A Note on Equal Proportional Sacrifice

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A NOTE ON EQUAL PROPORTIONAL SACRIFICE

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The "equal proportional sacrifice" concept is discussed in numerous texts (e.g., Bowers [1], Fromm and Taubman [2], Herber [3], Musgrave [4], and Winfrey [5]) and is quite familiar to students of public finance. The purpose of this Note is to extend the literature on the subject by determining the tax structure that would result under the equal proportional sacrifice principle if the marginal utility of income (hereafter MU_Y) were described by a rectangular hyperbola.

First refer to Figure 1, where the schedule for the MU_Y is shown as a rectangular hyperbola, with MU_Y plotted along the ordinate axis and dollars of income (Y) plotted along the abscissa. For illustrative purposes, consider a two person (A and B) case. Let A and B have initial before-tax incomes of Y_A and Y_B, respectively.

Since the MU_Y schedule is a rectangular hyperbola, it follows that

$$(MU_Y)(Y) = K \tag{1}$$

where K is a positive constant. Rewriting (1) yields

$$MU_Y = K/Y \tag{2}$$

Allowing Y to assume any value from $+1$ through $+\infty$, it follows that

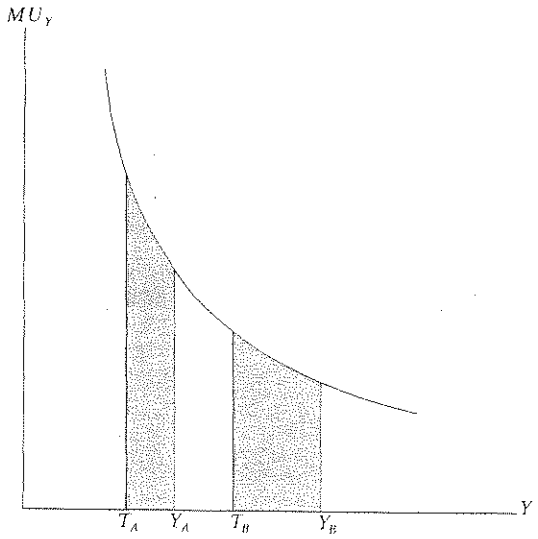

FIGURE 1.

$$\int_{+1}^{+\infty} MU_Y dY = K \ln Y \tag{3}$$

If the equal proportional sacrifice principle is met, we have

$$\frac{\ln Y_A - \ln (Y_A - T_A)}{\ln Y_A} = \frac{\ln Y_B - \ln (Y_A - T_A)}{\ln Y_B} \tag{4}$$

where T_A is A's tax liability and T_B is B's tax liability.

From (4), we can then proceed to

$$\frac{\ln Y_A - \ln (Y_A - T_A)}{\ln Y_A} = \frac{\ln [Y_A/(Y_A - T_A)]}{\ln Y_A} \tag{5}$$

and

$$\frac{\ln Y_B - \ln (Y_B - T_B)}{\ln Y_B} = \frac{\ln [Y_B/(Y_B - T_B)]}{\ln Y_B} \tag{6}$$

Furthermore, it is clear from (5) and (6) that

$$\frac{\ln Y_B}{\ln Y_A} = \frac{\ln [Y_B/(Y_B - T_B)]}{\ln [Y_A/(Y_A - T_A)]} \tag{7}$$

Since $\ln Y_B > \ln Y_A$ with $Y_B > Y_A$, it follows that

$$\ln [Y_B/(Y_B - T_B)] > \ln [Y_A/(Y_A - T_A)] \tag{8}$$

and

$$\frac{Y_B}{Y_B - T_B} > \frac{Y_A}{Y_A - T_A} \tag{9}$$

Finally we conclude from (9) that

$$\frac{Y_B - T_B}{Y_B} < \frac{Y_A - T_A}{Y_A} \tag{10}$$

That is, with MU_Y being a rectangular hyperbola, the tax structure is *progressive* since the lower income unit has a higher percentage of his income remaining after paying taxes.

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References

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