Comparisons and Characterizations of the Mean-Variance, Mean-VaR, Mean-CVaR Models for Portfolio Selection With Background Risk

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Abstract: This paper investigates the impact of background risk on an investor's portfolio choice in a mean-VaR, mean-CVaR and mean-variance framework, and analyzes the characterizations of the mean-variance boundary and mean-VaR efficient frontier in the presence of background risk. We also consider the case with a risk-free security.

Keywords: Background risk; Portfolio selection; VaR; CVaR

1 Introduction

Das et al. (2010) develop a model to incorporate features of both behavioral and mean-variance models by assuming that investors only faces portfolio risk. However, Jiang et al. (2010) and others find that when investors select their portfolios of financial assets, they face not only portfolio risk but also background risk.

Classical portfolio theory (Markowitz, 1952; Merton, 1969, 1971; Samuelson, 1969) do not include background risk because the market is assumed to be complete. Campbell (2006) shows that standard portfolio theory fails to explain household investment decisions in practice. To circumvent this limitation of the classical portfolio theory, academics introduce background risk in the study of portfolio compositions. For example, Rosen and Wu (2004), Berkowitz and Qiu (2006), Edwards (2008), and Fan and Zhao (2009) find that there are strong cross-sectional correlations between health and both financial and non-financial assets.

Cocco (2005) analyzes the impact of the housing investment on the composition of an investor's portfolio, and concludes that the investment in housing plays an important role in asset accumulation and in portfolio choice among stocks and Treasury bills. Lusk and Coble (2008) find that investors are more risk-averse in the presence of background risk than they are in the absence of background risk. Fan and Zhao (2009) document that there are strong cross-sectional correlations between health and both financial and non-financial assets, and that adverse health shocks discourage risky asset holdings. Cocco (2005) and Pelizzon and Weber (2009) analyze the impact of the housing investment on the composition of an investor's portfolio, and conclude that the investment in housing plays an important role in asset accumulation and in portfolio choice among financial assets.

Heaton and Lucas (2000), Viceira (2001), and others investigates the relation between labour income variations and investors' portfolio decisions and confirms the relevance of labour income risk to asset allocations. Using Australian data, Cardak and Wilkins (2009) further demonstrate that risky asset holdings are discouraged by both labour income risk and health risk. In addition, Shum and Faig's (2006) analysis of household stock holdings considers entrepreneurial risk as well as the impact of real estate investments, labour income, and other factors. Alghalith, et al. (2012) present two dynamic models of background risk. They first present a stochastic factor model with an additive background risk, and thereafter, present a dynamic model of simultaneous (correlated) multiplicative background risk and additive background risk. Guo, et al. (2013) investigate the impact of multiplicative background risk on an investor's portfolio choice in a mean-variance framework. They also study the efficient boundary frontiers with and without risk-free security.

Another area of study is to the mean-variance framework. For instance, Lajeri-Chaherli (2002) proves that proper risk aversion is equivalent to both quasi-concavity of a mean-variance utility function and DARA. Eichner and Wagener (2003) define the concept of variance vulnerability to characterize the property that an agent with mean-standard deviation preferences reduces his/her risky activities when facing an increase in the variance of an independent background risk. They derive the necessary and sufficient conditions for variance vulnerability, and provide connections between these mean-variance properties and those for risk vulnerability within the EU framework. Eichner (2008) transfers the concept of risk vulnerability into mean-variance preferences, and shows that risk vulnerability is equivalent to the slope of the mean-variance indifference curve being decreasing in mean and increasing in variance. Eichner and Wagener (2009) document the comparative statics with both an endogenous risk and a background risk for an agent with mean-variance preferences in a generic decision model, and confirm that the agent becomes less risk-averse in response to an increase in the expected value of the background risk or a decrease in its variability if the preferences exhibit DARA or variance vulnerability. On the other hand, Baptista (2008) explores optimal delegated portfolio management with background risk and provides conditions under which investors delegate their wealth to portfolio managers with mean and tracking error variance functions.

This paper investigates the impact of background risk on an investor's portfolio choice in a mean-VaR, mean-CVaR and mean-variance framework, and analyzes the characterizations of the mean-variance boundary and mean-VaR efficient frontier in the presence of background risk. We also consider the case with a risk-free security.

2 Mean-VaR/CVaR/variance Boundaries and Efficient Frontiers with Background Risk but without Risk-Free Security

We first assume no risk-free security and the return vector to be $r = (r_1, r_2, \cdots, r_n)^\tau$. We let $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^\tau$ represent a portfolio in which ω_i is the proportion (weight) of the portfolio invested in asset i with a positive (negative) weight represents a long (short) position and $\sum_{i=1}^n \omega_i = 1$. Thus, the return of the portfolio is $r_p = \omega^\tau r$. We denote r_b to be the return of the background asset. Then, the mean and variance of total return r_ω

$$r_\omega = \omega^\tau r + r_b \tag{2.1}$$

are given by $E(r_\omega) = \omega^\tau E(r) + E(r_b)$ and $\sigma^2(r_\omega) = \omega^\tau V \omega + 2\omega^\tau \mathrm{Cov}(r, r_b) + \mathrm{Var}(r_b)$ where V is non-singular.

We state the definition of VaR as follows:

Definition 2.1 *The VaR at the $100t\%$ confidence level of a risky portfolio for a specific time period is the rate of return $V[t, r_\omega]$ such that the probability of that portfolio having a rate of return of $-V[t, r_\omega]$ or less is $1 - t$. In other words, the VaR of the portfolio ω's return at the $100t\%$ confidence level is*

$$V[t, r_\omega] = -F_\omega^{-1}(1 - t) \ , \tag{2.2}$$

where $F_\omega(\cdot)$ is the cumulative distribution function of r_ω.

We state the definition of CVaR as follows:

Definition 2.2 *A portfolio's CVaR is the loss one expects to suffer at that confidence level by holding it over the investment period, given that the loss is equal to or larger than its VaR. Formally, the CVaR of the portfolio ω's return at the $100t\%$ confidence level is*

$$L[t, r_\omega] = -E\{r_\omega | r_\omega \le -V[t, r_\omega]\} \ . \tag{2.3}$$

2.1 Mean-Variance, Mean-VaR, Mean-CVaR Boundaries with Background Risk

For any $\bar{E} \in R$, we let $W(\bar{E}) = \{\omega \in W : E[r_\omega] = \bar{E}\}$ be the set of portfolios with expected return equal to $\bar{E}$ in which W is the set of portfolios. The definitions of mean-variance, mean-VaR, and mean-CVaR boundaries with background risk can then be defined as follows:

Definition 2.3 *A portfolio $\bar{\omega} \in W(\bar{E})$ is on the mean-variance boundary with background risk if and only if for $\bar{E} \in R$, $\bar{\omega}$ is the solution of solving $\min_{\omega \in W(\bar{E})} \sigma_\omega^2$ where σ_ω^2 is defined in (2.1).*

Definition 2.4 *A portfolio $\bar{\omega} \in W(\bar{E})$ is on to the mean-VaR boundary with background risk if and only if for $\bar{E} \in R$, $\bar{\omega}$ is the solution of solving $\min_{\omega \in W(\bar{E})} V[t, r_\omega]$ where $V[t, r_\omega]$ is defined in (2.2).*

Definition 2.5 *A portfolio $\bar{\omega} \in W(\bar{E})$ is on the mean-CVaR boundary with background risk if and only if for $\bar{E} \in R$, $\bar{\omega}$ is the solution of solving $\min_{\omega \in W(\bar{E})} L[t, r_\omega]$ where $L[t, r_\omega]$ is defined in (2.3).*

For the mean-variance boundary with background risk, we have the following proposition.

Proposition 2.1 *Portfolio ω is on the mean-variance boundary with background risk if and only if*

$$\frac{\sigma_\omega^2}{a} - \frac{(E(r_\omega) - E(r_b) - (A - EC + FA/C)^2}{Da/C} = 1 \ , \tag{2.4}$$

where $A = I^\tau V^{-1} E(r)$, $B = E(r)^\tau V^{-1} E(r)$, $C = I^\tau V^{-1} I$, $D = BC - A^2$, $E = \mathrm{Cov}(r, r_b)^\tau V^{-1} E(r)$, $F = \mathrm{Cov}(r, r_b)^\tau V^{-1} I$, $a = (1+F)^2/C - \mathrm{Cov}(r, r_b)^\tau V^{-1} \mathrm{Cov}(r, r_b) + \mathrm{Var}(r_b)$.

When the return of the background risk r_b is independent with the return of the financial assets, we have the following corollary:

Corollary 2.1 *When r_b is independent of r, portfolio ω is on the mean-variance boundary with background risk if and only if*

$$\frac{\sigma_\omega^2}{a} - \frac{(E(r_\omega) - E(r_b) - A/C)^2}{Da/C} = 1 \ , \tag{2.5}$$

where $A = I^\tau V^{-1}E(r)$, $B = E(r)^\tau V^{-1}E(r)$, $C = I^\tau V^{-1}I$, $D = BC - A^2$, $a = 1/C + \mathrm{Var}(r_b)$.

2.2 Mean-Variance, Mean-VaR, and Mean-CVaR Efficient Frontiers with Background Risk

We first provide the notions of efficiency associated with the mean-variance, mean-VaR, and mean-CVaR boundaries as shown in the following definitions:

Definition 2.6 *A portfolio* $\omega \in W$ *is on the mean-variance efficient frontier with background risk if and only if there is no portfolio* $\nu \in W$ *such that* $E(r_\nu) \geq E(r_\omega)$ *and* $\sigma(r_\nu) \leq \sigma(r_\omega)$ *with at least one of the inequalities holds strictly where* r_u *and* $\sigma(r_u)$ *are defined in (2.2) with* $u = \nu$ *or* ω.

Definition 2.7 *A portfolio* $\omega \in W$ *is on the mean-VaR efficient frontier with background risk if and only if there is no portfolio* $\nu \in W$ *such that* $E(r_\nu) \geq E(r_\omega)$ *and* $V[t, r_\nu] \leq V[t, r_\omega]$*, with at least one of the inequalities holds strictly where* r_u *is defined in (2.1) and* $V[t, r_u]$ *is defined in (2.2) with* $u = \nu$ *or* ω.

Definition 2.8 *A portfolio* $\omega \in W$ *is on the mean-CVaR efficient frontier with background risk if and only if there is no portfolio* $\nu \in W$ *such that* $E(r_\nu) \geq E(r_\omega)$ *and* $L[t, r_\nu] \leq L[t, r_\omega]$ *with at least one of the inequalities holds strictly where* r_u *is defined in (2.1) and* $V[t, r_u]$ *is defined in (2.3) with* $u = \nu$ *or* ω.

2.2.1 Characterizations of the Minimum VaR and Minimum CVaR Portfolios.

We begin by characterizing the minimum VaR portfolio with background risk.

Proposition 2.2

1. *If the minimum VaR portfolio exists, then it is both mean-variance and mean-CVaR efficient.*

2. *If the minimum CVaR portfolio exists, then it is mean-variance efficient.*

Assuming that both global minimum VaR portfolio and global minimum variance portfolio exist, we let $\omega_{V(t)} \in W$ denote the global minimum VaR portfolio and $\omega_\sigma \in W$ denote the global minimum variance portfolio at the $100t\%$ confidence level. We first establish the following proposition to describe the condition for the existence of the minimum VaR portfolio.

Proposition 2.3

1. *The minimum VaR portfolio exists if and only if* $z_t > \sqrt{D/C}$.

2. *Furthermore, if* $z_t > \sqrt{D/C}$, *then*

$$E(r_{\omega_{V(t)}}) = E(r_b) + \frac{A - EC + FA}{C} + \sqrt{\frac{D}{C}\Big(\frac{aCz_t^2}{Cz_t^2 - D} - a\Big)}.$$

From Proposition 2.3, we establish the following corollary:

Corollary 2.2

1. *If the minimum VaR portfolio exists, then* $E[r_{\omega_{V(t)}}] > E[r_{\omega_{L(t)}}]$, *and*

2. *if the minimum CVaR portfolio exists, then* $E[r_{\omega_{L(t)}}] > E[r_{\omega_\sigma}]$.

The above result infers that the minimum VaR portfolio lies above the minimum CVaR portfolio which, in turn, lies above the minimum variance portfolio on the mean-variance efficient frontier.

Corollary 2.3 *At any confidence level* $t < 1$,

1. *the minimum variance portfolio is mean-VaR inefficient,*

2. *the minimum variance portfolio is mean-CVaR inefficient, and*

3. *the minimum CVaR portfolio is mean-VaR inefficient.*

2.2.2 Characterization of Mean-VaR and Mean-CVaR Efficiency

We first state the characterization of mean-VaR efficiency with background risk in the following proposition:

Proposition 2.4

1. *If* $z_t > \sqrt{D/C}$*, then a portfolio* ω *is mean-VaR efficient if and only if it is on the mean-VaR boundary and* $E[r_\omega] \geq E[r_{\omega_{v(t)}}]$.

2. *If* $z_t \leq \sqrt{D/C}$*, then there is no mean-VaR efficient portfolio.*

We note that one could obtain the result for the mean-CVaR efficient frontier from Proposition 2.4 when z_t and $\omega_{v(t)}$ are replaced by k_t and $\omega_{L(t)}$, respectively. We state this result in the following corollary:

Corollary 2.4

1. *If* $k_t > \sqrt{D/C}$*, then a portfolio* ω *is mean-CVaR efficient if and only if it is on the mean-CVaR boundary and* $E[r_\omega] \geq E[r_{\omega_{L(t)}}]$.

2. *If* $k_t \leq \sqrt{D/C}$*, then there is no mean-CVaR efficient portfolio.*

From Propositions 2.3 and 2.4, one could also obtain the following corollary easily to compare the mean-variance, mean-VaR, and mean-CVaR efficient frontiers with background risk.

Corollary 2.5

1. *If* $k_t \leq \sqrt{D/C}$*, then both mean-VaR and mean-CVaR efficient frontiers are empty.*

2. *If* $z_t \leq \sqrt{D/C} < k_t$*, then the mean-VaR efficient frontier is empty but the mean-CVaR efficient frontier is a nonempty proper subset of the mean-variance efficient frontier.*

3. *If* $z_t > \sqrt{D/C}$*, then a portfolio is on the mean-VaR efficient frontier if and only if it is on the mean-CVaR efficient frontier and* $E[r_\omega] \geq E[r_{\omega_{v(t)}}]$*; that is, the mean-VaR efficient frontier is a nonempty proper subset of the mean-CVaR efficient frontier.*

3 Adding a Risk-free Security

3.1 Adding a Risk-free Lending but No Borrowing

Now, we turn to develop the theory by assuming that there is risk-free security with rate of return $r_f \geq 0$ at which agents can lend but cannot borrow. Let $W_f = \{(\omega, \omega_f) \in$

$R^n \times R : \sum_{j=1}^n \omega_j + \omega_f = 1\}$, the mean and variance of total return

$$r_\omega = \omega_f r_f + \omega^\tau r + r_b \tag{3.1}$$

become $E(r_\omega) = \omega_f r_f + \omega^\tau E(r) + E(r_b)$ and $\sigma^2(r_\omega) = \omega^\tau V \omega + 2\omega^\tau \mathrm{Cov}(r, r_b) + \mathrm{Var}(r_b)$.

We assume that the tangency portfolio associated with the risk-free lending rate, denoted by w_1, lies above the minimum variance portfolio in the absence of the risk-free security. We obtain the following proposition:

Proposition 3.1 *Portfolio ω is on the mean-variance boundary with both background risk and risk-free security if and only if*

$$\begin{aligned} &\frac{\sigma_\omega^2}{a} - \frac{(E(r_\omega) - E(r_b) - (A - EC + FA/C)^2}{Da/C} = 1 \quad \text{if} \quad E(r_\omega) > E(r_{\omega_1}) \ , \\ &\frac{\sigma_\omega^2}{a^*} - \frac{(E(r_\omega) - E(r_b) - (r_f + r_f F - E))^2}{Ha^*} = 1 \quad \text{if} \quad E(r_\omega) < E(r_{\omega_1}) \ , \end{aligned} \tag{3.2}$$

where $H = B - 2r_f A + r_f^2 C$ and $a^* = -\mathrm{Cov}(r, r_b)^\tau V^{-1} \mathrm{Cov}(r, r_b) + \mathrm{Var}(r_b) > 0$.

From Proposition 3.1, we obtain the following corollary:

Corollary 3.1 *When there is an additive background risk, the variance of the minimum variance portfolio without risk-free security is larger than that with risk-free security.*

The following proposition states the conditions needed for the existence of the minimum VaR portfolio when there is a risk-free security.

Proposition 3.2

1. *The minimum VaR portfolio exists if and only if $z_t > \sqrt{H}$.*

2. *Furthermore, if $z_t > \sqrt{H}$; then*

$$E(r_{\omega_{V(t)}}) = E(r_b) + (r_f + r_f F - E) + \sqrt{H\Big(\frac{a^* z_t^2}{z_t^2 - H} - a^*\Big)}.$$

From applying Proposition 3.2, we establish have the following two corollaries:

Corollary 3.2

1. *If the minimum VaR portfolio exists, then $E[r_{\omega_{V(t)}}] > E[r_{\omega_{L(t)}}]$, and*

2. *if the minimum CVaR portfolio exists, then $E[r_{\omega_{L(t)}}] > E[r_{\omega_\sigma}]$.*

Corollary 3.3 *At any confidence level $t < 1$,*

1. *the minimum variance portfolio is mean-VaR inefficient,*
2. *the minimum variance portfolio is mean-CVaR inefficient, and*
3. *the minimum CVaR portfolio is mean-VaR inefficient.*

The following is a characterization of mean-VaR efficiency when there is a risk-free security.

Proposition 3.3

1. *If $z_t > \sqrt{H}$, then a portfolio ω is mean-VaR efficient if and only if it is on the mean-VaR boundary and $E[r_\omega] \geq E[r_{\omega_{v(t)}}]$.*
2. *If $z_t \leq \sqrt{H}$, then no mean-VaR efficient portfolio exists.*

We note that a result similar to Proposition holds for the mean-CVaR efficient frontier when z_t and $\omega_{v(t)}$ are replaced by k_t and $\omega_{L(t)}$, respectively as shown in the following corollary:

Corollary 3.4

1. *If $k_t > \sqrt{H}$, then a portfolio ω is mean-CVaR efficient if and only if it is on the mean-CVaR boundary and $E[r_\omega] \geq E[r_{\omega_{L(t)}}]$.*
2. *If $k_t \leq \sqrt{H}$, then there is no mean-CVaR efficient portfolio.*

In addition, similar to Corollary 2.5, we establish the following corollary to compare the mean-variance, mean-VaR, mean-CVaR efficient frontier with both background risk and risk-free security:

Corollary 3.5

1. *If $k_t \leq \sqrt{H}$, then both mean-VaR and mean-CVaR efficient frontiers are empty.*
2. *If $z_t \leq \sqrt{H} < k_t$, then the mean-VaR efficient frontier is empty but the mean-CVaR efficient frontier is a nonempty proper subset of the mean-variance efficient frontier.*
3. *If $z_t > \sqrt{H}$, then a portfolio is on the mean-VaR efficient frontier if and only if it is on the mean-CVaR efficient frontier and $E[r_\omega] \geq E[r_{\omega_{v(t)}}]$; that is, the mean-VaR efficient frontier is a nonempty proper subset of the mean-CVaR efficient frontier.*

3.2 Allowing for Both Risk-Free Lending and Borrowing

Suppose now that both risk-free lending and borrowing are allowed and that the borrowing rate r_{fb} is higher than the risk-free lending rate r_{fl}. The set of portfolios with well-defined expected rates of return is then given by letting $W_f = \{(\omega, \omega_{fl}, \omega_{fb}) \in R^n \times R_+ \times R_- : \sum_{j=1}^n \omega_j + \omega_{fl} + \omega_{fb} = 1\}$, where ω_{fl} and ω_{fb} are the proportion of wealth lend and borrowed at r_{fl} and r_{fb}. Assuming that the tangency portfolio associated with the risk-free borrowing rate, denoted by ω_2, lies above the minimum variance portfolio in the absence of the risk-free security, we obtain the following proposition:

Proposition 3.4 *Portfolio ω is on the mean-variance boundary with both background risk and risk-free security if and only if*

$$\begin{aligned}
&\frac{\sigma_\omega^2}{a^*} - \frac{(E(r_\omega) - E(r_b) - (r_{fb} + r_{fb}F - E))^2}{H_2 a^*} = 1 \quad \text{if} E(r_\omega) > E(r_{\omega_2}) \ , \\
&\frac{\sigma_\omega^2}{a} - \frac{(E(r_\omega) - E(r_b) - (A - EC + FA/C)^2}{Da/C} = 1 \quad \text{if} E(r_{\omega_2}) > E(r_\omega) > E(r_{\omega_1}) \ , \\
&\frac{\sigma_\omega^2}{a^*} - \frac{(E(r_\omega) - E(r_b) - (r_{fl} + r_{fl}F - E))^2}{H_1 a^*} = 1 \quad \text{if} E(r_\omega) < E(r_{\omega_1}) \ ,
\end{aligned}$$

where $H_1 = B - 2r_{fl}A + r_{fl}^2 C$ and $H_2 = B - 2r_{fb}A + r_{fb}^2 C$.

4 Conclusion

This paper investigates the impact of background risk on an investor's portfolio choice in a mean-VaR, mean-CVaR and mean-variance framework, and analyzes the characterizations of the mean-variance boundary and mean-VaR efficient frontier in the presence of background risk. We also consider the case with a risk-free security.

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