, was developed in the works of Dasgupta and Heal (1974) and Solow (1974).

to constant per capita consumption over time. Dixit et al. (1980) extended the Hartwick rule by showing that, for a more general production function in a competitive economy with multiple assets, GI that is zero over time in present competitive prices[4] is a necessary and sufficient condition for a constant path of utility at maximum value. Dasgupta and Heal (1979, pp. 303-306), Hamilton and Hartwick (2005), and Hamilton et al. (2006) analyzed the link between GI in current prices and current change in per capita consumption. Hamilton and Withagen (2007) derived the result of Dixit et al. (1980), as well as the result of Hamilton and Hartwick (2005), in a more general setting (for multiple consumption good and internalized externalities), showing that instantaneous utility increases if and only if GI decreases in present prices.

Various forms of the indicator GI developed for efficient economies have been used for practical evaluations of sustainability. For example, Pearce and Atkinson (1993) offered a simple indicator of weak sustainability[5] based on the assertion that "an economy is sustainable if it saves more than the combined depreciation on the two forms of capital" (man-made and natural). A variant of this indicator, modified for open economies, has been developed in Proops et al. (1999). These indicators were used in both papers to classify a number of countries into sustainable and unsustainable. Hamilton and Clemens (1999) developed a theory of genuine saving by adding the investment in human capital to traditional net savings and subtracting the value of resource depletion and environmental damage. The value of genuine saving was used as an indicator of sustainability for a wide range of developing countries. A review of empirical work on sustainability evaluation can be found in Hamilton (2010). As Hamilton and Hartwick (2005) noted, "the magnitude of 'net investment' or 'genuine savings' has become a central focus in the measurement of the sustainability of an economy."

---

[4] Constant investment in present prices means that investment in current prices is growing with the rate of discount.

[5] Weak sustainability of growth (development) is defined in Pezzey (1992) as non-decreasing per capita consumption (utility).

While most of the theoretical results above were obtained for optimal or competitive economies, Dasgupta and Mäler (2000) and Arrow et al. (2003), developing a theory of sustainable resource use in imperfect economies, showed that the shadow price of a natural resource can be considerably higher than the market price, implying that the investment of the market resource rent and even the entire marked-valued output into man-made capital can be insufficient to compensate for damages caused by the resource extraction.[6] In other words, GI in shadow prices may be negative despite any effort in saving, suggesting that, for some inefficient economies, correction of institutions (e. g., property rights) leading to a more conservative resource policy is a prerequisite of sustainability.

Unfortunately, accurate shadow prices are not observable in real economies, in particular because of uncertainties in population growth, damages from economic activities, and future production possibilities (resource reserves estimates and rates of technical change). These uncertainties usually are dealt with by stochastic models in theory and optimistic-pessimistic scenarios in practice. Both approaches result in errors and, therefore, in inefficiencies. Asheim (2010), showing that the value of GI "cannot serve as a reliable indicator of sustainability," noted also that inefficiency may further loosen the link between GI and sustainability.

This paper shows *how* inefficiency loosens the link between GI and sustainability. Proposition 1 provides the dependence between "undistorted" GI, which was developed for an efficient economy, and current utility change (CUC) in an economy with inefficiencies. This result extends Proposition 1 of Hamilton and Hartwick (2005), and shows that CUC may be determined only by the influence of inefficiency when this influence is not close to zero.

---

[6] In real economies, the standard HR does not hold (e. g., Gaudet, 2007), implying deviations from the competitive efficient extraction path. Bazhanov (2007, 2008) showed that in the DHS economy under a modified HR and zero GI, the path of consumption may be either decreasing to zero or infinitely increasing depending on the path of extraction.

The examples of distortions (Section 3) include (i) misestimated dynamics of the natural resource stock that may lead either to a sustainable but dynamically inefficient or unsustainable economy; (ii) inadequate accounting system that ignores the damages from the resource use to utility and production yielding inefficiency and unsustainability; and (iii) insecure property rights that also cause inefficiency and unsustainability, unless corrected by institutional reforms.

This study shows that, for sustainability, it is preferable to underestimate future production possibilities and overestimate damages since this policy of extra caution can reduce irreversible losses. Of course, this policy may lead to dynamic inefficiency caused by an overly conservative resource policy, but with updates in knowledge, the policy can be corrected, and the economy can be asymptotically efficient. This study extends also (for general inefficiency) the effect shown by van der Ploeg (2011) that insecure property rights require more effort in investment to maintain constant per capita consumption. Corollary 2 adds, in particular, that underestimation of future production possibilities requires less investment than in the "undistorted" case.

Besides disregarded inefficiencies, the study shows how sustainability evaluation may depend on the specification of the same effects in the model and on the form of the indicator (Section 4). An indicator linked to CUC may be, by construction, insensitive to the changes in the long-term ability of an economy to maintain non-declining utility.[7] On the other hand, an indicator of long-term sustainability, also by construction, may be insensitive to CUC, also leading to unsustainability at the initial point. Therefore, various indicators may complement each other.

This paper is organized as follows: Section 2 describes the approach to modelling a dynamically inefficient "distorted" economy and derives the main theoretical results; Section 3 illustrates these results with the examples of various types of inefficient economies; Section 4

---

[7] Besides this paper, see, e.g., Asheim et al. (2003), Martinet (2007), and Cairns (2008).

shows the dependence of sustainability evaluation on the specification of the model and the form of indicator; and Section 5 concludes.

2. Investment and growth in inefficient economy

In order to define a distorted economy, it is instructive to introduce first a "perfect" or undistorted optimizing economy. Following Hamilton and Hartwick (2005, p. 618), assume that the economy is closed, time t is continuous, consumption is aggregated into a single good C, labour is fixed, so that output $Q(t) = F(K,R)$ depends on man-made capital $K(t)$ and the resource flow $R(t) = -\dot{S}(t)$, where $S(t)$ is the current resource stock ($\dot{S} := dS/dt$). The technology is stationary (F does not depend explicitly on t).

A number of studies, which results were used for practical evaluation of sustainability, assume that the economy satisfies the following:

- *resource productiveness:* $F(K,R)$ is a regular production function that (a) denotes the maximum output for the given K and R, and (b) satisfies the Inada conditions, in particular $F_R > 0$, where $F_R := \partial F / \partial R$;
- *static efficiency:* output Q equals $F(K,R)$[8]
- *non-wastefulness:* the balance equation holds: $C + \dot{K} = F(K,R) - \delta K$, where $\dot{K}$ is investment and δK with $\delta = const$ is capital decay;
- *dynamic efficiency:* the standard HR

  $$\dot{F}_R / F_R = r \quad (1)$$

  [9]

  holds as a necessary condition of dynamic efficiency;

---

[8] Conventionally, efficiency is defined via the Pareto-optimality. Some studies, e. g. Hurwicz (1960), called this notion non-wastefulness.

[9] Here, $r(t) := F_K(t) - \delta$ is the competitive interest rate.

- *optimality:* the economy (a planner) maximizes a (social) welfare function by choosing the paths of $\dot{K}$ and R, or the economy is competitive.

In the real world, however, the resource use can be

- non-productive ($F_R = 0$) or counter-productive ($F_R < 0$);[10]
- productive, but static-inefficient ($Q < F(K,R)$);
- productive, static-efficient, but wasteful ($C + \dot{K} < F(K,R) - \delta K$);
- productive, static-efficient, non-wasteful, but dynamically inefficient;
- productive, non-wasteful, efficient, but not optimal or under imperfect competition.

This paper assumes that there are *distortions* in the economy, which are represented by vector $\mathbf{D}$. The distortions may include imperfections, externalities, and any effects (including favourable for sustainability) that cause deviation of the intertemporal efficiency condition for the ratio $\dot{F}_R / F_R$ from the standard HR (1).[11] In resource economies, $\mathbf{D}$ may depend on the extracted amount of the resource when this extraction or the resource use causes irreversible losses to the environment, human health, and even production itself, e. g., as a result of oil spills or climate change due to the burning of fossil fuels. In some cases, the extracted amount may depend on institutional imperfections, e. g., insecure property rights. $\mathbf{D}$ may depend also on R, e. g., when damage includes opportunity cost (Gaudet et al., 2006), or when damage is partly reversible. Then the analysis becomes more complicated, which, however, does not alter the main approach and conclusions of the paper.

The main goal of the study below is to show how distortions modify intertemporal efficiency condition (1), and how inefficiencies, resulting from ignoring these distortions, affect the efficacy of investment policies. A particular goal is to show that, under large inefficiencies,

---

[10] The resource use is counter-productive when the decline in the natural resource stock results in the decline of output, e. g., as a result of a wildfire or oil spill.

[11] A review of studies with the modified HR is, e. g., in Gaudet (2007).

increasing investments in man-made or human capital may not help to restore sustainability and even may be harmful, e. g., to poor countries.[12] To illustrate the claims of this study, it is enough to assume that the components of $\mathbf{D}$ depend on the extracted amount $X(t)$ (or on the remaining stock $S(t)$)[13] or on the parameters that express distorted institutions.

In this paper, $\mathbf{D}$ has four components that are the distortions in

production: $$F = F(K, R, D_1), \tag{2}$$

social utility: $$U = U(C, D_2), \tag{3}$$

the balance equation: $$\dot{K} = F(K, R, D_1) - C - \delta K - D_3, \tag{4}$$

the dynamics of the stock: $$\dot{S} = -R + D_4. \tag{5}$$

Following the above, assume that D_4 either depends on $S(t)$ or on the parameters that do not depend on time explicitly, and $D_i = D_i[X(S(t))], i \in \{1,2,3\}$ depend on the extracted amount, which, by (5), is $X(t) = \int_0^t R(\xi)d\xi = S_0 - S(t) + \int_0^t D_4[S(\xi)]d\xi.$ Distortions D_1 and D_2 may represent irreversible damages caused by climate change resulted from the burning of fossil fuels, e.g., in the form $[\theta_1 X(t) + 1]^{\theta_2}$, where θ_1, θ_2 – parameters[14]; D_3 may stand for the growing cost of extraction ($\partial D_3 / \partial X > 0$ – the best-quality stock extracted first). As to D_4 , assume that this distortion is such that

(a) $D_4\big|_{S(t)=0} = 0$ (no resource reserve – no distortion);

(b) $\int_0^t D_4[S(\xi)]d\xi \geq S(t) - S_0$ (the productive extraction $X(t)$ is nonnegative).

---

[12] See also Cairns (2008).

[13] If $\mathbf{D}$ depends on the non-extracted resource, e.g., the stock has an amenity value (D'Autume, Schubert, 2008), then, expressing this value in terms of utility, the problem can be reformulated by introducing the damage from the resource extraction. The latter approach may be more precise, since the uncertainty in the extracted stock is less than in the remaining.

[14] See Bazhanov (2012) and Stollery (1998).

D_4 may take the form $D_4 = \delta_S S(t)$, where negative δ_S may stand for the rate of non-productive depreciation of the stock $S(t)$, e. g., in the cases of wildfires or leakage of fresh water. Positive δ_S may reflect explorations for a non-renewable resource, or may be the rate of natural appreciation of $S(t)$, e. g., in fishery or forestry. In the former case, for a plausible δ_S, the stock $S(t)$ must be monotonically decreasing starting from some $\bar{t} \geq 0$ (i.e., $D_4 \leq R$ for any $t > \bar{t}$ with $D_4 = R$ only for $S = R = 0$). Hence,

$$\frac{\partial X}{\partial S} = -1 + \int_0^t \frac{\partial D_4[S(\xi)]}{\partial S} d\xi.$$

Note that $\partial X / \partial S = -1$ implies $\partial D_i / \partial X = - \partial D_i / \partial S$. These equalities hold if and only if $\int_0^t \frac{\partial D_4[S(\xi)]}{\partial S} d\xi = 0$, e. g., for a non-renewable resource with no explorations and non-productive losses ($D_4 \equiv 0$), or for D_4 that does not depend on S and results only in a redistribution of the resource among generations (e.g., for imperfect institutions). Note also that, for any $t \geq 0$, $\partial X / \partial S < 0$ if $R > 0$ and $\dot{S} < 0$.

In the following lemma, Ψ_I is a deviation of $\dot{F}_R / F_R$ from a dynamically efficient path.

Lemma 1. *In economy (2)-(5),*

$$\dot{F}_R \big/ F_R = v(t) + \tau(t), \tag{6}$$

where $v(t) := F_K - \delta$,[15] $\tau(t)$[16] $= \tau[\mathbf{D}(t)] := \Psi_I - \partial D_4 / \partial S$

$$- \{[(U_{D_2} / U_C)\, \partial D_2 / \partial X + F_{D_1}\, \partial D_1 / \partial X - \partial D_3 / \partial X] / F_R\}\, \partial X / \partial S, \tag{7}$$

and $\Psi_I = 0$ *if the economy is dynamically efficient.*

---

[15] $v(t)$ is the market interest rate only with no distortion.

[16] $\tau(t)$ is the additive HR modifier or the influence of $\mathbf{D}$. This influence can be expressed in a multiplicative form: $\dot{F}_R = v\eta[\mathbf{D}]F_R$, where $\eta[\mathbf{D}] := 1 + \tau[\mathbf{D}]/v$. With no distortion, $\tau = 0$.

Proof is in Appendix 1.

Corollary 1. *If, under the conditions of Lemma 1,* D_4 *does not depend on* S *, then*

$$\tau(t) = \Psi_I - [(U_{D_2} / U_C)\, \partial D_2 / \partial S + F_{D_1}\, \partial D_1 / \partial S - \partial D_3 / \partial S] / F_R.$$

In this framework, dynamic efficiency is a relative notion. The planner's optimal path may be dynamically inefficient with respect to a first-best solution, e. g., because the planner underestimates future production possibilities and considers the first-best path as infeasible (Section 3.1). The planner's path may also be inefficient when the planner ignores some effects while estimating social progress. In the latter case, the planner may even consider the first-best path as inefficient due to the difference between the units of measure for utility in the planner's and the first-best solutions (Section 3.2).

Inefficiency Ψ_I depends on $\mathbf{D}$ when a simplified model for economy (2)-(5) results in $\tau = 0,$ ignoring the distortions. For example, $\Psi_I = \partial D_4 / \partial S$ when a planner misestimates the changes in the proven resource stock (Section 3.1). Ψ_I depends on D_1 and D_2 when the planner ignores the damages from the resource extraction (Section 3.2). In some cases, however, Ψ_I may not depend on $\mathbf{D},$ and instead, both Ψ_I and $\mathbf{D}$ may be determined by the same phenomena, e. g., imperfect institutions (Section 3.3).

Genuine investment (GI) defined in Hamilton and Hartwick (2005) is

$$G(t) := \dot{K}(t) + \dot{S}(t) F_R(t), \tag{8}$$

which is $\dot{K}(t) - R(t)F_R(t)$ without distortions. This measure includes not only the investment into man-made capital $\dot{K}$ but the value of the extracted natural resource $\dot{S}$ estimated in the marginal resource productivity $F_R,$ which, with no distortion, coincides with competitive price.

In the general case, GI is defined as $G(t) := \sum_i p_i \dot{K}_i(t)$ where K_i are various forms of man-made (including human and intangible) and natural capital, and p_i are the shadow prices equal to the marginal change in social welfare V resulting from a change in K_i: $p_i := \partial V / \partial K_i$ (Arrow et al. 2003).[17] By construction, G must coincide with the change in social welfare resulting from the combination of investments to (or extractions from) all essential for V assets at the current moment: $\dot{V}(K_1, K_2, \ldots) = \sum_i \left(\partial V / \partial K_i \right) \dot{K}_i$ (if V does not depend on time explicitly).[18] Then, if V reflects the ability of the economy to maintain non-declining utility, the indicator GI shows the change in sustainability of the economy.

As was mentioned above, sustainability of real economies is always evaluated under imperfections including imperfections in knowledge and in models that are used for this evaluation. Therefore, in practical sustainability evaluation, a model welfare function and the correspondent indicator GI never include all the factors that will affect the long-term path of utility in the real economy. In order to examine the effect of this discrepancy, this study considers indicator GI in the form of (8), which corresponds to the “undistorted” economy.

Due to the distortion D_2, utility may be decreasing in time while consumption is growing; therefore, consumption cannot always be a proxy for utility as a measure of well-being (see, e. g., Section 3.2). Hence, the proposition below establishes the link between G and U, which includes the link between G and C as a special case.

---

[17] For example, under the utilitarian criterion, $V(t) = \int_t^{\infty} U\left[K_1(s), K_2(s), \ldots\right] \pi(s) ds,$ where π is a discount factor, and, under the maximin, $V(t) = U^*\left[K_1(t), K_2(t), \ldots\right],$ where U^* is the maximum level of utility that can be maintained forever given the current assets $K_1(t), K_2(t), \ldots$.

[18] If social welfare depends on exogenous processes such as population growth or technical change, the time derivative is $\dot{V}(t, K_1, K_2, \ldots) = \partial V / \partial t + \sum_i \left(\partial V / \partial K_i \right) \dot{K}_i.$

Proposition 1. *Current utility change (CUC) in distorted economy (2)-(5) is*

$$\dot{U} = (v - \dot{G}^0 / G^0) G^0 U_C + \Psi_U , \qquad (9)$$

where G^0 *is the GI indicator for the undistorted economy, and*

$$\Psi_U := -U_C [F_R R(\Psi_I - \partial D_4 / \partial S) + (\dot{D}_3 - \dot{D}_1 F_{D_1} - \dot{D}_2 U_{D_2} / U_C)(1 + \partial X / \partial S)]$$

is the influence of the inadequate indicator G^0 *and the dynamic inefficiency* Ψ_I *.*

Proof is in Appendix 2.

When an economy is efficient ($\Psi_I = 0$), and GI is measured in competitive prices (with internalized externalities and $G \neq G^0$, implying $\Psi_U = 0$), formula (9) can be obtained from the results of Dixit et al. (1980) or Hamilton and Withagen (2007) (see the proof). With no distortion, Eq. (9) coincides with the result of Hamilton and Hartwick (2005). In the general case, Eq. (9) shows that investment (8) can indeed determine $\dot{U}$ if Ψ_U is relatively small. However, $\dot{U}$ can be also completely determined by Ψ_U when the term $(v - \dot{G}/G) G U_C$ is close to zero.

Of course, sharp changes in G can determine an instant sign of $\dot{U}$ despite the large values of Ψ_U. Formula (9) shows that if there is a $t = \bar{t}$, such that $\Psi(\bar{t})$ has a large positive (negative) value, $\dot{U}(\bar{t})$ can be negative (positive) if $G(\bar{t})$ is negative (positive) and $\dot{G}(\bar{t}) / G(\bar{t})$ has a large positive (negative) value. However, these cases are not relevant to sustainability due to the boundedness of investments, whereas distortions in general are less restricted. The boundedness of investment implies that the larger is Ψ_U, the shorter is the period of time when these cases are possible. Therefore, neglecting the short-run oscillations, assume, for determinateness, that $|\dot{G}/G| < v < \infty$,[19] and the current investment $\dot{K}$ is bounded by the current output Q. Then a feasible investment can be defined as follows.

Definition 1. Investment $\dot{K}(t) = w(t)Q(t)$ is *feasible* if $w(t) \in (0,1)$ and $|\dot{G}/G| < v$ for any $t \geq 0$.

Eq. (9) is essentially simplified if $\partial D_4 / \partial S \equiv 0$ while retaining the dependence of CUC on the interplay between investment and inefficiency. The following corollary uses this case to illustrate (I) how the sign of CUC ($\dot{U} \gtreqless 0$) depends on the values of investment (G or w) and inefficiency (Ψ_I or Ψ_U); and (II) that investment $\dot{K}$ can influence the sign of CUC if and only if inefficiency is sufficiently small.

Corollary 2. *If D_4 does not depend on S, Eq. (9) implies that*

(I) for a feasible investment $\dot{K}$, $\dot{U} \gtreqless 0$ if and only if

$$G \gtreqless RF_R \Psi_I / (v - \dot{G}/G) \quad \textit{or} \quad w \gtreqless [RF_R / Q][1 + \Psi_I / (v - \dot{G}/G)], \quad \textit{or} \tag{10}$$

$$\Psi_U \gtreqless -(v - \dot{G}/G) G U_C \quad \textit{or}$$

$$\Psi_I \lesseqgtr (v - \dot{G}/G)(\dot{K}/(RF_R) - 1). \tag{11}$$

(II) a feasible investment can change the sign of $\dot{U}$ if and only if

$$-(v - \dot{G}/G) < \Psi_I < (v - \dot{G}/G)(Q/(RF_R) - 1). \tag{12}$$

When $\Psi_I = 0$, condition (10) yields the Hartwick rule. The result of van der Ploeg (2011) can be obtained for $Q = F(K,R) = K^\alpha R^\beta$ and $\Psi_I = \xi^*(N-1)/K$, where N is the number of the owners of the common resource and $\xi^* > 0$ is the coefficient of violation of property rights. Then condition (10) is $w \gtreqless \beta[1 + \xi^*(N-1)/(\alpha Q - \delta K - \dot{Q}K/Q)]$. For $\delta = 0$ and under

---

[19] The analysis can be easily complemented with the case where $|\dot{G}/G| \geq v$.

requirement $\dot{Q} = 0$, it becomes $w = \beta[1 + \xi^*(N - 1)/(\alpha Q)]$, yielding the result of van der Ploeg. Condition (10) also shows that if $\Psi_I < 0$ (e.g., underestimated production possibilities), the growth of utility can be achieved with less investment than in the Hartwick rule.

The following examples show that, under Corollary 2, the impact of inefficiency on the efficacy of investment depends on the level of output and the share of the resource rent in output. Assume that $D_4 = 0$, $v = 0.06$, and $\dot{G}/G = 0.03$ at $\bar{t} \geq 0$.

(a) High-output resource-independent economy. Let $Q(\bar{t}) = 101$ and $R(\bar{t})F_R(\bar{t}) = 1$ (the output of the resource sector is relatively small). Then the investment $\dot{K}$ can change the sign of $\dot{U}(\bar{t})$ if and only if $-0.03 < \Psi_I < 3$.

(b) Low-output resource-dependent economy. For $Q(\bar{t}) = 11$ and $R(\bar{t})F_R(\bar{t}) = 10$, investment can affect the sign of $\dot{U}(\bar{t})$ if and only if $-0.03 < \Psi_I < 0.003$.

It is intuitive that a high-output economy has more opportunities in investment than a low-output one, and so the range for Ψ_I, in which investment is able to affect the sign of $\dot{U}$, is larger in case (a) than in case (b). Another difference between these two cases is that investment in case (a) can change the sign of $\dot{U}$ mostly when Ψ_I affects $\dot{U}$ negatively (positive Ψ_I reduces $\dot{U}$). This asymmetry is inverted in case (b).

The boundedness of investments implies that if $\partial D_4 / \partial S \equiv 0$, the current states of economies may be classified into the following types depending on the roles of investment and institutional changes in the current change of utility given the level of inefficiency Ψ_I.

(A) $\Psi_I \geq (v - \dot{G}/G)(Q/(RF_R) - 1)$: utility declines regardless of investment; non-negative values of $\dot{U}$ can be obtained only by reduction of the inefficiency.

(B) $0 < \Psi_I < (v - \dot{G}/G)(Q/(RF_R) - 1)$: utility growth can be achieved by investment policy alone; the optimal saving rate is higher than under $\Psi_U = 0$ (by the second inequality in (10)) in order to compensate not only for the shrinking natural capital but for the negative effect of inefficiency. Without a policy reducing Ψ_I, the level of utility may be lower than under $\Psi_U = 0$.

(C) $-(v - \dot{G}/G) < \Psi_I < 0$: utility growth can be achieved by investment policy alone; the optimal saving rate may be lower than under $\Psi_U = 0$ due to a positive effect from Ψ_I; decline in utility is still possible when $G < RF_R \Psi_I /(v - \dot{G}/G) < 0$.

(D) $\Psi_I < -(v - \dot{G}/G)$: utility grows regardless of investment; investment policy is important as a determinant of the level of utility along the growing path (Bazhanov 2008).

Types C and D may result from the underestimation of future production possibilities (Section 3.1).

Condition (9) shows that, for $\Psi_U < 0$, the minimum investment G that provides non-declining utility may be essentially higher than zero. The next section illustrates that a feasible value of G, guaranteeing $\dot{U} \geq 0$, may not exist.

3. Dynamic inefficiency and sustainability: examples

In the examples below, Ψ_I^0 denotes a deviation of the ratio $\dot{F}_R / F_R$ along the planner's optimal path from a first-best path and Ψ_I^* – a deviation of this ratio along the first-best path from the planner's path ($\Psi_I^* = -\Psi_I^0$). For succinctness, the planner's optimal paths and the models that do not take into account some of the distortions are called below “undistorted.”

3.1. Misestimating the resource stock

Assume that D_4, which may be either positive or negative, is the only distortion in economy (2)-(5). A planner ignores D_4 and works with the undistorted model. By Lemma 1, $\Psi_I = \partial D_4 / \partial S$ and the claim of Proposition 1 coincides with the efficient case since $\Psi_U = 0$:[20]

$$\dot{C} = (v - \dot{G}^0 / G^0) G^0 .$$

Hence, misestimations of S alone do not affect CUC. This fact, however, does not imply neither efficiency nor sustainability. Indeed, if $D_4 > 0$ (the stock is underestimated) and a sustainable path is feasible, the planner can implement it with the same policy G^0 without additional reserve. Inefficiency can be shown, e. g., when the planner follows a constant-consumption path. As is known from theory, a higher resource stock results in a higher optimal level of constant consumption. Therefore, an efficient planner recalculates the optimal path $C^*(t)$ with the updates in the stock, yielding a piecewise-constant path with growing levels. Hence, $C^*(t)$ will be Pareto-superior to the continuous path $C(t) \equiv C(0)$.

If $D_4 < 0$ is ignored, the actual reserve is $S_0 + \int_0^\infty D_4[S(\xi)]d\xi < S_0$ (overestimated stock) and there exists such $T > 0$ that $S(t) = 0$ for any $t \geq T$, yielding inefficiency and collapse regardless of G^0, e. g., for the DHS economy (Asheim et al., 2003, p. 138).

In this section, a discrepancy between theory and real life results either in a sustainable but inefficient, or in unsustainable path when $D_4 > 0$ or $D_4 < 0$. Inefficiency in the former case can be reduced only by stimulating extraction when the planner updates reserve estimates. This

---

[20] Utility is not distorted here ($U_{D_2} = 0$); hence, formula (9) becomes $\dot{C} = (v - \dot{G}^0 / G^0) G^0$ since $\dot{U} = U_C \dot{C} + U_{D_2} \dot{D}_2$ and $\Psi_U / U_C = 0$.

adjustment can result in sustainable and asymptotically efficient economy. Overestimated stock leads to unsustainability with a possible collapse of the economy if the resource is essential.

3.2. Inadequate accounting system

Assume that only production and social utility are negatively affected by the damage $D = D_1 = D_2$ caused by a stock externality ($D_X = -D_S > 0,\ U_D < 0,\ F_D < 0, U_C > 0$) resulted, e. g., from irreversible climate change due to the burning of fossil fuels.[21] If a planner uses an accounting system that disregards the damage, then, according to Corollary 1, the planner's paths are dynamically inefficient with $\Psi_I^0 = -(F_D + U_D / U_C) D_S / F_R > 0.$ The planner's problem reduces to the Solow (1974) - Hartwick (1977) case, where, under the maximin criterion, the path of extraction starts from a higher level than in the efficient case,[22] and the economy follows a constant-consumption path (due to $G = 0$) with a higher level than the initial level of the efficient path, which is measured in utility units. Since the planner assumes that $U_D = F_D = 0,$ formula (9) becomes $\dot{C} = (v - \dot{G}^0 / G^0) G^0.$

In reality, however, the change in well-being is[23]

$$\dot{U} = (v - \dot{G}^0 / G^0) G^0 U_C + (F_D U_C + U_D) \dot{D}, \tag{13}$$

which is negative for the planner's paths of investment and extraction ($G^0 = 0$) since $(F_D U_C + U_D)\dot{D} < 0.$ The investment G that provides $\dot{U} > 0$ does not exist here when the damage is large, namely, when $(F_D U_C + U_D)\dot{D} < -[v(Q - RF_R) - \dot{G}]U_C < 0.$ Hence, the undistorted policies result in inefficiency and unsustainability of this economy.

---

[21] See, e. g., Stollery (1998).

[22] When damage affects only production, Stollery (1998, p. 735) showed that the optimal extraction starts from a lower initial level and declines slower than in the case with no damage. The same result for a quasiarithmetic damage in utility was obtained in Bazhanov (2012, formula (33), Fig. 4) and for an exponential damage – in D'Autume et al. (2010).

[23] Eq. (13) results from Eq. (9) using the expression for Ψ_I^0 and the fact that $\dot{D} = -RD_S$ when $D_4 = 0.$

A non-declining path of the true quality of life (utility) can be achieved in this example only when the planner recognizes the damages and reconsiders the measure of progress in the society.[24] This done, the planner, situated in the Solow-Hartwick case, can switch to a sustainable and first-best optimal path by changing the resource policy alone, namely, by reducing extraction, while the investment rule remains the same.[25]

3.3. Insecure property rights

This section illustrates the case when inefficiency does not depend on $\mathbf{D}$ and instead, both Ψ_I and $\mathbf{D}$ are determined by the same phenomenon – imperfect institutions. Following Arrow et al. (2003, p. 664), assume that the owner i $(i = 1...N;\ N \geq 2)$ extracts a liquid resource from a pool with the stock S_i. All N owners are identical, non-cooperative, and the pools are separated by porous barriers. The resource diffuses from larger pools to smaller ones with the same rate $\lambda > 0$.[26] The depletion equations are

$$\dot{S}_i = \lambda \sum\nolimits_{j \neq i} (S_j - S_i) - R_i,\ i = 1 \ldots N,$$

where $R_i = R_i(t)$ is the owner i's rate of extraction at the moment t.

As is shown in Arrow et al. (2003), maximization of the each owner's payoff $\int_0^\infty U[R_i(t)]e^{-\rho t}dt$ with $U(R_i) = -R_i^{-(\eta-1)}$, where $\eta > 1$, yields Eq. (6) with $\tau = \Psi_I = (N-1)\lambda > 0$ (social efficiency requires $N = 1$). The distorted by the inefficiency path of extraction is $\tilde{R}(t) = \sum\nolimits_{i=1}^{N} R_i(t) = [(\rho + \Psi_I)/\eta]S_0 e^{-(\rho+\Psi_I)t/\eta}$ with a higher initial rate $\tilde{R}(0)$

---

[24] A review on the theory of social accounting is in Aronsson and Löfgren (2010). A practical illustration of the changes in the measure of social progress is the development of the Integrated Environmental and Economic Accounting, which was first offered as a handbook on environmental accounting in UN (1993) and eventually became a legal base in EU (2011).

[25] Stollery (1998) showed that the Hartwick rule ($G = 0$) is still optimal in this economy.

[26] No barriers corresponds to $\lambda \rightarrow \infty$.

and faster decline $\dot{\tilde{R}}(0)$ than for the efficient path $R(t)=[\rho/\eta]S_0e^{-\rho t/\eta}$. Hence, the distorted equation for the reserve is $\dot{S}=-R+D_4$, where $D_4=R-\tilde{R}$. In this example, neither D_4 depends on $S(t)$, nor Ψ_I depends on D_4. Both these deviations result from imperfect institutions, expressed in $N\geq 2$ and $\lambda>0$.

For illustration, assume that $v=0.06$ and $\dot{G}/G=-0.04$. By Corollary 2, a feasible investment $\dot{K}$ can change the sign of $\dot{U}$ if and only if $(N-1)\lambda<0.1[Q/(RF_R)-1]>0$. If this condition is not satisfied, only institutional changes can prevent decline in utility. Investment provides non-decreasing utility here if and only if

$G\geq 10(N-1)\lambda RF_R>0$ or $w\geq [10(N-1)\lambda+1](RF_R/Q)$,

which may be very restrictive for $N>1$. Consider two cases.

(a) High-output resource-independent economy ($Q=101; RF_R=1$). In this case, $\dot{K}$ can change the sign of $\dot{U}$ if and only if $(N-1)\lambda<10$, which means, e. g., that, for $\lambda=1$ and any $\dot{K}$, utility decreases if $N>10$. Let $N=5$. Then the saving rate, compensating for the shrinking resource and the inefficiency, should be no less than $w_{\min}=41/101$ (or $G/Q\geq 40/101$), whereas with no distortion, utility increases for any $w>1/101$ (or $G>0$).

(b) Low-output resource-dependent economy ($Q=11; RF_R=10$). $\dot{K}$ can change the sign of $\dot{U}$ if and only if $(N-1)\lambda<0.01$; e. g., for $\lambda\geq 0.01$ and $N\geq 2$, utility decreases regardless of any feasible $\dot{K}$. For $\lambda=0.009$ and $N=2$, non-decreasing utility is possible when almost all output is being invested: $w\geq 10.9/11$. Although, for this economy, even with $N=1$, the saving rate yielding constant utility is very high, namely, $w_{\min}=RF_R/Q\approx 0.91$.

The use of “undistorted” policy leads in this example to inefficiency and unsustainability; moreover, a feasible investment compensating for the inefficiency and providing non-decreasing utility may not exist. Therefore, recommendations to increase investments, implied by an undistorted model, may cause a drop in current utility below a subsistence minimum without reaching sustainability. This outcome implies that institutions must be corrected first in order to reduce inefficiency and only secondly the policies for the undistorted model can be applied.

Of course, a model that takes into account all the essential factors for utility does not necessarily imply a low level of inefficiency in practice. This level depends on the specifications of the factors, and practical sustainability requires moderately-pessimistic assumptions.[27]

4. A form of indicator and sustainability

As is known, the gap between sustainability evaluation and actual sustainability depends not only on disregarded inefficiencies. This section shows how sustainability evaluation may depend on the form of the indicator and on the specification of the same effects in the model.

Indicator GI that shows CUC may not reflect the change in the ability of an economy to maintain non-decreasing utility over time. For example, if $Q = F(K,R) = K^{\alpha}R^{\beta}$, indicator (8) expressed as a share of GDP takes the form $G/Q = w - \beta,$ where w is the rate of investment ($\dot{K} = wQ$). If utility monotonically increases with consumption, then, for $\alpha > \beta,$ an economic program with non-decreasing utility exists at $t = t_0$ if and only if the *level of potential sustainability* $G_{\infty}/Q\big|_{t=t_0} = w - KR\beta/[S_0Q(\alpha - \beta)]\big|_{t=t_0}$ is non-negative (Bazhanov 2011).[28] In this form of the indicator GI, the resource price shows the change in the maximum level of consumption that can be maintained forever, while the resource stock changes by one unit.

---

[27] Bazhanov (2011) showed that a model of IMF with oil and financial assets as perfect substitutes allows for sustainable development for oil-extracting countries, while the same oil extracting scenarios result in collapse for DHS model using Russian data.

[28] If α and β are to be determined by calibration (e. g., Bazhanov 2011, Section 3), a more convenient form of this indicator is $G_{\infty}/Q = \alpha - \beta - (\beta/w)KR/(S_0Q)\big|_{t=t_0}$. This expression is always negative when $\alpha < \beta,$ i.e., a non-decreasing path of consumption does not exist.

Hence, this economy is *potentially sustainable* at $t = t_0$ (a program with non-decreasing utility exists)[29] if and only if $R(t_0)/S_0 \leq wQ(\alpha - \beta)/[\beta K]\big|_{t=t_0}$, which means that potential sustainability can be achieved by a change in resource policy only, regardless of the value of $G/Q\big|_{t=t_0}$ and the sign of $\dot{C}(t_0)$. The indicators G and G_∞ coincide if and only if the state of the economy satisfies a "perfection" condition ($R/S_0 = Q(\alpha - \beta)/K$) with respect to a constant-consumption criterion (Bazhanov 2010, Eq. (6)).

On the other hand, the indicator G_∞ does not reflect CUC and current level of utility. As a result, a positive value of this indicator may "approve" a sharp decline in the current rate of the resource extraction, which may lead to a drop in the level of utility below a subsistence minimum, violating intergenerational justice. Of course, unsustainability of an economy ($G_\infty < 0$) calls for increases in investments into man-made and human capital and decreases in the rates of natural resource extraction to increase G_∞, which requires 1) to define the maximum value of G_∞ for an optimal growth; and 2) the optimal path of acceptable sacrifices in current utility for future sustainable development.

5. Conclusions

This paper examined the effects of unaccounted inefficiencies on credibility of sustainability indicators and efficacy of investment. The results imply that, for sustainability of real economies, 1) changes in institutions may be more important than investment into man-made

---

[29] This definition, offered in Bazhanov (2011), is partly equivalent to the following definition of Pezzey (2004): an economy is *sustainable at time* t_0, if $U(t_0) \leq U_{\max}$ (the economy is not overconsuming at t_0), where $U_{\max}$ is the maximum sustainable level of utility that can be maintained forever, given the stocks of man-made and natural capital at t_0. The equivalence is only partial because, as Pezzey noted, his definition works only for efficient economies. An overextracting inefficient economy may be underconsuming due to inefficient use of the resource. For such an economy, a sustainable program may not exist; therefore, for sustainability evaluation of real economies, it is preferable to use the tools that can work under inefficiencies since real economies are, as a rule, inefficient.

capital when the level of inefficiency is high; and 2) it is preferable that a resource policy is more conservative than is prescribed by a theory. In the latter conclusion, an overly conservative resource policy may result in dynamic inefficiency, but with updates in knowledge, the policy can be corrected, and the economy can be asymptotically efficient.

Besides the influence of disregarded inefficiencies, an indicator that is linked to CUC may not reflect sustainability, because, by construction, this indicator does not show the change in the long-term ability of the economy to maintain non-decreasing utility. However, the use of an indicator that shows this change may increase a long-term welfare at the cost of an unacceptable fall in the current utility, violating the principles of sustainability and intergenerational justice. Hence, various forms of indicators may complement each other in sustainability evaluation. Alternatively, another indicator that is consistent with a criterion of intergenerational justice at any moment in time can be constructed.

Despite all the difficulties with the use of indicator GI, the algorithm of its calculation undoubtedly provides useful information for policymakers by showing changes in factors that influence social welfare and sustainability. Apparently, the development of knowledge will further improve gathering this information and its use for sustainability policies.

Appendix A. Proof of Lemma 1

Since optimal paths are always efficient, a necessary condition of dynamic efficiency for economy (2)-(5) can be obtained from optimality conditions, e. g., in the problem of PV-maximization[30] of $\int_0^\infty U(C,D_2)\pi(t)dt$ with a discount factor $\pi(t)$. The Hamiltonian of this

---

[30] The maximin, formulated as $\max_{R,C} \int_0^\infty \overline{U}\delta e^{-\delta t}dt \equiv \overline{U} = const(R,C)$ with the additional constraint $U(C,D_2) = \overline{U}$, yields the same result (Leonard and Long (1992, 300–304)).

problem is $H = U(C, D_2)\pi(t) + \mu_K(F - C - \delta K - D_3) + \mu_S(D_4 - R)$,[31] and the Pontryagin-type necessary conditions are

$$H_C = U_C\pi(t) - \mu_K = 0, \tag{A.1}$$

$$H_R = \mu_K F_R - \mu_S = 0, \tag{A.2}$$

$$\dot{\mu}_K = -\partial H / \partial K = -\mu_K(F_K - \delta), \tag{A.3}$$

$$\begin{aligned} \dot{\mu}_S = {} & -\partial H / \partial S = -\{\pi(t)U_{D_2}\, \partial D_2 / \partial X \\ & + \mu_K(F_{D_1}\, \partial D_1 / \partial X - \partial D_3 / \partial X)\}\, \partial X / \partial S - \mu_S\, \partial D_4 / \partial S. \end{aligned} \tag{A.4}$$

Eq. (A.4) with μ_K from (A.1) becomes

$$\dot{\mu}_S = -\{\pi(t)U_{D_2}\frac{\partial D_2}{\partial X} + U_C\pi(t)(F_{D_1}\frac{\partial D_1}{\partial X} - \frac{\partial D_3}{\partial X})\}\frac{\partial X}{\partial S} - \mu_S\frac{\partial D_4}{\partial S}. \tag{A.5}$$

The time derivative of (A.2) is $\dot{\mu}_S = \dot{\mu}_K F_R + \mu_K \dot{F}_R$, which, combined with (A.5), gives

$$\dot{\mu}_K F_R + \mu_K \dot{F}_R = -\{\pi(t)[U_{D_2}\frac{\partial D_2}{\partial X} + U_C(F_{D_1}\frac{\partial D_1}{\partial X} - \frac{\partial D_3}{\partial X})]\}\frac{\partial X}{\partial S} - \mu_S\frac{\partial D_4}{\partial S}.$$

The last equation, after dividing through by F_R and substitutions for $\dot{\mu}_K$ from (A.3) and μ_S from (A.2), becomes

$$-\mu_K(F_K - \delta - \frac{\dot{F}_R}{F_R}) = -\{\frac{\pi(t)}{F_R}[U_{D_2}\frac{\partial D_2}{\partial X} + U_C(F_{D_1}\frac{\partial D_1}{\partial X} - \frac{\partial D_3}{\partial X})]\}\frac{\partial X}{\partial S} - \mu_K\frac{\partial D_4}{\partial S},$$

which, divided through by μ_K with substitution for $\mu_K = U_C\pi(t)$ from (A.1), yields $\dot{F}_R / F_R = F_K - \delta + \tau(\mathbf{D})$, where $\tau(\mathbf{D})$ is defined by (7)■

---

[31] Here and below, μ_K and μ_S are indexed dual variables for capital and resource stock unlike $H_C, H_R, U_C, U_{D_2}, F_K$, and F_R, which are the partial derivatives of H, U, and F.

Appendix B. Proof of Proposition 1

The proof follows the approach of Hamilton and Hartwick (2005, Proposition 1), which was first applied in Hartwick (1977). The differences are that the current proof uses: 1) utility as a measure of well-being (due to the distortion D_2); 2) a modified HR to substitute for $\dot{F}_R$ instead of the standard HR (1). Namely, differentiation of Eq. (4) using Eqs. (2) and (6) gives

$$\begin{aligned}\dot{C} &= \dot{K}F_K + \dot{R}F_R + \dot{D}_1 F_{D_1} - \delta\dot{K} - \ddot{K} - \dot{D}_3 = \dot{K}F_K + \dot{R}F_R + \dot{D}_1 F_{D_1} - \delta\dot{K} - \ddot{K} - \dot{D}_3 + R\dot{F}_R - R\dot{F}_R \\ &= (F_K - \delta)\dot{K} - (F_K - \delta + \tau)F_R R - \left[\ddot{K} - d(RF_R)/dt\right] + \dot{D}_1 F_{D_1} - \dot{D}_3.\end{aligned}$$

A planner uses the indicator GI for the undistorted model: $G^0 = \dot{K} - RF_R$. Then

$$\dot{C} = vG^0 - \dot{G}^0 + \dot{D}_1 F_{D_1} - \dot{D}_3 - \tau F_R R. \qquad \text{(B.1)}$$

By assumptions, $\dot{X} = R$ and $\dot{D}_i = \dot{X}\,\partial D_i / \partial X$, $i = 1,2,3.$ Then, by Lemma 1,

$$\tau F_R R = F_R R(\Psi_I - \partial D_4 / \partial S) - \{\dot{D}_1 F_{D_1} + \dot{D}_2 U_{D_2} / U_C - \dot{D}_3\}\,\partial X / \partial S.$$

Substitution of this expression into (B.1) results in

$$\dot{C} = vG^0 - \dot{G}^0 - F_R R(\Psi_I - \partial D_4 / \partial S) + (\dot{D}_1 F_{D_1} - \dot{D}_3)(1 + \partial X / \partial S) + (\dot{D}_2 U_{D_2} / U_C)\,\partial X / \partial S,$$

which, after substitution into $\dot{U} = U_C\dot{C} + U_{D_2}\dot{D}_2$, yields

$$\dot{U} = U_C[vG^0 - \dot{G}^0 - F_R R(\Psi_I - \partial D_4 / \partial S) - (\dot{D}_3 - \dot{D}_1 F_{D_1})(1 + \partial X / \partial S)] + \dot{D}_2 U_{D_2}(1 + \partial X / \partial S).$$

The use of the definition of Ψ_U results in Eq. (9) of the proposition.

When economy (2)-(5) is dynamically efficient ($\Psi_I = 0$) and the present value prices of utility, capital, and the resource are defined as $\pi(t), \mu_K,$ and μ_S in Lemma 1, Eq. (9) can be obtained from the result of Dixit et al. (1980, Theorem 1) or from a generalization of this result in Hamilton and Withagen (2007). Namely, these results claim that

$$\pi(t)\dot{U} = -\frac{d}{dt}\left(\mu_K\dot{K} + \mu_S\dot{S}\right),$$

which, using Eqs. (A.1)-(A.3), can be rewritten as follows:

$$\pi(t)\dot{U} = -\frac{d}{dt}\left[\mu_K\left(\dot{K} + \dot{S}\mu_S / \mu_K\right)\right] = -\frac{d}{dt}\left[\mu_K\left(\dot{K} + F_R\dot{S}\right)\right]$$
$$= -\frac{d}{dt}\left[\mu_K G\right] = -\mu_K\left(G\dot{\mu}_K / \mu_K + \dot{G}\right) = \mu_K\left[\left(F_K - \delta\right)G - \dot{G}\right].$$

Then, with the use of (A.1) and the notation $v := F_K - \delta,$ it becomes $\dot{U} = (v - \dot{G}/G)GU_C,$ which is Eq. (9) for $\Psi_U = 0$ ■

Acknowledgment. I am grateful to Geir Asheim and other participants of the 4th World Congress of Environmental and Resource Economists (Montreal, 2010) for very useful comments and advice.

References

1. Aronsson, Thomas and Karl-Gustaf Löfgren. 2010. An introduction to the theory of social accounting. In: *Handbook of Environmental Accounting*. Aronsson, Thomas and Karl-Gustaf Löfgren (eds). Edward Elgar, pp 1–27.
2. Arrow, Kenneth J., Partha Dasgupta, and Karl G. Mäler. 2003. Evaluating projects and assessing sustainable development in imperfect economies. *Environmental and Resource Economics* 26: 647–85.
3. Asheim, Geir, Wolfgang Buchholz, and Cees Withagen. 2003. The Hartwick Rule: Myths and Facts. *Environmental and Resource Economics* 25: 129–50.
4. Asheim, Geir. 2010. The relationship between welfare measures and indicators of sustainable development. In: *Handbook of Environmental Accounting*. Aronsson, Thomas and Karl-Gustaf Löfgren (eds). Edward Elgar, pp 237–56.
5. Bazhanov, Andrei. 2007. The transition to an oil contraction economy. *Ecological Economics* 64: 186–93.

6. Bazhanov, Andrei. 2008. Sustainable growth in a resource-based economy: the extraction-saving relationship. *MPRA paper* No. 23299.
7. Bazhanov, Andrei. 2010. Sustainable growth: compatibility between a plausible growth criterion and the initial state. *Resources Policy* 35: 116–25.
8. Bazhanov, Andrei. 2011. The dependence of the potential sustainability of a resource economy on the initial state: a comparison of models using the example of Russian oil extraction. *The Journal of the New Economic Association* 12: 1–20.
9. Bazhanov, Andrei. 2012. A closed-form solution to Stollery's problem with damage in utility. *Computational Economics* 39(4): 365–86.
10. Cairns, Robert D. 2008. Value and income. *Ecological Economics* 66: 417–24.
11. Dasgupta, Partha, and Geoffrey Heal. 1974. The optimal depletion of exhaustible resources. *Review of Economic Studies* 41: 3–28.
12. Dasgupta, Partha, and Geoffrey Heal. 1979. *Economic theory and exhaustible resources.* Cambridge University Press.
13. Dasgupta, Partha, and Karl G. Mäler. 2000. Net national product, wealth, and social well-being. *Environment and Development Economics* 5: 69–93.
14. D'Autume, Antoine, and Katheline Schubert. 2008. Hartwick's rule and maximin paths when the exhaustible resource has an amenity value. *Journal of Environmental Economics and Management* 56: 260–74.
15. D'Autume, Antoine, John M. Hartwick, and Katheline Schubert. 2010. The zero discounting and maximin optimal paths in a simple model of global warming. *Mathematical Social Sciences* 59: 193–207.

16. Dixit, Avinash, Peter Hammond, and Michael Hoel. 1980. On Hartwick's rule for regular maximin paths of capital accumulation and resource depletion. *Review of Economic Studies* 47: 551–56.

17. EU 2011. Regulation (EU) No 691/2011 of the European Parliament and of the Council of 6 July 2011 on European environmental economic accounts. *Official Journal of the European Union L 192* 54: 1–16.

18. Gaudet, Gerard. 2007. Natural resource economics under the rule of Hotelling. *Canadian Journal of Economics* 40(4): 1033–59.

19. Gaudet, Gerard, Michel Moreaux, and Cees Withagen. 2006. The Alberta dilemma: optimal sharing of a water resource by an agricultural and an oil sector. *Journal of Environmental Economics and Management* 52: 548–66.

20. Hamilton, Kirk. 2010. Genuine saving, social welfare and rules for sustainability. In: *Handbook of Environmental Accounting*. Aronsson, Thomas and Karl-Gustaf Löfgren (eds). Edward Elgar, pp 257–78.

21. Hamilton, Kirk, and Michael Clemens. 1999. Genuine savings rates in developing countries. *World Bank Economic Review* 13(2): 333–56.

22. Hamilton, Kirk, and John M. Hartwick. 2005. Investing exhaustible resource rents and the path of consumption. *Canadian Journal of Economics* 38: 615–21.

23. Hamilton, Kirk, Giovanni Ruta, and Liaila Tajibaeva. 2006. Capital accumulation and resource depletion: a Hartwick Rule counterfactual. *Environmental and Resource Economics* 34: 517–33.

24. Hamilton, Kirk, and Cees Withagen. 2007. Savings growth and the path of utility *Canadian Journal of Economics* 40: 703–13.

25. Hartwick, John M. 1977. Intergenerational equity and the investing of rents from exhaustible resources. *American Economic Review* 67: 972–974.

26. Hurwicz, Leonid 1960. Optimality and informational efficiency in resource allocation processes. In: *Mathematical Methods in the Social Sciences.* Arrow, Kenneth J., Samuel Karlin, and Patrick Suppes (eds). Stanford University Press, pp 27–46.

27. Leonard, Daniel, and Ngo V. Long. 1992. *Optimal control theory and static optimization in economics.* Cambridge University Press.

28. Martinet, Vincent. 2007. A step beside the maximin path: Can we sustain the economy by following Hartwick's investment rule? *Ecological Economics* 64: 103–08.

29. Pearce, David W., and Giles D. Atkinson. 1993. Capital theory and the measurement of sustainable development: an indicator of "weak" sustainability. *Ecological Economics* 8: 103–08.

30. Pezzey, John C. V. 1992. Sustainable development concepts: An economic analysis. *World Bank Environment Paper* No. 2. World Bank.

31. Pezzey, John C. V. 2004. One-sided sustainability tests with amenities, and changes in technology, trade and population. *Journal of Environmental Economics and Management*. 48: 613–31.

32. Proops, John L.R., Giles D. Atkinson, Burkhard Frhr. v. Schlotheim BFv, and Sandrine Simon. 1999. International trade and the sustainability footprint: a practical criterion for its assessment. *Ecological Economics* 28: 75–97.

33. Solow, Robert M. 1974. Intergenerational equity and exhaustible resources. *Review of Economic Studies* 41: 29–45.

34. Solow, Robert M. 1993. An almost practical step toward sustainability. *Resources Policy* 19: 162–72.

35. Stiglitz, Joseph E. 1991. The invisible hand and modern welfare economics. NBER Working Paper No. 3641.

36. Stollery, Kenneth R. 1998. Constant utility paths and irreversible global warming. *Canadian Journal of Economics* 31: 730–42.

37. UN 1993. *Handbook of National Accounting: Integrated Environmental and Economic Accounting*. Series F, No. 61. United Nations.

38. van der Ploeg, Frederick 2011. Rapacious resource depletion, excessive investment and insecure property rights: a puzzle. *Environmental and Resource Economics* 48: 105–28.