Selling a Cost Reducing Production Technique through Auction in a Duopolistic Industry

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Abstract

This paper considers a two-stage game, where in the first stage, two firms bid non-cooperatively for a production technique that leads to a reduction in cost. Following the auction in the second stage of the game these firms compete against each other in a duopolistic industry. The amount of cost reduction for every firm following the adoption of the production technique is a private information to the concerned firm. In the model, the auctioneer is the government. Before the auction, the government announces whether she will reveal the bids after the auction, which is her choice variable. This paper makes an attempt to figure out the welfare implications of the bid disclosure policies under different parametric and market conditions. Our findings suggest that for the Bertrand competition in the second stage the revelation of the bids does not have any impact on the level of social welfare. For the Cournot competition in the second stage, whether the disclosure of bids would lead to higher level of social welfare than when the bids are suppressed, is determined by parametric conditions.

Keywords: Auction, Market Structure, Oligopoly, Imperfect Competition
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1 Introduction

The sale of licenses in order to allow certain firms in an industry to use any particular production technique is a common phenomenon. The analysis of licensing of a particular technique is also quite common in economics literature. Kamien, Oren and Tauman (1992) discuss the licensing of a cost reducing innovation to an oligopolistic industry. They compare auctioning of a fixed number of licenses to a fixed license fee and to a per unit royalty in terms of the patentee's profit, licensees' profits, industry structure and product price. Their results suggest that auctioning licenses is the optimal strategy when the magnitude of innovation is not too small (although it does not hold for an arbitrary innovation). Cramton and Kerr (2002) suggest that an auction of carbon permits is the best way to achieve carbon caps set by international negotiation to limit global climate change.

With this backdrop we can think of a situation where there is an oligopolistic industry, in which the firms can use some production technique and only a limited number of licenses for using this technique are available. Therefore, only a limited number of firms can be permitted to use such a technique. If we consider the government to have the objective of maximizing social welfare and to be the authority that can sell these licenses, then the concerned licenses can actually be allocated through auction, with the government being the auctioneer. For example we can consider the auction of spectrum licenses in the United Kingdom. In the year 2000, the UK Government conducted an auction selling five licenses for different frequencies. There had been a larger number of bidders than the number of available licenses. Another example where the government sells a resource which is "limited" in amount through auction is that of land auction.

In this paper we look at a situation where the government is selling a limited number of licenses for a cost reducing production technique (due to its limited availability). We consider a two-stage game where, in the first stage, two firms bid non-cooperatively for the production technique that leads to a reduction in cost and the government offers to sell a single license for this technique through auction. Following the auction, in the second stage of the game, these firms compete against each other in an oligopolistic industry. The amount of cost reduction for every firm following the adoption of the cost reducing production technique is a private information to the concerned firm. Based on this private information, each firm submits a bid in the first stage auction. Therefore, the amount of cost reduction for each firm subject to the adoption of the technique constitutes its type. We consider a symmetric framework where the types are independently and identically distributed over the same interval and involving the same distribution and density functions. Here, the government is the auctioneer and its objective is to maximise social welfare.[1] Before the auction the government decides and announces whether to disclose or suppress the bids. This paper makes an attempt to figure out the welfare implications of the bid disclosure policies under different

[1] The Social welfare function has been explicitly specified in the subsequent analysis.

parametric conditions, when the oligopolistic competition in the second stage of the game is first of a Cournot type and then of a Bertrand type.

Many analysis of similar two-stage games are available in the existing literature. We discuss some of them, which are somewhat close to our work, here. Goeree (2003) considers a mature market in which competitors know one another's costs. The prevalent mode changes with the auction of a cost reducing innovation (which is protected by a patent) where the winner is granted the license to use the innovation. The competition in the second stage of the game is first of a Cournot type and then of a Bertrand type. The results of this paper capture the impact of the prevailing mode of competition in the aftermarket on the bidding behaviour in first and second price as well as English auctions. Two major findings of this paper are that first, the strategic equivalence between the English and the second price auctions breaks down and second, the revenue equivalence breaks down due to signaling effects.

Das Varma (2003) also analyzes a similar two-stage game and examines whether a first-price sealed-bid auction is efficient in allocating a process innovation amongst oligopolists engaged in either Cournot or Bertrand competition. The results he has derived ensure the existence of a unique equilibrium in strictly increasing strategies which is symmetric, and therefore allocatively efficient for the Cournot type competition but not for a Bertrand type competition in the in the downstream oligopolistic market. When the downstream market becomes perfectly competitive, the efficiency in allocation is restored. Thus the results in this paper capture the sensitivity of "efficient auction design to downstream market structures".

Molnar and Virag (2008) also consider a two-stage game that studies the effect of post-auction market interaction and asymmetric information on the design of a revenue maximizing mechanism. Here the bidders can have incentives to signal their types to influence the outcome of the post-auction market game. The results show that revealing all information about the winner is revenue-maximizing if the post-auction competition is Cournot and revealing no information about the winner is revenue-maximizing if the post-auction competition is Bertrand. Their paper also shows that if bidders have additively separable utility functions in their true and signaled types, then revealing information is revenue maximizing if the utility function is convex in the signaled type, while hiding information is revenue maximizing if the utility function is concave.

Katzman and Rhodes-Kroph (2008) discuss a two-stage game where the first stage involves an auction of right to enter production in a market with an existing monopolist and in the second stage, the winner of the first stage auction competes with the existing so far monopolist firm. So, in the first stage, they consider a set of firms who are competing against each other. The marginal cost of each firm is a private information and constitutes its type. Based on its type every firm submits a bid in the auction. The bids are decreasing functions of types. The types are assumed to be distributed over the same interval following the same distribution function for all participating firms. In this context after the first stage the auctioneer can choose to announce the bids completely, partially or not at all. The results suggest that depending on whether post auction profits bear a positive or inverse relation to rival beliefs, the firms either bid more aggressively or signal poorer types. So when post auction profits are negatively related to rival beliefs, announcement of bids leads to higher expected revenue to the auctioneer. For the opposite case, suppression of bids leads to higher expected revenue. Their model, however, didn't make any general statement regarding the welfare effects brought about by different announcement policies.

Scarpatetti and Wasser (2009) present a model involving a two-stage game where in the first stage, there is a multi-unit auction of a cost reducing technological innovation and in the following stage this innovation is used by firms in an oligopolistic market structure where each firm has a private information about its cost structure. The number of units of the cost reducing innovation is less than the number of competing firms. After the first stage auction, only the winning bids or all the bids can be revealed publicly which creates an opportunity for signaling. The existence of a truthful equilibrium in the auction, in which the bids truly represent the costs, depends on the type and aggressiveness of competition in the second stage, the chosen auction format and the policy regarding bid announcement. Their results suggest that, the existence of a separating equilibrium, however, depends on several factors, including the type of auction format chosen. The announcement of all bids for a discriminatory or a uniform-price auction poses the problem of non-credible signals becoming more grave, so that existence of separating equilibria is possible "*in the special case where the auction has only one loser*". The existence of a separating equilibrium becomes uncertain when there is Bertrand competition in the second stage. "The weight of the signaling incentive" gets reduced with the revelation of less information which makes the existence of a separating equilibrium under Bertrand competition possible. A separating equilibrium is likely to exist when there is only one winner in the discriminatory auction (i.e., if it is a first-price auction) and if only this winning bid is disclosed. The same result holds for all uniform-price auctions where only the highest losing bid is announced.

Our paper also concerns a two-stage game involving the auction of a cost-reducing technological innovation in the first stage and a duopolistic market structure in the second stage. We, however, have concerned ourselves with the social welfare implications rather than revenue maximization. We first consider first price sealed bid auction and then a second price sealed bid as well as an all-pay auctions of a single license for a cost reducing production technique in the first stage[2]. This paper analyzes a situation involving two firms in the second stage (who are bidding for the technological innovation in the first stage auction), first under a Cournot, and then under a

[2]Other auction formats e.g. All Pay Auction, Second Price Sealed Bid Auction etc, will not, however, change the parametric conditions of the results substantially.

Bertrand competition. For both the competitions we study and compare the levels of social welfare under different bid revelation policies. So this paper focuses on three key questions:

1. Generally it is observed that the bids are revealed after the auctions. We here try to see whether under any circumstances the suppression of the bids can be regarded optimal when the objective of the government is social welfare maximization.
2. We also try to check whether the optimality of suppression or revelation of bids is anyhow dependent on the types of the bidding firms and if so, how.
3. We finally try to figure out whether the type of competition prevailing in the market in the second stage has any impact on the choice of the bid disclosure policy.

We are, however, not considering how the government knows before the auction whether to reveal or suppress the bids. Therefore, the questions we are addressing in this paper are concerned with the strategic aspect of the problem[3]. The paper is organized as follows: section 2 specifies the structure of the model, section 3 and section 4 elaborate the cases of Cournot and Bertrand competitions respectively, and finally section 5 concludes the paper.

2 The Model: Structure

In this section we specify the structure of the model about the two-stage game, where an auction takes place in the first stage followed by a oligopolistic competition in the second stage.

There are two firms in the second stage oligopolistic industry. These firms are also the bidders in the first stage auction. The relative value of the innovation in each firm's technology is assumed to be private information to that firm in our paper. We parametrize this private information for firm i by the random variable $\hat{\theta}_i$ such that $\hat{\theta}_i \in [\underline{\theta}, \overline{\theta}]$ with $0 < \underline{\theta} < \overline{\theta}$. Thus $\hat{\theta}_i$ constitutes type of firm i here, where $i = 1, 2$. $\hat{\theta}_i$ is assumed to be distributed according to the continuously differentiable distribution function $F(.)$ with the corresponding density function $f(.)$ having full support. The distribution of each firm's private information is thus assumed to be symmetric. Furthermore, we assume that $\hat{\theta}_W$ and $\hat{\theta}_L$ (where W and L stand for the winning and the losing firms respectively) are independent random variables. $F(.)$, $\underline{\theta}$ and $\overline{\theta}$ are assumed to be common knowledge among the firms. The realization of the random variable $\hat{\theta}_i$, denoted by θ_i is therefore private information to firm i.

[3] It would be an interesting exercise to design a mechanism such that the government can optimally decide whether to reveal or suppress the bids after the first stage auction ex ante, which constitutes a separate research question that we are not addressing here.

Now consider the duopolistic industry with a downward sloping linear inverse demand function $P = a - bQ$, where P is the price, Q is the aggregate output and a, $b > 0$. The two firms are playing either a Cournot or a Bertrand game in the industry[4]. The cost function of the firms are identical and equal to $cq_i \quad \forall i = W, L$, where q_i is output produced by the i^{th} firm and $c > 0$ and constant, when no cost reducing production technique is available. On the other hand, when such a technique is available, the marginal costs are given by

$$c_i = \begin{cases} c - \theta_i & if\ i\ wins\ the\ auction \\ c & if\ i\ loses\ the\ auction \end{cases}$$

where $\underline{\theta} < \overline{\theta} < c < c + \overline{\theta} < a$[5]. The objective of both the firms is to maximize her own expected profit.

We assume that the government is selling the cost reducing production technique by conducting a first price sealed bid auction. The objective of the government is to maximize the social welfare, where social welfare is defined as the sum of consumer surplus, producer surplus, government surplus[6]. The government announces the bid disclosure policy before the auction takes place, so that while participating in the auction the firms already know whether the bids after the auction will be announced or not. In the whole analysis we are only interested in symmetric and increasing equilibrium bidding strategies in the auction.

Finally, we assume $\forall\theta_W \ \frac{dq_W^*}{d\theta_W} > 0$ with q_W being the output of firm W and θ_W being her type[7].

3 Case of Cournot Competition

3.1 Industry with undisclosed bids

Here we consider the case where the government will not disclose any bids. So, before the auction firms know that after the auction government will not announce any bids. We will begin our

[4]We are implicitly assuming that the type of the competition in the market will not change after the auction.

[5]The last inequality ensures that the innovation is not so drastic as to make the winning bidder the sole producer in the market.

[6]Government surplus is the amount of revenue that the government receives after the auction. Because we are considering first price sealed bid auction, the government surplus is nothing but the bid of the winning bidder.

[7]To motivate this assumption, note that higher type means lower marginal cost if the firm wins the auction. This assumption implies that in such a situation the firm should produce higher output in equilibrium in case of undisclosed bids. Note that in case of fully disclosed bids $\frac{dq_W^*}{dc} < 0$ holds always. So, in case of fully disclosed bids this assumption holds. Also reduction in marginal cost implies that the firm becomes more efficient if she wins the auction, so from that point of view also it is (generally) better for the firm to exploit the efficiency and produce more.

analysis from stage 2. Let us assume that in stage 1 both the firms bid according to their true types. Since the bids are not disclosed, the only information that the winning bidder W possesses about the losing bidder L is that the bid submitted by L is lower than that submitted by W. Now, since we are concerned about the symmetric equilibrium where each bidder bids truthfully, such that the bids are increasing functions of the types, therefore inverting the bids the types of the bidders can be calculated. Thus the lower bid would certainly imply a lower type. So, after the auction the winning firm knows that $\theta_L \in [\underline{\theta}, \theta_W]$. Similarly, the losing firm knows that $\theta_W \in \left[\theta_L, \overline{\theta}\right]$.

3.1.1 Characteristics of the equilibrium outputs and profits of the two firms

In stage 2 the objective of all the firms is to maximize her own expected profit by choosing an output level where the demand function of the industry is given in the equation $a - b(q_W + q_L)$ with q_W being the output of firm W. Assuming that both the firms bid truthfully in the first stage auction[8], the expected profit function of the winning firm is given below

$$
\begin{aligned}
\Pi_W &= \int_{\underline{\theta}}^{\theta_W} \left\{ [a - c + \theta_W] \, q_W - b \left(q_W + q_L \left(\theta_L \right) \right) q_W \right\} dF \left(\theta_L \left| \theta_L < \theta_W \right. \right) \\
&= [a - c + \theta_W] \, q_W - b q_W^2 - \tfrac{b q_W}{F(\theta_W)} \int_{\underline{\theta}}^{\theta_W} q_L \left(\theta_L \right) dF \left(\theta_L \right)
\end{aligned}
$$

Similarly the expected profit of the losing firm is given below

$$
\begin{aligned}
\Pi_L &= \int_{\theta_L}^{\overline{\theta}} \left\{ [a - c] \, q_L - b \left(q_W \left(\theta_W \right) + q_L \right) q_L \right\} dF \left(\theta_W \left| \theta_W > \theta_L \right. \right) \\
&= [a - c] \, q_L - b q_L^2 - \tfrac{b q_L}{(1 - F(\theta_L))} \int_{\theta_L}^{\overline{\theta}} q_W \left(\theta_W \right) dF \left(\theta_W \right)
\end{aligned}
$$

Our objective is to prove that bidding truthfully in the first stage and producing output according to true types in the second stage, constitute a perfect Bayesian equilibrium for our concerned two-stage game.

From the first order conditions for expected profit maximization we get the reaction functions for the winning and losing firms respectively, which are given below

$$
2q_W + \frac{1}{F\left(\theta_W\right)} \int_{\underline{\theta}}^{\theta_W} q_L \left(\theta_L \right) dF \left(\theta_L \right) = \frac{a - c + \theta_W}{b} \tag{3.1}
$$

[8] Later we show that bidding truthfully is indeed an equilibrium

and

$$\frac{1}{(1 - F(\theta_L))} \int_{\theta_L}^{\bar{\theta}} q_W(\theta_W)\, dF(\theta_W) + 2q_L = \frac{a-c}{b} \tag{3.2}$$

We denote the set of equilibrium outputs by q_W^* and q_L^*. Also note that the expression $\frac{1}{F(\theta_W)} \int_{\underline{\theta}}^{\theta_W} q_L(\theta_L)\, dF(\theta_L)$ gives us the expected output of the losing firm from the winning firm's perspective and similarly $\frac{1}{(1-F(\theta_L))} \int_{\theta_L}^{\bar{\theta}} q_W(\theta_W)\, dF(\theta_W)$ gives us the expected output of the winning firm from the losing firm's perspective. We define the sum of these two terms, given by $\frac{1}{F(\theta_W)} \int_{\underline{\theta}}^{\theta_W} q_L(\theta_L)\, dF(\theta_L) + \frac{1}{(1-F(\theta_L))} \int_{\theta_L}^{\bar{\theta}} q_W(\theta_W)\, dF(\theta_W)$, as the Total Expected Output of the Industry (TEOI). Now we are in a position to state our first theorem, which gives some relationships between $\frac{dq_W^*}{d\theta_W}$ and $\frac{dq_L^*}{d\theta_L}$.

Lemma 3.1. *(i) For all values of the winning type θ_W , the equilibrium output level of the winning firm q_W^* is an increasing function of θ_W , if and only if for all values of the losing type θ_L, the equilibrium output level of the losing firm, q_L^* is a decreasing function of θ_L, which means that*

$$\forall \theta_W \, \frac{dq_W^*}{d\theta_W} > 0 \; iff \; \forall \theta_L \, \frac{dq_L^*}{d\theta_L} < 0 \tag{3.3}$$

(ii) For all values of the winning type θ_W , if the equilibrium output level of the winning firm q_W^* is a decreasing function of θ_W, then the equilibrium output level of the losing firm, q_L^* is an increasing function of θ_L, which means that

$$if \; \forall \theta_W \, \frac{dq_W^*}{d\theta_W} < 0 \; then \; \forall \theta_L \, \frac{dq_L^*}{d\theta_L} > 0 \tag{3.4}$$

Proof. See Appendix A □

Once the equilibrium outputs are characterized by the above theorem, it is easy to characterize the expected profit functions of the firms. The next theorem exactly does this.

Theorem 3.2. *if $\forall \theta_W \, \frac{dq_W^*}{d\theta_W} > 0$ then $\forall \theta_W \, \frac{d\Pi_W^*}{d\theta_W} > 0$ and $\forall \theta_L \, \frac{d\Pi_L^*}{d\theta_L} < 0$.*

Proof. See Appendix B □

Before we move on to characterize the social welfare, we first show that given the lemma and theorem stated above, there exists an increasing and symmetric equilibrium bidding strategy for each of the firms in the auction.

3.1.2 Characteristics of the equilibrium bidding strategies of two firms

First we want to emphasize that we are interested in a symmetric and increasing equilibrium only. This ensures ex-post efficiency (i.e. the firm whose type is the highest gets the innovation). Consider firm i. The actual type for firm i is θ_i. But suppose firm i bids according to z_i. Let the symmetric bidding function be denoted by $h(.)$. For the time being assume that $h^{/}(.) > 0$. The expected profit of firm i is given below

$$\left[\Pi_{Wi}\left(\theta_i\right) - h\left(z_i\right)\right] F\left(z_i\right) + \Pi_{Li}\left(\theta_i\right)\left(1 - F\left(z_i\right)\right)$$

where $\Pi_{Wi}(.)$ is the profit of the firm in stage 2 if she wins the auction in stage 1 and $\Pi_{Li}(.)$ is the profit of the firm in stage 2 if she loses the auction in stage 1.

Theorem 3.3. *If $\forall i$ and $\forall \theta_i$ $\frac{dq^*_{Wi}}{d\theta_i} > 0$, where q^*_{Wi} is the output produced by firm i in the industry and if firm i wins this auction, then there exists a symmetric and increasing equilibrium in the above auction, given that in the second stage the firm i formulates its expectation regarding its contender's type based on its true type.*

Proof. See Appendix C □

Finally, we are going to show that in both the stages, both the firms bidding and producing according to their true types constitutes a perfect Bayesian Nash equilibrium of this two stage game.

Theorem 3.4. *In the first stage all the firms bidding according to their true types and in the second stage producing according to their true types, is a Perfect Bayesian-Nash equilibrium.*

Proof. We have shown in the above Proposition 3.3 that if all the firms are producing according to their true types in the second stage, then in the first stage auction they will also bid according to their true types. Similarly, we have also shown that if in the first stage they are bidding according to their true types, then it is optimal for them to produce according to their true types in the second stage. Therefore, bidding in the first stage auction and producing the output in the second stage, both based on their true types, are sequentially rational. Hence the theorem. □

3.2 Industry with fully revealed bids

Here we consider the case where government will announce all the bids after the auction takes place. So, before the auction firms know that after the auction government will announce all the bids. We will again begin our analysis from stage 2 when after the auction we assume that firm 1 wins the auction and gets the cost reducing production technique. Assume in stage 1 both the firms bid according to their true types. So after the auction the winning firm knows the actual value of θ_L. Similarly, the losing firm knows the actual value of θ_W. Since the marginal costs before the auction are common knowledge to the firms, so in this case after the auction, both the firms can calculate the marginal cost of the rival firm.

3.2.1 Equilibrium Outputs and Profits

We assume that in stage 2 the objective of each firm is to maximize her own profit by choosing an output level. The profit function of the winning firm is given by $\Pi_W = [a - c + \theta_W]\, q_W - b\,(q_W + q_L)\, q_W$. Similarly the profit function of the losing firm is given by $\Pi_L = [a - c]\, q_L - b\,(q_W + q_L)\, q_L$.

Reaction functions of the firms are given by

$$\begin{aligned} 2q_W^* + q_L^* &= \tfrac{a-c+\theta_W}{b} \\ q_W^* + 2q_L^* &= \tfrac{a-c}{b} \end{aligned} \tag{3.5}$$

From the equation 3.5, we get $q_W^* = \frac{a-c+2\theta_W}{3b}$ and $q_L^* = \frac{a-c-\theta_W}{3b}$. Also note that $\Pi_W^* = bq_W^{*2} = \frac{(a-c+2\theta_W)^2}{9b}$ and $\Pi_L^* = bq_L^{*2} = \frac{(a-c-\theta_W)^2}{9b}$[9].

3.2.2 Equilibrium Bidding Strategies

Again we are interested in symmetric and increasing equilibrium only. Consider firm i. The actual θ for firm i is θ_i. But let firm i bid according to z_i (i.e. firm i signals its type to be z_i instead of θ_i). Let the symmetric bidding function be denoted by $h(.)$. For the time being assume that $h^{/}(.) > 0$. The expected profit of firm i is given below

$$\left[\frac{(a-c+2\theta_i)^2}{9b} - h(z_i)\right] F(z_i) + \left[\int_{z_i}^{\bar{\theta}} \frac{(a-c-\theta_j)^2}{9b} dF(\theta_j)\right] (1 - F(z_i))$$

[9]Note that here $\forall \theta_W \ \frac{d\Pi_W^*}{d\theta_W} > 0$ holds.

From the first order condition of the expected profit maximization we get

$$h(\theta_i) = \frac{1}{F(\theta_i)} \int\limits_{\underline{\theta}}^{\theta_i} \left[\frac{(a-c+2z_i)^2}{9b} - \int_{z_i}^{\overline{\theta}} \frac{(a-c-\theta_j)^2}{9b} dF(\theta_j) - \frac{(a-c-z_i)^2(1-F(z_i))}{9b} \right] dF(z_i)$$

From the above expression of $h(.)$, one can routinely check that $h^{/}(.) > 0$ and the second order condition for the expected payoff maximization holds.

Now we are in a position to compare the social welfare between fully disclosed and undisclosed bid regimes. After the comparison we will provide two examples to illustrate the results.

3.3 Comparison of social welfare for limited information and full information

Theorem 3.5. *If* $\forall i$ *and* $\forall \theta_i$ $\frac{dq^*_{Wi}}{d\theta_i} > 0$ *then industry output under undisclosed bids* ($Q_L = q_W^{L*} + q_L^{L*}$) *and industry output under fully disclosed bids* ($Q_F = q_W^{F*} + q_L^{F*}$) *have the following relationship*

$$Q_F \gtreqqless Q_L \quad iff \quad \frac{2(a-c)+\theta_W}{3b} \lesseqqgtr TOEI$$

Proof. See Appendix D □

The above theorem basically suggests that the ranking between the total industry output under fully disclosed bids, Q_F and $Q_L$depends on the value of TEOI. If the value of TEOI exceeds a certain threshold value[10] then the case with fully disclosed bids yields a higher industry output than the case under undisclosed bids. This result is quite intuitive because if TOEI is very high then both the firms are expecting that her rival firm will be going to produce a high output so that it is optimal for her to produce a relatively low output than the case of fully revealed bids where she knows for certain how much her rival is going to produce.

The condition holds true for the ranking of consumer surpluses under fully disclosed and undisclosed bids regimes which is formally stated in the following theorem.

Theorem 3.6. *If* $\forall i$ *and* $\forall \theta_i$ $\frac{dq^*_{Wi}}{d\theta_i} > 0$ *then consumer surplus under undisclosed bids* (CS^L) *and industry output under fully disclosed bids* (CS^F) *have the following relationship*

[10]Surprisingly this threshold value turns out to be equal to the industry output under fully disclosed bids.

$$CS^F \gtreqqless CS^L \quad iff \quad \frac{2(a-c)+\theta_W}{3b} \lesseqqgtr TOEI$$

Proof. See Appendix E $\square$

The net profit of the winning firm is its profit earned in the industry (in the second stage) net of its payment as its bid in the auction, whereas the net profit of the losing firm is simply its profit earned in the industry. The profit (surplus) of the government (GS) is the revenue earned from the auction (which is the winning bid). Total Producer Surplus (PS) is the sum of the net profits of both the firms.

Note that,$\Pi_W^F > \Pi_L^F$, $\Pi_W^L > \Pi_L^L$, $\Pi_W^F > \Pi_L^F \rightarrow q_W^F > q_L^F$ and $\Pi_W^L > \Pi_L^L \rightarrow q_W^L > q_L^L$. Here, Π_w^F and Π_W^L denote the profits of the winning firm under fully disclosed and undisclosed bids respectively, q_W^F and q_W^L denote the output of the winning firm under fully disclosed and undisclosed bids respectively, Π_L^F and Π_L^L denote the profit of the losing firm under fully disclosed and undisclosed bids respectively, and q_L^F and q_L^L denote the output of the losing firm under fully disclosed and undisclosed bids respectively. So the following four cases are possible.

Case 1: $q_W^F > q_W^L$ and $q_L^F > q_L^L$ (here industry output under fully disclosed bids is greater than industry output under undisclosed bids). This case happens when $\frac{2(a-c)+\theta_W}{6b} < \frac{1}{F(\theta_W)} \int\limits_{\underline{\theta}}^{\theta_W} q_L^* (\theta_L) \, dF(\theta_L) + \frac{1}{(1-F(\theta_L))} \int\limits_{\theta_L}^{\overline{\theta}} q_W^* (\theta_W) \, dF(\theta_W)$

Case 2: $q_W^F > q_W^L$ and $q_L^F < q_L^L$ (here the relation between industry output under fully disclosed bids and industry output under undisclosed bids is ambiguous)

Case 3: $q_W^F < q_W^L$ and $q_L^F > q_L^L$ (here the relation between industry output under fully disclosed bids and industry output under undisclosed bids is ambiguous)

Case 4: $q_W^F < q_W^L$ and $q_L^F < q_L^L$ (here industry output under fully disclosed bids is less than industry output under undisclosed bids). This case happens when $\frac{2(a-c)+\theta_W}{6b} > \frac{1}{F(\theta_W)} \int\limits_{\underline{\theta}}^{\theta_W} q_L^* (\theta_L) \, dF(\theta_L) + \frac{1}{(1-F(\theta_L))} \int\limits_{\theta_L}^{\overline{\theta}} q_W^* (\theta_W) \, dF(\theta_W)$

Let us define the gross profit of a firm by the profit earned by the firm in the industry. So, the gross profit of the winning firm is the sum of the net profit and the payment of its bid to the government, whereas the gross profit of the losing firm is simply its net profit. Let GP be the gross profit of the industry. So, $GP = GP_W + GP_L = NP_W + NP_L + GS = PS + GS$. Now, $GP^F = b\left(q_W^F\right)^2 + b\left(q_L^F\right)^2$ and $GP^L = b\left(q_W^L\right)^2 + b\left(q_L^L\right)^2$, where GP denotes gross profit, GP_W

denotes the gross profit for the winning firm, GP_L denotes the the gross profit for the losing firm, NP_W denotes the net profit for the winning firm, NP_L denotes the net profit for the losing firm, GS denotes the government's surplus, PS denotes the producer surplus, GP^F denotes the gross profit of the industry under fully disclosed bids, GP^L denotes the gross profit of the industry under undisclosed bids. Let us define $\Delta GP = GP^F - GP^L = b\left[\left(q_W^F\right)^2 + \left(q_L^F\right)^2 - \left(q_W^L\right)^2 - \left(q_L^L\right)^2\right]$ as the difference between the gross profit under fully disclosed bids and that under undisclosed bids. The following theorem characterizes ΔGP for the above three cases.

Theorem 3.7. *If $\forall i$ and $\forall \theta_i$ $\frac{dq^*_{Wi}}{d\theta_i} > 0$ then for Case 1 we have $\Delta GP > 0$, and for Case 2 and Case 3 the sign of ΔGP is ambiguous i.e. we have $\Delta GP \gtreqless 0$ and finally for case 4 we have $\Delta GP < 0$.*

Proof. See Appendix F □

The next theorem shows that gross profit under fully disclosed bids (i.e. full information) is an increasing function of the winning type.

Theorem 3.8. *GP is a strictly increasing function of the winning type under fully disclosed bids, and doesn't depend on the losing type.*

Proof. Trivial. □

The next theorem in this section compares the levels of social welfare (SW) under the two different information structures. Let us define $\triangle SW = SW^F - SW^L$ and $\triangle CS = CS^F - CS^L$, where SW^F and SW^L are the social welfare levels under fully disclosed and undisclosed bids respectively.

Theorem 3.9. *If $\forall i$ and $\forall \theta_i$ $\frac{dq^*_{Wi}}{d\theta_i} > 0$ then for Case 1 we have $\Delta SW > 0$, and for Case 2 and Case 3 we have $\Delta SW \gtreqless 0$ and finally for Case 4 $\Delta SW < 0$.*

Proof. Note that $SW = CS + GP$, therefore $\Delta SW = \Delta CS + \Delta GP$. The rest of the proof is trivial. □

The above theorem implies that depending on the types of the firms it is optimal for the government to disclose any information (in our case it is the bid of the winning firm). Also note that in our case it is the difference between the industry outputs under different information structures that determines the differences of social welfare levels under different industrial structures. So our final theorem for this section illustrates the condition which decides the differences in levels of social welfare under different industrial structures.

Theorem 3.10. *If* $\forall i$ *and* $\forall \theta_i$ $\frac{dq^*_{Wi}}{d\theta_i} > 0$ *then*

$$\Delta SW \gtreqless 0 \quad iff \quad \frac{2(a-c)+\theta_W}{3b} \lesseqgtr TEOI$$

Proof. Trivial □

First, TEOI plays a crucial role to determine whether government should reveal the bids of the firms. Second, the same condition guiding the theorems, 3.5 and 3.6 is also impacting the ranking of the levels social welfare under the two different bid disclosure regimes. We illustrate the above discussed cases with the following example.

Example 3.11. We suppose that $q^*_i = \alpha_i + \beta_i\theta_i$ where $i = W, L$ (where W stands for winner and L for loser) and also that θ_is are uniformly distributed over the interval $[0,1]$. Here our objective is to illustrate an equilibrium under undisclosed bids. For simplicity we are interested in an equilibrium for which the quantity produced by each firm is a linear function of its type[11]. So the profit function under undisclosed bids for the winning bidder can be written as $\Pi_W = (a-c+\theta_W)\, q_W - bq_W^2 - bq_W\left(\alpha_L + \frac{1}{2}\beta_L\theta_W\right)$. Therefore, $q^*_W = \frac{a-c+\theta_W}{2b} - \frac{1}{2}\left(\alpha_L + \frac{1}{2}\beta_L\theta_W\right)$. From this we can easily verify that $\frac{dq^*_W}{d\theta_W} = \frac{1}{2b} - \frac{1}{4}\beta_L$.

Similarly, for the losing bidder, the profit function can be written as $\Pi_L = (a-c)\, q_L - bq_L^2 - bq_L\left[\alpha_W + \frac{1}{2}\beta_W(1+\theta_L)\right]$. Therefore, $q^*_L = \frac{a-c}{2b} - \frac{1}{2}\left[\alpha_W + \frac{1}{2}\beta_W(1+\theta_L)\right]$. From this we can easily verify that $\frac{dq^*_L}{d\theta_L} = -\frac{1}{4}\beta_W$.

Again, from our assumptions for this specific example, we have $\frac{dq^*_W}{d\theta_W} = \beta_W$ and $\frac{dq^*_L}{d\theta_L} = \beta_L$. From these two sets of relations we can surely infer that $\forall \beta_L < 0 \; iff \; \forall \beta_W > 0$. So this confirms the results in Theorem 3.1. Now since $\frac{d\Pi^*_W}{d\theta_W} = 2bq^*_W\frac{dq^*_W}{d\theta_W}$ and $\frac{d\Pi^*_L}{d\theta_L} = 2bq^*_L\frac{dq^*_L}{d\theta_L}$, therefore, the results in Theorem 3.2 are also confirmed.

From the reaction functions for undisclosed bids (equations 3.1 and 3.2) we can clearly see that $q_W(\theta_i)$ and $q_L(\theta_i)$ are not explicitly solvable, since the lack of knowledge about the functional forms of $q_W(\theta_i)$ and $q_L(\theta_i)$ makes it impossible for us to integrate them. For our case with assumptions of linear functions (we assume $q_L(\theta_L) = \alpha_L + \beta_L\theta_L$ and $q_W(\theta_W) = \alpha_W + \beta_W\theta_W$), we get $2\alpha_W + \alpha_L = \frac{a-c}{b} - \left(2\beta_W + \frac{\beta_L}{2} - \frac{1}{b}\right)\theta_W$ and $\alpha_W + 2\alpha_L = \frac{a-c}{b} - \frac{\beta_W}{2} - \left(\frac{\beta_W}{2} + 2\beta_L\right)\theta_L$.

Now, by assumption, α_W and α_L are independent of θ_W and θ_L. Therefore we must have, $2\beta_W + \frac{\beta_L}{2} - \frac{1}{b} = 0$ and $\frac{\beta_W}{2} + 2\beta_L = 0$. Solving these two equations we obtain, $\beta_W = \frac{8}{15b}$ and $\beta_L = -\frac{2}{15b}$. Thus we also obtain $\alpha_W = \frac{1}{45b}[15(a-c)+4]$ and $\alpha_L = \frac{1}{45b}[15(a-c)-8]$.

[11] Note that our original analysis does not require this simplified assumption. We are assume linearity here first of all only to illustrate our main results, and secondly to show that their at least exists an equilibrium possibly in linear form.

Therefore we get our intended equilibrium quantities for the winning and the losing firms respectively as $q_W^L(\theta_W) = \frac{15(a-c)+4+24\theta_W}{45b}$ and $q_L^L(\theta_L) = \frac{15(a-c)-8-6\theta_L}{45b}$.

The next example shows that depending on θ_W and θ_L, we may have any one of the above mentioned cases.

Example 3.12. We know that $q_W^F = \frac{a-c+2\theta_W}{3b} = \frac{a-c}{3b} + \frac{2}{3b}\theta_W$, $q_L^F = \frac{a-c-\theta_W}{3b} = \frac{a-c}{3b} - \frac{1}{3b}\theta_W$, $q_W^L(\theta_W) = \left(\frac{a-c}{3b} + \frac{4}{45b}\right) + \frac{8}{15b}\theta_W$ and $q_L^L(\theta_L) = \left(\frac{a-c}{3b} - \frac{8}{45b}\right) - \frac{2}{15b}\theta_L$.
Note that $q_W^F(\theta_W) \gtreqless q_W^L(\theta_W)\ iff\ \theta_W \gtreqless \frac{2}{3}$, $q_L^F(\theta_W) \gtreqless q_L^L(\theta_L)\ iff\ \theta_L \lesseqgtr \left(-\frac{4}{3} + \frac{5}{2}\theta_W\right)$ and $q_W^L(\theta_W) + q_L^L(\theta_L) = \frac{2(a-c)}{3b} + \frac{8}{15b}\theta_W - \frac{2}{15b}\theta_L - \frac{4}{45b}$, therefore,

$$\begin{aligned}&\tfrac{2(a-c)+\theta_W}{3b} \lesseqgtr \left[\tfrac{1}{F(\theta_W)}\int_{\underline{\theta}}^{\theta_W} q_L^*(\theta_L)\,dF(\theta_L) + \tfrac{1}{(1-F(\theta_L))}\int_{\theta_L}^{\bar{\theta}} q_W^*(\theta_W)\,dF(\theta_W)\right]\\ &\Leftrightarrow \tfrac{2}{13}\theta_L + \tfrac{4}{39} \gtreqless \theta_W\end{aligned}$$

First, note that if $\theta_W \leqq \frac{4}{39}$, then it is better for the government to suppress the information before the auction. Again if $\theta_W > \frac{2}{13}\theta_L + \frac{4}{39}$, then it is better to disclose the information before the auction. Finally in case of $\frac{2}{13}\theta_L + \frac{4}{39} = \theta_W$, the government is indifferent between suppressing or disclosing the information.

Remark 3.13. In case of Second Price Sealed Bid Auction the expected profit function for the winning firm is given by $\Pi_W = (a-c+\theta_W)\,q_W - b\,(q_W + q_L(\theta_L))\,q_W$ and that for the losing firm is $\Pi_L = \int\limits_{\theta_L}^{\bar{\theta}} [(a-c)\,q_L - b\,(q_W(\theta_W;\theta_L) + q_L)\,q_L]\,dF(\theta_W|\theta_W > \theta_L)$. From these two equations we can find the reaction functions as $2q_W + q_L(\theta_L) = \frac{a-c+\theta_W}{b}$ and $2q_L + \frac{1}{1-F(\theta_L)}\int\limits_{\theta_L}^{\bar{\theta}} q_W(\theta_W;\theta_L)\,dF(\theta_W) = \frac{a-c}{b}$. Proceeding analogously as the previous analysis it can be shown that

$$\Delta SW \gtreqless 0 \quad iff \quad \frac{2(a-c)+\theta_W}{3b} \lesseqgtr \left[q_L^*(\theta_L) + \frac{1}{(1-F(\theta_L))}\int_{\theta_L}^{\bar{\theta}} q_W^*(\theta_W;\theta_L)\,dF(\theta_W)\right]$$

The equilibrium bidding strategy in the first stage for undisclosed bids is given by $h(\theta_i) = \Pi_W^i(\theta_i;\theta_i) - \Pi_L^i(\theta_i)$ and that for fully disclosed bids is given by $h(\theta_i) = \frac{(a-c+2\theta_i)^2}{9b} - \int\limits_{\theta_i}^{\bar{\theta}} \frac{(a-c-\theta_j)^2}{9b}\,dF(\theta_j) - (1-F(\theta_i))\,f(\theta_i)\,\frac{(a-c-\theta_i)^2}{9b}$.

Remark 3.14. For an all-pay auction, the analysis of the second stage game will be the same as that for the first price sealed-bid auction. Only the bidding strategies will be different. $h(\theta_i) = \int\limits_{\underline{\theta}}^{\theta_i}\left[\frac{(a-c+2z_i)^2}{9b} - \int\limits_{z_i}^{\bar{\theta}} \frac{(a-c-\theta_j)^2}{9b}\,dF(\theta_j) - \frac{(a-c-z_i)^2}{9b}(1-F(z_i))\right]dF(z_i)$ is the equilibrium bidding strategy in the first stage for fully disclosed bids and that for undisclosed bids is given by $h(\theta_i) = \int_{\underline{\theta}}^{\theta_i} [\Pi_{Wi}(z_i) - \Pi_{Li}(z_i)]\, dF(z_i)$.

4 Case of Bertrand Competition

4.1 Bidding Strategies

Blume (2003) has analyzed Bertrand competition with homogeneous products and different marginal costs and has established an equilibrium in undominated strategies. In such an equilibrium, the "*conventional outcome*" in which "*the low-cost firm charges a price equal to the high-cost firm's cost is supported by an equilibrium under standard rationing rule that both firms split the market if their prices coincide. Moreover, any equilibrium that supports this outcome has the appealing property that it does not rely on the use of dominated strategies.*" Here, it seems a good idea to elaborate a little bit on Blume's analysis.

Blume (2003) considers two firms with commonly known marginal costs c_1 and c_2 respectively, where $c_1 < c_2$. Both the firms produce identical products and face a strictly decreasing and differentiable market demand function $D(p)$ on $[0, \overline{p}]$ with $c_2 < \overline{p} \leq \infty$. The price that the low-cost firm would charge, in case it were a monopolist, $p^m(c_1)$, satisfies $c_2 < p^m(c_1)$. Blume (2003) claims that, for small enough $\eta > 0$, the low-cost firm charging a price $c_2$and the high-cost firm randomizing uniformly over $[c_2, c_2 + \eta]$, constitutes an equilibrium.

Thus, from the analysis presented by Blume (2003), we can straightaway find out the prevailing price and thus quantity levels for the second stage of our concerned game. Very evidently the price in the second stage will be $P = c$ which is the cost of both the firms before the sale of the cost-reducing technological innovation and thus it remains the cost of the losing firm in the first stage auction (as a consequence the losing firm becomes the high-cost firm). The winning firm in the first stage experiences a positive profit in the second stage since its cost in the second stage is $(c - \theta_W)$ and it charges a price c. Here the disclosure of the bids plays no role since the very information that the losing firm's marginal cost is higher than that of the winning firm is sufficient to ensure an outcome where the winning and therefore the low-cost firm charges a price $P = c$. The knowledge of the losing firm's type does not have any impact on the price decision of the winning firm. Therefore, the outcome will be the same irrespective of the bid being disclosed after the auction in the first stage. The profit earned by the winning firm in the second stage is

$$\Pi_W = \theta_W q(c)$$

The losing firm, however, always earns a zero profit in the second stage. Therefore, the expected payoff in the first stage for a first price sealed-bid auction for any firm i , when it submits a bid $b(z_i)$, pretending to have a type z_i when its true type is θ_i , can be written as

$$E[\Pi_i] = F(z_i)(\theta_i q(c) - b(z_i))$$

From the first order condition for maximization of this expected payoff function, we obtain

$$\begin{aligned}&\tfrac{\partial E[\Pi_i]}{\partial z_i} = 0\\ &\Rightarrow \theta_i q(c) f(z_i) - [b(z_i) F(z_i)]^{/} = 0\end{aligned}$$

At a symmetric equilibrium $z_i = \theta_i$, so that we can write the above equation as

$$\begin{aligned}&\theta_i q(c) f(\theta_i) - [b(\theta_i) F(\theta_i)]^{/} = 0\\ &\Rightarrow b(\theta_i) = q(c)\left[\theta_i - \tfrac{1}{F(\theta_i)} \int\limits_{\underline{\theta}}^{\theta_i} F(y)\,dy\right]\end{aligned}$$

To check first whether $b(\theta_i)$ is indeed increasing in θ_i , we differentiate both sides of the above equation with respect to θ_i , and thus obtain $b^{/}(\theta_i) > 0$. To check for a maximum, we proceed to check the second order condition as $\frac{\partial^2 E[\Pi_i]}{\partial z_i^2}|_{z_i=\theta_i} = -q(c) f(\theta_i) < 0$. Thus the second order condition is also satisfied. Therefore, $b(\theta_i)$ indeed constitutes an equilibrium for this first price sealed-bid auction.

4.2 Level of Social Welfare

Here the level of social welfare is simply the sum of the consumer surplus and the producer surplus since the payment of the winning firm is equal to the revenue earned by the government. So in the calculation of the total social surplus, these payments and revenue terms will cancel each other out. The total social welfare will therefore be equal to the sum of consumer surplus (CS) and producer surplus (PS) in all the three types of auctions[12]. In each case CS and PS are the same since the price, and therefore quantity, are decided independent of the information regarding type or the payment rule in the first stage auction. We can easily verify that $CS = \frac{1}{2}\frac{(a-c)^2}{b}$ and $PS = \frac{(a-c)}{b}\theta_W$. Thus the total social welfare would be $SW = CS + PS = \frac{(a-c)(a-c+2\theta_W)}{2b}$. It is evident from the above analysis, that the level of social welfare under the Bertrand type of competition in the second stage is independent of the bid disclosure policy and the type of auction.

[12] We have considered the same market demand function as that in Cournot.

Remark 4.1. In case of Second Price Sealed Bid Auction, the expected payoff function can be written as $E[\Pi_i] = F(z_i)(\theta_i q(c) - E(b(\theta_j)|\theta_j < z_i)) = F(z_i)\theta_i q(c) - \int\limits_{\underline{\theta}}^{z_i} b(\theta_j) f(\theta_j) d\theta_j$. Following the routine procedure we can calculate the corresponding bidding strategy in a truthful equilibrium as $b(\theta_i) = \theta_i q(c)$.

$E[\Pi_i] = F(z_i)\theta_i q(c) - b(z_i)$ is the expected payoff functions of the firms for the all-pay auction. Again we can show that the bidding strategy in this case is given by $b(\theta_i) = q(c)\int\limits_{\underline{\theta}}^{\theta_i} yf(y)\,dy$.

For both the types of auctions the levels of social welfare remain unaffected by the bid disclosure policy.

5 Conclusion

In this paper we have analyzed a two-stage game where in the first stage a cost-reducing production technique is sold through a first-price sealed bid auction. We have considered only two bidders for the present analysis who are competitors in an oligopolistic market in the second stage. Since there is only one unit of the cost-reducing production technique, therefore only one firm can make use of it in the second stage. Before the adoption of this cost reducing production technique, the cost structures of the firms have been assumed to be identical and common knowledge. The cost reduction resulting from the adoption of this production technique has been assumed to be different for the firms and a private information for each firm. Thus the amount of cost reduction constitutes the type of each firm in our analysis. The types of the firms have been assumed to be distributed over the same interval following the same distribution function and both of these have been assumed to be common knowledge. Based on this type, each firm submits a bid in the first period auction.

For the second stage, where the firms compete in an oligopolistic market, we have considered two alternative market structures, one is a Cournot competition and the other one is a Bertrand competition. Depending on these two alternative structures the bid functions in a symmetric equilibrium in the first stage auction have been calculated. We have assumed the auctioneer to be a social welfare maximizing agent whom we have termed as the government. The government chooses to announce or not to announce the bids after the first stage when the auction gets over and before the competition in the second stage starts. Whether the bids would be disclosed or suppressed after the first stage auction is announced before the auction itself. Depending on whether the information regarding bids is revealed, the output levels of the firms in a Cournot type of competitive market vary. We have compared the total social welfare levels for both these cases under a Cournot competition and have calculated the outcomes under different parametric restrictions. Our results suggest that the industry output, consumer surplus and the level of social welfare in a Cournot type of industry is higher when the government chooses to announce the bids after the first stage auction, provided the total expected output of the industry under the undisclosed bid regime is lower than the total industry output in the Cournot competition with fully disclosed bids. We have also provided illustrative examples for all the above mentioned cases. Finally, we have shown that the prevailing price will always remain equal to the marginal cost of the firms before the adoption of the cost-reducing production technique even though only two firms are competing in a Bertrand type of competition. Therefore the bid revelation policy will have no impact on the level of social welfare when the the second stage oligopolistic competition is of a Bertrand type.

These results have significant policy implications. First, they show that it is not always optimum for the government to reveal the bids as most of the governments do after the auction in reality. Second, as the choice of the government does not have any impact on the social welfare under Bertrand competition, government now decides whether to reveal the bids or not by looking at the after-market type. Finally, since the paper fully characterizes the parametric conditions of bid revelation, one only needs to find a mechanism that reveals these parametric conditions before the auction takes place.

An interesting extension of this analysis will be to study the case of Cournot competition involving more than two firms when the bids are not disclosed after the first stage auction. Another interesting exercise would be to design a mechanism such that the government can optimally decide whether to reveal or suppress the bids after the first stage auction ex ante. Both of these questions can be taken up for future research.

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Appendix

A Proof of the Theorem 3.1

First, differentiating equation 3.1 with respect to $\theta_W$we get

$$
\begin{aligned}
&2\tfrac{dq_W^*}{d\theta_W} - \tfrac{f(\theta_W)}{F^2(\theta_W)} \int_{\underline{\theta}}^{\theta_W} q_L^*(\theta_L)\, dF(\theta_L) + \tfrac{1}{F(\theta_W)} q_L^*(\theta_W)\, f(\theta_W) = \tfrac{1}{b}\\
&\tfrac{dq_W^*}{d\theta_W} = \tfrac{1}{2b} + \tfrac{f(\theta_W)}{2F(\theta_W)} \left[\tfrac{1}{F(\theta_W)} \int_{\underline{\theta}}^{\theta_W} q_L^*(\theta_L)\, dF(\theta_L) - q_L^*(\theta_W) \right]\\
&\tfrac{dq_W^*}{d\theta_W} = \tfrac{1}{2b} + \tfrac{f(\theta_W)}{2F(\theta_W)} \left[\tfrac{1}{F(\theta_W)} \left[q_L^*(\theta_W)\, F(\theta_W) - \int_{\underline{\theta}}^{\theta_W} q_L^{*/}(\theta_L)\, F(\theta_L)\, d\theta_L \right] - q_L^*(\theta_W) \right]\\
&\tfrac{dq_W^*}{d\theta_W} = \tfrac{1}{2b} - \tfrac{f(\theta_W)}{2F(\theta_W)^2} \int_{\underline{\theta}}^{\theta_W} q_L^{*/}(\theta_L)\, F(\theta_L)\, d\theta_L
\end{aligned}
$$

So the change in equilibrium output of the winning firm with respect to θ_W is given by the equation below

$$
\frac{dq_W^*}{d\theta_W} = \frac{1}{2b} - \frac{f(\theta_W)}{2F(\theta_W)^2} \int_{\underline{\theta}}^{\theta_W} q_L^{*/}(\theta_L)\, F(\theta_L)\, d\theta_L \tag{A.1}
$$

Next, differentiating equation 3.2 with respect to $\theta_L$we get

$$
\begin{aligned}
&2\frac{dq_L^*}{d\theta_L} + \frac{f(\theta_L)}{(1-F(\theta_L))^2}\int_{\theta_L}^{\overline{\theta}} q_W^*\left(\theta_W\right)dF\left(\theta_W\right) - \frac{1}{1-F(\theta_L)}q_W^*\left(\theta_W\right)f\left(\theta_W\right) = 0\\
&\frac{dq_L^*}{d\theta_L} = -\frac{f(\theta_L)}{2(1-F(\theta_L))}\left[\frac{1}{1-F(\theta_L)}\int_{\theta_L}^{\overline{\theta}} q_W^*\left(\theta_1\right)dF\left(\theta_W\right) - q_W^*\left(\theta_L\right)\right]\\
&\frac{dq_L^*}{d\theta_L} = -\frac{f(\theta_L)}{2(1-F(\theta_L))}\left[q_W^*\left(\theta_L\right) + \frac{1}{1-F(\theta_L)}\left[\left[q_W^*\left(\overline{\theta}\right) - q_W^*\left(\theta_L\right)\right] - \int_{\theta_L}^{\overline{\theta}} q_W^{*/}\left(\theta_W\right)F\left(\theta_W\right)d\theta_W\right] - q_W^*\left(\theta_L\right)\right]\\
&\frac{dq_L^*}{d\theta_L} = -\frac{f(\theta_L)}{2(1-F(\theta_L))^2}\left[\left[q_W^*\left(\overline{\theta}\right) - q_W^*\left(\theta_L\right)\right] - \int_{\theta_L}^{\overline{\theta}} q_W^{*/}\left(\theta_W\right)F\left(\theta_W\right)d\theta_W\right]
\end{aligned}
$$

So the change in equilibrium output of firm the losing firm with respect to θ_L is given by the equation below

$$
\frac{dq_L^*}{d\theta_L} = -\frac{f\left(\theta_L\right)}{2\left(1-F\left(\theta_L\right)\right)^2}\left[\left[q_W^*\left(\overline{\theta}\right) - q_W^*\left(\theta_L\right)\right] - \int_{\theta_L}^{\overline{\theta}} q_W^{*/}\left(\theta_W\right)F\left(\theta_W\right)d\theta_W\right] \tag{A.2}
$$

From the equations A.1 and A.2 the theorem is trivial.

B Proof of the Theorem 3.2

We will prove this theorem by using Theorem 3.1.

Firstly note that from equation 3.1 we get

$$
\begin{aligned}
&2q_W + \frac{1}{F(\theta_W)}\int_{\underline{\theta}}^{\theta_W} q_L\left(\theta_L\right)dF\left(\theta_L\right) = \frac{a-c+\theta_W}{b}\\
&\int_{\underline{\theta}}^{\theta_W} q_L\left(\theta_L\right)dF\left(\theta_L\right) = F\left(\theta_W\right)\left(\frac{a-c+\theta_W}{b} - 2q_W\right)
\end{aligned} \tag{B.1}
$$

and from equation 3.2 we get

$$
\begin{aligned}
&\frac{1}{(1-F(\theta_L))}\int_{\theta_L}^{\overline{\theta}} q_W\left(\theta_W\right)dF\left(\theta_W\right) + 2q_L = \frac{a-c}{b}\\
&\int_{\theta_L}^{\overline{\theta}} q_W\left(\theta_W\right)dF\left(\theta_W\right) = \left(1-F\left(\theta_L\right)\right)\left(\frac{a-c}{b} - 2q_L\right)
\end{aligned} \tag{B.2}
$$

Now, substituting equation B.1 in Π_W we get

$$
\begin{aligned}
&\Pi_W^*\\
&= \left[a-c+\theta_W\right]q_W^* - bq_W^{*2} - \frac{bq_W^*}{F(\theta_W)}\int_{\underline{\theta}}^{\theta_W} q_W^*\left(\theta_W\right)dF\left(\theta_W\right)\\
&= \left[a-c+\theta_W\right]q_W^* - bq_W^{*2} - \frac{bq_W^*}{F(\theta_W)}F\left(\theta_W\right)\left(\frac{a-c+\theta_W}{b} - 2q_W\right)\\
&= bq_W^{*2}\left(\theta_W\right)
\end{aligned} \tag{B.3}
$$

Similarly, substituting equation B.2 in Π_L we get

$$
\begin{aligned}
&\Pi_L^* \\
&= [a-c]\, q_L - bq_L^2 - \tfrac{bq_L}{(1-F(\theta_L))} \int_{\theta_L}^{\overline{\theta}} q_W(\theta_W)\, dF(\theta_W) \\
&= [a-c]\, q_L - bq_L^2 - \tfrac{bq_L}{(1-F(\theta_L))} (1-F(\theta_L)) \left(\tfrac{a-c}{b} - 2q_L\right) \\
&= bq_L^{*2}(\theta_L)
\end{aligned}
\tag{B.4}
$$

Finally, note that

$$
\frac{d\Pi_W^*}{d\theta_W} = 2bq_W^*(\theta_W) \frac{dq_W^*}{d\theta_W} \tag{B.5}
$$

and

$$
\frac{d\Pi_L^*}{d\theta_L} = 2bq_L^*(\theta_L) \frac{dq_L^*}{d\theta_L} \tag{B.6}
$$

From equations B.5 and B.6 and from Theorem 3.1 we can trivially prove this theorem.

C Proof of the Theorem 3.3

From the first order condition of the expected profit maximization we get

$$
\begin{aligned}
&\Pi_{Wi}(\theta_i) f(z_i) - h^{/}(z_i) F(z_i) - f(z_i) h(z_i) - \Pi_{Li}(\theta_i) f(z_i) = 0 \\
&[\Pi_{Wi}(\theta_i) - \Pi_{Li}(\theta_i)] f(z_i)\, dz_i = d[F(z_i) h(z_i)] \\
&h(\theta_i) = \tfrac{1}{F(\theta_i)} \int_{\underline{\theta}}^{\theta_i} [\Pi_{Wi}(z_i) - \Pi_{Li}(z_i)]\, dF(z_i) \\
&h(\theta_i) = \tfrac{1}{F(\theta_i)} \left[[\Pi_{Wi}(\theta_i) - \Pi_{Li}(\theta_i)] F(\theta_i) - \int_{\underline{\theta}}^{\theta_i} \left[\Pi_{Wi}^{/}(z_i) - \Pi_{Li}^{/}(z_i)\right] F(z_i)\, dz_i \right] \\
&h(\theta_i) = [\Pi_{Wi}(\theta_i) - \Pi_{Li}(\theta_i)] - \tfrac{1}{F(\theta_i)} \int_{\underline{\theta}}^{\theta_i} \left[\Pi_{Wi}^{/}(z_i) - \Pi_{Li}^{/}(z_i)\right] F(z_i)\, dz_i
\end{aligned}
\tag{C.1}
$$

From the above equation we get

$$
h^{/}(\theta_i) = \frac{f(\theta_i)}{F(\theta_i)^2} \int_{\underline{\theta}}^{\theta_i} \left[\Pi_{Wi}^{/}(z_i) - \Pi_{Li}^{/}(z_i)\right] F(z_i)\, dz_i > 0 \tag{C.2}
$$

The last inequality follows from the assumption $\forall i$ *and* $\forall \theta_i$ $\frac{dq_{Wi}^*}{d\theta_i} > 0$. Let us now check the second order condition of the expected payoff maximization. Suppose the actual type of the bidder i is θ_i but she bids according to the type $\tilde{\theta}_i$. Her expected payoff is given below.

$$
\begin{aligned}
&E\Pi_i\left(\theta_i,\tilde{\theta}_i\right)=F\left(\tilde{\theta}_i\right)\left[\Pi_{Wi}\left(\theta_i\right)-h\left(\tilde{\theta}_i\right)\right]+\left(1-F\left(\tilde{\theta}_i\right)\right)\Pi_{Li}\left(\theta_i\right)\\
&=F\left(\tilde{\theta}_i\right)\left[\Pi_{Wi}\left(\theta_i\right)-\left[\left[\Pi_{Wi}\left(\tilde{\theta}_i\right)-\Pi_{Li}\left(\tilde{\theta}_i\right)\right]-\frac{1}{F\left(\tilde{\theta}_i\right)}\int_{\underline{\theta}}^{\tilde{\theta}_i}\left[\Pi_{Wi}^{/}\left(z_i\right)-\Pi_{Li}^{/}\left(z_i\right)\right]F\left(z_i\right)dz_i\right]\right]\\
&\qquad+\left(1-F\left(\tilde{\theta}_i\right)\right)\Pi_{Li}\left(\theta_i\right)\\
&=\Pi_{Li}\left(\theta_i\right)+F\left(\tilde{\theta}_i\right)\left[\left[\Pi_{Wi}\left(\theta_i\right)-\Pi_{Li}\left(\theta_i\right)\right]-\left[\Pi_{Wi}\left(\tilde{\theta}_i\right)-\Pi_{Li}\left(\tilde{\theta}_i\right)\right]\right]\\
&\qquad+\int_{\underline{\theta}}^{\tilde{\theta}_i}\left[\Pi_{Wi}^{/}\left(z_i\right)-\Pi_{Li}^{/}\left(z_i\right)\right]F\left(z_i\right)dz_i
\end{aligned}
$$

Therefore, $E\Pi_i\left(\theta_i,\theta_i\right)=\Pi_{Li}\left(\theta_i\right)+\int_{\underline{\theta}}^{\theta_i}\left[\Pi_{Wi}^{/}\left(z_i\right)-\Pi_{Li}^{/}\left(z_i\right)\right]F\left(z_i\right)dz_i$

Our objective is to show that given θ_i, the expression $\left[E\Pi_i\left(\theta_i,\theta_i\right)-E\Pi_i\left(\theta_i,\tilde{\theta}_i\right)\right]$ has a minimum at $\tilde{\theta}_i=\theta_i$, and that minimum value is equal to zero. Note that at $\tilde{\theta}_i=\theta_i$, $\left[E\Pi_i\left(\theta_i,\theta_i\right)-E\Pi_i\left(\theta_i,\tilde{\theta}_i\right)\right]=0$.

Note that

$$
\begin{aligned}
&\left[E\Pi_i\left(\theta_i,\theta_i\right)-E\Pi_i\left(\theta_i,\tilde{\theta}_i\right)\right]\\
&=\int_{\tilde{\theta}_i}^{\theta_i}\left[\Pi_{Wi}^{/}\left(\theta_j\right)-\Pi_{Li}^{/}\left(\theta_j\right)\right]F\left(\theta_j\right)d\theta_j\\
&\qquad-F\left(\tilde{\theta}_i\right)\left[\left\{\Pi_{Wi}\left(\theta_i\right)-\Pi_{Li}\left(\theta_i\right)\right\}-\left\{\Pi_{Wi}\left(\tilde{\theta}_i\right)-\Pi_{Li}\left(\tilde{\theta}_i\right)\right\}\right]
\end{aligned}
\tag{C.3}
$$

we want to minimize the above equation by choosing $\tilde{\theta}_i$, given θ_i.

The first order condition of the minimization problem yields

$$
\begin{aligned}
&-\left[\Pi_{Wi}\left(\theta_i\right)-\Pi_{Li}\left(\theta_i\right)\right]f\left(\tilde{\theta}_i\right)+\left[\Pi_{Wi}\left(\tilde{\theta}_i\right)-\Pi_{Li}\left(\tilde{\theta}_i\right)\right]f\left(\tilde{\theta}_i\right)=0\\
&OR,\quad\left[\Pi_{Wi}\left(\theta_i\right)-\Pi_{Li}\left(\theta_i\right)\right]=\left[\Pi_{Wi}\left(\tilde{\theta}_i\right)-\Pi_{Li}\left(\tilde{\theta}_i\right)\right]
\end{aligned}
$$

The above equation is satisfied at $\tilde{\theta}_i=\theta_i$. So $\left[E\Pi_i\left(\theta_i,\theta_i\right)-E\Pi_i\left(\theta_i,\tilde{\theta}_i\right)\right]$ is optimized at $\tilde{\theta}_i=\theta_i$. To check whether it is minimized or not we proceed to check the second order condition at this point. The second order condition is given below

$$
\begin{aligned}
&-\left[\Pi_{Wi}\left(\theta_i\right)-\Pi_{Li}\left(\theta_i\right)\right]f^{/}\left(\tilde{\theta}_i\right)\\
&\quad+\left[\Pi_{Wi}\left(\tilde{\theta}_i\right)-\Pi_{Li}\left(\tilde{\theta}_i\right)\right]f^{/}\left(\tilde{\theta}_i\right)\\
&\qquad+\left[\Pi_{Wi}^{/}\left(\tilde{\theta}_i\right)-\Pi_{Li}^{/}\left(\tilde{\theta}_i\right)\right]f\left(\tilde{\theta}_i\right)|_{\tilde{\theta}_i=\theta_i}\\
&=\left[\Pi_{Wi}^{/}\left(\theta_i\right)-\Pi_{Li}^{/}\left(\theta_i\right)\right]f\left(\theta_i\right)>0
\end{aligned}
$$

The last inequality follows from the assumption $\forall i \; and \; \forall \theta_i \; \frac{dq^*_{Wi}}{d\theta_i} > 0$.

So, from equation C.2, Theorem 3.1 and 3.2, we can prove this theorem.

D Proof of the Theorem 3.5

Note that

$$\begin{array}{l} 2q^*_W + \frac{1}{F(\theta_W)} \int_{\underline{\theta}}^{\theta_W} q^*_L(\theta_L)\, dF(\theta_L) = \frac{a-c+\theta_W}{b} \\ 2q^*_W = \frac{a-c+\theta_W}{b} - \frac{1}{F(\theta_W)} \int_{\underline{\theta}}^{\theta_W} q^*_L(\theta_L)\, dF(\theta_L) \end{array}$$

and

$$\begin{array}{l} \frac{1}{(1-F(\theta_L))} \int_{\theta_L}^{\overline{\theta}} q^*_W(\theta_W)\, dF(\theta_W) + 2q^*_L = \frac{a-c}{b} \\ 2q^*_L = \frac{a-c}{b} - \frac{1}{(1-F(\theta_L))} \int_{\theta_L}^{\overline{\theta}} q^*_W(\theta_W)\, dF(\theta_W) \end{array}$$

Adding both the equations we get

$$q^*_W + q^*_L = \frac{2(a-c)+\theta_W}{2b} - \frac{1}{2}\left[\frac{1}{F(\theta_W)} \int_{\underline{\theta}}^{\theta_W} q^*_L(\theta_L)\, dF(\theta_L) + \frac{1}{(1-F(\theta_L))} \int_{\theta_L}^{\overline{\theta}} q^*_W(\theta_W)\, dF(\theta_W)\right]$$

Therefore,

$$\begin{array}{l} Q_F \gtreqless Q_L \\ \Leftrightarrow \frac{2(a-c)+\theta_W}{3b} - \frac{2(a-c)+\theta_W}{2b} + \frac{1}{2}\left[\frac{1}{F(\theta_W)} \int_{\underline{\theta}}^{\theta_W} q^*_L(\theta_L)\, dF(\theta_L) + \frac{1}{(1-F(\theta_L))} \int_{\theta_L}^{\overline{\theta}} q^*_W(\theta_W)\, dF(\theta_W)\right] \gtreqless 0 \\ \Leftrightarrow \frac{2(a-c)+\theta_W}{6b} \lesseqgtr \frac{1}{2}\left[\frac{1}{F(\theta_W)} \int_{\underline{\theta}}^{\theta_W} q^*_L(\theta_L)\, dF(\theta_L) + \frac{1}{(1-F(\theta_L))} \int_{\theta_L}^{\overline{\theta}} q^*_W(\theta_W)\, dF(\theta_W)\right] \end{array}$$

E Proof of the Theorem 3.6

Consumer Surplus (CS) under fully disclosed bids is given by

$$CS^F = \frac{1}{2} b \left(q^F_W + q^F_L\right)^2$$

Consumer surplus under undisclosed bids can be written as

$$CS^L = \frac{1}{2}b\left(q_W^L + q_L^L\right)^2$$

The rest of the proof trivially follows from the above theorem (Theorem:3.5).

F Proof of the Theorem 3.7

Note that

$$\begin{aligned}
&\Delta GP \\
&= b\left[\left(q_W^F\right)^2 + \left(q_L^F\right)^2 - \left(q_W^L\right)^2 - \left(q_L^L\right)^2\right] \\
&= b\left[\left(q_W^F + q_W^L\right)\left(q_W^F - q_W^L\right) + \left(q_L^F + q_L^L\right)\left(q_L^F - q_L^L\right)\right]
\end{aligned}$$

Case 1: $(q_W^F > q_W^L$ and $q_L^F > q_L^L)$ The proof is trivial.

Case 2: $(q_W^F > q_W^L$ and $q_L^F < q_L^L)$

$$\begin{aligned}
&\Delta GP \\
&= b\left[\left(q_W^F + q_W^L\right)\left(q_W^F - q_W^L\right) + \left(q_L^F + q_L^L\right)\left(q_L^F - q_L^L\right)\right] \gtreqless 0
\end{aligned}$$

Since $\left(q_W^F + q_W^L\right) \gtreqless \left(q_L^F + q_L^L\right)$and note that

$$\begin{aligned}
&\left(q_W^F + q_L^F\right) \gtreqless \left(q_W^L + q_L^L\right) \\
&\Rightarrow \left(q_W^F - q_W^L\right) \gtreqless \left(q_L^L - q_L^F\right) > 0
\end{aligned}$$

Case 3: $(q_W^F < q_W^L$ and $q_L^F > q_L^L)$ The proof is similar to the proof of Case 2.

Case 4: $(q_W^F < q_W^L$ and $q_L^F < q_L^L)$ The proof is trivial.