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Cebula, Richard

Jacksonville University

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A NOTE ON EQUAL PROPORTIONAL SACRIFICE

by Richard J. Cebula*

The “equal proportional sacrifice” concept is discussed in numerous texts (e.g., Bowers [1], Fromm and Taubman [2], Herber [3], Musgrave [4], and Winfrey [5]) and is quite familiar to students of public finance. The purpose of this Note is to extend the literature on the subject by determining the tax structure that would result under the equal proportional sacrifice principle if the marginal utility of income (hereafter \(MU_y\)) were described by a rectangular hyperbola.

First refer to Figure 1, where the schedule for the \(MU_y\) is shown as a rectangular hyperbola, with \(MU_y\) plotted along the ordinate axis and dollars of income (\(Y\)) plotted along the abscissa. For illustrative purposes, consider a two person (\(A\) and \(B\)) case. Let \(A\) and \(B\) have initial before-tax incomes of \(Y_A\) and \(Y_B\), respectively.

Since the \(MU_y\) schedule is a rectangular hyperbola, it follows that

\[
(MU_y)(Y) = K
\]

where \(K\) is a positive constant. Rewriting (1) yields

\[
MU_y = K/Y
\]

Allowing \(Y\) to assume any value from \(+\infty\) through \(+\infty\), it follows that

\[
\int_{+\infty}^{+\infty} MU_y dy = K \ln Y
\]  

If the equal proportional sacrifice principle is met, we have

\[
\ln Y_A - \ln (Y_A - T_A) = \ln Y_B - \ln (Y_A - T_A) \quad \ln Y_A \quad \ln Y_B
\]

where \(T_A\) is \(A\)'s tax liability and \(T_B\) is \(B\)'s tax liability.

From (4), we can then proceed to

\[
\ln Y_A - \ln (Y_A - T_A) = \frac{\ln [Y_A/(Y_A - T_A)]}{\ln Y_A}
\]

and

\[
\ln Y_B - \ln (Y_B - T_B) = \frac{\ln [Y_B/(Y_B - T_B)]}{\ln Y_B}
\]

Furthermore, it is clear from (5) and (6) that

\[
\frac{\ln Y_B}{\ln Y_A} = \frac{\ln [Y_B/(Y_B - T_B)]}{\ln [Y_A/(Y_A - T_A)]}
\]

Since \(\ln Y_B > \ln Y_A\) with \(Y_B > Y_A\), it follows that

\[
\ln [Y_B/(Y_B - T_B)] > \ln [Y_A/(Y_A - T_A)]
\]

and

\[
\frac{Y_B}{Y_B - T_B} > \frac{Y_A}{Y_A - T_A}
\]

Finally we conclude from (9) that

\[
\frac{Y_B - T_B}{Y_B} < \frac{Y_A - T_A}{Y_A}
\]

That is, with \(MU_y\) being a rectangular hyperbola, the tax structure is progressive since the lower income unit has a higher percentage of his income remaining after paying taxes.

References